

ANGULAR MOMENTUM OF A BLACK HOLE IN A DENSE STELLAR SYSTEM

PETER J. YOUNG

Department of Astronomy, University of Texas at Austin

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ABSTRACT

The capture of stars from plunge orbits by a massive black hole in a dense stellar system provides a mechanism to reduce the angular momentum of the hole to what is effectively a Schwarzschild state as the mass increases from $M = 10^5 M_\odot$ (spin $a^* = a/M = 0.998$, efficiency of energy conversion in accretion disk $\eta = 0.32$) to $10^8 M_\odot$ ($a^* = 0.05$, $\eta = 0.06$), cutting the luminosity from the disk by a factor of 5 compared with that expected from a Kerr hole ($a^* = 0.998$). This spin-down is due to the greater cross section of a Kerr hole for orbits with negative angular momentum with respect to the spin of the hole.

A small black hole ($M = 10^3 M_\odot$) in a globular cluster is also likely to be Schwarzschild in nature provided sufficient time has elapsed for significant growth of the hole. A growth in mass by a factor of 2 will reduce $a^* = 0.998$ ($\eta = 0.32$) to $a^* = 0.10$ ($\eta = 0.06$).

The presumption is made that an accretion disk supplied by the tidal disruption of stars attempts to increase the angular momentum of the hole. The disk competes with the plunge-orbit flux which is successful in reducing the angular momentum of the hole to $a^* \lesssim 0.6$ ($\eta \lesssim 0.1$) if it is supplied with mass at a rate $\gtrsim 50\%$ of the disk flux.

The conclusions may be destroyed if a strong source of flux to the disk is provided which is independent of the relaxation of the nucleus (which controls both the plunge orbit and the tidal disruption fluxes).

Subject headings: black holes — clusters: globular — stars: accretion — stars: stellar dynamics

I. INTRODUCTION

Bardeen (1970) has suggested that black holes formed by collapse will be close to extreme Kerr ($a^* = a/M \approx 1$) and that a black hole accreting matter from a disk of material rotating in the same sense as the hole will evolve to an extreme Kerr state ($a^* = 1$, $\eta = 0.42$) as the mass of the hole increases by a factor of $6^{1/2} = 2.449$. This result has been modified by Thorne (1974) to take into account the capture of photons with negative angular momentum from the luminous disk; the result is that the limiting value of the angular momentum of the hole is $a^* = 0.998$ ($\eta = 0.32$) asymptotically attained with increasing mass, and effectively attained as the mass increase by a factor of 3.

Another effect which may be important for large black holes embedded in dense galactic nuclei (Hills 1975) is the direct capture of stars in plunge orbits. The cross-section of a Kerr hole is higher for negative-angular-momentum particle orbits as well as for photons, and this process can exert a considerable back-torque on the hole and may cause it to spin down to values of a^* such that the efficiency η becomes close to the Schwarzschild hole value of $\eta = 0.06$ (Godfrey 1970).

We shall consider an accretion disk around the black hole fed by tidal disruption of stars, with a plunge orbit flux supplied by star-hole encounters.

Section II reviews the evolution of the hole as determined by an accretion disc, § III derives rates of accretion, § IV describes numerical integrations, and § V provides a discussion.

II. THE ACCRETION DISK

Bardeen, Press, and Teukolsky (1972) give the following formulae relevant to orbits in the equatorial plane of a Kerr black hole:

Radius of last stable circular orbit:

$$r_{\text{ms}} = M[3 + Z_2 \mp ((3 - Z_1)(3 + Z_1 + 2Z_2))^{1/2}], \quad (\text{II-1})$$

where

$$Z_1 = 1 + (1 - a^2/M^2)^{1/3}[(1 + a/M)^{1/3} + (1 - a/M)^{1/3}], \quad Z_2 = (3a^2/M^2 + Z_1^2)^{1/2},$$

and the upper and lower symbols refer to orbits of positive angular momentum (prograde) and negative angular momentum (retrograde), respectively. The energy per unit mass in this orbit is

$$E/\mu = (r_{\text{ms}}^{3/2} - 2Mr_{\text{ms}}^{1/2} \pm aM^{1/2})/r_{\text{ms}}^{3/4}(r_{\text{ms}}^{3/2} - 3Mr_{\text{ms}}^{1/2} \pm 2aM^{1/2})^{1/2}; \quad (\text{II-2})$$

the angular momentum per unit mass,

$$L/\mu = \pm M^{1/2}(r_{\text{ms}}^2 \mp 2aM^{1/2}r_{\text{ms}}^{1/2} + a^2)/r_{\text{ms}}^{3/4}(r_{\text{ms}}^{3/2} - 3Mr_{\text{ms}}^{1/2} \pm 2aM^{1/2})^{1/2}; \quad (\text{II-3})$$

efficiency of energy conversion in accretion disk,

$$\eta = 1 - E/\mu. \quad (\text{II-4})$$

Denoting E/μ by $E^+(a)$ and L/μ by $L^+(a)$ (both for prograde orbits), the evolution of the hole, as mass dm is accreted, is

$$dM = E^+(a)dm, \quad dJ = d(aM) = L^+(a)dm. \quad (\text{II-5})$$

If the accretion rate from the disk is \dot{M}_0 then, including the effects of photons captured from the disk,

$$dM/dt = \dot{M}_0 E^+(a) + (dM/dt)_{\text{rad}}, \quad dJ/dt = \dot{M}_0 L^+(a) + (dJ/dt)_{\text{rad}}, \quad (\text{II-6})$$

where these equations, and the rather involved expressions for $(dM/dt)_{\text{rad}}$ and $(dJ/dt)_{\text{rad}}$, are given by Thorne (1974).

III. FLUX RATES FOR DISK AND PLUNGE ORBITS

The mathematical formulation of the rates at which stars encounter a black hole in a dense galactic nucleus is, at present, in a developmental stage. Recent papers by Hills (1975), Frank and Rees (1976), Bahcall and Wolf (1976), and Lightman and Shapiro (1977) discuss this phenomenon.

The flux of matter into the black hole may be supplied from a one-component stellar population by (i) tidally disrupted stars, the debris from which is assumed to form an accretion disk, and (ii) stars in "plunge orbits" that would lead to their being swallowed directly by the black hole. It should be noted that if the tidal disruption radius lies outside the event horizon, then "plunge orbit" stars will suffer tidal disruption before actually passing through the event horizon. However, since the debris will have the same average angular momentum per unit mass as the original star and the change in orbital energy is slight (for black hole $M > 10^2 M_\odot$, i.e., orbit is still nearly "parabolic"), most of this material should proceed to "plunge" (assuming it is not exactly in the equatorial plane and does not have to plow through an accretion disk).

We shall see that although both "tidal" and "plunge" fluxes exist when the tidal disruption limit r_t lies outside the event horizon, the "plunge" flux is not important when $r_t \gg 4GM/c^2$ since the cross section for "plunge" is far less than for tidal disruption, but that the plunge flux gradually grows in importance as the mass of the hole grows, becoming comparable to the tidal flux when $r_t \sim 4GM/c^2$. When $r_t < 4GM/c^2$, only the plunge flux exists since a star which passes within the tidal disruption radius is doomed to plunge through the event horizon. We will discuss the following cases:

i) In the first half of this section and in § IV a black hole growing in a galactic nucleus is considered. Initially, with $M = 10^5 M_\odot$, we have $r_t \gg 4GM/c^2$, and only tidal flux is important; but as the hole grows, and $r_t \sim 4GM/c^2$, the plunge flux progressively robs the tidal flux of stellar debris, and this causes the black hole to spin down. Eventually, as $r_t < 4GM/c^2$, the tidal flux is cutoff completely, and the hole spins down to a final Schwarzschild state.

ii) In the latter half of this section we consider a black hole of mass $10^3 M_\odot$ growing in a globular cluster. Here we have $r_t \gg 4GM/c^2$ at all times (since the growth rate of the hole is modest), so, although a plunge flux will exist, it is negligible compared to the tidal flux (and will be ignored).

For our purpose the flux rates into the disk and from plunge orbits are coupled to each other via the relative cross sections of the black hole. This may be confirmed by noting that (Lightman and Shapiro 1976) the flux of tidally disrupted matter is

$$F \propto r_t / \ln [r_{\text{crit}}/(r_a r_t)^{1/2}] \propto r_t \quad (\text{if } r_{\text{crit}} > r_a),$$

where r_{crit} is the critical radius beyond which stars can scatter out of the "loss-cone" in one orbital period, r_a is the accretion radius at which the potential energy in the gravitational field of the hole equals the stellar kinetic energy due to velocity dispersion in the cluster, and r_t is the tidal disruption radius. We ignore the dependence of F on r_t via the logarithm. The above formula applies to a black hole in a galactic nucleus with mass $M < 2 \times 10^9 M_\odot$ when, with $r_{\text{crit}} > r_a$, it is mainly stars which are not bound to the black hole that contribute to the plunge and tidal fluxes. When the mass is $M > 2 \times 10^9 M_\odot$, and $r_{\text{crit}} < r_a$, the flux is supplied by stars bound to the black hole (see Lightman and Shapiro 1976), in which case the following formula is applicable:

$$F \propto r_t^{4/9} / \ln [r_{\text{crit}}/r_t] \quad (\text{if } r_{\text{crit}} < r_a).$$

This formula is also applicable to globular clusters, containing a black hole of mass $10^3 M_\odot$, when again it is the stars bound to the hole that contribute to the flux.

We shall now discuss the case of a black hole growing in a galactic nucleus. Black holes in globular clusters will be considered in the latter half of this section.

For a black hole in a galactic nucleus of mass $M < 2 \times 10^9 M_\odot$, when $r_{\text{orbit}} > r_a$, we shall consider the cross section to both plunge and tidal fluxes, for which the dependence $F \propto r_t$ is crucial.

The cross section of a Newtonian mass M for stars to have periastra $\leq q$ is

$$\sigma(q) = \pi q^2 + 2GM\pi q/v_\infty^2 \approx 2GM\pi q/v_\infty^2, \quad (\text{III-1})$$

where the approximation is valid for $M > 10 M_\odot$ with $v_\infty < 200 \text{ km s}^{-1}$. Then $\sigma(q) \propto q$, and the flux $F \propto \sigma(r_t)$.

We define a reference flux F_0 as the flux due to tidal disruption around a Newtonian object of mass M , taking $r_t = 3 \times 10^{-8} (M/M_\odot)^{1/3} \text{ pc}$. As will be shown, it is not necessary to have explicit knowledge of F_0 .

The cross section of a Schwarzschild hole to plunge orbits is

$$\sigma = (\pi R_g^2) 4c^2/v_\infty^2; \quad R_g = 2GM/c^2 \quad (\text{III-2})$$

(Novikov and Thorne 1973). (An object entering the region $r < 4GM/c^2$ in a "parabolic" orbit, $E/\mu = 1$, is doomed to plunge into the hole.) The cross section of a Kerr hole is reduced by not more than 11% as has been verified by computation (Young 1976b).

Defining the "plunge radius" r_p by

$$r_p/r_t = \sigma/\sigma(r_t), \quad (\text{III-3})$$

we find the flux of material in plunge orbits is

$$F_{p1} = [\sigma/\sigma(r_t)]F_0 = 2(M/6 \times 10^7)^{2/3}F_0 = 1.3 \times 10^{-5}M^{2/3}F_0. \quad (\text{III-4})$$

The cross section of a Schwarzschild hole for a star to have periastron $\leq q$ is

$$\sigma^*(q) = \frac{\sigma(q)}{1 - 2GM/qc^2} \quad (q \geq 4GM/c^2), \quad (\text{III-5})$$

and so

$$\sigma(q) < \sigma^*(q) \leq 2\sigma(q).$$

This takes account of the relativistic shift of the periastron, but does not correct for the changed tidal forces in the Schwarzschild geometry (however, see Misner, Thorne, and Wheeler 1973, p. 821) which are neglected here. The use of Schwarzschild formulae will be justified by the results presented later, where it is clear that the hole is Schwarzschild in nature when relativistic effects become important.

The flux into the disk,

$$F_{\text{disk}} = (\sigma^*(r_t)/\sigma(r_t))F_0 - F_{p1},$$

or

$$F_{\text{disk}} = [(\sigma^*(r_t) - \sigma)/\sigma(r_t)]F_0, \quad (\text{III-6})$$

where the relativistic correction has been made and the flux into plunge orbits subtracted. We take $F_{\text{disk}} = 0$ when $M \geq 6 \times 10^7 M_\odot$ (this is smaller than the Hills 1975 cutoff because, for a Schwarzschild hole, the cutoff occurs when the gravitational radius is half the tidal radius).

We may add the contribution due to red giant stars which do not begin to suffer a cutoff until $M \geq 5 \times 10^8 M_\odot$,

$$F_{\text{rg}} = 0.06 F_0,$$

as calculated in Young, Shields, and Wheeler (1977). Then equation (III-6) becomes

$$F_{\text{disk}} = [(\sigma^*(r_t) - \sigma)/\sigma(r_t) + 0.06]F_0. \quad (\text{III-7})$$

The fluxes given in equations (III-4) and (III-7) rely on having stars incident upon the hole uniformly in angular momentum space. The mechanism that provides the flux is diffusion into "loss-cones" (Lightman and Shapiro 1976); and should it turn out that the expected distribution is biased toward higher angular momentum (preferential population of the edge of the "loss-cone"), then the strength of F_{disk} will be raised relative to F_{p1} .

We now consider the angular momentum of stars captured in plunge orbits. For "parabolic" orbits in the equatorial plane of a Kerr hole, plunge orbits lie in the range

$$-2[M + (M + a)^{1/2}M^{1/2}] < L/\mu < 2[M + (M - a)^{1/2}M^{1/2}], \quad (\text{III-8})$$

giving an average swallowed flux of

$$L(a) = M^{1/2}[(M - a)^{1/2} - (M + a)^{1/2}]. \quad (\text{III-9})$$

For an isotropic flux (noting that the average angular momentum from particles incident along the poles is, by symmetry, zero),

$$L_{\text{pl}}(a) \approx \frac{2}{3}M^{1/2}[(M - a)^{1/2} - (M + a)^{1/2}], \quad (\text{III-10})$$

and numerical computations verified this approximation (Young 1976*b*). The greater cross section of a black hole to negative angular momentum particles is exhibited in the fact that $L_{\text{pl}}(a) \leq 0$.

The evolution of the hole after accretion of mass dm from plunge orbits alone is

$$dM = dm, \quad dJ = d(aM) = L_{\text{pl}}(a)dm. \quad (\text{III-11})$$

Setting $a^* = a/M$; $L_{\text{pl}}^* = L_{\text{pl}}/M$, equations (III-11) reduce to

$$da^*/d \ln M = L_{\text{pl}}^* - 2a^* \approx (2/3)[1 - a^{*1/2} - (1 + a^*)^{1/2}] - 2a^*. \quad (\text{III-12})$$

Thus

$$\ln(M/M_0) = -\frac{3}{2} \int \frac{da'^*}{(1 + a'^*)^{1/2} - (1 - a'^*)^{1/2} + 3a'^*}, \quad (\text{III-13})$$

where the integration is from a_0^* to a^* . For small a_0^* this has the approximate solution

$$a^*/a_0^* = (M_0/M)^{8/3}. \quad (\text{III-14})$$

The numerical integration of equation (III-13) is displayed in Figure 1.

A black hole accreting material with zero average angular momentum will evolve according to

$$da^*/d \ln M = -2a^*,$$

or

$$a^*/a_0^* = (M_0/M)^2. \quad (\text{III-15})$$

The evolution of the spin of the hole represented by this is also plotted in Figure 1.

Now, in consideration of a black hole ($M \approx 10^3 M_\odot$) in a globular cluster, it should be noted that the "plunge flux" is negligible ($r_i \gg 4GM/c^2$) and also that the stars bound to the hole are responsible for the tidal disruption

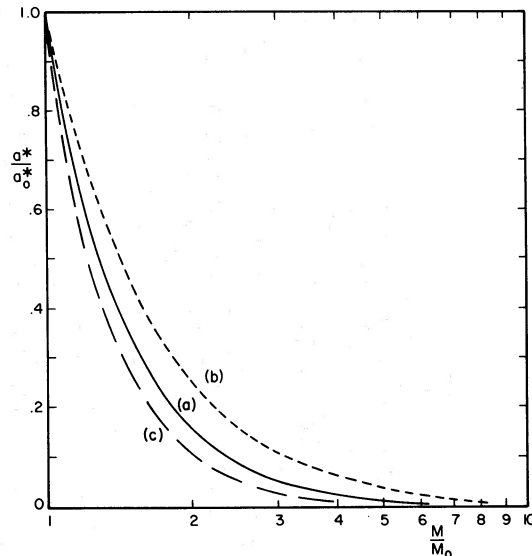


FIG. 1.—Evolution of the spin a^* of a black hole against mass M , when (a) matter is captured from plunge orbits alone, (b) matter with zero mean angular momentum is captured, (c) matter is accreted from disks at random orientations to the spin axis.

flux into an accretion disk. For a small enough black hole (and this also applies to a small black hole in a galactic nucleus), the hole has time between stellar disruptions to eat one stellar mass at the Eddington limit ($t_{\text{encounter}} \propto M^{4/3}$; $t_{\text{eat}} \propto M$). The concept of a permanently present accretion disk should then be replaced by a series of disks at random orientations to the spin axis of the hole, caused by the discrete stellar encounters at random orbital inclinations, followed by consumption of the disk at that random angle. The evolution of the hole is then independent of the absolute value of the flux F , and we no longer require $F \propto r_t$ (since we neglect plunge fluxes). It will proceed according to

$$dM = \frac{1}{2}[E^+(a) + E^-(a)]dm, \quad (\text{III-16a})$$

$$d(aM) = \frac{2}{3}[L^+(a) + L^-(a)]dm, \quad (\text{III-16b})$$

where L^+ , E^+ refer to the last stable circular prograde orbit, and L^- , E^- to the retrograde equivalent. The factor of $\frac{2}{3}$ allows for the fact that polar disks will accrete zero angular momentum around the spin axis.

Then

$$\frac{da^*}{d \ln M} = \frac{4}{3} \frac{[L^+(a) + L^-(a)]}{M[E^+(a) + E^-(a)]} - 2a^*, \quad (\text{III-17})$$

where $a^* = a/M$. Numerical integration yields the curve c on Figure 1, the spin-down being more rapid than that given by (III-15) or (III-13). For small spin a^* the evolution proceeds as

$$a^*/a_0^* = (M_0/M)^{10/3}. \quad (\text{III-18})$$

We have ignored the tendency of a disk around a rapidly spinning hole to settle into the equatorial plane; this would increase the factor $\frac{2}{3}$ in (III-16b) to a value nearer to unity, which would enhance the rapidity of the spin-down. Furthermore we have assumed "isotropic formation of disks," i.e., isotropic tidal disruption of stars. This is valid for $M < 10^6 M_\odot$ when relativistic effects are negligible.

IV. NUMERICAL INTEGRATIONS

In this section we consider (i) the equilibrium spin of a black hole accreting matter from a disk and also from a plunge flux (the sources of the fluxes are unspecified, it simply being assumed that fraction f of the flux is from accretion disk); (ii) the black hole in a galactic nucleus. Here we compute, from the cross sections of § III, the ratio of tidal to plunge flux for a given black hole mass and then integrate the equations describing the evolution. It will be seen as the hole grows that the plunge flux becomes more important than the tidal flux; this causes the black hole to spin down. Let fraction f of the total flux of material eaten by the hole be from the accretion disk:

$$f = F_{\text{disk}}/(F_{\text{disk}} + F_{\text{pl}}). \quad (\text{IV-1})$$

Setting the total flux $(F_{\text{disk}} + F_{\text{pl}}) = \dot{M}_0$, by combining equations (II-6) and (III-11) the evolution of the hole is described by

$$\begin{aligned} \frac{dM}{dt} &= f \left[\dot{M}_0 E^+(a) + \frac{dM}{dt_{\text{rad}}} \right] + (1-f)\dot{M}_0 \\ \frac{dJ}{dt} &= \frac{d(aM)}{dt} = f \left[\dot{M}_0 L^+(a) + \frac{dJ}{dt_{\text{rad}}} \right] + (1-f)L_{\text{pl}}(a)\dot{M}_0 \end{aligned} \quad (\text{IV-2})$$

Setting $a^* = a/M$; $L_{\text{pl}}^* = L_{\text{pl}}/M$; $E^*(a^*) = E^+(a)$; $L^*(a^*) = L^+(a)/M$; $(dJ^*/dt)_{\text{rad}} = (dJ/dt)_{\text{rad}}/M$, we obtain

$$\frac{dM}{dt} = f \left[\dot{M}_0 E^*(a^*) + \frac{dM}{dt_{\text{rad}}} \right] + (1-f)\dot{M}_0, \quad (\text{IV-3})$$

$$\frac{da^*}{d \ln M} = \frac{f[L^*(a^*) + \dot{M}_0^{-1} dJ^*/dt_{\text{rad}}] + (1-f)L_{\text{pl}}^*(a^*)}{f[E^*(a^*) + \dot{M}_0^{-1} dM/dt_{\text{rad}}] + (1-f)}. \quad (\text{IV-4})$$

For given f the equilibrium angular momentum of the hole is given by the solution of $da^*/d \ln M = 0$. These solutions have been computed numerically, and the results are presented in Table 1 and Figure 2, for a disk emission corresponding to an electron scattering atmosphere as prescribed by Thorne (1974). The effect of radiation was slight except for f very close to unity. Flux fractions $f \lesssim 0.8$ are sufficient to wind down the hole significantly ($a^* \lesssim 0.89$; $\eta \lesssim 0.15$).

The value of f as a function of the mass of the hole M is shown in Figure 3, and the numerical integration of equation (IV-4) is shown in Figure 4. Also shown in Figure 4 is the equilibrium spin corresponding to the values of f from Figure 3; this shows the slight lag of the hole behind its equilibrium spin.

TABLE 1
BLACK HOLE EQUILIBRIUM SPINS

Fraction of Flux into Disk, f	Equilibrium Spin a^*	Efficiency η	Radius of Last Stable Circular Orbit r_{ms}/M	Angular Momentum of Last Stable Circular Orbit L^+/M
0.0.....	0.000	0.057	6.000	3.464
0.1.....	0.128	0.062	5.573	3.338
0.2.....	0.255	0.067	5.139	3.205
0.3.....	0.377	0.074	4.699	3.063
0.4.....	0.495	0.082	4.254	2.910
0.5.....	0.606	0.092	3.802	2.745
0.6.....	0.711	0.105	3.343	2.566
0.7.....	0.806	0.124	2.872	2.365
0.8.....	0.891	0.151	2.383	2.132
0.9.....	0.959	0.200	1.852	1.837
1.0.....	0.998	0.321	1.237	1.392

V. DISCUSSION

The back-torque exerted by material captured from plunge orbits is sufficient to wind down a hole in a galactic nucleus as its mass increases from 10^5 to $10^8 M_\odot$ (Fig. 4), and thereafter the hole remains in a nearly Schwarzschild state as was considered by Lynden-Bell (1969). This reduces the luminosity available from the accretion disk fed by tidal disruption of stars by a factor of 5, assuming it to be initially sub-Eddington, since the efficiency of conversion of matter to energy in the disk becomes $\eta = 0.06$ ($a^* = 0$) instead of $\eta = 0.32$ ($a^* = 0.998$).

A small black hole in a globular cluster, $M \approx 10^3 M_\odot$ (Bahcall and Ostriker 1975; Silk and Arons 1975), is also likely to be effectively Schwarzschild since the time between stellar encounters is $\sim 3 \times 10^8$ yr (Lightman and Shapiro 1976) and the time required to eat a star at the Eddington limit is 3×10^9 yr (in the unfavourable case when $a^* = 0.998$). Thus the discrete encounters with stars from random directions will reduce the angular momentum of the hole according to the evolution described by equation (III-17). An increase in mass by a factor of 2 would require $\sim 3 \times 10^{10}$ yr and would reduce $a^* = 0.998$ ($\eta = 0.32$) to $a^* = 0.10$ ($\eta = 0.07$); and an increase in mass by a factor of 1.5 would require $\sim 1.5 \times 10^{10}$ yr and would reduce the spin to $a^* = 0.26$ ($\eta = 0.07$). This also applies to a small black hole in a galactic nucleus. Rotation of the star system to a maximum ellipticity $\epsilon \approx 0.5$ will fail to distort the distribution function of the stellar population enough to bias the flux toward positive angular momentum; the distribution will be approximately Maxwellian in rotating axes, but the angular velocity of these axes will be small, corresponding to a rotation period \sim free-fall time (5×10^6 yr for a globular cluster and 5×10^8 yr for a galaxy).

If the flux to the disk is supplied from sources other than tidal disruption of stars, enough to overwhelm the plunge orbit flux, then it is still possible to wind up the hole to a nearly Kerr state provided that the plunge orbit flux is not comparable to the Eddington limit of the disk representing the approximate maximum accretion rate possible from the disk.

Other points to note are: (i) Fluxes from bound stars in a galactic nucleus (important for $M > 2 \times 10^9 M_\odot$) may never be important since the black hole will not grow much beyond the nucleus mass ($\sim 3 \times 10^8 M_\odot$; Young 1976a). (ii) When the event horizon is close to filling the Roche tidal limit, the accretion disk picture for the debris

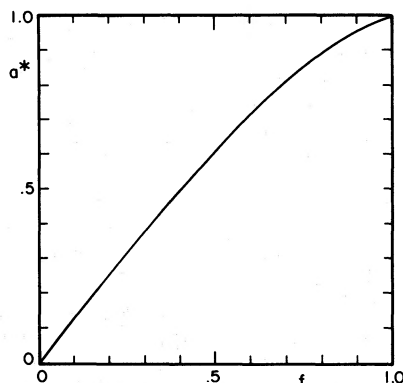


FIG. 2.—Equilibrium spin a^* of a black hole accreting fraction f of the captured mass from a disk and fraction $(1 - f)$ from plunge orbits.

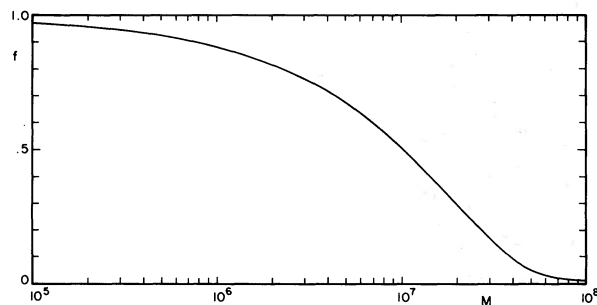


FIG. 3

FIG. 3.—Fraction of mass accreted from the disk, f , for a black hole of mass M (in M_{\odot}). The accreted flux is supplied by both tidally disrupted stars and capture of plunge orbit stars.

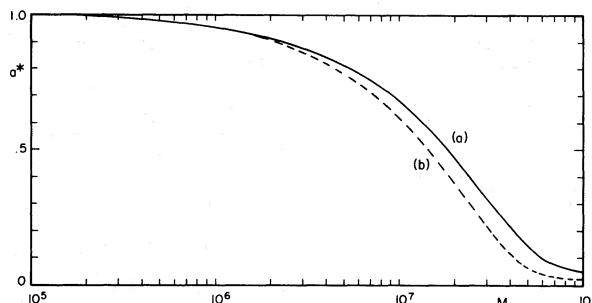


FIG. 4

FIG. 4.—Evolution of spin, a^* , of a black hole of mass M (in M_{\odot}). (a) Integration of eq. (IV-4) with f from Fig. 3. (b) The equilibrium spin at each stage corresponding to fraction f .

of tidally disrupted stars may become invalid to be replaced by a “gas-collision” scenario near the event horizon which is capable of releasing greater energies (Fabian *et al.* 1976).

In considerations of the evolution of a hole growing in a dense stellar system, the evolution of the spin may be an important factor since it can affect the energy output and structure of a surrounding accretion disk. The spin of the hole will affect the growth rate if the disk is close to the Eddington limit in its accretion rate, or if the hole is massive ($\geq 10^7 M_{\odot}$) and relativistic effects are important in the tidal disruption of stars.

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REFERENCES

- Bahcall, J. N., and Ostriker, J. P. 1975, *Nature*, **256**, 23.
 Bahcall, J. N., and Wolf, R. A. 1976, *Ap. J.*, **209**, 214.
 Bardeen, J. M. 1970, *Nature*, **226**, 64.
 Bardeen, J. M., Press, W. H., and Teukolsky, S. A. 1972, *Ap. J.*, **178**, 347.
 Fabian, A. C., Maccagni, D., Rees, M. J., and Stoeger, W. R. 1976, *Nature*, **260**, 683.
 Frank, J. H., and Rees, M. J. 1976, *M.N.R.A.S.*, **176**, 633.
 Godfrey, B. 1970, *Phys. Rev.*, **D1**, 2721.
 Hills, J. G. 1975, *Nature*, **254**, 295.
 Lightman, A. P., and Shapiro, S. L. 1977, *Ap. J.*, **211**, 244.
 Lynden-Bell, D. 1969, *Nature*, **223**, 690.
 Misner, C. W., Thorne, K. S., and Wheeler, J. A. 1973, *Gravitation* (San Francisco: Freeman).
 Novikov, I. D., and Thorne, K. S. 1973, *Black Holes*, ed. C. deWitt and B. deWitt (New York: Gordon & Breach).
 Silk, J., and Arons, J. 1975, *Ap. J. (Letters)*, **200**, L131.
 Thorne, K. S. 1974, *Ap. J.*, **191**, 507.
 Young, P. J. 1976a, *A.J.*, in press.
 ———. 1976b, *Phys. Rev.*, in press.
 Young, P. J., Shields, G. A., and Wheeler, J. C. 1977, *Ap. J.*, **212**, in press.

PETER J. YOUNG: Robinson Laboratory, California Institute of Technology, Pasadena, CA 91109