

TIME-DEPENDENT EFFECTS IN THE NEBULAR SHELL OF FG SAGITTAE

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ABSTRACT

Theoretical models have been constructed of planetary nebulae illuminated by central stars which are evolving on a time scale comparable with the cooling time of the nebular gas. The emission line spectra of the models have been compared with the spectrum of the planetary nebula surrounding FG Sge. The observed spectrum can be reproduced by that of a nebular shell of density $n_H = 160 \text{ cm}^{-3}$, illuminated by a central star which has decreased linearly in temperature at constant luminosity from an initial value of 55,000–75,000 K. The nebula appears to have a N/O ratio which is anomalously high even for planetary nebulae.

Subject headings: nebulae: abundances — nebulae: planetary — stars: individual

I. INTRODUCTION

The behavior of the peculiar variable FG Sge has inspired considerable theoretical effort and speculation in recent years. The star has steadily increased in brightness by about 4 mag since 1895 (Richter 1960). Spectroscopic observations by Herbig and Boyarchuk (1968) and by Langer, Kraft, and Anderson (1974) (hereinafter referred to as HB and LKA, respectively) show a progressive change in spectral type from B4 I in 1955 to F6 Ip in 1973. Around 1970 the increase in apparent magnitude ceased. LKA point out that this occurred at the time when the star passed through the spectral type having the smallest bolometric correction and suggest that the star is moving along a horizontal track in the H-R diagram. Most intriguing of all, over the last decade LKA have noted the appearance and increase of *s*-process elements in the absorption spectrum to a value of approximately 25 times solar abundances.

The star appears to be the central excitation source of the faint planetary nebula PK 60°–7°1. Examination of H α and [N II] nebular emission lines by Flannery and Herbig (1973) indicate (1) an expansion velocity of 34 km s^{-1} , a value quite typical for normal planetaries, and (2) a mean radial velocity of $+38 \text{ km s}^{-1}$, in close agreement with the value of $+37 \text{ km s}^{-1}$ of FG Sge, leaving little doubt as to the intrinsic association of the star and the nebula. Based upon the reddening in this region of the sky, HB estimate a distance of 2.5 kpc. The 14" angular radius of the nebula then implies an expansion age of about 5000 years. Estimates of the bolometric magnitude of the star by LKA yield a luminosity of $\log_{10} (L_*/L_\odot) = 3.55$, consistent with typical planetary nuclei.

The scenario we wish to investigate is that of a planetary nebula which has evolved near thermal and ionization equilibrium for 5000 years, at which time it is subjected to a rapidly changing radiation field resembling the behavior of FG Sge over the last 75 years. Since the number of ionizing photons available to the nebula is diminishing more rapidly than the recombination time scale of the gas, nonequilibrium effects will soon dominate. The present nebula remains as a fossil record of the past behavior of the central excitation source, while the nonequilibrium response of the gas can also reveal something of the nature of the nebular material.

Numerical models with various densities and chemical abundances are evolved, starting with a number of initial stellar temperatures and luminosities. Comparison of the calculated emission line intensities with the observed nebular lines allows constraints to be placed upon the stellar temperature of FG Sge prior to 1895 and upon the density of the nebular gas, as well as predicting the future behavior of the nebular spectrum. The models also suggest that the nebula has an abnormally high N/O ratio.

II. THEORY

That the nebula is now visible is in itself evidence of the nonequilibrium condition of the gas, for in 1955 FG Sge was already too cool to produce appreciable ionizing radiation. The time scale for the recombination of ions is

$$\tau_R = \frac{1}{n_e \alpha}.$$

At an electron density of 200 cm^{-3} (consistent with the observed [O III] $\lambda 3726/\lambda 3729$ ratio) $\tau_R \sim 600$ years

for hydrogen and ~ 100 years for a doubly ionized atom like He^{2+} . There is no simple expression for the cooling time scale of an ionized gas, but it is generally about one-fifth the hydrogen recombination time, i.e., ~ 120 years. Thus it is clear that the nebula can remain ionized and indeed preserve higher stages of ionization that it may have had for the required length of time. It is also clear that any quantitative interpretation of the nebular spectrum must take into account the cooling of the nebula.

The problem of nonequilibrium ionization was considered long ago by Grotrian (1937) in connection with the light curve of nova DQ Herculis, and later Aller (1956) considered the case of a filament suddenly deprived of a source of excitation. These authors did not, however, evaluate the temperature change of the gas.

a) Heating and Cooling

The equation governing the thermodynamic properties of the gas as a function of time may be written

$$\frac{dP}{dt} - \frac{\gamma P}{\rho} \frac{d\rho}{dt} + (\gamma - 1)\rho\mathcal{L} = 0, \quad (1)$$

where P is the total gas pressure, ρ the density, γ the ratio of specific heats, and $\mathcal{L}$ the net rate of energy loss per unit mass. For a perfect gas, $P = (n_i + n_e)kT$, where n_e is the electron density and n_i the density of ions and atoms per unit volume. Since we are interested in behavior over times too short for motions of the gas to be important, we may regard n_i as a constant (but n_e will vary because of recombinations). Then equation (1) becomes

$$(n_i + n_e)k \frac{dT}{dt} = \frac{2}{3}(G - L) - kT \frac{dn_e}{dt}, \quad (2)$$

where G and L are, respectively, the energy gain and loss rates per unit volume. The last term in this equation represents temperature changes which result from distributing a given amount of energy among a changing number of particles; if the gas is recombining, this term is nearly canceled by the cooling due to recombinations which is included in L . (The cancellation is not exact, since the lower energy electrons are more likely to be captured. If a gas could recombine with no other energy losses, it would increase in temperature.)

The energy gain, due to photoionization, may be written

$$G = \sum_{i=1}^I \sum_{j=1}^{J(i)-1} N_{ij} \Psi_{ij}, \quad (3)$$

where

$$\Psi_{ij} = \int_{\nu_{ij}}^{\infty} 4\pi J_{\nu} a_{ij}(\nu) \left(1 - \frac{\nu_{ij}}{\nu}\right) d\nu. \quad (4)$$

Here, I is the number of elements, $J(i)$ the number of stages of ionization of a given element, N_{ij} the number

of ions cm^{-3} , J_{ν} the mean intensity of radiation, $a_{ij}(\nu)$ the photoionization cross section, and ν_{ij} the threshold for photoionization. The energy lost by the gas can be written as

$$L = \sum_{i=1}^I \sum_{j=1}^{J(i)} n_e N_{ij} \left[\beta(z, T) + \beta_{ff}(z, T) + \sum_{k=2}^K \sum_{l=1}^{k-1} C_{ijlk}(n_e, T) \right], \quad (5)$$

where β and β_{ff} are the coefficients for loss by recombination and by free-free encounters, and C_{ijlk} represents the energy lost by radiative de-excitation following collisional excitation of the $l \rightarrow k$ transition of the ij ion. The ions included as coolants are H I, C II–C IV, N I–N V, O I–O VI, Ne II–Ne V, and S II–S VI. Fine-structure infrared transitions were included. The values of C_{ijlk} are obtained by solving the set of equations for the level populations, so that collisional de-excitation is explicitly included. (Collisional de-excitation is not very important, however, at the densities considered here; 100–400 particles cm^{-3} .)

The values of $a_{ij}(\nu)$ are from Flower (1968), Henry (1970), and Chapman and Henry (1971). The values of the collision cross sections used are the same as those collected in Osterbrock (1974) with the exception of the more recent values for N^+ , O^{2+} , and Ne^{2+} given by Seaton (1975) and the addition of values for C^+ and N^{2+} by Jackson (1973).

b) Ionic Abundances

The rate of change of a particular ion in the nebula is given by the expression

$$\begin{aligned} \frac{dN_{ij}}{dt} = & \Phi_{i,j-1} N_{i,j-1} - (n_e \alpha_{ij} + \Phi_{ij}) N_{ij} \\ & + n_e \alpha_{i,j+1} N_{i,j+1}, \end{aligned} \quad (7)$$

for $i = 1, I$; $j = 1, J(i)$, except that for $j = 1$ the first term of the right-hand side is omitted and for $j = J(i)$ the last is omitted. Here,

$$\Phi_{ij} = \int_{\nu_{ij}}^{\infty} \frac{4\pi J_{\nu}}{h\nu} a_{ij}(\nu) d\nu \quad (8)$$

is the rate of photoionization per ion and α_{ij} is the recombination rate coefficient summed over all levels. Dielectronic recombination has not been included. The recombination rates were taken from Tarter (1971, 1973) except for helium, sulfur, and some higher ionic states which were assumed hydrogenic. The equations for O^0 , O^+ , N^0 , and N^+ were modified to include the charge-exchange reactions $\text{O}^0 + \text{H}^+ \rightleftharpoons \text{O}^+ + \text{H}^0$ and $\text{N}^0 + \text{H}^+ \rightleftharpoons \text{N}^+ + \text{H}^0$. The rates of Field and Steigman (1971) and of Steigman, Werner, and Geldon (1971) were used.

For each element i there is a redundant equation in the set given by (7). One of the $J(i)$ equations can be

eliminated by the expression for the conservation of particles:

$$A_i N_H = \sum_{j=1}^{J(i)} N_{ij}, \quad (9)$$

where A_i is the abundance of the element relative to hydrogen and N_H the total hydrogen number density. We have, however, integrated the full set of equations (7); equation (9) then provides a check on numerical accuracy.

c) The Radiation Field

The mean intensity of the radiation in equations (4) and (8) is the sum of a stellar component and a diffuse component originating in the gas itself; $J_\nu = J_\nu^* + J_\nu^D$. The stellar component at a point r in the nebula can be expressed as

$$J_\nu^* = \left(\frac{F_\nu^*}{4}\right) \left(\frac{R_\odot}{r}\right)^2 \left(\frac{T_\odot}{T_*}\right)^4 \left(\frac{L_*}{L_\odot}\right) \exp[-\tau_\nu(r)], \quad (10)$$

where L_* and T_* are the time dependent stellar luminosity and effective temperature, F_ν^* is the flux emitted per unit area at the stellar surface, and $\tau_\nu(r)$ is the optical depth through the nebular material. The only significant absorption will be by H^0 , He^0 , and He^+ :

$$\tau_\nu(r) = \int_{r_0}^r [N_H^0 a_{H^0}(\nu) + N_{He^0} a_{He^0}(\nu) + N_{He^+} a_{He^+}(\nu)] dr'. \quad (11)$$

Thus provided that we know the stellar temperature, luminosity and some approximation to its flux distribution, and know the ionic abundances interior to the point r , we can evaluate J_ν^* .

The diffuse radiation presents more of a problem. Direct evaluation would require an iterative approach which does not seem worthwhile. If the nebula is optically thin, we can neglect J_ν^D entirely, while if it is optically thick, the on-the-spot approximation can be used. However, in the models constructed here, the shell is initially optically thin in the Lyman continuum (since this nebula is an old, low-density object) but after the stellar temperature drops below $\sim 25,000$ K, sufficient hydrogen quickly recombines to make the nebula optically thick, although the overall degree of ionization remains high for a long time. If the optical depth through the nebular shell at the absorption edge of H^0 is $\tau(H^0)$, then the quantity $f(H^0) = 1 - 0.5\{\exp[-0.5\tau(H^0)] + \exp[-1.5\tau(H^0)]\}$ is a rough measure of the amount of diffuse radiation absorbed near the Lyman limit (i.e., this expression is just the average of the fractions of radiation which H^0 absorbs from rays emitted at the center of the shell and traveling inward or outward, respectively). The approximation we then have made is to represent the diffuse field as

$$J_\nu^D(r) = f J_{\nu}^{\text{on-the-spot}}(r), \quad (12)$$

where we use $f(H^0)$ if $13.6 < h\nu < 24.6$ eV, $f(He^0)$ if $24.6 < h\nu < 54.4$ eV, and $f(He^+)$ if $h\nu < 54.4$ eV.

The on-the-spot value is just the ratio of emission to absorption coefficients at point r , and includes emission by recombination to the ground states of H^0 , He^0 , and He^+ , to the $n = 2$ level of He^+ and by two-photon radiation from He^+ and He^0 . This procedure seems justifiable as a well-defined means of switching from $J_\nu^D = 0$ to the on-the-spot approximation in a smooth fashion at the proper time in the nebular evolution.

d) Integration of the Equations

To start the evolution, an equilibrium model is constructed using a program developed for the analysis of normal planetary nebulae. This procedure is well known, and the code used is an extension of that described elsewhere (Harrington 1968, 1969). The equilibrium model provides the initial nebular temperature and the abundances of the ions, N_{ij} , at each point r_n in the shell. The number of radial zones varied between 11 and 16.

The simultaneous integration of equations (7) and (2) proceeds by taking each radial zone in turn one step forward in time, starting with the innermost radial point. Thus the radiation field, which is defined by the stellar parameters (assumed known as a function of time) and by the optical depth $\tau_\nu(r)$, can be determined from ionic abundances interior to r which have been interpolated to the proper time. It is necessary, however, to use values of $f(H^0)$, $f(He^0)$, and $f(He^+)$ determined at the previous time step. This treatment reduces what is really a partial differential system in space and time to the sequential solution of ordinary differential equations in time. A fourth-order Runge-Kutta formula is used for the integration.

If a radiation field J_ν is zero, the integration of the recombining gas proceeds well with any reasonable time step. However, with a slowly varying radiation field of intensity comparable to that which established the equilibrium model, it was found that the system was stable only for very small integration time steps. The difficulty stems from the trace of neutral material always present. If, for example, the recombination time scale for hydrogen is ~ 600 years, and if $N(H^0)/N(H^+) = 5 \times 10^{-5}$, this ratio could increase to $N(H^0)/N(H^+) = 10^{-4}$ in ~ 0.03 years. Thus there is a short time scale for neutrals and other highly ionized species over which their fractional abundance could in principle change substantially, and it was found that if the time step exceeded this value, they would grow exponentially in an oscillatory fashion. This occurs even though there may be no change in the radiation field, for there is always some numerical noise which can be amplified. The maximum time step can be expressed as

$$\Delta t \lesssim [\Phi_{ij}^{-1}]_{\text{minimum}}, \quad (13)$$

where Φ_{ij} is defined by equation (8). In fact, Δt was set by the Φ value of S I, which is the element whose neutral atoms have the smallest abundance relative to the total abundance of the element. In some cases, this limited the time step to ~ 0.015 years. With such a short time step, the evaluation of G , L , and the Φ and

Ψ integrals would lead to unreasonably long running times. To circumvent this difficulty, a second time step $\Delta t' \sim 3$ years was introduced. Φ_{ij} and Ψ_{ij} were evaluated for the stellar radiation field corresponding to $T_*(t_n)$ and $T_*(t_n + \Delta t')$, and values interpolated for intermediate times. The values of β , β_{eff} , C_{ijlk} in equation (5) and α_{ij} in equation (7) were held constant over the interval $\Delta t'$ while the system of equations was integrated from t_n to $t_n + \Delta t'$ using the small time step Δt .

III. THE OBSERVED NEBULAR SPECTRUM

The spectrum of the nebulosity around FG Sge has been discussed by HB. The Balmer series through H8 is observed and strong lines of [O II], [O III], [N II], [Ne III], and [S II] are present. Unfortunately, no measurements of the line intensities are available. It is necessary, however, to have some estimates if we are to make any comparison between theory and observation. Thus we will attach numerical values to these lines, but it must be stressed that these could be greatly in error.

Upon examination of plate 9 in HB, we can make some comparisons between the forbidden lines and the Balmer lines. Thus the [O III] $\lambda 4959$ line appears comparable in intensity to H β . The [Ne III] $\lambda 3868$ line seems weaker than H γ but a bit stronger than H δ . The stronger of the [N II] lines, $\lambda 6583$, is slightly stronger than H α , as noted by Flannery and Herbig (1973); and this can be seen more clearly on the spectrum reproduced in their paper. The [O II] $\lambda \lambda 3726-3729$ doublet presents the most serious problem. The lines are clearly quite strong, but there are no H lines nearby for comparison. However, in addition to the two spectra of the FG Sge nebulosity, HB present for comparison the spectrum of the planetary nebula IC 2149 taken with the same equipment, and it appears that the [O II] doublet is substantially stronger in FG Sge than it is in IC 2149. The interstellar absorption for IC 2149 is comparable to that estimated by HB for FG Sge, and photoelectric measurements of the line intensities in IC 2149 have been given by O'Dell (1968) who finds that [O II]/H β = 1.2 (corrected for reddening). Thus we guess that [O II]/H β $\sim$ 2 for FG Sge. Finally, HB do not mention He II $\lambda 4686$, but since they claim that H8 is visible, we can put an upper limit on this important line. In this fashion we obtain the values

TABLE 1
ESTIMATED LINE INTENSITIES FOR THE NEBULAR SHELL
OF FG SAGITTAE

Line	FG Sge	IC 2149*	IC 418†
[O II] $\lambda \lambda 3726 + \lambda 3729$	200?	120	162
[Ne III] $\lambda 3869$	30	37	2
He II $\lambda 4686$	< 10	...	...
[O III] $\lambda 4959$	100	160	41
[N II] $\lambda 6584$	300	...	166
[S II] $\lambda 6716$	30?	3	6

* O'Dell 1968.

† Peimbert and Torres-Peimbert 1971a.

listed in Table 1. The estimates are relative to H β = 100, and the spectra of two planetary nebulae are listed for comparison.

Two remarks might be made. First, the [N II] lines are unusually strong. Second, although the nebula has been cooling for some time, the total energy radiated in the forbidden lines is greater than in the low-excitation planetary IC 418 and seems greater than in the intermediate-excitation object IC 2149, indicating that the nebular shell of FG Sge must have had a very substantial energy input at sometime in the past.

HB state that the [O II] doublet is resolved on a spectrogram of Henize and that $I(\lambda 3729) > I(\lambda 3726)$. This indicates a low density which has been estimated to be $\sim 100 \text{ cm}^{-3}$ by Henize (1961) and $\sim 500 \text{ cm}^{-3}$ by HB. (See also Faulkner and Bessell 1970.)

IV. MODELS CONSTRUCTED

The nebulosity around FG Sge appears as a rather uniform disk with a radius of $12''-14''$, with fainter material extending out to $18''$ (HB). We have represented this nebula by thick, spherically symmetric shells. The first series of models assumed uniform density, while later models were constructed with a more realistic density variation through the shell. High-density condensations were not, however, taken into consideration. The choice of a distance will fix the radius of the model and also determine the present stellar luminosity. We have generally assumed a distance of 2.5 kpc (HB) which implies a luminosity of $\sim 4000 L_{\odot}$; but a few models were constructed with other assumed distances, and they show that this choice is not crucial to within about a factor of 2.

Since spectra of the star were first obtained in 1955, FG Sge has shown a steady decrease in temperature, while the bolometric magnitude shows no definite change over this period (LKA). For the interval 1894-1955, we have only the photographic apparent magnitude, so that we cannot separate possible variations in luminosity from the magnitude change due to the temperature variation. We have restricted our calculations to the assumption of constant luminosity. We will argue below that a slow change in stellar luminosity will not alter our conclusions.

Having assumed constant luminosity, we could obtain the stellar temperature as a function of time from the photographic light curve if we had knowledge of the bolometric correction. Such a detailed treatment would be somewhat academic because the nature of the flux distribution from this unique object is so uncertain. We have simply assumed that from an equilibrium state at some initial epoch t_0 , the temperature decreases linearly:

$$T_*(t) = T_*^i, \quad t \leq t_0$$

$$= T_*^i - R(t - t_0), \quad t > t_0. \quad (14)$$

For the various flux distributions used for the models, this linear formula can produce a variation in the photographic magnitude close to the observed light curve during the first half of this century. The rate R

for such a fit is greater than the presently observed value of $\sim 250 \text{ K yr}^{-1}$; but when the stellar temperature has dropped below $\sim 25,000 \text{ K}$, no ionizing radiation is produced and the behavior of the star is no longer relevant.

The choice of the stellar flux distribution for a given temperature is a major difficulty, since there are no stellar atmosphere calculations which can be applied with any confidence to this star. The fact that in 1955 the spectrum showed absorption lines with P Cygni characteristics indicates that plane-parallel models may be inadequate for earlier epochs. Under these circumstances it seemed reasonable to use blackbody fluxes for an initial analysis.

We first discuss a grid of seven such models. It is clear from the observed strength of $[\text{Ne III}] \lambda 3846$ that the initial stellar temperature must be high enough to produce substantial doubly ionized neon. The basic model, which we will call B, has a uniform hydrogen density of $N_{\text{H}} = 200 \text{ cm}^{-3}$ through a shell extending from 0.087 pc to 0.129 pc, while the stellar parameters are $T_*^i = 56,000 \text{ K}$ with $R = 540 \text{ K yr}^{-1}$ and $L_* = 4000 L_{\odot}$ (corresponding to a distance of 2.5 kpc). Variations on this model are A and C, which have $T_*^i = 49,000 \text{ K}$ and $T_*^i = 63,000 \text{ K}$, respectively; models D and E with $N_{\text{H}} = 100 \text{ cm}^{-3}$ and $N_{\text{H}} = 400 \text{ cm}^{-3}$, respectively; and models F and G with luminosities of $L_* = 1500 L_{\odot}$ and $10,000 L_{\odot}$ (corresponding to distances of 1.5 and 4 kpc). The chemical

abundances for all models were $\text{He}/\text{H} = 0.11$, $\text{C}/\text{H} = 0.0002$, $\text{N}/\text{H} = \text{O}/\text{H} = 0.0003$, and $\text{Ne}/\text{H} = 0.000075$. The rather high nitrogen abundance was needed to produce $[\text{N II}]$ line strengths comparable to observed values. Emission line spectra were calculated at intervals of 15 years or less, and the results for some important lines are shown in Figures 1–6. The intensities are expressed relative to $\text{H}\beta = 100$. (It should be noted that the $\text{H}\beta$ flux itself is not constant, but increases somewhat as the nebula cools.)

First, consider the behavior of the $\text{He II } \lambda 4686$ line compared with the $[\text{Ne III}] \lambda 3846$ line in Figure 1 and with $[\text{O III}] \lambda 4959$ in Figure 4. The He II line is produced by recombinations, and the rate at which the intensity declines indicates the rate of recombination of doubly ionized species in the nebula. The much more rapid decrease of the $[\text{Ne III}]$ and $[\text{O III}]$ lines after $t = 50 \text{ yr}$ (when the star has ceased to produce ionizing radiation) shows that the major effect here is not depletion of the ions but the decreasing temperature of the gas. The behavior of the $[\text{O II}]$ doublet (Fig. 4) also illustrates this point: the intensity first increases as the O^{2+} ions recombine, but then declines as the temperature falls, even though—as can be seen from the plot of ionic abundances for model B in Figure 7—the number of O^+ ions continues to increase throughout the calculation. The eventual rate of decrease for the

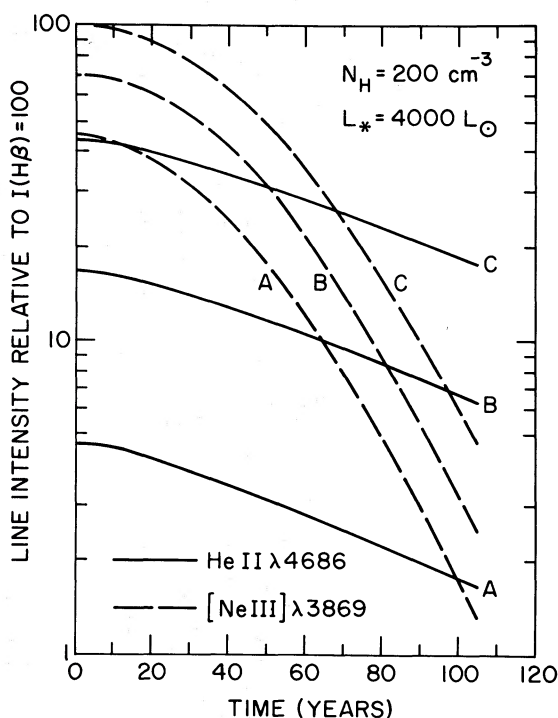


FIG. 1.—The effect of variations in the initial stellar temperature on $\text{He II } \lambda 4686$ and $[\text{Ne III}] \lambda 3869$. The intensities in this and subsequent figures are relative to $\text{H}\beta = 100$. Models A, B, and C have initial stellar blackbody temperatures of 49,000, 56,000, and 63,000 K, respectively.

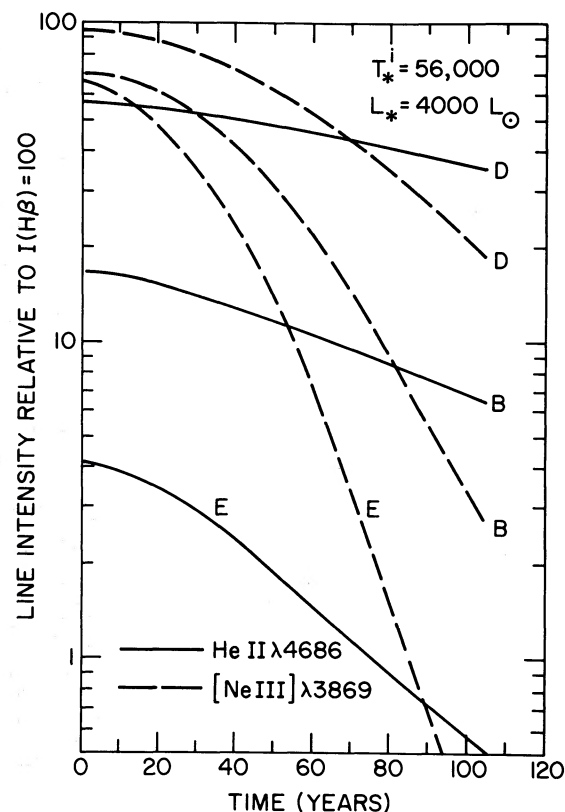


FIG. 2.—The effect of variations in the density on $\text{He II } \lambda 4686$ and $[\text{Ne III}] \lambda 3869$. Models D, B, and E have number densities of hydrogen of 100, 200, and 400 cm^{-3} , respectively.

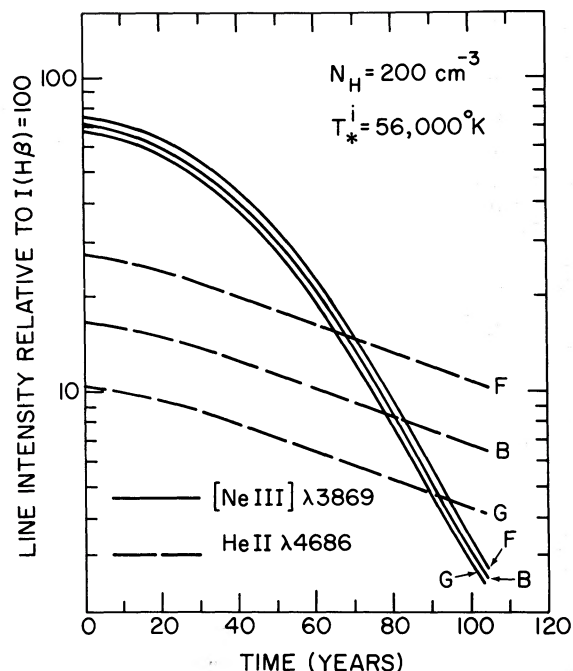


FIG. 3.—The effect of variations in stellar luminosity on He II $\lambda 4686$ and [Ne III] $\lambda 3869$. Models F, B, and G have luminosities of 1500, 4000, and 10,000 $L_{\odot}$, respectively.

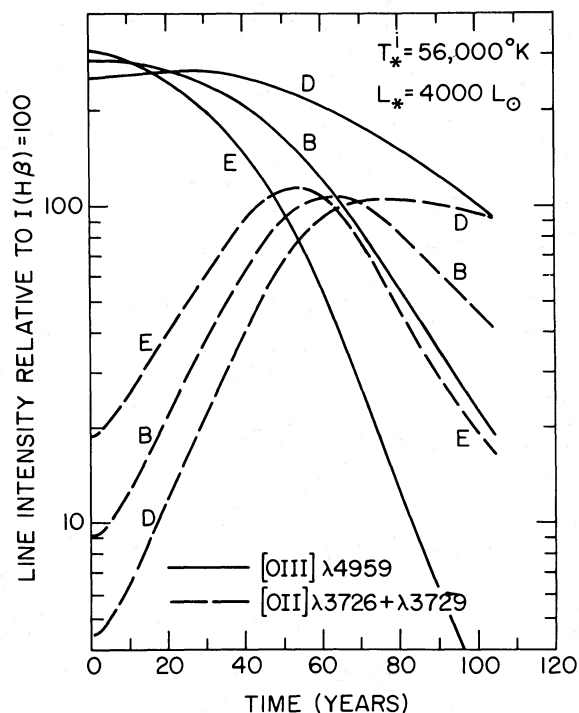


FIG. 5.—The effect of variations in the density on [O II] $\lambda 3726-3729$ and [O III] $\lambda 4959$. Models D, B, and E are the same as defined in Fig. 2.

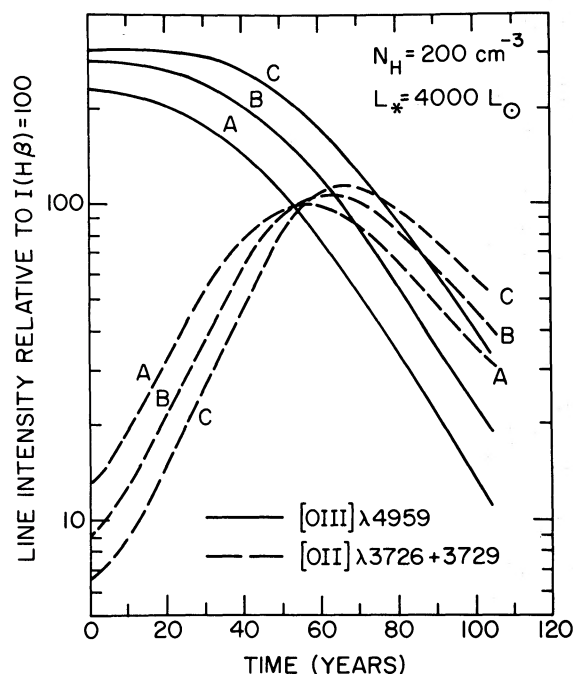


FIG. 4.—The effect of variations in the initial stellar temperature on [O II] $\lambda 3726-3729$ and [O III] $\lambda 4959$. Models A, B, and C are the same as defined in Fig. 1.

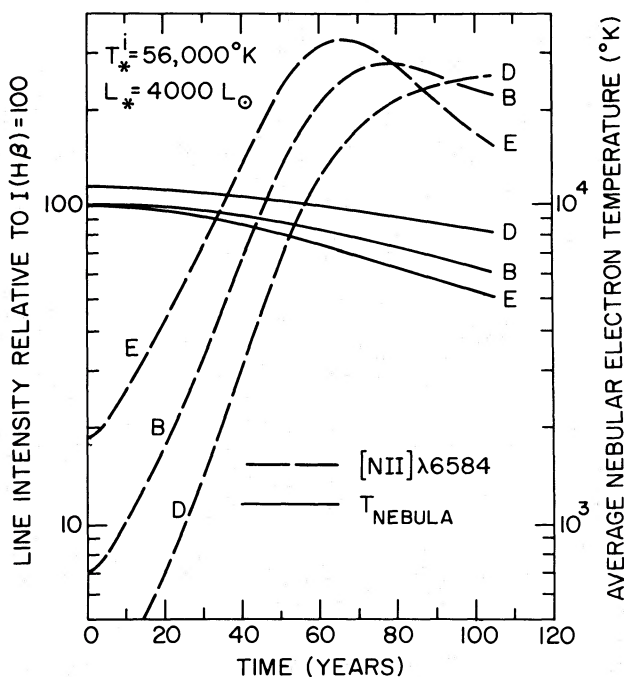


FIG. 6.—The effect of variations in the density on [N II] $\lambda 6584$ and on the average value of the electron temperature in the nebula. Models D, B, and E are the same as defined in Fig. 2.

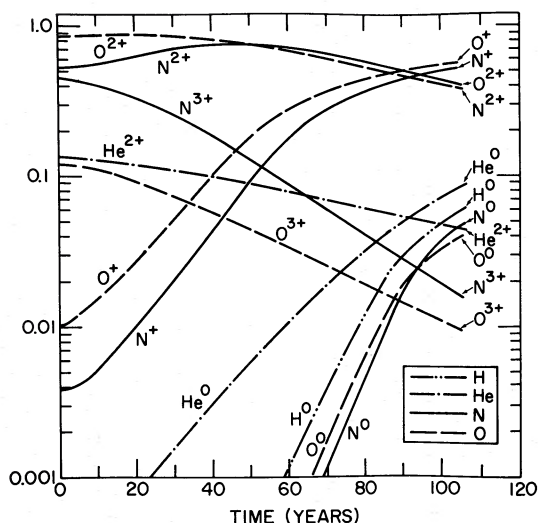


FIG. 7.—The time variation of the relative abundances of the ions, integrated over the nebular volume, for model B.

[O II] intensity is less than for [O III] because the recombinations partially compensate for the falling temperature.

The effects of density in Figures 2 and 5 are easily understood. The rates of recombination and cooling both scale as n_e^2 (collisional de-excitation of the major coolants is not very important at such low densities), and thus the density determines the time scale of the nebula's response to the stellar evolution. It would appear that densities higher than those considered here would lead to unacceptably rapid cooling. A lower density for a given radiation field will also produce a higher degree of ionization, and this accounts for the variation of the initial He II and [O II] intensity with N_H . The initial enhancement of [Ne III] in model D, however, is due to the higher temperature associated with the more extensive He²⁺ zone.

Figure 3 shows the effects of assuming different distances for the nebula. The [Ne III] line shows little change, while we have not even graphed the [O II], [O III], or [N II] lines because models F, B, and G would be nearly identical on this scale. There is a considerable difference in the He II lines, however. In models F, B, and G the stellar luminosity and the dilution factor within the nebula scale so that the radiation field at the inner edge of the nebula remains unchanged. Since the models are all initially optically thin in the Lyman continuum, the conditions throughout the models are nearly identical. There is, however, a region of doubly ionized helium at the inner edge of the shell whose thickness is determined by the optical depth in the He⁺ continuum, and the fraction of the total volume occupied by this region thus decreases for the more distant models. But even in the case of the He II $\lambda 4686$ line, the influence of the assumed distance is less pronounced than that of changes in the density or stellar temperature.

The choice of the initial stellar temperature T_* has a twofold effect. First, the amount of doubly ionized

helium is quite sensitive to the stellar temperature, and because of the relatively long recombination time the effect of this initial ionization persists throughout the evolution. Second, the higher temperature star produces higher initial electron temperatures, both in the innermost region where the presence of oxygen as O³⁺ rather than O²⁺ results in less effective cooling, and throughout the bulk of the shell due to the flatter spectrum which yields more energetic photoelectrons. The higher electron temperature leads to more intense forbidden lines, especially the higher excitation lines such as [Ne III]. If we wish to compare models A, B, and C at the time when they have the *same stellar temperature*, we must displace curve A forward by 14 years and C backward by the same amount. Then the [O III] and [Ne III] lines will nearly coincide. Thus it appears that only the He II line has a strong "memory" of the initial stellar temperature.

Let us now compare the intensities predicted by the models with our estimates of the nebular line intensities. Since the spectra were obtained in 1960 and 1967, the comparison should be made at $t = 70$ –90 years (corresponding to $t_0 = 1895$ –1875). It can be seen that most of the general features have been reproduced: [O III] at about the right intensity, strong [O II] lines and very strong [N II] coexisting with reasonably prominent [Ne III]. The density is critical in obtaining the right [O III] line strength; the value of N_H would seem to lie between 100 and 200 cm⁻³ on the basis of these models. An actual measurement of the change in [O III] line intensity with time should provide an accurate determination of N_H and place a strong constraint on the models. In Table 2 we compare a model interpolated between B and D at $t = 80$ yr with the estimated FG Sge spectrum. The [Ne III] intensity agrees fairly well and could be increased to the estimated value by adjusting the Ne abundance. The [N II] lines were of course brought up to near the observed value by employing an unusually high nitrogen abundance. The critical test is the agreement of the [O III] and [O II] lines, since here we have a ratio of two ions of the same element. Also, because it is the main coolant, adjustment of the oxygen abundance changes the temperature of the initial model in such a way that the [O III] intensity tends to remain about the same. In this respect the model is only partially successful, for while the [O II] radiation is near its maximum when [O III]

TABLE 2
COMPARISON OF MODEL SPECTRA WITH THE
ESTIMATED INTENSITIES

Line	FG Sge	Model B-D	Model Z
[O II] $\lambda 3726 + \lambda 3729$	200?	95	152
[Ne III] $\lambda 3869$	30	20	35
He II $\lambda 4686$	< 10	20	2.5
[O III] $\lambda 4959$	100	100	102
[N II] $\lambda 6584$	300	280	270
[S II] $\lambda 6716$	30?	Not calculated	25

reaches the right intensity, it is still a factor of 2 below the estimated value. Finally, the He II line is far too strong—since it is not observed, we would like to have it very weak, say less than 3, rather than the value of 20 given by this model.

The problem with He II $\lambda 4686$ is of course a direct result of using blackbody stellar fluxes. While we have no adequate model for the ultraviolet spectrum of FG Sge, an examination of the available model atmospheres for stars in the 40,000–75,000 K temperature range shows that two features are common to all of them: (1) a very large drop in flux at the absorption edge of He⁺ at 228 Å and (2) a spectral energy distribution between 912 and 228 Å which is flatter than the corresponding blackbody curve. This is true of the plane-parallel LTE models of Hummer and Mihalas (1970), the extended LTE models of Cassinelli (1971), and the recent extended non-LTE models of Kunasz, Hummer, and Mihalas (1975). The strong He II line in our models is the result of neglecting the drop in flux at 228 Å. A series of models were evolved illuminated by a blackbody spectrum for $\lambda > 228$ Å, but with the blackbody flux reduced by a factor of 50 for $\lambda < 228$ Å. These models showed only very weak He II lines, while the general behavior of the other lines was unchanged. The suppression of the high-frequency radiation did, however, lower the temperature of the inner edge of the nebula somewhat, thus slightly lowering the intensities of the forbidden lines and aggravating the difference between the models and the estimated nebular spectrum.

Figures 4 and 5 show that changes in the initial stellar temperature and gas density do not greatly change the maximum [O II] intensity. What is necessary is a higher initial electron temperature, so that after the gas has cooled and recombined to the point where $I(\lambda 4959) \approx I(H\beta)$, the electron temperature is still high enough to excite the [O II] lines strongly. By adjusting the total metal abundance of the nebula, we change the electron temperature and distribute the radiation loss differently among the various cooling ions. Thus if the abundance is very high, the gas is cool and most of the energy is lost via infrared fine-structure transitions (which are not collisionally de-excited at these densities) with the result that the visible forbidden lines are all weak. Conversely, if the metal abundance is very low, the gas is so hot that ultraviolet transitions of ions like C²⁺, C³⁺, N²⁺, and N³⁺ carry off much of the energy, and the visible lines are again weakened. A series of models were evolved with a wide range of total abundances, and it was found that the [O II] lines reached their greatest intensities for O/H $\approx$ 0.0001. While this dependence is not critical enough to fix the metal abundance with any accuracy, we can say that, on the one hand, O/H $\leq$ 0.0004, while on the other, very low values, say less than 0.00004, can be excluded.

While adjustment of the metal abundances to channel most of the energy loss into the proper forbidden lines is helpful in bringing the model spectra closer to the estimated values, the fit can also be improved by considering a stellar flux distribution

which will produce greater heating for a given effective temperature. As pointed out above, model atmospheres have this characteristic because of their flatter spectra. The only extensive grid of models is that of Hummer and Mihalas (1970); and while they are not necessarily appropriate to FG Sge, they do provide a definite way of testing the importance of a more realistic flux distribution. We made use of the sequence of models 205, 223, 217, 219, 209, 213, and 208 which have $\log_{10} g = 4.0$, $10^{-4}T_{\text{eff}} = 3, 3.5, 4, 4.5, 5, 5.5$; and $\log_{10} g = 4.75$ at $T_{\text{eff}} = 75,000$ K. For each stellar temperature—determined by the time step through equation (14)—a flux distribution was generated by interpolation in this sequence.

In Figure 8 we plot the spectral line intensities for two runs using these model atmosphere fluxes. We have included some weak lines which have not yet been observed, but which could be of interest. The chemical abundances used were as follows: He/H = 0.11, C/H = 0.000075, N/H = 0.00015, O/H = 0.0001, Ne/H = 0.00003, and S/H = 0.00001. The two runs were identical except that in one case (model Y) $T_*^i = 55,000$ K and in the other (model Z) $T_*^i = 75,000$ K. (In Fig. 7 the Y curves are displaced so that times with the same stellar radiation field coincide.) In both runs $R = 659$ K yr⁻¹. To ensure that important effects were not overlooked by the use of constant-density shells, these models had a density which varied quadratically through the shell, from minimum values of 40 cm⁻³ at the edges through a maximum value of 212 cm⁻³ at $r = 0.16$ pc. The root mean square density was 162 cm⁻³. As a result the shell cools nonuniformly, with the center cooling quickly while the edges remain quite hot.

It is clear from Figure 7 that the forbidden lines have little memory of the initial stellar temperature. Because ionization structures of optically thin nebulae with 50,000 K < T_* < 75,000 K are very similar, the difference arises only because as model Z passes $T_* = 55,000$ K it is not in thermal equilibrium and thus is somewhat hotter than the equilibrium state in which model Y starts. This results in stronger lines which are maintained for the rest of the evolution. But this similarity of the models does not apply to the He II $\lambda 4686$ line, which is 6.8 times stronger in model Z than in model Y (the model Y line is off scale in Fig. 7). While even in model Z the $\lambda 4686$ line is so weak as to be unobservable, it can be seen that if T_*^i were much above 75,000 K, say 90,000 K, this line would initially be strong and would persist at an observable level to the present epoch.

In Table 2 the intensities of model Z at $t = 95$ years have been listed, and it can be seen that they come quite close to our estimates for FG Sge, although the [O II] lines are still 50 percent less than we were aiming at. If it should turn out that the [O II] intensity is even greater than our estimate, then it would seem, on the basis of the preceding discussion, that we should consider further the problem of the flux distribution—especially since the extended model atmospheres tend to have even flatter spectra than the models used here. But without firm observational data, there would seem

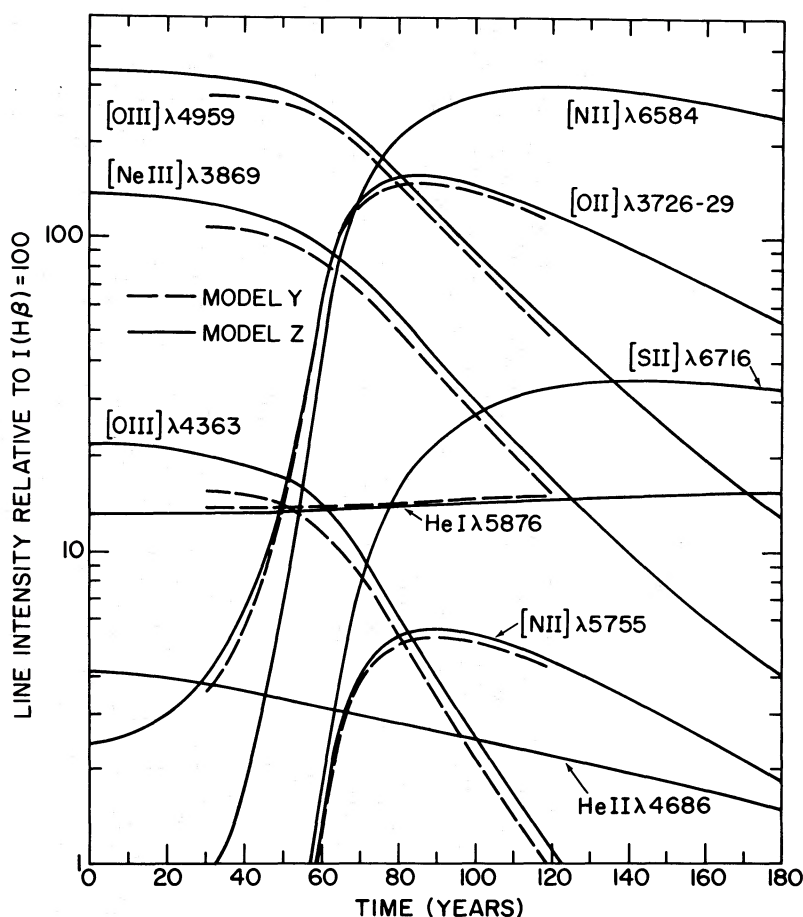


FIG. 8.—Line intensities for two cases evolved with model atmosphere stellar fluxes. Models Z and Y have initial stellar effective temperatures of 75,000 and 55,000 K, respectively. Model Y was evolved only for 90 years, starting from equilibrium at $t = 30$ years, at which time model Z has the same stellar temperature. The $[\text{N II}]$ and $[\text{S II}]$ intensities are not shown for model Y, as they are nearly coincident with the values for model Z.

to be little point in further refinement of these calculations.

The high nitrogen abundance required by our models merits further comment, for while the $[\text{N II}]$ lines of FG Sge are strong, such line intensities are not rare among planetary nebulae. Furthermore, there have been longstanding difficulties in obtaining the N abundance in planetaries because N can only be observed as N^+ , while in intermediate- and high-excitation nebulae most of the N is N^{2+} or even more highly ionized and only trace amounts of N^+ are present. Thus it is necessary to include a very large correction factor for unobserved stages of ionization based either on theoretical models or on the ratios of other ions like O^+/O^{2+} . Under these conditions, isolated regions of low-excitation material ("condensations") can contribute more $[\text{N II}]$ radiation than the background gas and invalidate any analysis based on a smooth model. That this in fact occurs is shown by the highly nonuniform appearance of many nebulae on monochromatic photographs taken in the light of $[\text{N II}]$. The problems this entails for abundance analysis is discussed in some detail for NGC 7009 by

Aller and Epps (1975). The situation is not hopeless, however, for low-excitation objects like IC 418 have a large proportion of their N in the form of N^+ . Analysis of this nebula by Buerger (1973) gave $\text{N/O} \sim 1/6$, while analysis using newer observations and a more likely He/H ratio of 0.11 yields $\text{N/O} \sim \frac{1}{3}$ (Harrington, unpublished). The later value is quite similar to the average values for planetaries obtained by Peimbert and Torres-Peimbert (1971b), and indicates that commonly quoted chemical abundances may not be too badly biased by the effects of condensations.

Turning to our models of FG Sge, one may ask whether the high N abundance that we have employed only reflects our neglect of condensations. We do not believe this is the case. The line profiles of $\text{H}\alpha$ and $[\text{N II}] \lambda 6584$ obtained by Flannery and Herbig (1973) have a fair amount of spatial resolution. Aside from the greater thermal broadening of $\text{H}\alpha$, the lines look very similar and show no strong clumping of $[\text{N II}]$ such as that which is so evident in the profiles of a "problem object" like NGC 7009 (Ford and Rubin 1971). The high N/O ratio is necessary in our models

in spite of the fact that N^+ is *not* a scarce ionization stage. At $t = 95$ years model Z has the following distribution of ionization: 22 percent N^+ , 55 percent N^{2+} , 22 percent N^{3+} ; 30 percent O^+ , 68 percent O^{2+} , 1 percent O^{3+} . If we were to use a "normal" N/O ratio of $\sim \frac{1}{3}$, the [N II] lines would not be sufficiently intense unless essentially 100 percent of the N were singly ionized. Under such conditions it would not seem possible to explain the strong [O III] and [Ne III] lines which are observed.

V. DISCUSSION

In the preceding section we have shown that a planetary nebula illuminated by a central star which is cooling rapidly at constant luminosity can give rise to a spectrum which is consistent with the information available about the nebular shell around FG Sge. Since the observational data are so uncertain, we have discussed the influence of various parameters on the theoretical spectrum in some detail so that readers may be able to judge the relevance of future observations to the models presented here.

While the star must have begun the phase of rapid evolution with $T_* \geq 40,000$ K to produce the required Ne^{2+} and strong heating, it appears that the later evolution of the nebula is not sensitive to the initial temperature over the 50,000–75,000 K range. Above this temperature, however, the extensive He^{2+} zone originally present would lead to a strong, persistent He II $\lambda 4686$ line which is not observed. To put this another way: When the central star began its phase of rapid cooling, the planetary nebula was not a high-excitation object.

We find that the models indicate a density of 160 cm^{-3} , which is consistent with the [O II] doublet ratio. At the adopted distance of 2.5 kpc, the mass of the nebula is then $\sim 0.1 M_\odot$, typical for a planetary. The time rate of change of the [O III]/H β ratio is a sensitive measure of the density of the nebula, and future measurements of this ratio should permit the construction of more definitive models.

While we have assumed a constant luminosity, an evolutionary track which is not quite horizontal will not significantly change the models presented here. For while $T_* \geq 35,000$ K, the models are optically thin and thus insensitive to the luminosity; and when T_* drops below $\sim 25,000$ K, the star is effectively off regardless of luminosity. In fact, the main result of a luminosity variation should be the effect on the inferred rate of change of the stellar temperature. For example, if the luminosity had been decreasing since 1900, then the effective temperature would have had to drop more rapidly to produce the observed increase in apparent magnitude. (Actually, a more rapid decline in T_* would make the models fit more easily,

since the nebula would have less time to cool from its initial state before recombination set in.)

Perhaps the most interesting result of this study is the inferred chemical composition of the nebula. The N/O ratio appears to be very high; to reproduce the strong N II lines, model Z required $N/O = 1.5$. Even if this result were in error by a factor of 2, the nebula would be anomalous. The models do not seem to require unusual Ne or S abundances. While our models point to a rather low overall metal abundance, not too great significance need be attached to this result inasmuch as it should be possible to double the metal abundance and compensate for the increased cooling by a more energetic stellar flux distribution. It should be remembered, however, that because of the high N abundance, the total CNO abundance used here is not as low as it would appear from the O/H ratio alone.

Peimbert and Torres-Peimbert (1971*b*) find that in H II regions $N/O \approx 0.07$, while for planetary nebulae the average value is $N/O \approx 0.3$. They suggest that the relative overabundance of nitrogen in planetary nebulae is the result of a partial mixing of CNO bi-cycle products with the other layers of the parent star before these layers are ejected to form the planetary nebula. They further maintain that planetary nebulae can enrich the interstellar medium and are responsible for the observed N/O gradient across the disks of galaxies. It is therefore extremely suggestive to find that the envelope of FG Sge—a star which has just shown a dramatic surface enrichment of *s*-process elements—should have an N/O ratio considerably higher than a normal planetary nebula. The *s*-process elements presumably were produced in connection with a helium shell flash and have just recently reached the surface. Any connection between the mixing of these elements to the surface of the remnant and the mixing of products of hydrogen burning during the pre-planetary nebula red-giant phase is obscure, since the phase of nitrogen enrichment would necessarily have occurred more than 5000 years ago. It is nevertheless tempting to regard FG Sge as an object which, for whatever reason, is especially able to mix the products of its interior nuclear reactions to its surface.

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