

A THREE-COLOR PHOTOELECTRIC INVESTIGATION OF THE
ECLIPSING BINARY SYSTEM AR LACERTAE

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AR Lac was observed on 22 nights during the autumns of 1972, 1973, and 1974. Totals of 650-700 observations each were obtained in yellow, blue, and ultraviolet. The period of AR Lac, which is 1^d9832, has undergone substantial changes during the past 40 years. Residuals are calculated for all published times of minimum light using three different least squares procedures. The light curves of AR Lac indicate that the intrinsic variation has a total amplitude of about 0^m04. The eclipses are complete, the primary being an occultation. The primary minimum displays less scatter than does the secondary minimum, and this is consistent with the hypothesis that the intrinsic variation arises chiefly from star A. Star B may well be nonvariable. Photometric elements are obtained using the method of Russell and Merrill. Good agreement was found between the observed light curves and the theoretical light curves for primary minimum, but the agreement was poor for secondary minimum. Orbital eccentricity is considered, but it is found to be very small, if present at all. Radii of 1.54 $R_{\odot}$ and 2.81 $R_{\odot}$ are obtained for star A and star B, respectively. The masses of the components are nearly identical. Their effective temperatures are estimated to be 5600° K and 4700° K, respectively, and these are in agreement with their spectral types of G2 IV and K0 IV. AR Lac is one of very few eclipsing binaries which have been observed to show pronounced flare activity at radio wavelengths. The radio outbursts do not appear to be correlated with phase or with instabilities in the light curves at optical wavelengths.

Key words — eclipsing binaries — intrinsic variables — subgiants — radio sources

I. Introduction

AR Lacertae was discovered to be variable by Leavitt in 1907. The nature of its variability was not recognized until 1929 by Jacchia (1929), who found AR Lac to be an eclipsing binary. This delay was due in part to the fact that the period of AR Lac is very nearly two days which makes it difficult to obtain a complete light curve at one site. This has always been a complicating factor to investigators of this system.

Photometric investigations of AR Lac have been made by several observers including Jacchia, who obtained a visual light curve. Photographic light curves were published by Rügemer (1931) and by Schneller and Plaut (1932). Gainullin (1943) undertook a two-color photographic investigation of the system and derived photometric elements for it. In 1938-39 Wood (1946) obtained 143 photoelectric observations in blue light and published a limiting solution for the photometric elements, i.e., one in which the eclipses are assumed to be central. Wood's observations were subsequently used by Kopal and Shapley (1956) to derive a new set of photometric elements. During 1938-47, the Krons (1947, 1974) obtained a total of 2249 photoelectric observations of AR Lac in the blue and 705 in the infrared. All photoelectric investigations of AR Lac have shown it to be intrinsically variable.

AR Lac belongs to a small group of detached eclipsing binary systems in which both components are subgiants. The members of this group, of which

RS Canum Venaticorum can be regarded as the prototype, are characterized by intrinsic variability, substantial changes of period, and the presence of strong Ca II emission lines in their spectra. AR Lac is the brightest member of this class of stars, and it is one of the very few eclipsing binaries known to display outbursts at radio wavelengths.

II. Observations

AR Lac was observed by this investigator on 22 nights from September 1972 to November 1974. All observations were made with the 46-cm Cassegrain reflector of the Kutztown State College Observatory. An unrefrigerated EMI 6256 SA (S-13 surface) photomultiplier was used in conjunction with standard *UBV* filters. A total of 2008 observations (679 in yellow, 676 in blue, and 653 in ultraviolet) was obtained. These observations, corrected for light time and extinction, have been placed on permanent file in the Variable Star Archives of the Royal Astronomical Society as file no. 40.

The comparison star used for all observations was HD 210731, a star of spectral type F8. As the spectral types of the components of AR Lac are later than this, the comparison is not too well matched in color, but HD 210731 has been found by previous observers to be constant in light and this was also the finding of this investigator. Wood had used the star HD 209813 as a comparison, but subsequently Blanco and Catalano (1970) have found this star to be in-

trinsically variable with an amplitude of 0^m12. As comparison stars these observers used HD 208728 and HD 210731. This investigator has also observed HD 208728, finding it to be 0^m571 brighter in yellow, 0^m040 brighter in blue, and 0^m740 fainter in ultraviolet than HD 210731.

The values for V , $(B-V)$, and $(U-B)$ of AR Lac and HD 210731, as standardized on the UBV system by this investigator, are listed in Table I. The fact that the wavelength bands in which the AR Lac observations were made differ somewhat from those of the UBV system was not realized until this investigation was well underway, and it was decided to make all analyses of the light curves in their observed colors. Approximate transformation equations are as follows:

$$Y^* = V + 0.04 (B - V)$$

$$B^* = B - 0.07 (B - V)$$

$$U^* = U - 0.04 (U - B)$$

In this case $Y^* = V$, $B^* = B$, and $U^* = U$ for A0 V stars. In subsequent tables Y^* , B^* , and U^* are referred to as Y , B , and U , respectively.

Since the primary minimum is a complete occultation, the color indices observed at that phase are those of the cooler component. For the hotter component, the values of $(B-V)$ and $(U-B)$ can be estimated as +0.68 and +0.23, respectively. These values are in agreement with the findings of Eggen (1955), who regarded AR Lac A to be similar to the G0 IV star ζ Herculis A and AR Lac B to be similar to the K0 IV star η Cephei.

III. Ephemeris and Period Study

On the basis of photographic and visual observations made between 1900 and 1931, Rügemer (1931) determined the following ephemeris for AR Lac:

$$\text{Hel. Min I} = \text{JD}2426624.378 + 1^d983244 E$$

Although this ephemeris is satisfactory for the earlier observations, Wood (1946) found that it failed completely to predict his times of minimum light. He found that his observations were satisfied by the ephemeris

$$\text{Hel. Min I} = \text{JD}2426624.378 + 1^d983216 E$$

Plavec, Smetanova, and Pekny (1961) found that times of minimum light observed between 1938 and 1959 could be represented by the ephemeris

$$\text{Hel. Min I} = \text{JD}2426624.3762 + 1^d9832155 E$$

This period is in very good agreement with that found by Wood. Further variations in the period of AR Lac were noted by Cester (1967). This investigator (Chambliss 1974), using times of minimum light observed between 1966 and 1973, obtained the following ephemeris:

$$\text{Hel. Min I} = \text{JD}2439376.4955 + 1^d9831987 E$$

The period of AR Lac clearly has decreased over the years of this century, but it has not done so in a uniform fashion. As previous investigators have noted, the earlier times of minimum light, i.e., those obtained between 1900 and 1932, show no evidence for a change of period. There seems to be definite evidence that the period decreased by about 2:7 between 1932 and 1938.

The author has made least squares solutions on all published times of minimum light. Times of minimum light which refer to the same night were averaged. A linear least squares solution using the weights given in Table III yields the ephemeris

$$\text{Hel. Min I} = \text{JD}2426624.3628 + 1^d9832219 E \pm 22 \pm 4 \text{ p.e.}$$

This ephemeris results in large residuals for some times of minimum light, particularly the earliest ones, where the residuals are as large as two hours. The sum of the squares of the residuals is $0.0425 d^2$.

A quadratic least squares solution results in the ephemeris

$$\begin{aligned} \text{Hel. Min I} = & \text{JD}2426624.4362 + 1^d9832302 E - 1.24 \times 10^{-9} E^2 \\ & \pm 19 \pm 9 \pm 0.12 \text{ p.e.} \end{aligned}$$

For this ephemeris the sum of the squares of the residuals is $0.0244 d^2$, but large nonrandom residuals are still encountered, particularly for some of the data of 1948 to 1965.

TABLE I
AR LACERTAE AND ITS COMPARISON STAR

	α (1950)	δ	V	$B-V$	$U-B$
HD 210731	22 ^h 09 ^m 21 ^s	+46° 54'	7.37	+0 ^m 57	+0 ^m 07
AR Lac (max.)	22 ^h 06 ^m 41 ^s	+45° 30'	6.09	+0 .81	+0 .32
AR Lac (pri.)	—	—	6.75	+0 .93	+0 .58
AR Lac (sec.)	—	—	6.41	+0 ^m 79	+0 ^m 29

Finally, separate linear least squares solutions were made for the data of 1900–32, 1938–40, 1948–59, 1960–65, and 1966–73. This treatment greatly reduces the residuals, as the sum of the squares of the residuals is now only $0.0067 d^2$. The periods of AR Lac together with their probable errors and differences are listed in Table II. The large probable error for the period of 1960–65 can be attributed to the fact that most of these observations are not very precise, and the time interval over which the period was calculated was relatively short.

Table III lists the times of minimum light for AR Lac which have been published. The epoch numbers follow the scheme used by previous investigators, and the weights are those assigned by the author. The residuals $(O-C)_1$, $(O-C)_2$, and $(O-C)_3$ refer to the linear ephemeris, the quadratic ephemeris, and the five separate linear ephemerides, respectively. In Figure 1 the residuals as calculated from the linear ephemeris are shown plotted against epoch number. The phases of the observations made by this author have all been calculated using the ephemeris

$$\text{Hel. Min I} = \text{JD}2439376.4955 + 1^d9831987 E$$

Chambliss (1974) noted that for the recent times of minimum light the secondary was shifted by 0^m0036 with respect to the primary minimum. This effect has been noted on a much larger scale by Catalano and Rodono (1974) in the similar eclipsing binary system RS CVn, where distortions of the light curve can lead to erroneous times of minimum light.

Catalano and Rodono have proposed a long-term variation in the period of RS CVn (ca. 70 years) with an amplitude in the residuals of about 0^d10 or 0^d11 . At present insufficient data are available to make similar predictions for AR Lac. The fact that its period has been constant for long intervals of time in the past makes it seem probable that the changes of period in AR Lac are more abrupt than cyclical in nature.

TABLE II
PERIODS OF AR LACERTAE

Date Interval	Mean Date	Period	Difference
1900–32	1921.4	$1^d9832437$ ± 6	—
1938–40	1939.3	$1^d9832125$ ± 11	-2^s70
1948–59	1956.5	$1^d9832244$ ± 36	$+1^s03$
1960–65	1963.5	$1^d9832122$ ± 133	-1^s05
1966–73	1970.1	$1^d9831981$ ± 5	-1^s22

IV. Light Curves and Rectification

The major problem which the observer of AR Lac encounters in obtaining a complete light curve is that the period of this system is very nearly equal to two days. As a consequence the observations made during any one season show strong selectivity with respect to phase, and it is possible to obtain a complete light curve at a single site only after several seasons of observing. The observations made by the author were made in the autumns of 1972, 1973, and 1974 and are distributed by phase as follows.

$$1972 \quad 0.86 - 0.10, 0.34 - 0.57$$

$$1973 \quad 0.91 - 0.14, 0.44 - 0.63$$

$$1974 \quad 0.02 - 0.19, 0.48 - 0.64, 0.68 - 0.74$$

The light curve of AR Lac in yellow is shown in Figure 2, and in Figures 3 and 4 the primary and secondary minima, respectively, in blue light are shown.

As is obvious from these figures, AR Lac shows a significant amount of intrinsic variability. As has been noted by Wood (1946), the primary minimum shows less inconsistency than does the secondary. The opposite situation is found in RS CVn, where the primary minimum is observed to be greatly variable in depth. In AR Lac the intrinsic variability appears to be associated primarily with the hotter component.

The observations of the author made in September 1972, at phase 0.34–0.41, are higher than any others on the curve, while those at phase 0.68–0.74 made in November 1974, are unusually low considering the fact that the ellipticity effect should make this portion of the light curve high in comparison with the other portions. Other variations are also noticeable, e.g., the ingress of primary minimum at phase 0.92 is lower in September 1973 than in September 1972.

The total amplitude of these variations is about 0^m04 . Wood (1946) found a larger amount of variation, but as has been previously noted the comparison star used by him (HD 209813 = HK Lacertae) has itself been reported to be variable in light. Kron (1947) attributed the intrinsic variations in AR Lac to patches on the hotter component. For the similar system RS CVn, Hall (1972) finds that the amplitude of the intrinsic variation varies in a sinusoidal fashion from 0^m05 to 0^m19 with a period of about 23.5 years. He attributes this to a sunspot cycle of that period on the cooler component. He also indicated a migration of the phase of maximum light in a period of 9–10 years, and this was confirmed by Catalano and Rodono (1974), who represent the intrinsic variability of RS CVn as a wave-like distortion, whose period is slightly less than the orbital period of the system. As a result, the wave moves slowly backward in phase with a

TABLE III. TIMES OF MINIMUM LIGHT OF AR LACERTAE

JD Hel.	E	(O-C) ₁	(O-C) ₂	(O-C) ₃	Wt.	Method	Ref.
2415300.059	-5710	-0. ^d 1064	-0. ^d 0183	-0. ^d 0002	1	v	DW
7005.649	-4850	-0.0873	-0.0176	+0.0002	1	v	DW
8889.729	-3900	-0.0682	-0.0167	-0.0013	1	v	DW
2420991.973	-2840	-0.0394	-0.0056	+0.0044	1	v	DW
2995.038	-1830	-0.0286	-0.0089	-0.0067	1	v	DW
5196.444	-720	+0.0010	+0.0080	-0.0012	1	v	DW
5555.428	-539	+0.0219	+0.0271	+0.0157	1	pg	Ja
5573.278	-530	+0.0229	+0.0280	+0.0165	1	v	Lo
5712.086	-460	+0.0053	+0.0098	-0.0026	5	pg	SP
5801.328	-415	+0.0023	+0.0064	-0.0066	1	v	Pa
5803.320	-414	+0.0111	+0.0151	+0.0022	1	v	Pa
6146.405	-241	-0.0013	+0.0012	-0.0140	1	pg	Ru
6489.526	-68	+0.0223	+0.0233	+0.0059	1	pg	Ru
6592.651	-16	+0.0198	+0.0203	+0.0022	1	pg	Ru
6604.548	-10	+0.0175	+0.0179	-0.0003	1	v	Zv
6622.395	-1	+0.0155	+0.0158	-0.0025	1	pg	Ru
6624.378	0	+0.0152	+0.0156	-0.0027	1	pg	Ru
6626.361	1	+0.0150	+0.0154	-0.0030	1	pg	Ru
6842.542	110	+0.0248	+0.0243	+0.0045	1	v	DW
9178.761	1288	+0.0084	+0.0002	+0.0008	10	pe	Wo
9186.693	1292	+0.0075	-0.0008	-0.0001	10	pe	Wo
9188.675	1293	+0.0063	-0.0020	-0.0013	10	pe	Wo
9404.847	1402	+0.0071	-0.0017	+0.0006	10	pe	Wo
9535.739	1468	+0.0064	-0.0027	+0.0006	10	pe	Wo
9692.411	1547	+0.0039	-0.0056	-0.0012	5	pg	Ga
2432889.336	3159	-0.0249	-0.0382	-0.0002	1	v	PSP
4646.470	4045	-0.0256	-0.0382	-0.0031	1	v	PSP
4888.429	4167	-0.0196	-0.0321	+0.0026	1	v	PSP
4890.406	4168	-0.0259	-0.0383	-0.0037	1	v	PSP
4902.323	4174	-0.0082	-0.0206	+0.0140	1	v	PSP
6080.334	4768	-0.0310	-0.0418	-0.0103	1	v	PSP
6082.322	4769	-0.-262	-0.0370	-0.0055	1	v	PSP
6084.303	4770	-0.0285	-0.0392	-0.0078	1	v	PSP
6766.525	5114	-0.0348	-0.0442	-0.0149	1	v	PSP
6774.474	5118	-0.0187	-0.0280	+0.0012	1	v	PSP
6778.446	5120	-0.0131	-0.0225	+0.0067	1	v	PSP
6784.410	5123	+0.0012	-0.0081	+0.0210	1	v	PSP
7018.414	5241	-0.0150	-0.0238	-0.0336	1	v	Ce
7569.7977	5519	+0.0330	+0.0256	+0.0171	5	pe	Ka
7623.332	5546	+0.0203	+0.0131	+0.0047	1	v	DL
7958.473	5715	-0.0032	-0.0094	-0.0172	1	v	DL
8315.452	5895	-0.0042	-0.0093	-0.0164	1	v	Ob
8321.393	5898	-0.0128	-0.0179	-0.0250	1	v	Ob
8323.369	5899	-0.0200	-0.0251	-0.0322	1	v	Ob
8670.481	6074	+0.0281	+0.0242	+0.0177	1	v	Ce
8672.444	6075	+0.0079	+0.0040	-0.0026	1	v	Ce
8680.382	6079	+0.0130	+0.0091	+0.0026	1	v	Ce
9019.525	6250	+0.0251	+0.0224	+0.0163	1	v	PK
9027.435	6254	+0.0022	-0.0005	-0.0066	1	v	Gu
9029.413	6255	-0.0030	-0.0058	-0.0118	1	v	Gu

TABLE III (Continued)

JD Hel.	E	(O-C) ₁	(O-C) ₂	(O-C) ₃	Wt.	Method	Ref.
9259.492	6371	+0.0223	+0.0205	+0.0058	1	v	Gu
9376.4926	6430	+0.0127	+0.0114	-0.0036	10	pe	Ce
9383.4386	6433.5	+0.0174	+0.0161	+0.0012	5	pe	Ce
9386.404	6435	+0.0082	+0.0066	-0.0080	1	v	Gu
9876.268	6682	+0.0162	+0.0168	+0.0058	5	pe	PK
2440932.3168	7214.5	-0.0007	+0.0048	+0.0016	5	pe	Ba
1592.7219	7547.5	-0.0085	+0.0003	+0.0017	5	pe	Ch
1593.7124	7548	-0.0096	-0.0008	+0.0006	10	pe	Ch
1604.6224	7553.5	-0.0074	+0.0016	+0.0031	5	pe	Ch
1936.8022	7721	-0.0172	-0.0065	-0.0028	10	pe	Ch
1938.7874	7722	-0.0153	-0.0045	-0.0008	10	pe	Ch

References: DW = Dugan and Wright (1939), Ja = Jacchia (1929),
 Lo = Loreta (1930), SP = Schneller and Plaut (1932),
 Pa = Parenago (1930), Ru = Rügemer (1931), Zv = Zverev
 (1936), Wo = Wood (1946), Ga = Gainullin (1943),
 PSP = Plavec, Smetanova, and Pekny (1961), Ce =
 Cester (1967), Ka = Karle (1962), DL = Dueball and
 Lehmann (1965), Ob = Oburka (1964), Gu = Guarnieri et
 al. (1975), PK = Pohl and Kizilirmak (1966, 1970),
 Ba = Battistini et al. (1973), Ch = Chambliss (1974).

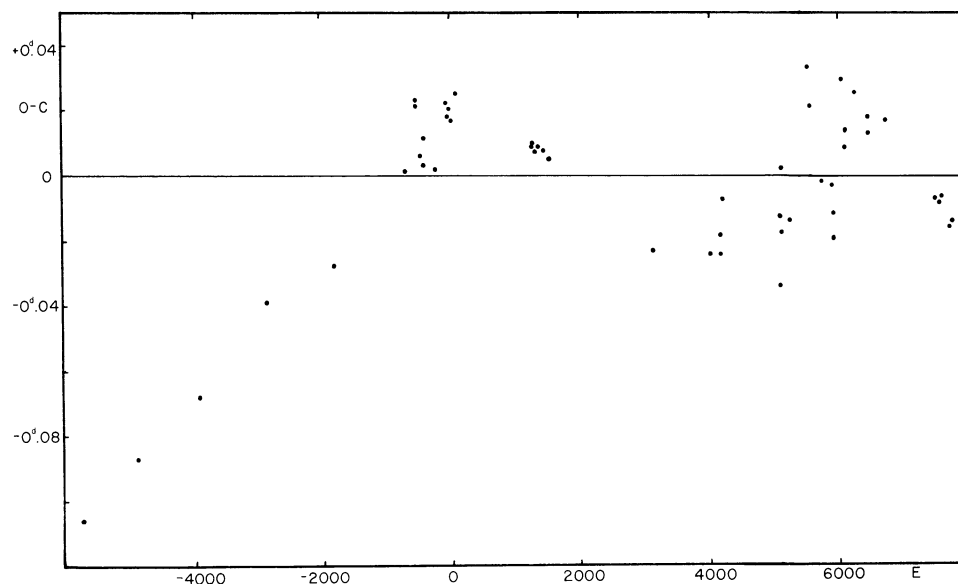


FIG. 1 — Residuals for times of minimum light of AR Lac. The residuals listed in Table III as (O-C)₁ are shown plotted against epoch number. The residuals of 1900-32 can be fitted to a linear ephemeris, but the later residuals indicate substantial changes of period.

modulation period of 10.2 years.

For AR Lac there are insufficient published observations to determine if the amplitude of the intrinsic variation itself varies with time. As observations made

in one season do not always agree with observations at the same phase made in a different season, it does appear that the maxima and minima of these variations shift in phase.

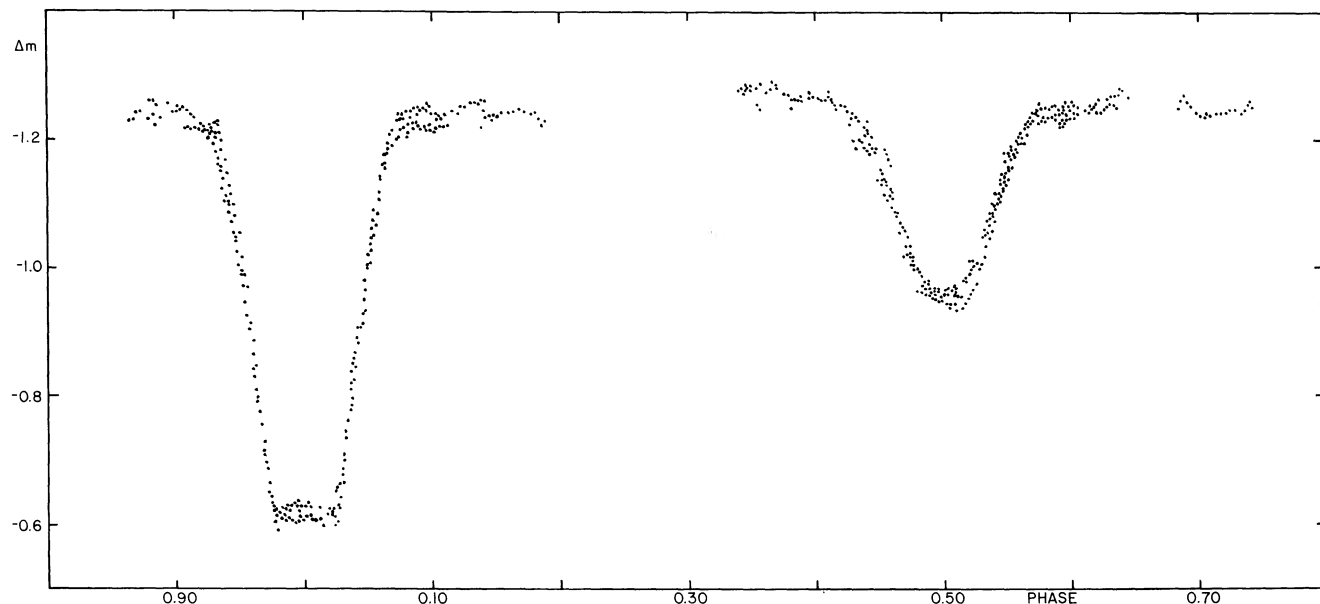


FIG. 2 — The light curve of AR Lac in yellow. The quantity Δm (AR Lac — HD 210731) is shown plotted against phase. The substantial scatter shown in the portions of the light curve outside eclipses is due chiefly to the intrinsic variation of AR Lac.

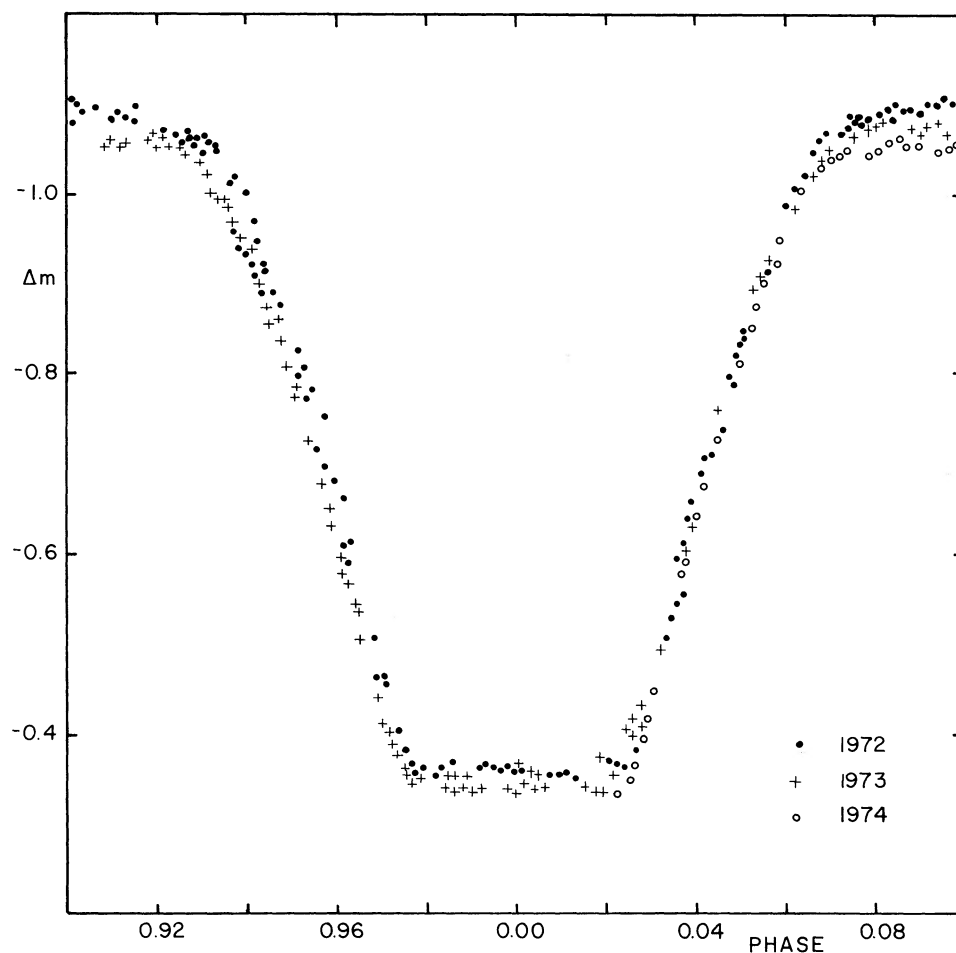


FIG. 3 — The primary minimum of AR Lac in blue light. Different symbols are used for the three seasons in which this minimum was observed. Little or no variation appears in the brightness of AR Lac at mid-eclipse, but the brightness at the shoulders of the eclipse varies from season to season. This is consistent with the hypothesis that the K0 IV component shows little or no intrinsic variation, but the G2 IV component is intrinsically variable.

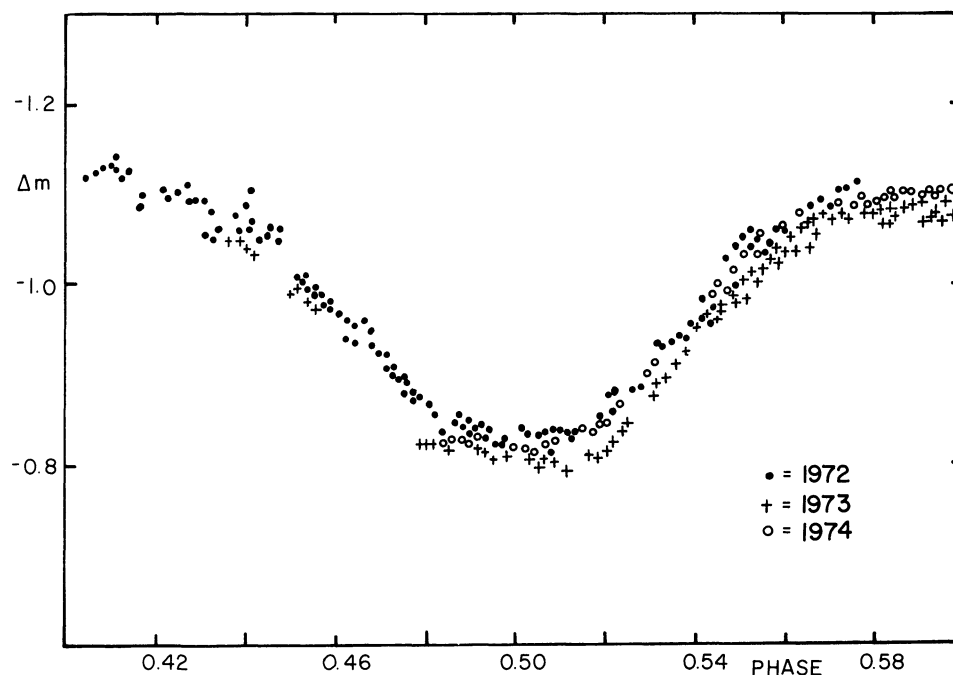


FIG. 4 — The secondary minimum of AR Lac in blue light. The same symbols for seasons are used as in Figure 3. The secondary minimum displays substantially greater scatter at mideclipse than does the primary minimum.

The strong selectivity with respect to phase displayed by the light curves obtained in this investigation poses problems when one undertakes Fourier analyses of the observations outside eclipses. In making these analyses, observations with phases of 0.08–0.42 and 0.58–0.92 were used, normalized to unity at maximum light. For unit light, Δm values (AR Lac — HD 210731) of -1.270 , -1.120 , and -0.925 in yellow, blue, and ultraviolet, respectively, were chosen. Least squares solutions were then made on the expression

$$\begin{aligned} \ell = & A_0 + A_1 \cos \theta + A_2 \cos 2\theta + B_1 \sin \theta \\ & + B_2 \sin 2\theta \end{aligned} \quad (1)$$

As the coverage for some seasons in the portions of the light curves outside eclipses was very fragmentary, not all Fourier coefficients obtained were meaningful.

The coefficients obtained from an analysis of all observations outside eclipses and for those of the 1972 and 1973 seasons only are listed together with their probable errors in Table IV.

Although the ellipticity and reflection effects (see section VI of this paper) are largely responsible for the cosine terms, the sine terms are generally assumed to arise from such effects as intrinsic variability, gas streaming, or peculiar distributions of light on the components. Although the values of the coefficients B_1 and B_2 differ from one season to another, their difference, $B_1 - B_2$, is fairly close to 0.02 for all cases listed in Table IV. This difference can be regarded as the half-amplitude of the intrinsic variability of AR Lac, which was about 0^m.04 in the seasons during which the author's observations were made.

The light curves of AR Lac were rectified according to the expression

$$\ell_r = \frac{\ell - B_1 \sin \theta - B_2 \sin 2\theta + D_0 - A_1 \cos \theta + D_2 \cos^2 \theta}{(A_0 - A_2 + D_0) + (2A_2 + D_2) \cos^2 \theta} \quad (2)$$

The values of the rectification coefficients for reflection, D_0 and D_2 , were determined from the relative depths of the eclipses and the angle of external tangency in the manner described by Russell and Merrill (1952). The phases were rectified according to the expression

$$\sin^2 \theta_r = \frac{\sin^2 \theta}{1 - z \cos^2 \theta} \quad (3)$$

where the value of z , the photometric ellipticity, is given by

$$z = \frac{10(3-x)(D_2 - 2A_2)}{(15+x)(1+y)(A_0 - A_2 - D_0)} \quad (4)$$

For the limb darkening a value of $x = 0.8$ was used, and for the gravity brightening the values given by Hosokawa (1957) were used.

TABLE IV
COEFFICIENTS FOR LIGHT OUTSIDE ECLIPSES OF AR LACERTAE

	A_0	A_1	A_2	B_1	B_2
Y (1972-73 obs.)	0.9838 ± 8	-0.0066 ± 7	-0.0248 ± 20	+0.0116 ± 9	-0.0053 ± 6
Y (all obs.)	0.9808 ± 6	-0.0122 ± 8	-0.0093 ± 13	+0.0061 ± 8	-0.0072 ± 6
B (1972-73 obs.)	0.9845 ± 9	-0.0081 ± 7	-0.0311 ± 22	+0.0157 ± 10	-0.0056 ± 7
B (all obs.)	0.9799 ± 6	-0.0124 ± 8	-0.0111 ± 14	+0.0095 ± 9	-0.0094 ± 7
U (1972-73 obs.)	0.9772 ± 11	-0.0135 ± 9	-0.0253 ± 28	+0.0170 ± 13	-0.0028 ± 8
U (all obs.)	0.9726 ± 9	-0.0187 ± 12	-0.0086 ± 21	+0.0090 ± 13	-0.0074 ± 10

Two rectifications were made, the first using the coefficients obtained from the 1972 and 1973 observations only (R1) and the second using the coefficients obtained from all observations (R2). In Table V the unrectified and rectified depths of the minima are listed as well as the values of the photometric ellipticity used in the rectifications. The depths of primary minimum are somewhat deeper for R1 than for R2, and the depths of the secondary minimum are thus somewhat shallower. This is due to the fact that the values of the A_1 coefficients are substantially greater for R2 than they are for R1.

V. Solution and Photometric Elements

Normal points were formed for the observations made between 0.92-0.08 phase and between 0.42-0.58 phase. From four to nine observations were used for each normal point, and about 40 normal points were formed for the primary minimum in each of the three colors and about 30 for the secondary in each color. Rectifications were then made using both sets of coefficients listed in Table V. For R2 substantial asymmetry existed in all light curves; the ascending branches were higher than the descending branches in all cases. For R1 there was little asymmetry between the ascending and descending branches, and the depths of the secondary minima were less than for R2. Although R1 predicts units intensity at external tangency for the secondary eclipses in all three colors, for the primary eclipses the intensity at external tangency was only 0.985 in all three cases. Thus the intensities for the primary eclipses were normalized to this value. The rectified depths of the primary minimum are then 0.411, 0.459, and 0.552 in yellow, blue, and ultraviolet, respectively, while the depths of the secondary minimum are 0.229, 0.215, and 0.193 in the same three colors.

TABLE V
DEPTHS OF MINIMA OF AR LACERTAE

	Y	B	U
Depth I (unrect.)	0.454	0.508	0.607
Depth II (unrect.)	0.253	0.242	0.220
z (R1)	0.0386	0.0464	0.0410
Depth I (R1)	0.420	0.467	0.558
Depth II (R1)	0.229	0.215	0.193
z (R2)	0.0276	0.0272	0.0272
Depth I (R2)	0.408	0.462	0.551
Depth II (R2)	0.232	0.222	0.202

Solutions for the photometric elements of AR Lac were obtained with the use of the method of Russell and Merrill and the tables of Merrill (1953). For systems with complete eclipses the parameter

$$q_0 = \frac{1 - l_0^{\text{tr}}}{\alpha_0^{\text{oc}} - (1 - l_0^{\text{oc}})} \quad (5)$$

reduces to

$$q_0 = \frac{1 - l_0^{\text{tr}}}{l_0^{\text{oc}}} \quad (6)$$

since $\alpha_0^{\text{oc}} = 1.00$ in these cases. The values of q_0 were calculated to be 0.388, 0.397, and 0.431 in yellow, blue, and ultraviolet, respectively. The parameter q_0 is also functionally related to the ratio of radii, k , and to the geometric depth at mideclipse, p_0 . Using tabulated values of $k(x, q_0, p_0)$ approximate values of k could be estimated.

Tables of the shape function $\psi^{\text{oc}}(x_1, k, \alpha)$ were then used to derive theoretical light curves for the primary minimum. (In this paper the subscript 1 and the term star A refer to the G2 IV component, while star B and the subscript 2 refer to the K0 IV component.) For each of the colors various values of k and x_1 were tried

until good fits were obtained. The combinations which gave the best fits to the observations were $k = 0.55$, $x_1 = 0.5$ for yellow, $k = 0.54$, $x_1 = 0.6$ for blue, and $k = 0.55$, $x_1 = 0.6$ for ultraviolet.

These solutions are constrained by the requirement that they must satisfy the depth relation for secondary minimum. The parameter q_0 is also equivalent to

$$q_0 = \tau(x_2, k) \times \alpha_0^{\text{tr}}(x_2, p_0, k) \quad (7)$$

Using the observed values of q_0 , the adopted values of k , and the tabulated values of $\tau(x_2, k)$, the values of α_0^{tr} and x_2 could be estimated. It was found that consistent solutions were possible only if a high limb-darkening coefficient was assumed for star B. For this component, values of x_2 equal to 0.9, 1.0, and 1.0 in yellow, blue, and ultraviolet, respectively, were adopted. The parameter α_0^{tr} could then be calculated. For yellow light the adopted value of α_0^{tr} was 1.11, and for blue light it was 1.16. For ultraviolet light consistent solutions were possible only if the depth of secondary minimum was assumed to be somewhat less than actually observed. For this case a limiting solution was obtained in which the eclipses were assumed to be central and the depth of the secondary was assumed to be 0.187. Thus the observed depth of the secondary minimum in ultraviolet is greater than that permitted for a consistent solution. Since R2 yields deeper secondary minima than does R1, it is also less satisfactory for this reason.

Once these parameters have been adopted, the other elements can be readily calculated according to the procedures given by Russell and Merrill. The adopted elements are listed in Table VI. The tabulated phase angles of external and internal tangency are unrectified. The parameters L_1 , L_2 , and J_1/J_2 refer to the luminosities and surface brightness of the heated hemispheres of the components, while L_1' , L_2' , and J_1'/J_2' refer to the unheated hemispheres. The latter values were obtained using the procedure given by Koch, Plavec, and Wood (1970). If the hotter component is assumed to have an effective temperature of 5600°K , then the temperature of its companion can be estimated as 4700°K . This is in harmony with spectral types of G2 IV for star A and K0 IV for star B. AR Lac does not show an inconsistency frequently encountered in semidetached systems where the temperature derived for the cooler component is often substantially greater for ultraviolet light than it is for yellow light with the result for blue falling between these two values. In AR Lac the temperatures derived from all three light curves are very nearly the same.

The values for blue light listed in Table VI are in good agreement with the photometric elements published by Kopal and Shapley (1956) and in fair agree-

TABLE VI
PHOTOMETRIC ELEMENTS OF AR LACERTAE

	Y	B	U
x_1	0.5	0.6	0.6
x_2	0.9	1.0	1.0
k	0.55	0.54	0.55
a_1	0.169	0.167	0.170
a_2	0.307	0.310	0.309
b_1	0.165	0.163	0.166
b_2	0.300	0.302	0.302
p_0	-1.41	-1.52	-1.82
j	$86^\circ 1$	$86^\circ 9$	$90^\circ 0$
ϵ	0.020	0.025	0.022
θ_e	$27^\circ 6$	$27^\circ 8$	$28^\circ 1$
θ_i	$8^\circ 0$	$8^\circ 1$	$8^\circ 0$
α_0^{oc}	1.00	1.00	1.00
α_0^{tr}	1.11	1.16	1.18
L_1	0.411	0.459	0.552
L_2	0.589	0.541	0.448
J_1/J_2	2.30	2.91	4.06
L_1'	0.416	0.469	0.571
L_2'	0.584	0.531	0.429
J_1'/J_2'	2.35	3.03	4.40

ment with the limiting solution published by Wood (1946). The results of this investigation indicate somewhat more central eclipses and slightly larger angles of internal tangency than does the solution of Kopal and Shapley. The observational material of this investigation is far more extensive than that used by Kopal and Shapley and by Wood, and on that basis, one should expect the elements listed in Table VI to be somewhat more reliable than those previously published. The fits between the observed and theoretical light curves for primary minimum were good, the mean residuals being ± 0.0037 in yellow, ± 0.0028 in blue, and ± 0.0048 in ultraviolet.

Theoretical light curves were also calculated for the secondary minimum. For these the angles of external tangency derived from determinations based on the primary minimum were used. The secondary eclipse light curves in yellow and in blue were also required to have the observed depths, i.e., 0.229 in yellow and 0.215 in blue. For ultraviolet a depth of 0.187 was adopted as required by the limiting solution obtained for this color. The theoretical light curves were obtained with the use of the $\psi^{\text{tr}}(x_2, k, \alpha)$ tables listed by Merrill. For circular orbits the values of external and of internal tangency should be the same for both primary and secondary eclipses. In this case the angles of internal tangency were found to average about $0^\circ 7$ less for the secondary minimum than for the primary minimum. For secondary minimum the fits between the observed normal points and the theoretical light curves were poor. The

residuals were systematically negative, having mean values of -0.0129 in yellow, -0.0109 in blue, and -0.0095 in ultraviolet. Clearly these theoretical light curves do not represent the best possible fits to the normal points. The theoretical light curves for secondary minimum were required to yield the same angles of external tangency as did those for the primary minimum. The same elements were used in both cases, and the secondary curves were required to have the depths calculated for secondary minimum. Further constraints were not placed on them, however, and they do not pass through the majority of the normal points. Much better fits to the data could be obtained, if the mean rectified angle of external tangency were taken as $32^\circ.2$ for secondary minimum. For primary minimum the corresponding angle was $28^\circ.4$. An examination of Figures 3 and 4 indicates that a difference as large as this in the angles of external tangency is untenable.

Unequal durations of minimum light imply that the orbit of an eclipsing binary is eccentric. Using the notation of Kopal (1959) the radial component of the eccentricity is given by the approximation

$$e \sin \omega = \frac{d_2 - d_1}{d_2 + d_1} \frac{\alpha^2 - \cos^2 i}{\alpha^2 - 2\cos^2 i}, \quad (8)$$

where

$$\alpha = (1 - \sin^2 i \cos^2 \phi_0)^{1/2} \quad (9)$$

and

$$d_1 + d_2 = 4\phi_0. \quad (10)$$

If one uses values of $\Theta = 28^\circ.4$ for the primary eclipse and $32^\circ.2$ for the secondary eclipse and sets $i = 87^\circ$, $e \sin \omega$ can be calculated to be 0.0634 . As has previously been mentioned, this investigator found from his own observations that the secondary minimum occurred at a phase of 0.5018 . If one assumes that this phase shift arises from orbital eccentricity, then the tangential component of the eccentricity can be calculated from the expression (Binnendijk 1960)

$$e \cos \omega = \frac{\pi(D - 0.5)}{1 + \operatorname{cosec}^2 i}, \quad (11)$$

where $D - 0.5$ is the phase shift, which in this case equals 0.0018 . Thus $e \cos \omega$ can be calculated to be 0.0028 . Combining the two components one obtains $e = 0.0635$ and $\omega = 87^\circ.5$.

It is highly improbable that these values are realistic. As Catalano and Rodono (1974) point out in their study of the period variations of RS CVn, substantial phase shifts are to be expected in systems of this kind where intrinsic variability is pronounced. As has already been mentioned there does not appear

to be any apparent difference in the durations of the primary and secondary eclipses in the observed light curves, and this was also the finding of Wood. If $e \sin \omega$ were as large as 0.06 , a visual inspection of the observed light curves would readily detect the unequal durations of the eclipses. Thus it is the conclusion of this investigator that orbital eccentricity in AR Lac is extremely small, if it is present at all.

It is interesting to compare the values of external and internal tangency which have been reported by previous investigators of AR Lac. From his visual study Jacchia (1929) found $\theta_e = 33^\circ$ and $\theta_i = 6^\circ.4$ for the primary eclipse. Insufficient observations were made to determine the duration of secondary minimum. The same situation occurred with the photographic investigation of Schneller and Plaut (1932), who found $\theta_e = 32^\circ$ and $\theta_i = 6^\circ.0$. From his photographic study Rügemer (1931) found $\theta_e = 36^\circ$ and $\theta_i = 10^\circ.2$ for primary, but $\theta_e = 29^\circ$ for secondary. In his photographic determination of the light elements Gainullin (1943) found $\theta_e = 33^\circ$ and $\theta_i = 9^\circ.2$ for primary minimum, and $\theta_e = 29^\circ$ for secondary minimum. He also determined $e \sin \omega = -0.04$ and $e \cos \omega = +0.004$. All published photoelectric investigations of AR Lac have indicated that the eclipses are of equal duration. Wood found $\theta_e = 30^\circ.0$ and $\theta_i = 6^\circ.9$, while Kopal and Shapley redetermined these to be $28^\circ.5$ and $7^\circ.1$, respectively. The mean values listed in Table VI of this investigation are $27^\circ.8$ for θ_e and $8^\circ.0$ for θ_i .

If the values of $e \sin \omega$ and $e \cos \omega$ found by Gainullin are compared with the values considered in this investigation, one can see that the relative durations of the eclipses have reversed themselves, implying an apsidal rotation period of ca. 70–80 years. Apsidal rotation, however, produces a characteristic sinusoidal variation in the residuals of the times of minimum light. The fact that this variation is not observed is all the more reason to believe that the orbital eccentricity in AR Lac is negligible. In considering the results of the earlier visual and photographic investigations of AR Lac, one must recall that the precision of the observations is much lower than in photoelectric studies, and hence any results derived from these investigations must be regarded only as approximations.

The conclusion of Wood (1946) and of Kron (1947) that the intrinsic variability of AR Lac is associated primarily with the hotter component is supported by this investigation. A comparison of Figure 3 with Figure 4 indicates that the primary minimum is much less distorted than is the secondary. Most of the difficulties encountered with the secondary minimum undoubtedly are due to these distortions. Since at least one of the components of AR Lac is intrinsically vari-

able, the fractional luminosities L_1 , L_2 , and L_1' , L_2' also vary with time. Kron concluded that AR Lac A has dark spots. If this is the case, assumption of the standard cosine law of limb darkening for this component is not very realistic. Taking the results listed in Table VI at face value, this investigation indicates that the limb darkening is relatively low for star A but high for star B.

VI. Comparison of Results with Theory

Since the empirical Fourier coefficients listed in Table IV are related to the photometric elements, a comparison can be made between these and the theoretical Fourier coefficients calculated with the derived photometric elements listed in Table VI.

The A_2 coefficient results from both the ellipticity and reflection effects. The contribution of the ellipticity effect to this coefficient is given approximately by the following expression (Merrill 1970)

$$A_2 = \frac{-3(15 + x_1)}{20(3 - x_1)} (1 + y_1) \sin^2 i \frac{\mathfrak{M}_2}{\mathfrak{M}_1} L_1 r_1^3 + \frac{-3(15 + x_2)}{20(3 - x_2)} (1 + y_2) \sin^2 i \frac{\mathfrak{M}_1}{\mathfrak{M}_2} L_2 r_2^3 \quad (12)$$

In calculating A_2 the mass ratio has been assumed to be unity (see section VII of this paper), values of $i = 87^\circ$, $r_1 = 0.167$, and $r_2 = 0.305$ have been used. The values of x_1 , x_2 , L_1' and L_2' listed in Table VI have been used. The value of A_2 also depends upon the gravity-brightening coefficients of the components. These can be calculated from the expression

$$y = \frac{\beta c_2 e^{c_2/\lambda T}}{\lambda T(e^{c_2/\lambda T} - 1)} \quad (13)$$

where $c_2 = 1.43879$ cm deg and β is the exponential dependence of temperature on local gravity. Lucy (1967, 1968) has found that $\beta = 0.08$ for stars with convective envelopes. Since both components of AR Lac can be assumed to have deep convective envelopes, use of this value seems justified. In that case y_1 can be calculated to be 0.38, 0.47, and 0.56 and y_2 to be 0.45, 0.56, and 0.66 in yellow, blue, and ultraviolet, respectively. The values of A_2 are then -0.0297 , -0.0312 , and -0.0282 , respectively.

The reflection effect will reduce somewhat the absolute value of the A_2 coefficient. The $\cos 2\theta$ coefficient of the reflection effect is given by Russell and Merrill as

$$C_2 = +0.10(L_1 r_2^2 + L_2 r_1^2) \sin^2 i \quad (14)$$

Using the adopted elements C_2 can be calculated to be $+0.0055$, $+0.0058$, and $+0.0065$ in yellow, blue, and ultraviolet, respectively. Combining the effects of ellipticity and reflection, one obtains theoretical values

of A_2 equal to -0.0242 , -0.0254 , and -0.0217 , respectively.

Comparing these values with those listed in Table IV, one finds fairly good agreement between the theoretical values of A_2 and the first set of empirical coefficients (R1). The agreement with the second set (R2) is very poor. The latter values of A_2 are undoubtedly much too small, because the observations made between 0.68 and 0.74 phase have been included in their determination. As has previously been noted, these observations were unusually low.

The A_1 coefficient results largely from the reflection effect. This is given by Russell and Merrill as

$$A_1 = -0.40(L_1 r_2^2 - L_2 r_1^2) \sin i \quad (15)$$

Using the adopted elements, one obtains values of -0.0090 , -0.0115 , and -0.0164 , respectively, for this coefficient. There is also a small contribution to the $\cos \theta$ term from the ellipticity effect. This is given by the following expression (Merrill 1970)

$$A_1 = \frac{-15x_1}{32(3 - x_1)} (2 + y_1)(5 \sin^3 i - 4 \sin i) \frac{\mathfrak{M}_2}{\mathfrak{M}_1} L_1 r_1^4 + \frac{15x_2}{32(3 - x_2)} (2 + y_2)(5 \sin^3 i - 4 \sin i) \frac{\mathfrak{M}_1}{\mathfrak{M}_2} L_2 r_2^4 \quad (16)$$

Using the adopted elements, respective values of $+0.0023$, $+0.0025$, and $+0.0021$ are obtained for this coefficient. When the effects of ellipticity and reflection are combined, one obtains theoretical values of A_1 equal to -0.0067 , -0.0090 , and -0.0143 in yellow, blue, and ultraviolet. When these results are compared with the coefficients listed in Table IV, fairly good agreement is noted between them and both sets of the empirical A_1 coefficients. In all three cases the theoretical values lie between the two sets of observed coefficients.

Both sets of Fourier coefficients include large values of B_1 and B_2 . As has been previously mentioned, the difference $B_1 - B_2$ is approximately equal to the half-amplitude of the intrinsic variation, which is estimated to be about 0.04. Although the intrinsic variation of the system results in appreciable $\sin \theta$ and $\sin 2\theta$ terms, it also affects the values of the A_1 and A_2 coefficients. The A_1 and A_2 coefficients resulting from the ellipticity and reflection effects should be nearly independent of time and should show little or no variation from season to season. Ideally, one should remove the effects of intrinsic variation prior to making any Fourier analyses or rectifications of the light curves. The B_1 and B_2 coefficients should then be negligibly small, and the A_1 and A_2 coefficients should arise entirely from the ellipticity and reflection effects. With such a procedure it is likely that far fewer difficulties

would be found with the secondary eclipse than were found in this investigation. Unfortunately, such a procedure can be used only if the nature of the intrinsic variation is known. Not only must its amplitude be known, but also its period, if there is one, and the form of its modulation. At present too little is known about the intrinsic variation of AR Lac to make reliable corrections for it.

The use of limb-darkening coefficients presupposes that the surface brightness of a star varies in a uniform manner with the angle from the line of sight, while the use of gravity-brightening coefficients presupposes that the flux varies in some fashion with the local gravity. If Kron is correct in assuming that AR Lac A has extensive regions of spots, then neither the use of x nor of y is very realistic for this component. However, the contributions of AR Lac B to the coefficients A_1 and A_2 are much larger than those of AR Lac A. AR Lac B certainly has less intrinsic variation than does AR Lac A, and it may well be nonvariable. Consequently one can probably meaningfully speak of limb darkening and gravity brightening for this component.

The Russell-Merrill method assumes that the components are similar ellipsoids. After rectification the system can be treated as consisting of two noninteracting spheres. In AR Lac the two components will not be distorted to an equal extent. Wood (1971) has given the following expressions for the ellipsoidal axes of the components:

$$\text{Star A } a_1 = r_1 \left[1 + \frac{1}{6}(1 + 7q)r_1^3 \right] \quad (17)$$

$$b_1 = r_1 \left[1 + \frac{1}{6}(1 - 2q)r_1^3 \right] \quad (18)$$

$$c_1 = r_1 \left[1 - \frac{1}{6}(2 + 5q)r_1^3 \right] \quad (19)$$

$$\text{Star B } a_2 = r_2 \left[1 + \frac{1}{6} \left(1 + \frac{7}{q} \right) r_2^3 \right] \quad (20)$$

$$b_2 = r_2 \left[1 + \frac{1}{6} \left(1 - \frac{2}{q} \right) r_2^3 \right] \quad (21)$$

$$c_2 = r_2 \left[1 - \frac{1}{6} \left(2 + \frac{5}{q} \right) r_2^3 \right] \quad , \quad (22)$$

where q is the mass-ratio, $\mathcal{M}_2/\mathcal{M}_1$. These expressions assume that the polytropic index $n = 5$ (i.e., all mass concentrated at the center). If one assumes $r_1 = 0.167$, $r_2 = 0.305$, and $q = 1.00$, then the axes a , b , c are 0.168, 0.167, and 0.166 for star A and 0.315, 0.304, and 0.295 for star B. Oblatenesses of 0.006 for star A and 0.035 for star B are thus obtained. The mean oblateness of the system, weighted according to the fractional luminosities of the components, is then

0.022 in yellow, 0.021 in blue, and 0.018 in ultraviolet. These values compare favorably with those listed in Table VI.

AR Lac is a detached system. For a system with a mass ratio equal to unity, the axes of the Roche lobe are 0.405, 0.374, and 0.356 according to the tables of the Roche model by Plavec and Kratochvil (1964). Thus both components lie well within their Roche lobes. Yet neither component is on the main sequence. Kopal (1959) has referred to systems such as AR Lac as having "undersize" subgiant secondaries. The stage of evolutionary development of these systems is uncertain. Although most investigators have assumed that the components of systems like AR Lac and RS CVn are stars which are evolving off the main sequence, Hall (1972) has considered the possibility that the components of such systems may still be in the pre-main-sequence contraction stage, but he no longer regards this as a likely hypothesis (Hall 1976).

Period changes are to be expected in systems in which one or both of the components fill their Roche lobes. Yet substantial period changes have occurred in AR Lac, although it is dynamically a detached system. This is also true of the similar binary system RS CVn. In systems such as these the interactions between the components must be much greater than they are in more normal detached systems.

VII. Dimensions of AR Lac

AR Lac is a double-lined spectroscopic binary. Thus far two radial velocity curves of AR Lac have been published; these are by Harper (1933) and by Sanford (1951). The orbital elements obtained by these investigators are listed in Table VII. Both investigators find the mass ratio to be nearly unity; Harper found $\mathcal{M}_2/\mathcal{M}_1 = 1.007$, while Sanford found 1.004. Thus star B appears to be very slightly the more massive, and it also has a somewhat higher bolometric luminosity. Harper found an orbital eccentricity of 0.041, but Sanford assumed the orbit to be circular. An orbital eccentricity as large as 0.041 would readily manifest itself in the appearance of the

TABLE VII
SPECTROSCOPIC ORBITAL ELEMENTS OF AR LACERTAE

	Harper (1933)	Sanford (1951)
γ	-36.3 km s^{-1}	-33.7 km s^{-1}
K_1	120.0 km s^{-1}	116.1 km s^{-1}
K_2	119.2 km s^{-1}	115.6 km s^{-1}
$a_1 \sin i$	$3.263 \times 10^6 \text{ km}$	$3.167 \times 10^6 \text{ km}$
$a_2 \sin i$	$3.247 \times 10^6 \text{ km}$	$3.151 \times 10^6 \text{ km}$
$\mathcal{M}_1 \sin^3 i$	$1.39 \mathcal{M}_\odot$	$1.29 \mathcal{M}_\odot$
$\mathcal{M}_2 \sin^3 i$	$1.40 \mathcal{M}_\odot$	$1.30 \mathcal{M}_\odot$
e	0.041	0.00

light curves, and it seems unlikely that the eccentricity can be as large as this. The orbital elements derived by the two investigators are very similar, those obtained by Harper being somewhat larger than the elements calculated by Sanford.

If one takes the means of the orbital elements obtained by Harper and by Sanford and assumes that $i = 87^\circ$, $r_1 = 0.167$, and $r_2 = 0.305$, then the semi-major axis of AR Lac can be calculated to be 6,420,000 km or $0.22 R_\odot$. The dimensions of the components are as follows.

	AR Lac A	AR Lac B
R	$1.54 \odot$	$2.81 \odot$
$\mathfrak{M}$	$1.35 \odot$	$1.35 \odot$
$\langle \rho \rangle$	$0.369 \odot$	$0.060 \odot$

To two significant figures the masses of the components are equal. Thus the assumption made in the previous section of this paper that $q = 1.00$ is justified. Popper (1967) quotes masses (in solar units) of 1.32 ± 0.06 for star A and 1.31 ± 0.07 for star B. His determinations were based on Sanford's plate material. Although it is unlikely that pulsation has anything to do with the intrinsic variability of AR Lac, periods of pulsation equal to 0^d067 for star A and 0^d167 for star B can be calculated assuming a pulsation constant $Q = 0.041$, which is typical of Population I variables. The intrinsic variability observed for AR Lac is definitely on a much longer time scale than this.

Harper regarded the components of AR Lac as being of spectral types G2 and G5, while Sanford classed them as G5 and K0. These spectral classes were also assigned to the components by Wyse (1934), who carried out an extensive spectroscopic investigation of AR Lac. Additional spectroscopic work on AR Lac has been undertaken by Joy and Wilson (1949) and by Struve (1952). All of these investigators have noted that the H and K lines of Ca II appear strongly in emission. Popper (1970) regards this as a characteristic feature of the group of eclipsing binaries of which AR Lac and RS CVn are members. Sanford found a half-width of 39 km s^{-1} for the Ca II emission lines of AR Lac A and a half-width of 84 km s^{-1} for the same lines of AR Lac B. Using the dimensions given in this section and assuming synchronous rotation, one finds that the equatorial rotational velocity is 39 km s^{-1} for star A and 72 km s^{-1} for star B. Thus the broadening of the spectral lines in AR Lac is clearly associated with the synchronous rotation of its components.

Combining the values of the dimensions of AR Lac with the data of Tables I and VI, absolute visual

magnitudes of $+4.01$ for star A and $+3.64$ for star B are obtained. The corresponding absolute bolometric magnitudes are $+3.91$ and $+3.37$, respectively. Although both components are subgiants, AR Lac B lies about 2^m5 above the main sequence, while AR Lac A lies only about 1^m1 above it. The absolute visual magnitude of the system is $+3.06$, and its distance is 40 parsecs. Although its galactic latitude is only -8° , one can expect no significant reddening at this distance. In fact, AR Lac is one of the closest of the bright eclipsing binary systems.

VIII. Radio Outbursts of AR Lac

AR Lac is one of very few eclipsing binaries which have thus far been observed to display radio outbursts. Radio emission from AR Lac was first detected by Hjellming and Blankenship (1973), and more extensive data were published by Gibson and Hjellming (1974). The observations were made with the NRAO interferometer at Green Bank, West Virginia, at frequencies of 8085 MHz (3.7 cm) and 2695 MHz (11.1 cm).

Flare activity was detected on 1973 February 23 and on 1973 October 16–17 and 17–18. Later observations have indicated some flare activity on 1975 January 17–18 and 1975 March 6 (Gibson 1975). By far the strongest flare activity thus far observed occurred on 1973 October 17–18 when fluxes as high as 74 mJy at 8085 MHz and 30 mJy at 2695 MHz were detected. Radio observations were undertaken on this night between 22^h00^m and 5^h25^m UT. This investigator made optical observations on the same night from 2^h08^m to 4^h32^m UT. No evidence of unusual activity was detected by this investigator, however; thus the radio outbursts of AR Lac do not seem to be correlated with instabilities in its light curve at optical wavelengths.

The large flare observed at radio wavelengths appears to have begun shortly before 4^h00^m UT on 1973 October 17. This time was just shortly after fourth contact of the primary eclipse. On the following night the flare of AR Lac was observed to undergo a steady decline at a time when the system was going through secondary minimum. If this flare arose from the surface of only one component, one should expect that component to be star A, since the outburst was observed to begin only after star A had emerged from behind star B, and on the second night the flare was observed to decline steadily as star A transited the disk of star B. If star B were responsible for the flare activity, then the observations on the second night would more likely have shown a decline in this activity followed by a subsequent increase. The flare activity of AR Lac does not seem to be correlated with phase, however, and some flare activity was observed on 1975 March 6 at a time when AR Lac was at pri-

mary minimum. Thus the start of the outburst of 1973 October 17–18 at very nearly the time of fourth contact of the primary eclipse seems purely coincidental.

Radio outbursts have also been detected from Algol, and with these there appears to be no correlation with phase or with optical instability. The outbursts observed for Algol seem to be very similar to those of AR Lac. Although they may arise from the surfaces of the components, the flares rapidly expand in an envelope which engulfs the entire system. In the case of Algol the approximate size of the region undergoing flare activity has been estimated as 0.2 AU (Clark, Kellermann, and Shaffer 1975).

For AR Lac very long baseline interferometry may solve this problem. The semimajor axis of AR Lac is 0.0429 AU; its corresponding angular size at a distance of 40 parsecs is 0''.00107. Interferometry at resolutions substantially better than this may be needed to determine the extent of the regions responsible for flare activity in this system.

IX. Concluding Remarks

Because of its intrinsic variation and the complications present in its light curve, AR Lac definitely needs to be more extensively observed. Since its period is so close to two days, a complete light curve of AR Lac cannot be obtained at one site in only one season. Thus observations of AR Lac should be undertaken by several observatories well spaced with regard to longitude, so that complete light curves can be assembled in relatively short periods of time. Coordinated observing programs will require that the wavelength bands of the photometry be consistent at all sites. Intermediate-band photometry will probably insure greater uniformity than will wide-band photometry. Observations are needed in as wide a range of wavelengths as is possible. At the long wavelength end of the spectrum, the secondary minimum will be deeper and hence can be observed with greater precision. Yet the greatest complications are to be expected at the shortest wavelengths, so observations are needed here also. An internationally coordinated observing program of AR Lac conducted for several years should be able to determine if the amplitude of the intrinsic variation varies with time, if the phases of the maximum and minimum of this variation are time variable, and what modulations, if any, are present in the intrinsic variation of the system.

Despite the fact that AR Lac is two magnitudes brighter than RS CVn and has a period less than half as long, it has been far less extensively observed than the latter system. The light curve of AR Lac shows much less intrinsic variation than does that of RS CVn, but both systems display many similarities, and these common properties are shared by several other sys-

tems, whose special characteristics have been noted by Popper (1970) and by Hall (1972). In addition to the observations made in this study, there are the nearly 3000 observations made by the Krons during 1938–47, which are now on permanent file in the Variable Star Archives of the Royal Astronomical Society as file no. 27, and the recently published study of Babaev (1974), and 850 unpublished observations in *UBV* made by Blanco and Catalano in 1967–68 (Catalano 1976).

Once the nature of the intrinsic variability of AR Lac has been resolved, its effects can be removed from the observed light curves. Underlying the observed light curves of AR Lac there should be a light curve which remains constant with time and whose variations are caused solely by the geometric and photometric effects of the eclipses themselves and by the ellipticity and reflection effects. An analysis of such a light curve should yield Fourier coefficients which do not vary from season to season, and the agreement between the theoretical and observed light curves for secondary minimum should also be much better than was found to be the case in this study.

Despite the complications in the light curves of AR Lac which arise from its intrinsic variability, AR Lac is geometrically a relatively simple system. It is unlikely that future investigations of AR Lac will yield values of k , r_1 , r_2 , and i which differ significantly from 0.55, 0.167, 0.305, and 87° , respectively. The geometry of AR Lac is well enough defined for one to make fairly reliable estimates of the limb darkening and gravity brightening of the components. The gravity-brightening index in AR Lac appears to be consistent with the values expected for stars with deep convective envelopes. Although the surface brightness on star A may be distributed in a nonuniform manner, the bulk of the ellipticity and reflection effects arises from the contribution of star B, and for this component a more regular distribution of surface brightness seems likely.

At the present time too little is known about the radio outbursts of AR Lac to make many specific statements. They do not appear to be correlated with phase or with instabilities in the light curves at optical wavelengths. Further observations will be necessary to determine how frequent they are, and if they display any type of periodicity. Thus far AR Lac is the only eclipsing binary aside from Algol from which strong radio outbursts have been detected.

Many questions about AR Lac remain as yet unanswered or inadequately explained. For instance, why does the period of AR Lac undergo such substantial changes despite the fact that the system is detached? Is the intrinsic variation observed in AR Lac restricted to star A, or is star B also variable? If

the latter is the case, does star B have extensive regions of spots as seems to be the case with star A? Is the intrinsic variation observed in the light curve of AR Lac subject to the same sort of modulations in amplitude and in phase that have been proposed for RS CVn? At what stages of stellar evolution are each of the components? Why are the Ca II emission lines so pronounced in their spectra? Further investigations of AR Lac and of the systems similar to it will be needed before these questions, and many others as well, can be answered.

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