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ASTRONOMY AND ASTROPHYSICS

Equation of State for Hot and Dense n, p, e-Mixture with Zero Charge Density*

M. F. El Eid and E. R. Hilf

Institut für Kernphysik, Technische Hochschule Darmstadt, Schloßgartenstr. 9, D-6100 Darmstadt, Federal Republic of Germany

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Summary. An equation of state ($P=P(\varrho, T)$ of matter (homogeneous charge-neutral mixture of electrons and interacting neutrons and protons) at high temperatures ($3\,10^9 \lesssim T \lesssim 4\,10^{11}$ K) and densities ($10^{10} \lesssim \varrho \lesssim 5\,10^{14}$ g/cm^3) as expected in a supernova core is calculated in a Thomas-Fermi approximation which simultaneously describes correctly the saturation properties of nuclear matter.

It is shown that in the region $10^{13} \lesssim \varrho \lesssim 10^{14}$ g/cm^3 and $T \lesssim 2\,10^{11}$ K the attractive nuclear interaction is lowering the pressure by a factor up to five compared to the results of previous works for pure neutron matter.

Key words: hot and dense matter — supernova — equation of state

1. Introduction

During the collapse of a massive star with a mass $M \gtrsim 7\,M_\odot$ (Schramm and Arnett, 1975), the core of the star becomes a hot and dense region ($T \gtrsim 10^{10}$ K, $\varrho \gtrsim 10^{10}$ g/cm^3).

Due to the high temperatures and densities, processes such as nuclear photodisintegration and neutronization (electron capture) will have converted the matter to a mixture of neutrons, protons and electrons as much as to assure charge neutrality (Schramm and Arnett, 1975; El Eid, 1976). Our aim is to determine for that mixture the equation of state (EOS) which is of use in the hydrodynamics of a supernova explosion. The EOS plays the key role whether or not the supernova core will collapse to high densities and at which density it will bounce, to explode and finally whether it might leave a neutron star.

The calculations have been performed for a homogeneous mixture consisting of electrons (e), interacting neutrons (n) and protons (p). The nuclear part, i.e. the strong interaction between nucleons is treated in a Thomas-Fermi approach which describes correctly the saturation properties (density, binding and compressibility) of nuclear matter (see Sect. 4 for details) by using an effective nuclear potential as suggested by Seyler and Blanchard (1963).

The contribution of the electrons to the total pressure is calculated by applying the Fermi-Dirac statistics.

The formalism used in this work is presented in Section 2. We then describe the numerical evaluation in Section 3. Section 4 contains the discussion of our results for the equation of state of the n,p,e-mixture and a comparison with the results of previous works.

The main results of this work may be summarized as follows:

a) With the exception of the limit of low temperatures and densities our equation of state differs from the one of Buchler and Coon (1976) (see Fig. 1 and the discussion in Sect. 4).

b) The contribution of the electrons to the total pressure is quite important for low densities and high temperatures (Fig. 2).

c) For high densities the percentage of protons in β-equilibrium increases rapidly due to the degeneracy of the neutrons (Fig. 3). The presence of protons reduces the pressure due to the attractive n-p-interaction (Fig. 4).

d) The pressure of neutron matter at finite temperature is, compared to Buchler and Coon (1976) much lower (up to a factor of five) for $\varrho \lesssim \varrho_0/2$ (ϱ_0 being the nuclear matter density $\simeq 2.5\,10^{14}$ g/cm^3) and $T \lesssim 2\,10^{11}$ K, but higher for $\varrho \gtrsim \varrho_0$ (Fig. 5). The corresponding energy per particle E/A is in our case higher than the perfect non-interacting neutron gas for $\varrho > \varrho_0$, whereas with Buchler and Coon (1976) E/A stays for all densities below the perfect gas (Fig. 6).

e) Neutron matter is unbound for all densities in the Thomas-Fermi model (Fig. 7). We do not get phase transition at any density (Fig. 8).

2. Method

Our aim is to determine in a simple way the grand canonical potential for a homogeneous mixture of neu-

Send offprint requests to: M. F. El Eid

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trons, protons and electrons in local thermostatic equilibrium by taking care of charge neutrality. Thus

$$\Omega = -PV = E - TS - \sum_i \mu_i N_i . \tag{1}$$

The method which we used was developed by Küpper et al. (1974) who applied it to finite excited nuclei. In what follows we shall present the expressions of the energy E and of the entropy S. Starting from the expression of the total energy E in Hartree-Fock approximation, it can be rewritten in phase space (Theiss, 1956; Küpper, 1975), in the form

$$E = \frac{2}{(2\pi\hbar)^3} \int d^3\boldsymbol{r} \sum_\tau \int d^3\boldsymbol{p} \left[\frac{P^2}{2m} + \tfrac{1}{2} V_\tau(\boldsymbol{p}, \boldsymbol{r})\right] \varrho_\tau(\boldsymbol{p}, \boldsymbol{r}), \tag{2}$$

where τ is the isospin of the nucleon, ϱ_τ is the probability of a state with momentum $\boldsymbol{p}$ being occupied, and V_τ denotes a single nucleon potential with isospin τ.

Since we are using Thomas-Fermi approximation it is reasonable and consistent to take for ϱ_τ the Fermi-Dirac distribution function

$$\varrho_\tau(\boldsymbol{p}, \boldsymbol{r}) = \left\{1 + \exp\left[\beta\left(\frac{P^2}{2m} + V_\tau(\boldsymbol{p}, \boldsymbol{r}) - \mu_\tau\right)\right]\right\}^{-1}, \tag{3}$$

where $\beta := 1/K_B T$; T is the temperature and μ_τ represents the chemical potential of the nucleon with isospin τ. The single-particle potential V_τ is given (Myers and Swiatecki, 1969; Küpper et al., 1974) in the form

$$\begin{aligned} V_\tau(\boldsymbol{p}, \boldsymbol{r}) = {} & \frac{2}{(2\pi\hbar)^3} \int d^3\boldsymbol{r}' \int d^3\boldsymbol{p}' \\ & \cdot [V_l(|\boldsymbol{r}-\boldsymbol{r}'|, |\boldsymbol{p}-\boldsymbol{p}'|)\varrho_\tau(\boldsymbol{p}', \boldsymbol{r}') \\ & + V_u(|\boldsymbol{r}-\boldsymbol{r}'|, |\boldsymbol{p}-\boldsymbol{p}'|)\varrho_{-\tau}(\boldsymbol{p}', \boldsymbol{r}')] . \end{aligned} \tag{4}$$

Here, $V_{l,u}$ denotes an effective interaction between nucleons with parallel ("like") and opposite ("unlike") isospin respectively and is given by (Seyler and Blanchard, 1963; Myers and Swiatecki, 1969)

$$V_{l,u} = -C_{l,u} \frac{\exp(-|\boldsymbol{r}-\boldsymbol{r}'|/r_D)}{(|\boldsymbol{r}-\boldsymbol{r}'|/r_D)} \cdot [1 - (|\boldsymbol{p}-\boldsymbol{p}'|/p_D)^2] . \tag{5}$$

This potential contains a simple repulsive quadratic momentum dependence and has four parameters: $C_{l,u}$ the strengths of the effective interaction, r_D its range and p_D characterizes the momentum dependence. The force in Equation (5) has been applied by Myers and Swiatecki (1969) to nuclei in equilibrium at $T=0$, where they got reasonable account for the nucleon distributions.

For simplicity we omitted a possible implicit dependence of the interaction parameters $V_{l,u}$ on the density and temperature. Applying then the previous equations to homogeneous matter and assuming within the Thomas-Fermi model an isotropic momentum distribution, one gets (Küpper et al., 1974) for V_τ after performing the angular integration

$$V_\tau(K) = V_{0\tau} + K^2 V_{1\tau}, \tag{6}$$

where

$$V_{0\tau} = -2 \int_0^\infty dK' K'^2 (1 - K'^2)[a\varrho_\tau(K') + b\varrho_{-\tau}(K')] \tag{7}$$

$$V_{1\tau} = 2 \int_0^\infty dK' K'^2 [a\varrho_\tau(K') + b\varrho_{-\tau}(K')] , \tag{8}$$

with the definitions

$$K := p/p_D; \quad a := (4\pi)^2 (r_D p_D/h)^3 C_l; \quad b := a C_u/C_l . \tag{9}$$

With the aid of Equation (6) the distribution function as given in (3) now takes the form

$$\varrho_\tau(K) = [1 + \exp(y - \eta_\tau)]^{-1}, \tag{10}$$

where

$$y := \beta(t_d + V_{1\tau})K^2; \quad \eta_\tau := \beta(\mu_\tau - V_{0\tau}); \quad t_d := p_D^2/2m . \tag{11}$$

It is now easy to write the previous equations in terms of the Fermi-integrals (Stoner, 1939; Wrubel, 1958) by changing the integration variables. The following relation can be obtained.

$$\int_0^\infty dK\, K^{2n} \varrho_\tau(K) = \frac{1}{2[\beta(t_d + V_{1\tau})]^{(2n+1)/2}} F_{(2n-1)/2}(\eta), \tag{12}$$

with $n = 0, 1, 2, \ldots$ and the usual definition of the Fermi-integrals has been taken

$$F_{(2n-1)/2}(\eta) = \int_0^\infty dy \frac{y^{(2n-1)/2}}{1 + \exp(y - \eta)} . \tag{13}$$

Throughout the rest of the paper Equation (12) is used as a tool to get the quantities we need for the numerical evaluations. The number density ϱ_τ of a nucleon with isospin τ can be obtained as

$$\varrho_\tau = 8\pi \left(\frac{p_D}{h}\right)^3 \int_0^\infty dK\, K^2 \varrho_\tau(K) = \frac{1}{2\pi^2} \left(\frac{p_D}{\hbar}\right)^3 \frac{F_{1/2}(\eta_\tau)}{[\beta(t_d + V_{1\tau})]^{3/2}} . \tag{14}$$

The quantities V_{0n}, V_{0z} can be calculated by using Equation (7), (10) and (12)

$$\begin{aligned} V_{0n} = {} & a\{F_{3/2}(\eta_n)/[\beta(t_d + V_{1n})]^{5/2} \\ & - F_{1/2}(\eta_n)/[\beta(t_d + V_{1n})]^{3/2}\} \\ & + b\{F_{3/2}(\eta_z)/[\beta(t_d + V_{1z})]^{5/2} \\ & - F_{1/2}(\eta_z)/[\beta(t_d + V_{1z})]^{3/2}\} , \end{aligned} \tag{15}$$

where the subscripts n, z denote neutrons and protons respectively. The equation for V_{0z} is similar to Equation (15) but the subscripts n, z have to be interchanged. Furthermore the equations for V_{1n}, V_{1z} are determined from the Equations (8), (10) and (12)

$$\begin{aligned} V_{1n} = {} & aF_{1/2}(\eta_n)/[\beta(t_d + V_{1n})]^{3/2} \\ & + bF_{1/2}(\eta_z)/[\beta(t_d + V_{1z})]^{3/2} \\ V_{1z} = {} & aF_{1/2}(\eta_z)/[\beta(t_d + V_{1z})]^{3/2} \\ & + bF_{1/2}(\eta_n)/[\beta(t_d + V_{1n})]^{3/2} . \end{aligned} \tag{16}$$

The baryon density ϱ_b, that is the sum of ϱ_n and ϱ_z is given by using Equations (14) and (16) as

$$\varrho_b = \frac{1}{2\pi^2}\left(\frac{p_D}{\hbar}\right)^3 (V_{1n}+V_{1z})/(a+b)\,. \tag{17}$$

The energy density $\varepsilon = E/V$ for the homogeneous nucleon mixture is obtained from the Equations (2), (6), (10) and (12) to be

$$\begin{aligned}\varepsilon = \frac{1}{2\pi^2}\left(\frac{p_D}{\hbar}\right)^3 & \{(t_d+\tfrac{1}{2}V_{1n})\cdot F_{3/2}(\eta_n)/[\beta(t_d+V_{1n})]^{5/2}\\ &+(t_d+\tfrac{1}{2}V_{1z})\cdot F_{3/2}(\eta_z)/[\beta(t_d+V_{1z})]^{5/2}\\ &+\tfrac{1}{2}[V_{0n}\cdot F_{1/2}(\eta_n)/[\beta(t_d+V_{1n})]^{3/2}\\ &+V_{0z}\cdot F_{1/2}(\eta_z)/[\beta(t_d+V_{1z})]^{3/2}]\}\,. \end{aligned}\tag{18}$$

In order to calculate the entropy density s Landau's quasiparticle approximation (Küpper et al., 1974 and references therein) is applied, where the entropy per nucleon S/A has the form

$$\begin{aligned}\frac{S}{A} = -\frac{K_B}{\varrho_b}\frac{2}{(2\pi\hbar)^3}\sum_\tau \int d^3\boldsymbol{K}\\ \cdot[\varrho_\tau(K)\ln\varrho_\tau(K)+(1-\varrho_\tau(K))\ln(1-\varrho_\tau(K))]\,.\end{aligned}\tag{19}$$

This equation is formally identical to the entropy of a free Fermi gas. However the spectrum of free nucleons $t(K)=\hbar^2K^2/2m$ is now replaced by the single-nucleon energies

$$t(K)\Rightarrow t(K)+V_\tau(K)\,.$$

As already mentioned by Küpper et al. (1974) the Equation (19) is based on the assumption of weakly interacting quasiparticles in a nonsuperfluid Fermi liquid at sufficiently low temperatures and might be a too simple approximation at high temperatures. However, in our case, the expression (19) for the entropy is correct at low densities ($\varrho < 10^{12}$ g/cm^3) since the nucleon interaction is small (dilute gas) and thus (19) reduces to the expression for a perfect gas. For medium densities ($12 \lesssim \log\varrho \lesssim 14$) the expression (19) will be an underestimate of the entropy. Thus the lowering of the pressure might be even somewhat bigger compared to a perfect gas. For higher densities the degeneracy of neutrons is large, thus the product $T\cdot s$ is small compared to the internal energy.

In our calculations we need an expression for the entropy density. By writing $s=(S/A)\varrho_b$ and using the Equations (19), (10) and (12) we get

$$\begin{aligned}s = \frac{1}{2\pi^2}\left(\frac{p_D}{\hbar}\right)^3 & \{\tfrac{5}{3}[(t_d+V_{1n})F_{3/2}(\eta_n)/[\beta(t_d+V_{1n})]^{5/2}\\ &+(t_d+V_{1z})F_{3/2}(\eta_z)/[\beta(t_d+V_{1z})]^{5/2}]\\ &-[(\mu_{0n}-V_{0n})F_{1/2}(\eta_n)/[\beta(t_d+V_{1n})]^{3/2}\\ &+(\mu_{0z}-V_{0z})F_{1/2}(\eta_z)/[\beta(t_d+V_{1z})]^{3/2}]\}\,T \end{aligned}\tag{20}$$

where μ_n, μ_z are the chemical potentials of the neutrons and protons respectively.

Finally we determine the pressure $P=P(\varrho_b, T)$ as a function of temperature T and baryon density ϱ_b. The pressure of the interacting protons and neutrons is obtained from

$$P_{n,p} = -\varepsilon + Ts + \mu_n\varrho_n + \mu_z\varrho_z\,, \tag{21}$$

where ε is given in Equation (18), s in (20), ϱ_n and ϱ_z in Equation (14).

The total pressure is then calculated by adding to $P_{n,p}$ in Equation (21) the contribution of the electrons as a perfect quantum gas: $P=P_{n,p}+P_e$.

3. Numerical Evaluations

We used for the parameters of the potential [see Eqs. (5) and (9)] the results which have been obtained from a Thomas-Fermi calculation to nuclear masses by von Groote (1973)

$r_D = 0.557$ fm; $\quad p_D = 409.29$ MeV/C (C = velocity of light)

$C_1 = 353.69$ MeV; $\quad C_u = 516.53$ MeV.

These quantities lead to values for a, b, t_d [Eqs. (9) and (11)]

$a = 347.26$ MeV; $\quad b = 507.14$ MeV; $\quad t_d = 89.21$ MeV.

To determine the pressure P according to Equation (21) we solved iteratively the coupled equations for V_{1n} and V_{1z} as given in (16) with the constraints of charge neutrality $\varrho_e=\varrho_z$ with ϱ_e, ϱ_z being the densities of electrons and protons respectively. The numerical procedure for a fixed temperature T works as follows: with a given η_z [Eq. (11)] the Fermi-integral $F_{1/2}(\eta_z)$ is evaluated according to Equation (13). An iteration loop depending on the quantity η_n is performed in order to fulfil the condition of charge neutrality ($\varrho_e=\varrho_z$). At each iteration step the coupled equations for V_{1n}, V_{1z} are solved and the quantities V_{0n}, V_{0z} [Eq. (15)], μ_n, μ_z [Eq. (11)] are then known.

The number density of protons ϱ_z is calculated according to Equation (14).

The electron number density ϱ_e is determined, as a function of temperature T and μ_e the chemical potential, by evaluation of the Fermi-integrals (El Eid, 1976). The chemical potential μ_e is calculated under the condition that $\mu_e=\mu_n-\mu_z$. The calculations have been done for several temperatures and over the range of densities as shown in Figures 1 and 2.

4. Results and Discussion

An equation of state (EOS) is computed for hot ($3\ 10^9 \lesssim T \lesssim 4\ 10^{11}$ K) and dense ($10^{10} \lesssim \varrho \lesssim 5\ 10^{14}$ g/cm^3) matter expected in a supernova core which is considered in this work to be a homogeneous, charge-neutral mixture of electrons (e), interacting neutrons (n) and protons (p)

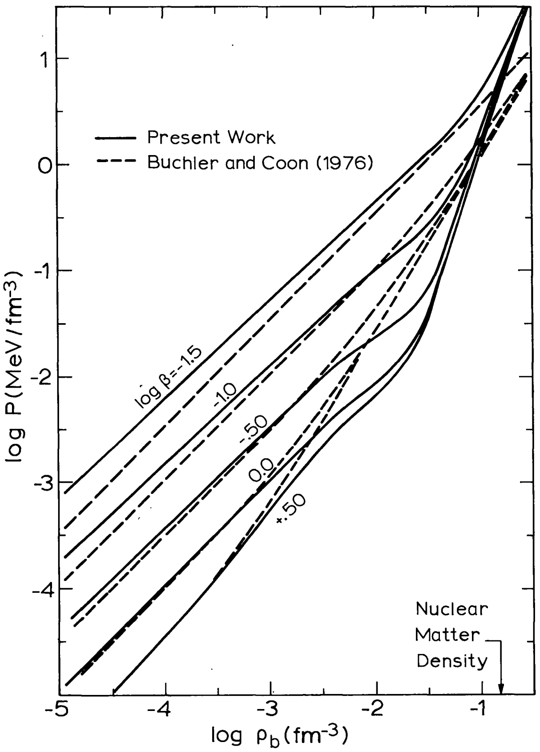

Fig. 1. The pressure $P(\varrho_b, \beta)$ as a function of baryon density ϱ_b for various temperatures $T=1.161\ 10^{10}/\beta$ in Kelvin. (——): this work (charge-neutral mixture of electrons and interacting neutron and protons). (– – –): Buchler and Coon (1976) (pure neutron matter). With the exception of the limit of low T and ϱ_b both equations of state are quite different

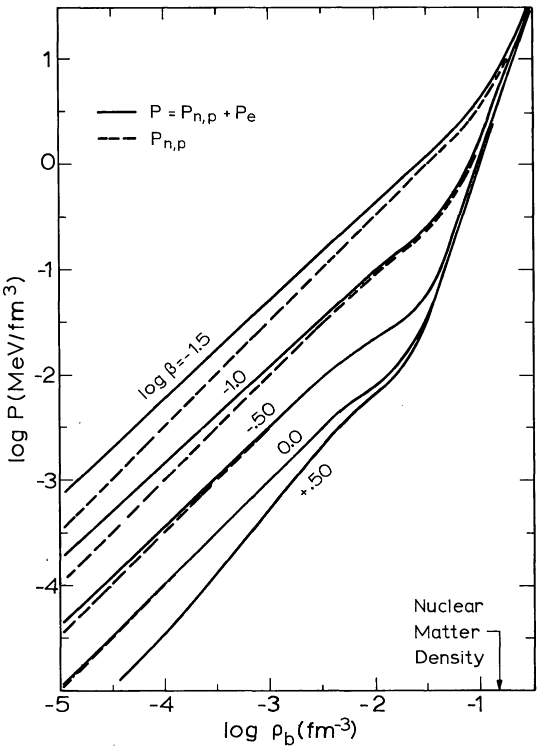

Fig. 2. Contribution of the electrons to the total pressure $P=P_{n,p}+P_e$ as a function of ϱ_b for various temperatures. (——): P. (– – –): $P_{n,p}$ (only protons and neutrons). The contribution of the electrons is important for low densities and high temperatures

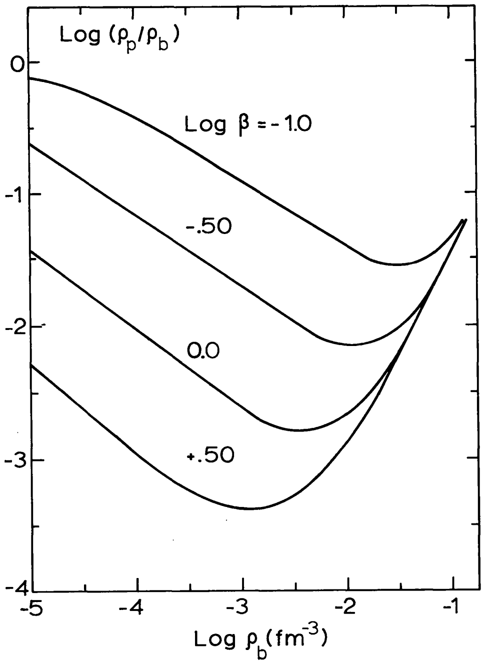

Fig. 3. The ratio of proton density to the total baryon density ϱ_p/ϱ_b in the *n,p,e*-mixture as a function of baryon density ϱ_b for various temperatures. For high densities the percentage of protons increases rapidly due to the degeneracy of the neutrons

in β-equilibrium (see Sect. 3). The contribution of the partially degenerated electrons to the total pressure is calculated by using the Fermi-Dirac statistics. The nuclear part (interaction between nucleons) is treated with the Thomas-Fermi model, which we presented in Section 2.

In Figure 1 a comparison is made of our EOS to that one recently presented by Buchler and Coon (1976) who considered pure neutron matter and used a many-body technique of Bloch and Dominicis (1958) and the isospin singlet part of the Reid sof-core potential (Reid, 1968).

The electrons which assure charge neutrality in the *n, p, e*-mixture have a quite important contribution to the total pressure in the high temperature-low density region ($T>10^{10}$ K, $\varrho<10^{12}$ g/cm^3) since the neutrons are still non-degenerate. This can be seen from Figure 2 for the present EOS.

At higher densities ($10^{12} \lesssim \varrho \lesssim \varrho_0$ g/cm^3, where $\varrho_0 \simeq 2.5\ 10^{14}$ g/cm^3 is the saturation density of nuclear matter) and $T \lesssim 10^{11}$ K the pressure compared with that of Buchler and Coon (1976) is systematically lower by a factor of two to five depending on temperature while for $\varrho \gtrsim \varrho_0$ our method gives higher pressure.

The homogeneous matter contains a certain fraction of protons. The ratio of the proton number density to the total baryon number density ϱ_p/ϱ_b is displayed in Figure 3 as a function of ϱ_b for various T. In the low density region ($\varrho \lesssim 10^{12}$ g/cm^3) ϱ_p/ϱ_b decreases for all temperatures as ϱ_b increases. At higher densities the

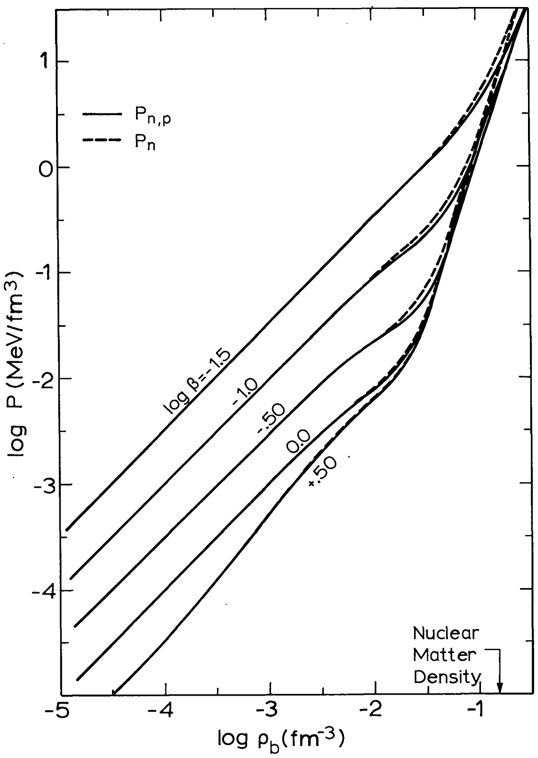

Fig. 4. Influence of protons on the equation of state. (–––): pure neutron matter.(——): protons and neutrons. The presence of protons reduces the pressure, the *n*-*p*-interaction being attractive

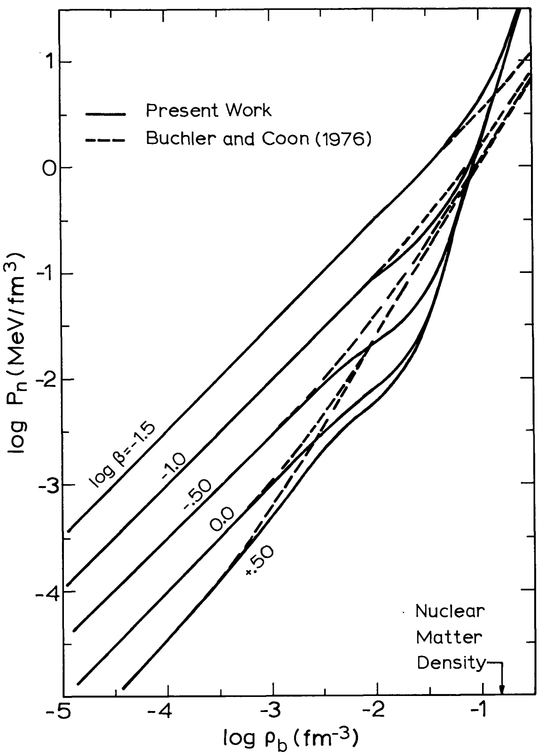

Fig. 5. The pressure of pure neutron matter at finite temperature. (——): this work using Thomas-Fermi approach. (–––): Buchler and Coon (see text for details). The nucleon-nucleon interaction in this work is more attractive for $\varrho \lesssim \varrho_{0/2}$ (ϱ_0: nuclear matter density) and $T \lesssim 10^{11}$ K, but repulsive for $\varrho \gtrsim \varrho_0$

neutrons become partially degenerate and the compensation of their rising Fermi-energy causes the protons to be more abundant (Fig. 3). The effect of the inclusion of protons on the EOS, illustrated in Figure 4, is a certain reduction of the pressure due to the attractive $p-n$ interaction.

In order to compare the nuclear part of our method with that of Buchler and Coon (1976) we calculated the EOS of pure neutron matter. The results are shown in Figure 5. Of couse, both methods yield the same pressure as long as the nucleon-nucleon interaction can be neglected compared with the kinetic energy. One can clearly see that the Thomas-Fermi method compared to Buchler and Coon leads to lower pressure for neutron matter up to $\varrho \sim 10^{14}$ g/cm^3 ($\sim \varrho_0/2$), but for $\varrho \gtrsim \varrho_0$ to higher pressure. From Figure 6 it can be seen that the energy per particle E/A for neutron matter is in our model lower than the perfect non-interacting gas up to $\varrho \sim \varrho_0$ but higher for $\varrho > \varrho_0$, while Buchler and Coon obtained E/A staying always below that of a perfect gas.

Figure 7 summarizes E/A calculated by the Thomas-Fermi method which includes the results for symmetric nuclear matter ($\varrho_p = \varrho_n$) and neutron matter at $T=0$ obtained by the method of Myers and Swiatecki (1969) but for the potential parameter readjusted by von Groote (1973) (see Sect. 3). In order to study the behaviour of these curves for E/A we have calculated the free energy per particle for neutron matter at various tem-

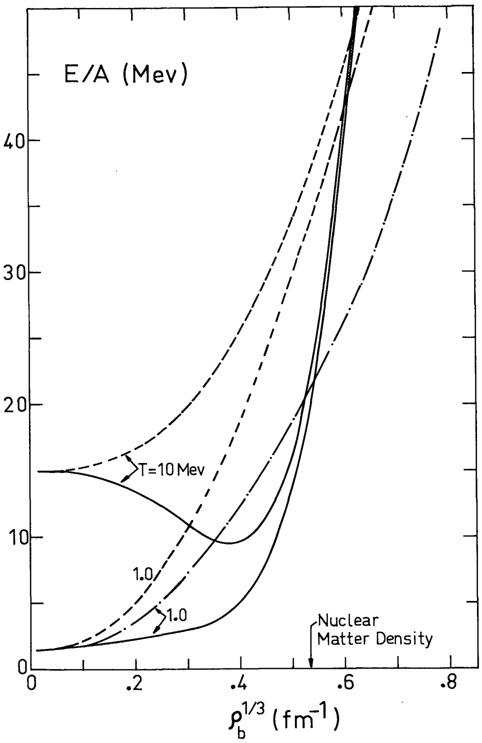

Fig. 6. Comparison of the energy per particle E/A for neutron matter (——) to the perfect non interacting gas (–––) as a function of $\varrho_b^{1/3}$ for $T=1$ and 10 MeV. The dashed-dotted line, taken for comparison from Buchler and Coon for $T=1$ MeV, stays always below the perfect gas

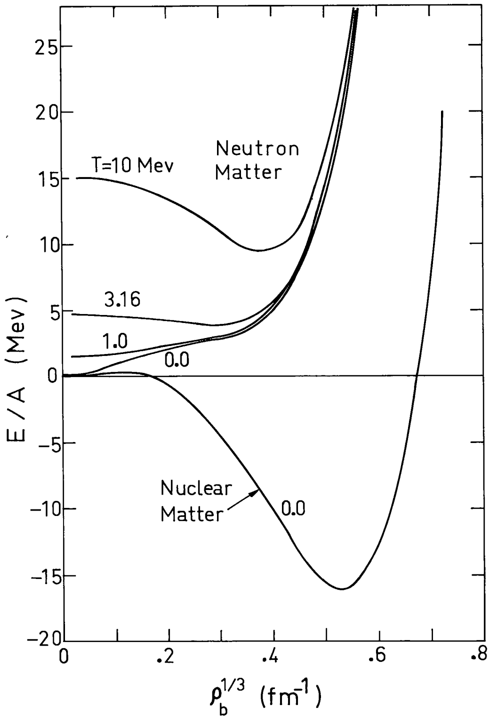

Fig. 7. Energy per particle E/A for neutron matter of various temperatures as a function of $\varrho_b^{1/3}$. The curves of cold neutron matter and cold nuclear matter are taken from von Groote (1973). Neutron matter is unbound for all densities

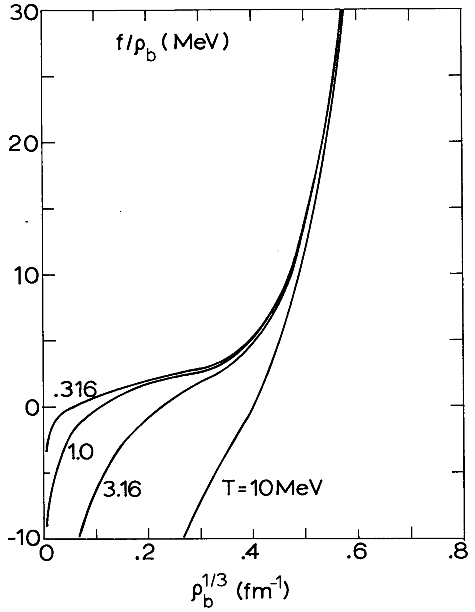

Fig. 8. Free energy per particle f/ϱ_b for neutron matter as a function of $\varrho_b^{1/3}$ for various temperatures. No phase transition occurs

peratures. It can be seen from Figure 8 that no phase transition occurs at any density (critical temperature of neutron matter: $T_c=0$).

Summing up, we may emphasize some main features of the method used in this paper for the treatment of the nuclear interaction.

(1) The semi-empirical macroscopic Thomas-Fermi approach is easy to compute, and not only describes correctly the saturation properties of nuclear matter but allows for the treatment of a mixture with any n/p-ratio.

(2) The two-body effective force of Seyler and Blanchard (1963) has not only an attractive but also a repulsive part being simply proportional to the square of the relative momentum. For low temperatures and densities the relative momentum is on the average small and an attractive part of the effective force lowers the pressure (Fig. 1). At high densities the relative momentum gets larger with higher degeneracy and thus increases the pressure (Fig. 1).

(3) The free parameters of the Seyler-Blanchard force (Sect. 3) have been adjusted (van Groote, 1973) to reproduce the binding energies of finite nuclei. They give the values for the energy per particle, the density and the compressibility of saturated nuclear matter as:

$$E/A=-16.1\ \text{MeV}, \quad K_f=1.31\ \text{fm}^{-1}, \quad K=306\ \text{MeV}$$

which are in good agreement with macroscopic mass formula values from the Droplet model (Myers and Swiatecki, 1969) fitted to all known nuclear masses (von Groote, 1973; von Groote et al., 1976).

(4) The application of the Thomas-Fermi approach used in this paper to hot symmetric nuclear matter and finite excited nuclei has been done by Küpper et al. (1974) and Küpper (1975). Their results for the critical temperature $T_c=17.35$ MeV of nuclear matter (no phase transition vapour/liquid for $T>T_c$ at any ϱ) agree well with the corresponding states approach to nuclear matter (Palmer and Anderson, 1974).

The microscopic method used by Buchler and Coon (1976) is based on a general many-body technique for the treatment of interacting fermions at finite temperatures developed by Bloch and Dominicis (1958). This method with the isospin singlet part of the Reid sof-core potential has to our knowledge not been applied to nuclear matter or finite nuclei. It may be noted, however, that most of the many-body calculations based on nucleon-nucleon potentials yield underbound nuclear matter.

5. Conclusion

The behaviour of the equation of state calculated with the Thomas-Fermi method presented in Section 2 is strongly dependent on the interaction between nucleons. While the attractive part of the interaction dominates (see Fig. 1) in the region of $13 \lesssim \log \varrho \lesssim 14$, it is repulsive in the higher density region.

A further step in calculating the EOS for hot matter is the inclusion of heavy and possibly other elementary particles as mesons and hyperons.

A main application of the present EOS will be the incorporation in a hydrodynamic calculation of supernova explosion.

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References

Bloch, C., de Dominicis, C.: 1958, *Nucl. Phys.* **7**, 459
Bloch, C., de Dominicis, C.: 1959, *Nucl. Phys.* **10**, 181
Bloch, C., de Dominicis, C.: 1959, *Nucl. Phys.* **10**, 509
Buchler, J. R., Coon, S. A.: 1976, (preprint) "The hot interacting neutron gas"
El Eid, M. F.: 1976, Thermostatic properties of hot and dense matter (to be published in *Nucl. Phys.*)
von Groote, H.: 1973, report presented at the meeting on gross properties of nuclei, Hirschegg, Austria
von Groote, H., Hilf, E. R., Takahashi, K.: 1976, *Atomic Data and Nucl. Data Table* **17**, 418
Küpper, W. A.: 1975, report of the Institut für Kernphysik TH Darmstadt, IKDA 75/3
Küpper, W. A., Wegmann, G., Hilf, E. R.: 1974, *Ann. Phys.* **88**, 454
Myers, W. D., Swiatecki, W. J.: 1969, *Ann. Phys.* **55**, 395
Palmer, R. G., Anderson, P. W.: 1974, *Phys. Rev.* D **9**, 3281
Schramm, D. N., Arnett, W. D.: 1975, *Astrophys. J.* **198**, 629
Seyler, R. G., Blanchard, C. H.: 1963, *Phys. Rev.* **131**, 355
Stoner, E. C.: 1939, *Phil. Mag.* **28**, 257
Theis, W. R.: 1955, *Z. Phys.* **142**, 503
Wrubel, M. H.: 1958, Handbuch der Physik, Vol. LI, Astrophys. II