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Spin and Atmospheric Tides of Venus

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Summary. Venus should not form an exception to the law that the solar system secondaries were formed with an initial prograde spin rate of order $2\,10^{-4}\,s^{-1}$. Reviving Gold's idea of 1964, it is argued that braking by tidal torques and possibly magnetospheric friction can account for a spindown of Venus within some 10^9 years. Subsequent retrograde spinup via thermally driven atmospheric tides can explain the present slow retrograde rotation of the planet. Spin reversal has been achieved by a delicate cooperation of one-sided solar heating, longitude-dependent solar attraction, and atmospheric mass asymmetry by means of which the present 4-day atmospheric circulation was switched on just in time to reverse the rotation. The present spin period of 243.1 d cannot be understood as a stable spin-orbit resonance.

Key words: Venus — planetary spins — atmospheric tides — Q factor

Introduction

The spin of the planet Venus and the circulation of its atmosphere are exceptional within the solar system: The planet spins in a retrograde sense, i.e. against the angular momentum of the solar system, with a (long) period of 243.1 days (Shapiro, 1976). Its atmosphere circulates in the same (retrograde) sense, but with velocities that approach the velocity of sound in parts of the upper stratosphere [a typical measured wind speed is $10^2\,m\,s^{-1}$; cf. Stone (1975)]. If it is true that all solar system bodies were formed with prograde spin periods between 5 and 10 h—a law which appears to hold at least for masses between several 10^{30} and 10^{18} g (cf. Alfvén and Arrhenius, 1970)—these facts are important clues to the history of Venus and its atmosphere. In essential agreement with Goldreich and Peale (1968) and Gold and Soter (1969, 1971), I am going to argue that they ought to be understood as caused by tidal interactions with the Sun. (Other seeming counter examples to this "law of isochronism", like Uranus, Pluto, Mercury and Mars, can likewise be explained as spun down, and tilted by the action of tidal or magnetic couples.)

Suggested History

The following evolution and angular momentum balance will be used to explain the spin history of Venus: Assume, according to the above-quoted "law of isochronism", that Venus formed with an initial spin angular velocity Ω_i of order $\Omega_i \approx 2\,10^{-4}\,s^{-1}$. The sweating planet will have developed oceans on a time-scale between 10^7 and 10^9 years (Walker, 1975), and its convective interior will have generated magnetic fields on a much shorter time scale. The present dense atmosphere will have formed on a similar time scale as the oceans (Walker, 1975); its retrograde rotation tells (within our model) that it cannot have degassed before considerable spindown, i.e. not much faster than within some 10^9 years. During its history, Venus is essentially acted upon by the following five (external) torques:

1) the braking torque $T_{\odot B}$ of the Sun by solid body and ocean tides,

2) the braking torque T_M of a corotating magnetosphere "rubbing" against the solar wind,

3) a torque $T_{\oplus P}$ exerted by Earth due to a longitude dependent permanent deformation, or transverse quadrupole moment of Venus which varies sinusoidally,

4) an analogous, much larger torque $T_{\odot P}$ exerted by Sun which averages out to zero except temporarily near corotation, and

5) an accelerating torque $T_{\odot A}$ via the horizontally oscillating stratosphere (driven by solar heating) whose second harmonic has a non-zero mass quadrupole moment.

All other torques—such as a Sun-induced tidal quadrupole acted upon by Earth attraction, an Earth-induced tidal quadrupole acted upon by Sun attraction, a braking solar torque on a gravitationally induced atmospheric tide, or an accelerating solar radiation pressure acting upon the ground harmonic of the periodically heated (and hence expanded) atmosphere —can be seen to be ignorable on times scales below 10^{10} years.

In our model, the prograde spin of Venus has been slowed down soon after formation by a combination of the solar tidal and magnetospheric torques: Oceans, originally near resonant motion, a fluid interior, and possibly a strong magnetic moment have to be invoked to spin the planet down to corotation before its atmosphere has become sufficiently dense to interfere (via $T_{\odot A}$). This should not have taken longer than some 10^9 years. A delicate situation arises when Venus nears corotation: Torques 1), 2) and 5) go to zero whereas the strong oscillatory torque $T_{\odot P}$ forces its orientation into a motion analogous to that of a rotating, weakly damped pendulum. The pendulum will eventually change from rotation to libration, and come to rest. The orientation of Venus will likewise change from rotation to libration; i.e. there will be a last forward swing after which the sign of the synodic rotation rate changes. At this moment, Venus experiences the longest day in its history [of order several 10^5 to 10^6 years, see Inequation (18) below], which for certain longitudes can be some ten times longer than the preceding days. Dayside heating, and nightside cooling give rise to a strong density contrast in the (forthcoming) atmosphere, and the resulting (thermally driven) atmospheric torque $T_{\odot A}$ will be stronger on the swing back so that instead of libration, the synodic rotation is (so far temporarily) reversed. Here it has been assumed that at the time of synodic spin reversal, the accelerating atmospheric torque $T_{\odot A}$ happened to catch up with the braking tidal torque $T_{\odot B}$. This coincidence is not as improbable as it might look because inhomogeneous heating and cooling (during long days and nights) results in a faster cooling rate of the planet, a faster evaporation of the oceans, and faster degassing. Oceanic tides thereby lose at importance whereas atmospheric tides gain. Moreover and most importantly, $T_{\odot A}$ has a resonant feedback behaviour near the observed 4-day circulation of the stratosphere (for which tidal acceleration is balanced by atmospheric friction): either the stratosphere corotates with the planet, and $T_{\odot A}$ nearly vanishes, or the length of its day approaches its thermal time scale (of order days) in which case $T_{\odot A}$ can become quite large. I suggest that this resonance was switched on soon after spin reversal, by the extraordinary thermal asymmetry built up during the longest day.

With this assumption granted, spinup in the retrograde sense will ensue prograde spindown, on a time scale of some 4 10^9 years, until some day the (frequency-dependent) tidal torque $T_{\odot B}$ catches up with the (at first frequency-independent) atmospheric torque $T_{\odot A}$. The maximum available 4.5 10^9 years (since formation of the planets) seem to rule out that this torque-equilibrium has already been reached, see constraint (25) below. In the meantime, the small couple $T_{\oplus P}$ of Earth upon the frozen-in transverse quadrupole moment of Venus can wobble its rotation like shallow hills and troughs wobble the speed of a rolling ball. Such wobbling has a negligible long term effect unless the spin of Venus is in resonance with the Earth's orbit—i.e. Venus performs a half-odd number of revolutions during successive inferior conjunctions—which happens at the (inertial) spin periods $P_r = P_V P_\oplus / [r P_V - (r-1) P_\oplus]$, where r = halfodd, and $P_\oplus$, P_V are the orbital periods of Earth and Venus respectively. Starting from corotation (P_o = 224.7 d), they are: P_r/day = 225, 278, 365, 531, 975, 5922, −1455, −648, −417, −307, −243.16, −172,... (for $-r$ = 0, 0.5, 1, ...). A formerly prograde Venus has thus traversed eleven resonances uncaptured before it reached the present spin period of −243.1 days. Based on the earlier measured spin period of 243.05 d, Goldreich and Peale (1968) and Gold and Soter (1969) have discussed the possibility that Venus was presently caught in the $P_{-5} = -243.16$ d—resonance. Such a capture would demand a period coincidence to within a few 10^{-5}, see (15). The present model rules it out.

The existence of the torque $T_{\odot A}$—which we hold responsible for a retrograde spinup from corotation—has already been suggested by Gold and Soter in 1969, and used in 1971 to explain the observed 4-day circulation of the Venus stratosphere; the torque $T_{\odot A}$ acts on the atmosphere, essentially between 40 and 70 km altitude, and sets up a large velocity gradient because of the small laminar viscosity in layers with Richardson number larger than unity[1]. Several other models have been put forward to explain this impressive 4-day circulation, cf. Stone (1975), such as the "moving flame" mechanism, rising Hadley cells, and the like; they cannot easily explain the global velocity gradient—which demonstrates itself most impressively by patterns wrapped around the equator up to three times, cf. Hunten (1975)—nor the two layers of small-scale turbulence at altitudes of 45 and 60 km measured by Woo (1975). Such a global velocity gradient seems to imply the action of an external torque, i.e. cannot be maintained by forces inside a closed system.

Goldreich and Peale (1970) have argued that a retrograde spin state is unstable unless there acts an accelerating atmospheric torque, or a dissipative torque between a rigid mantle and a differentially rotating liquid core. I favour the former mechanism, for the reasons given above and below.

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[1] This manuscript has been written in April 1976. Meanwhile, new measurements by Venera 9 and 10 show a different velocity profile from that by Venera 8. The new result may tell us that laminar and convective viscosity can dominate in turns. This does not invalidate our estimates below which are based on the (handier) laminar data

Notation and Numbers

Definition of a quantity A by an expression B: $A:=B$.

We calculate in CGS units, and use the convention that a physical quantity A of dimension a is written $A=:A_\alpha 10^\alpha a$, where $A_\alpha = A/10^\alpha a$ is a pure number. Examples: $\Omega_{-4}:=\Omega/10^{-4}\,\mathrm{s}^{-1}$, $B_2:=B/10^2$ Gauss, $(\Delta I/I)_{-5}:=10^5\,\Delta I/I$.

Most of the following numbers are taken from Alfvén and Arrhenius (1970): M=mass of Venus$=4.87\;10^{27}$ g, R=radius of Venus $=0.605\;10^9$ cm, $M_\oplus$=mass of Earth$=5.97\;10^{27}$ g, $R_\oplus$=radius of Earth$=0.638\;10^9$ cm, $M_\odot$=mass of Sun$=1.99\;10^{33}$ g, G=Newton's constant$=0.667\;10^{-7}$ cm^3 g^{-1} s^{-2}, $a_\odot$=distance Sun-Venus$=1.08\;10^{13}$ cm, ρv^2=(solar wind ram pressure at $r=a_\odot$)$=4\;10^{-8}$ g cm^{-1} s^{-2}.

Braking Tidal Torque

We now come to a quantitative estimate of the torques introduced above. The solar gravitational monopole induces a prolate quadrupole moment upon the solid body of Venus whose symmetry axis lags behind the connecting vector by an angle χ due to imperfect elasticity; this inclined mass quadrupole couples back upon the solar monopole, and yields the torque (Goldreich and Peale, 1968)

$$T_{\odot B}=(3kGM_\odot^2R^5/2a_\odot^6)\sin 2\chi$$
$$=0.51\;10^{25}\,\mathrm{erg}\,(4k)\sin 2\chi, \tag{1}$$

where G=Newton's constant, $M_\odot$=mass of the Sun, R=radius of Venus, $a_\odot$=distance of Venus from the Sun,

$$k=(3/2)\,(1+19\,\mu/2\,g\,\rho\,R)^{-1}, \tag{2}$$

(μ, g, ρ=shear modulus, gravity field strength, mass density), is the "tidal Love number" whose value for Earth is 0.3, and

$$\mathrm{tg}\,2\chi=\Omega/(\Omega_0^2-\Omega^2)\tau=:Q^{-1} \tag{3}$$

describes the phase lag 2χ of a harmonic oscillator of resonance angular frequency Ω_0, damping time τ, driven by a periodic excitation of angular frequency Ω. The quality factor Q is sometimes defined as 2π times the tidal energy divided by the energy dissipation per period, and sometimes by 2π times the number of cycles in an e-folding time of the decreasing intensity of a free oscillation.

Our definition (which corresponds to the first alternative, and seems to be common in planetary physics) suggests that for a solid planet, Q^{-1} should behave as Ω for $\Omega \ll \Omega_0$. In reality, plastic flow can to some extent compensate this Ω-dependence, cf. Goldreich and Soter (1966). In the mantle of Earth, $\Omega_0\tau$ takes values between 10^2 and 10^3, and

$$\Omega_0 \approx 2\,c_s/R \gtrsim 10^{-3}\,\mathrm{s}^{-1} \tag{4}$$

(c_s=sound speed).

Present best estimates for oceanic tidal dissipation (cf. Zahel, 1976) yield an effective Q-factor of 12 for Earth, which is related to a growth rate of $2\;10^{-5}$ s/year of the length of the day. We conclude that oceans near resonance can be more efficient in braking rotation than a solid mantle by a factor 10^2 and more.

When a torque T is applied to a rotator of moment of inertia $I\approx MR^2/3$, its angular frequency Ω changes on the time scale

$$\tau:=I\Omega/T. \tag{5}$$

In particular, insertion of (1) for Venus yields the spindown time

$$\tau_{\odot B}=3.7\;10^8\,y\,\Omega_{-4}Q/4k \tag{6}$$

with $\Omega_{-4}:=\Omega/10^{-4}\,\mathrm{s}^{-1}$. The uncertainty in this spindown time is hidden in the factor $Q/4k$: $4k$ may range from 1 (for a solid Venus) to 6 (for a fluid, or unsettled Venus, with $\Omega_0\tau=10^2$), cf. Equation (2); and Q may range from $10^2\,\Omega_0/\Omega$ (for a rather solid Venus) down to order unity (for a rather viscous [unsettled] Venus, possibly with oceans near resonance). (Q will decrease with amplitude due to plastic flow). As a result, $\tau_{\odot B}$ for the primordial Venus is expected to take values between $10^8\,\Omega_{-4}$ y and 10^9 y. In the first case, spindown would proceed linearly with time, i.e. within some 10^8 years, whereas in the second case, spindown would proceed exponentially, and would take more than 6 e-folding times to reduce Ω from the initial $\Omega_i=2\;10^{-4}\,\mathrm{s}^{-1}$ to the corotation value $\Omega_c=3\;10^{-7}\,\mathrm{s}^{-1}$. Whether or not $T_{\odot B}$ has been strong enough to brake the rotation of Venus within less than some 10^9 y depends on its rather uncertain viscous structure.

Magnetospheric Torque

Next we consider the possibility of a corotating magnetosphere. Note that planetary dynamos can generate equipartition magnetic fields B of order (cf. Kippenhahn und Möllenhoff, 1975)

$$B\approx(4\pi\rho u^2)^{1/2}=10^2\,u_1\,\mathrm{Gauss}, \tag{7}$$

where u is a typical convective velocity of the fluid interior, and $u_1:=u/10$ cm. The extent R_M of a primordial Venusian magnetosphere would be determined by the equality of solar wind ram pressure ρv^2 and magnetic pressure $B^2/8\pi$, where $B\approx B_0(R/R_M)^3$ is the assumed magnetic dipole field strength of surface value B_0. (Deviation from a dipole geometry can be accounted for by doubling B_0).

We find

$$R_M/R=(B_0^2/8\pi\rho v^2)^{1/6}=53\,B_2{}^{1/3}. \tag{8}$$

The highly conducting solar wind bends magnetic field lines towards a near-toroidal direction. Integration of Maxwell's stresses over a surface near the magnetopause at which magnetic field lines are inclined at $\lesssim 45°$ yields the torque

$$T_M \lesssim (4\pi R_M^3)\,(B^2/8\pi) = \\ = \sqrt{2\pi}\, B_0 R^3 (\rho v^2)^{1/2} = 10^{25}\ \text{erg}\ B_2\,, \quad (9)$$

and the related spindown time [according to Eq. 5]

$$\tau_M \approx 2\ 10^8\, y\, \Omega_{-4} B_2^{-1} (\rho_0 v_0^2/\rho v^2)^{1/2}, \quad (10)$$

where an index "0" denotes present day values. Allowing for a stronger solar wind in the past, I find it likely that magnetospheric friction has been of comparable importance to tidal friction in the past.

Note however that Hirshberg (1972) derived a much smaller upper limit on T_M from the objectionable assumption that solar wind plasma be accelerated to at most corotational velocities. Note, on the other hand, that (i) the Earth-Moon system may have lost angular momentum in the past (Weinstein and Keeney, 1973), as well as Mars, that (ii) even the Moon may have had a primordial magnetic field (Runcorn, 1975) and that (iii) neutron stars in binary systems seem to lose angular momentum by magnetospheric friction (Kundt, 1976).

Permanent Moment Torque

Next we estimate the longitude-dependent torque exerted by Earth on Venus due to the existence of highlands and lowlands (or ocean basins), using Earth as a model. (An upper bound to the height of mountain ranges above sea floor level is set by the visco-elastic properties of the underlying material, which are expected to be similar on Venus.) In the case of Earth, such a permanent inhomogeneity in the mass distribution has a measured relative transverse quadrupole moment $\Delta I/I = 2.2\ 10^{-5}$, cf. Gold and Soter (1969), corresponding to a difference ΔR of principal axes of the equivalent triaxial ellipsoid of $\Delta R = 0.14$ km. For Venus with its slightly weaker surface gravity, but probably warmer crust we expect a similar value. The resulting torque-amplitude $T_{\oplus P}$ exerted by Earth, averaged over a synodic period near the r^{th} spin-orbit resonance, has been calculated by Goldreich and Peale (1968):

$$T_{\oplus P} = 3GM_{\oplus} \Delta I K(r)/2a^3 = 3\ 10^{21}\ \text{erg}\ (\Delta I/I)_{-5}\,, \quad (11)$$

where a is the astronomical unit, $M_{\oplus}$ = mass of Earth, and the dimensionless $K(r)$ take values between 1 and 4.5 for $-7 \lesssim r \lesssim -1.5$; we have inserted $K(-5) = 2.53$ above. The time scale $\tau_{\oplus P}$ related to $T_{\oplus P}$ [Eq. (5)] is

$$\tau_{\oplus P} = 2\ 10^9\, y\, \Omega_{-6.5} (\Delta I/I)_{-5}^{-1}. \quad (12)$$

On the other hand, if Venus were trapped in the $r = -5$ resonance, its semi-period $\tilde{\tau}_{\oplus P}$ for small oscillations (of the orientation angle ψ) would be given by

$$\tilde{\tau}_{\oplus P} = \pi (I/2T_{\oplus P})^{1/2} = 3\ 10^4\, y\, (\Delta I/I)_{-5}^{-1/2}, \quad (13)$$

corresponding to an equation of motion

$$I\ddot{\psi} + T_{\oplus P} \sin 2\psi = T_{\odot B} - T_{\odot A}\,. \quad (14)$$

$\tilde{\tau}_{\oplus P}$ measures the effective time for trapping whereas $\tau_{\oplus P}$ is the characteristic time for spindown (or spinup); in the r^{th} resonance, Ω must coincide with Ω_r within

$$|\Omega - \Omega_r|/\Omega_r \lesssim \tilde{\tau}_{\oplus P}/\tau_{\oplus P} = 1.5\ 10^{-5}\ (\Delta I/I)_{-5}^{1/2}. \quad (15)$$

A measured $3\ 10^{-4}$ disagreement between Ω and Ω_r would demand $\Delta I/I$ to be of order $4\ 10^{-3}$, an improbably high value within our model.

The permanent transverse quadrupole of Venus also feels a solar torque $T_{\odot P}$ given [in analogy to Eq. (11)] by

$$T_{\odot P} = (3GM_{\odot} \Delta I/2a_{\odot}^3) \sin 2\psi \\ = 10^{27}\ \text{erg}\ (\Delta I/I)_{-5} \sin 2\psi \quad (16)$$

corresponding to a characteristic time

$$\tau_{\odot P} \gtrsim 0.6\ 10^4\ \text{y}\ \Omega_{-6.5} (\Delta I/I)_{-5}^{-1}. \quad (17)$$

This strong torque modulates the rotation of Venus, predominantly near corotation. Transition from rotation to libration occurs when the minimum synodic angular velocity Ω (during one tide) is less than its dissipational change $\Delta\Omega$ during one tide, the latter lasting almost $\pi/\Omega_{\min}$. From this condition, viz. $\Omega_{\min} < \Delta\Omega = \dot{\Omega}\pi/\Omega_{\min}$, and $\dot{\Omega} = \Omega/\tau_{\odot B}$ we find $\Omega_{\min}^2 < \pi \langle \Omega/\tau_{\odot B} \rangle$, where $\langle \ldots \rangle$ denotes averaging over a tide, whence

$$\Delta t := \pi/\Omega_{\min} > \langle \Omega/\pi\tau_{\odot B} \rangle^{-1/2} = 10^5\ \text{y}\ \langle 4k/Q \rangle_{-5}^{-1/2}. \quad (18)$$

$2\,\Delta t$ is the duration of the longest day on Venus. Our a posteriori estimates (25), (28), (31) below suggest $\langle 4k/Q \rangle_{-5} \gtrsim 1$, so that Δt is expected to be of order several 10^5 years or longer.

Atmospheric Torque

The last torque to be calculated is the solar couple upon the thermally induced semidiurnal stratospheric tide; cf. Gold and Soter (1971). It is known that the solar radiation is (nowadays) essentially absorbed by clouds above 35 km altitude, and that convective motions force the lower part of the atmosphere to essentially corotate with the planet, within velocities of order 1 m/s at least below 10 km. This permits us to ignore gravitational tidal forces upon the atmosphere (because its bottom part is glued to the planet, i.e. behaves like an outer mantle, and its top part is light), and to ignore daily temperature variations in the troposphere (at least below 35 km).

Once the atmosphere is set into rotation, the daily heating by the Sun causes a temperature maximum somewhere between noon and evening. On Earth, the phase of maximal temperature in the upper atmosphere is at $(45 \pm 15)°$. In comparison, the longer duration of an atmospheric day and the larger relative heat input into the stratosphere of Venus will raise the importance of radiative cooling, and thereby tend to decrease the phase lag; on the other hand, a larger heat capacity (below 50 km) will tend to increase the lag. For adiabatic motion, the thermal energy stored in some piece of atmospheric column changes in proportion to $(f/2+1)\int(\Delta p/T)\,dz$, where f = number of molecular degrees of freedom, Δp = pressure difference and z = height coordinate. Radiative cooling of an atmospheric layer proceeds in proportion to $\sigma\Delta(T^4)$ (σ = Stefan-Boltzmann's constant). The (radiative) cooling time scale $\tau_{\rm rad}$ is given by their ratio:

$\tau_{\rm rad}=(f/2+1)\int(\Delta p/\Delta T)dz/8\sigma T^3$. The phase lag of daily atmospheric heating will grow from 0 to $\pi/2$ when the relative cooling time $\delta := \tau_{\rm rad}/\tau_{\rm day}$ ranges from small to large values (compared with unity), where $\tau_{\rm day}$ is the length of an atmospheric day; for small fluctuations we have:

$$\delta=(f+2)\int p\,dz/8\sigma T^4\tau_{\rm day}. \qquad (19)$$

When the integral is taken over a scale height, δ becomes a rapidly varying function of z which ranges through comparable values (of typical order 10) for Earth and for Venus (at its present circulation period of 4 days). We shall give an independent argument below that the phase of maximal temperature in the stratosphere of Venus should lie somewhere in the afternoon interval.

A zone of high temperature gives rise to atmospheric expansion and circulation that turns it into a zone of low density. In this way, a diurnal horizontal density oscillation is excited: the driving agent is a temperature input of rectified cosine behaviour. All Fourier components of a rectified cosine have a maximum where the rectified cosine peaks. In particular, the second harmonic has its minima at right angles to that peak. It excites the second harmonic of a horizontal density oscillation of phase lag between 0 and $\pi/2$ (depending on the ratio of exciting to resonance frequency), whose first maximum occurs in the morning sector, at more than $\pi/2$ with respect to the temperature maximum. For Earth, the second harmonic is near resonance: $\pi R/c_s=0.7$ d, where c_s is the atmospheric velocity of sound, and damping lowers the resonance frequency; the observed oscillation has a ground pressure amplitude Δp of 1 mb and phase lead of 34°. For Venus, the second harmonic is a factor of 4 slower than the stratospheric eigen frequency; when compared with Earth, its (relative) amplitude should be reduced by roughly a factor 4^2 = squared ratio of exciting to resonant frequency, and its density minima should be roughly at right angles (because of negligible phase lag).

Only the second harmonic (in a Fourier decomposition of the horizontal density oscillation) gives rise to a non-zero mass quadrupole moment; the corresponding tidal torque reads (Gold and Soter, 1971):

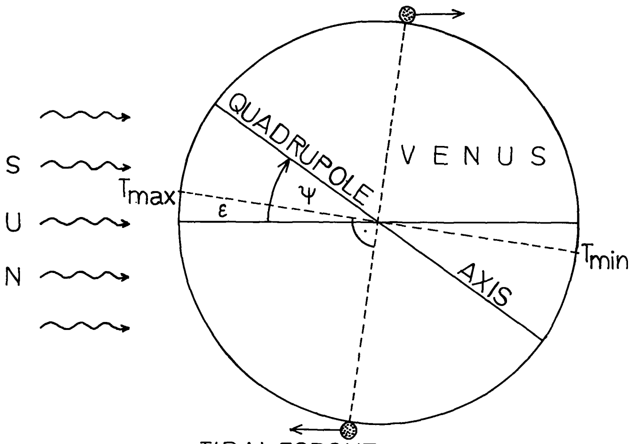

Fig. 1. Solar couple upon the (thermally induced) mass quadrupole of the Venusian atmosphere. Retarded warming up (cooling) causes a temperature maximum (minimum) at an angle ε in the afternoon sector which in turn causes an atmospheric density distribution whose second harmonic has its maxima (symbolized by dots) roughly at right angles. The resultant tidal torque propels the atmosphere and the planet. This effect caught up with solid body tidal braking (it is claimed) soon after (synodic) spin reversal, during the longest afternoon of the planet, and is held responsible for the 4-day atmospheric circulation

$$T_{\odot A}=(15\pi^2 M_\odot R^6\Delta p/32 M a_\odot^3)\sin 2\varepsilon$$
$$=0.7\;10^{22}\ {\rm erg}\ (\Delta p)_2 \sin 2\varepsilon, \qquad (20)$$

where Δp is the pressure amplitude of the second harmonic at the bottom of that part of the atmosphere which partakes in the oscillation, $(\Delta p)_2 := \Delta p/10^2$ dyn cm^{-2}, and $\pi/2-\varepsilon$ is its phase lead. For Earth, $T_{\odot A}$ amounts to some 10% of the tidal torque by the Moon. See Figure 1.

We have guessed above that $\sin 2\varepsilon$ should not be much smaller than unity. As an independent argument, let us invoke stability: When the atmosphere of Venus was spun up from corotation with the planet, a sizable cumulative torque was necessary to set up considerable atmospheric circulation (of spin rate Ω_A). As will be shown around Equation (29), this torque had to be stronger than some 10^{21} erg $(p/at)\,(4k/Q)^{1/2}_{-5}\,(\Delta t)^{-1}_{13.5}$. The second atmospheric density harmonic caused by a hot afternoon sector of the librating planet is held responsible for this initial torque, which changed sign and grew in magnitude during (synodic) spin reversal. As a result of atmospheric spinup, the ratio δ of relevant time scales grows in proportion to Ω_A, which in turn implies a growing angle ε. Now differential atmospheric rotation creates a resistent shear torque T_A which increases in proportion to $\Omega_A-\Omega$ until $T_{\odot A}$ and T_A get into equilibrium:

$$T_{\odot A}=T_A. \qquad (21)$$

This turned-on $\delta-\varepsilon$-generator appears to work as a stable feed-back system: a decrease in the circulation rate Ω_A decreases δ and thereby turns ε towards noon; at the same time it reduces T_A; the result is an increase in $T_{\odot A}-T_A$ which raises Ω_A. The optimal Ω_A corresponds to the observed 4-day period; it falls short by a factor 4 of oscillatory resonance. Near stable equilibrium, we expect sin 2ε and δ to be of order unity.

Equation (21) gives us a more reliable estimate of the torque $T_{\odot A}$. In almost agreement with Gold and Soter (1971) we find

$$T_A = 4\pi R^3 \eta \partial_z v \langle \sin \vartheta \rangle = 3.4\ 10^{21}\ \text{erg}\ (\partial_z v)_{-2} \qquad (22)$$

with η = dynamic viscosity coefficient (of CO_2 at 300 K, $=1.5\ 10^{-4}$ cgs), $\partial_z v$ = average laminar velocity gradient, and $\langle \sin \vartheta \rangle :=$ spherical average of $\sin \vartheta = \pi/4$. The (planetary) spinup time belonging to T_A is

$$\tau_A = 1.8\ 10^9\ \text{y}\ \Omega_{-6.5} (\partial_z v)^{-1}_{-2}.$$

A comparison of Equations (20)–(22) gives

$$(\Delta p)_2 \sin 2\varepsilon = 0.5, \qquad (23)$$

a reasonable value.

If our suggested history is correct, Venus has been spun up by $T_A - T_{\odot B}$ (from corotation) throughout some 4 10^9 years. Indeed, the condition

$$(T_A - T_{\odot B})\ \tau_{\text{spinup}} = I(\Omega_{\text{now}} + \Omega_c) \qquad (24)$$

and Equations (1), (3), (22) yield $\tau_{\text{spinup}} = 3.5\ 10^9$ y for $T_{\odot B} \ll T_A$, a remarkable check on the model. Moreover, the condition $\tau_{\text{spinup}} = 4.5\ 10^9\ \text{y} - \tau_{\text{spindown}} \leq 4.3\ 10^9$ y implies $T_{\odot B} \leq T_A/4$, whence

$$Q/4k \geq 0.6\ 10^4 \qquad (25)$$

for the average Q-factor-by-tidal-Love-number ratio since (synodic) spin reversal; (at present, $4k$ is held to be of order unity). Equilibrium between $T_{\odot A}$ and $T_{\odot B}$ would demand $Q/4k = 2\ 10^3$. We conclude that spinup is still going on.

The Longest Day of Venus

Whereas all the big planetary satellites are presently caught in synchronous rotation by tidal couples, Venus succeeded to escape, driven by its atmospheric generator. This remarkable mechanism may have operated as follows.

A degassing atmosphere will tend to corotate with its planet, held by convective friction. An exception to this tendency has been discussed above: thermally driven atmospheric tides can give rise to a mass quadrupole whose tidal couple propels the atmosphere. For a small spin rate of Venus, this atmospheric generator seems to act with a period of 4 days. Had a considerable atmosphere already existed when the spin period of Venus was still of order 4 days in the prograde sense, Venus would never have reached corotation: the atmospheric generator would have been started in the prograde sense, and would have balanced solid body tidal friction at some equilibrium spin period of order week to day, depending on $Q/4k$. We conclude that the Venusian atmosphere has come into existence at a time when the spin period had grown considerably beyond 4 days.

For long spin periods of the planet, the atmospheric generator cannot easily get started. A new situation occurs, however, when rotation changes into libration: now the longitude-dependent solar heat input shows a much stronger asymmetry. Its time integral over the longest day resembles remotely a rectified cosine. Dayside heat diffuses into the upper planetary crust whereas the nightside cools considerably. On the swing back, the temperature distribution of the planetary surface has its maximum somewhat in the new afternoon sector. This temperature distribution is communicated to the atmosphere. For constant pressure, the density distribution of the atmosphere is inversely proportional to its temperature. Fourier expansion of the longitude-dependent mass distribution results in a non-zero second harmonic with (one) maximum in the (new) morning sector.

The solar torque $T_{\odot A}$ upon this second harmonic was found in Equation (20):

$$T_{\odot A} = 0.7\ 10^{22}\ \text{erg}\ (\Delta p)_2 \sin 2\varepsilon, \qquad (26)$$

where the ground layer pressure amplitude Δp is the equivalent of a density amplitude Δn, and where $(\Delta p)_2$ sin 2ε is expected to rise from values below $0.7\ 10^{-2}$ $\langle 4k/Q \rangle_{-5}$ to values above it during the longest afternoon, in order that spinup via $T_{\odot A}$ overcome spindown via the braking solid body torque $T_{\odot B}$ [given in (1) with (3)]:

$$T_{\odot B} = 0.51\ 10^{20}\ \text{erg}\ (4k/Q)_{-5}. \qquad (27)$$

The quantity $(\Delta p)_2$ sin 2ε should tend asymptotically towards its present value 0.5, cf. Equation (23). We therefore face the constraint $0.7\ 10^{-2} \langle 4k/Q \rangle_{-5} \lesssim 0.5$ for avoiding prograde spinup, i.e.

$$\langle 4k/Q \rangle \lesssim 0.6\ 10^{-3}. \qquad (28)$$

On the other hand, $T_{\odot A}$ should have been strong enough to start the 4-day atmospheric circulation within essentially the afternoon duration Δt of the longest day. In order to check this, let us ignore atmospheric friction, and estimate the atmospheric spin rate Ω_A resulting from a constant torque $T_{\odot A}$ acting during Δt, Equation (18), upon the moment of inertia $I_A = (8\pi/3)$ $R^4 p/g$ of the (upper transparent part of the) atmosphere:

$$\Omega_A = T_{\odot A} \Delta t / I_A = 10^{-1}\ \text{s}^{-1} (\Delta p/p)_{-1} \sin 2\varepsilon (\Delta t)_{13}. \qquad (29)$$

For a hypothetical atmospheric density contrast $\Delta p/p = 10^{-1}$ of the second harmonic, a small effective heat asymmetry $\varepsilon \geq 10^{-4} (\Delta t)^{-1}_{13}$ turns out to be sufficient to start the δ-ε-generator, i.e. $\Omega_A \to 2\ 10^{-5}\ \text{s}^{-1}$, during the longest afternoon, corresponding to

$$(\Delta p)_2 \sin 2\varepsilon \geq 0.3\ (p_6/\alpha) \langle 4k/Q \rangle^{1/2}_{-5} \qquad (30)$$

for $\Delta t =: \alpha\, 2\, 10^5$ y $\langle 4k/Q \rangle_{-5}^{-1/2}$, $\alpha > 1$, cf. Inequation (18). In order to keep this constraint (for starting the generator) compatible with the threshold constraint below Equation (26), $0.3 \cdot (p_6/\alpha)$ should not be much larger than $0.7\, 10^{-2} \langle 4k/Q \rangle_{-5}^{1/2}$, suggesting

$$\langle 4k/Q \rangle \gtrsim 10^{-5} (p_6/0.2\alpha)^2. \tag{31}$$

Constraints (25), (28) and (31) are all compatible with a "present" $Q/4k \gtrsim 10^5$. More detailed calculations should bring this out without a posteriori conclusions; but there seems to be a reasonably large class of atmospheric models for which the atmospheric circulation was switched on no earlier than, and also no later than during the longest afternoon of Venus.

Concluding Remarks

My treatment of the history of Venus differs partially from earlier treatments in the following three points:

i) A small initial Q-factor and possibly magnetic friction are involved in order to reduce a large initial Ω to a synchronous one. Plastic flow in the interior and temporary oceans (Walker, 1975) lend support to this suggestion.

ii) The torque exerted by the 4-day atmospheric circulation is held responsible for subsequent spinup (from corotation to the present retrograde value). This circulation has been started during (synodic) spin reversal, driven by a thermally induced atmospheric mass quadrupole moment, and is strong enough to have prevented Venus from getting caught in synchronous rotation.

iii) The present Q-factor must be rather large, of the order of that for the outer planets. Again, this is understandable because the interior has partially solidified, the oceans have evaporated, the magnetic moment has decayed, and Ω has fallen by a large factor below internal resonances.

If correct, this explanation has important implications for the thermal history and evaporation of the oceans because of the intermediate existence of extremely long lasting days. It also rules out models of atmospheric degassing on too short time scales.

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