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GAMMA RAYS FROM PRIMORDIAL BLACK HOLES*

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ABSTRACT

This paper examines the possibilities of detecting hard γ-rays produced by the quantum-mechanical decay of small black holes created by inhomogeneities in the early universe. Observations of the isotropic γ-ray background around 100 MeV place an upper limit of 10^4 pc^{-3} on the average number density of primordial black holes with initial masses around 10^{15} g. The local number density could be greater than this by a factor of up to 10^6 if the black holes were clustered in the halos of galaxies. The best prospect for detecting a primordial black hole seems to be to look for the burst of hard γ-rays that would be expected in the final stages of the evaporation of the black hole. Such observations would be a great confirmation of general relativity and quantum theory and would provide information about the early universe and about strong-interaction physics.

Subject headings: gamma rays: bursts — stars: black holes

I. INTRODUCTION

The aim of this paper is to discuss the possibilities of detecting high-energy γ-rays produced by the quantum-mechanical decay of small black holes created in the early universe. Recently it has been shown (Hawking 1974, 1975*a*, *b*; Wald 1975; Parker 1975; DeWitt 1975) that the strong gravitational fields around black holes cause particle creation and that the black holes emit all species of particles thermally with a temperature of about $1.2 \times 10^{26} M^{-1}$ K, where M is the mass in grams of the black hole. One can think of this emission as arising from the spontaneous creation of pairs of particles near the event horizon of the black hole. One particle, having a positive energy, can escape to infinity. The other particle has negative energy and has to tunnel through the horizon into the black hole where there are particle states with negative energy with respect to infinity. Equivalently, one can regard the particles as coming from the singularity inside the black hole and tunneling out through the event horizon to infinity (Hartle and Hawking 1975). As black holes emit particles, they lose mass and so will evaporate completely and disappear in a time of the order of $10^{-26} M^3$ s (Page 1976). (For $M < 10^{14}$ g this lifetime may be shortened by strong interaction effects discussed in § III.)

It would be practically impossible to detect particle emission from black holes of stellar mass because the temperature would be less than 10^{-7} K. One does not know of any process that could produce black holes in the present epoch with mass substantially less than a stellar mass and therefore with higher temperatures. However, one would expect that small black holes would have been created in the early universe if at these epochs the universe was chaotic or had a soft equation of state (Hawking 1971; Carr and Hawking 1974; Carr 1976). Such black holes will be referred to as primordial. If their original mass was less than $M_* \approx 5 \times 10^{14}$ g (Page 1976), they would have completely evaporated by now. Primordial black holes of slightly greater initial mass would by now have decayed to a mass of around 5×10^{14} g and would have a temperature of about 2.5×10^{11} K $= 20$ MeV. Calculations by Page (1976) indicate that such a black hole would radiate energy at the rate of 2.5×10^{17} ergs s^{-1} of which 1 percent is in gravitons, 45 percent is in neutrinos, 45 percent is in electrons and positrons, and 9 percent is in photons. (At this temperature there will also be some emission of muons and pions which is not included in the energy rate above.) It would be very difficult to detect the gravitons or neutrinos because they have such small interaction cross sections. The charged particles would be deflected by magnetic fields and so would not propagate freely to Earth. On the other hand, the photons, whose number spectrum would be peaked at about 120 MeV, could reach us from anywhere in the observable universe. There are three possibilities for detecting these photons.

1. One could look in the isotropic γ-ray background for the integrated emission of all the primordial black holes in the universe. As shown in § II, a uniform distribution of primordial black holes would give a background

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number spectrum of γ-rays with a logarithmic slope of -3 above 120 MeV. Below 120 MeV the spectrum may be flatter, depending on the slope of the number spectrum of black holes. Observations of background γ-rays (Fichtel *et al.* 1975) show no indication of a break in the spectrum at 120 MeV. This puts an upper limit of about 3×10^{-52} cm^{-3} or about 10^4 pc^{-3} on $dn/d \ln M$ at M_*, where n is the original number per comoving volume of primordial black holes with original masses less than M. (Similar upper limits have been placed by Chapline 1974 and Carr 1976). The upper limit on the local number density might be increased by a factor of up to 10^6 if the black holes were clustered in the halos of galaxies rather than being uniformly distributed throughout the universe.

2. One might hope to detect the steady emission from a primordial black hole sufficiently near the Earth. However, the upper limit from the γ-ray background indicates that the nearest primordial black hole is probably not closer than 10^{15} cm, about $1\frac{1}{2}$ times the distance to Pluto. To obtain a counting rate of one photon per thousand seconds would require a detector with directional resolution (to overcome background) and an effective area of at least 10^8 cm^2.

3. As the black hole loses mass, its temperature will rise and the black hole will begin emitting particles of higher rest mass. In the statistical bootstrap (Hagedorn 1973; Frautschi 1971) or dual resonance models of strong interactions (Huang and Weinberg 1970) the number of species of particles rises exponentially with mass. This might cause a black hole to emit all its remaining mass in a very short time when it got down to a mass of about $M_H = 6.6 \times 10^{13}$ g corresponding to the Hagedorn limiting temperature of about 160 MeV. The heavy hadrons emitted by the hole would decay rapidly, and one might expect about 10–30 percent of their energy or about 10^{34} ergs to emerge as a short burst of hard γ-rays between 100 and 1000 MeV. These bursts cannot be connected with those reported by Klebesadel, Strong, and Olson (1973) which were very soft ($\sim$150 keV). If the number density of primordial black holes were near the upper limit set by the γ-ray background, one would expect one burst per month within a distance of 200 pc if the black holes were uniformly distributed or about 2 pc if they were clustered in the halos of galaxies. To detect such a burst, one would need a detector with an effective area of greater than 4×10^5 cm^2 in the former case and 40 cm^2 in the latter. The burst could be distinguished from the γ-ray background by the arrival of several photons in a short period of time or from within a small solid angle. Even if black holes do not explode at M_H, they will have a very rapid final burst of emission when their mass gets down to some value M_B between M_H and 10^{10} g. In this case one would expect the number of photons to be reduced by a factor q^{-2} and the energy of these photons to increase by a factor q where $q = M_H/M_B$. To observe such bursts would require detectors with areas q^2 times the areas given above.

A definite observation of γ-rays from a primordial black hole would be a tremendous vindication of general relativity and quantum theory and would give us important information about the early universe and strong interactions at high energies, information that probably could not be obtained in any other way. On the other hand, negative observations which placed a strong upper limit on the density of primordial black holes would also give us valuable information because they would indicate that the early universe was probably nearly homogeneous and isotropic with a hard equation of state. The best experimental prospect would seem to be to look for bursts using large-area wide-angle detectors with either good time resolution or good angular resolution. Such detectors could be flown on constant-pressure balloons or on the space shuttle. If the particles and photons from the burst were of sufficiently high energy, it might be possible to detect them from the ground either by air showers or by Cerenkov radiation in the upper atmosphere.

In § II we compute the background γ-ray spectrum that would be produced by a uniform distribution of primordial black holes with a power-law spectrum of masses. In § III we consider the final burst of emission on the basis of various theories of strong interactions. Where convenient, we use dimensionless units in which $G = c = \hbar = k = 1$.

II. THE GAMMA-RAY BACKGROUND

In this section we shall calculate the present number flux $dJ/d\omega_0$ of gamma rays of frequency ω_0 from primordial black holes. (Henceforth we shall use the abbreviation PBH.) To do this, we must integrate the contributions over the cosmological time t and at each t integrate over all PBH masses M the emission at the blueshifted angular frequency

$$\omega = (1 + Z)\omega_0 = (R_0/R)\omega_0 \,, \tag{1}$$

where R is the expansion parameter of the universe at time t, and the subscript 0 denotes the value of a quantity at the present epoch. The interactions of the γ-rays with the other matter of the universe will be taken into account by putting in a factor $e^{-\tau}$ for the probability of a photon's propagating without energy loss through absorption optical depth τ from t to t_0, but the effect of the absorbed radiation will not be considered.

Consider a uniform distribution of PBHs created shortly after $t = 0$ in a nearly Friedmann universe. Let $n(M_i)$ denote the original number per comoving volume of PBHs with original masses less than M_i. One can express n as

$$n(M_i) \equiv \mathcal{N} \int_0^{M_i/M_*} s(y) dy \,, \tag{2}$$

where s is a dimensionless function with $s(1) = 1$ and M_* is the original mass of a PBH that would just have evaporated by the present time t_0. We shall assume that, apart from statistical fluctuations, the PBHs are uncharged and nonrotating. Any charge would be rapidly neutralized by the preferential emission of electrons or positrons (Carter 1974; Gibbons 1975). PBHs would also lose angular momentum, but only slowly. One would not expect them to be formed with large angular momenta.

A PBH will emit photons at a rate

$$f(x) \equiv \frac{dN\gamma}{dtd\omega} = \frac{1}{2\pi}\sum_{l,m,p}\frac{\Gamma_{lmp}(x)}{e^{8\pi x}-1}\,. \tag{3}$$

Here $\Gamma_{lmp}(x)$ is the absorption probability for photons of total angular momentum l, axial angular momentum m, polarization or helicity p, and frequency $\omega = M^{-1}x$ where $M = M(M_i, t)$ is the mass to which a PBH of original mass M_i has been reduced by time t.

In terms of these functions the specific number flux today of γ-rays from PBHs is

$$\frac{dJ}{d\omega_0} \equiv \frac{d(\text{number of photons})}{d(\text{area})d(\text{time})d(\text{solid angle})d\omega_0} = \frac{\mathcal{N}}{4\pi}\int_0^{t_0} dt(1+Z)e^{-\tau}\int dys(y)f(x)\,. \tag{4}$$

Here $y \equiv M_i/M_*$ is to be integrated over all values of the initial mass of PBHs that do not disappear by time t, and x is the value of $M\omega$ at that t and y.

To calculate $dJ/d\omega_0$, one needs a specific model for $M(M_i, t)$, $s(y)$, $R(t)$, and $\tau(\omega_0, t)$, as well as the numerical results for $f(x)$ (Page 1976). As long as a PBH emits predominantly a fixed number of particle species at ultrarelativistic energies (i.e., with negligible effects from the rest mass),

$$dM/dt \approx -\alpha/M^2\,. \tag{5}$$

Page (1976) showed that for an uncharged, nonrotating hole emitting only known particles, $\alpha = 2.011 \times 10^{-4}$ for $M \gg 10^{17}$ g (emitting massless particles only), and $\alpha = 3.6 \times 10^{-4}$ for $5 \times 10^{14}\text{ g} \ll M \ll 10^{17}$ g (emitting predominantly massless particles and ultrarelativistic relectons and positrons). This implies that

$$M_* = (3\alpha t_0)^{1/3} \approx 2.1 \times 10^{19} = 5 \times 10^{14}\text{ g}\,. \tag{6}$$

Since the important part of the spectrum comes from $M \approx M_*$, and since α is not known for $M \lesssim M_*$ anyway, we shall take $\alpha = 3.6 \times 10^{-4}$. Then our model for the mass evolution is

$$M = (M_i^3 - 3\alpha t)^{1/3} = M_*(y^3 - t/t_0)^{1/3}\,. \tag{7}$$

If we use this expression to solve for y in terms of x at some t, we find that

$$\frac{dJ}{d\omega_0} = \frac{\mathcal{N}}{4\pi M_*^3}\omega_0^{-3}\int_0^{t_0} dtr^2e^{-\tau}\int_0^\infty dxx^2f(x)\left(\frac{t}{t_0}+\frac{r^3x^3}{M_*^3\omega_0^3}\right)^{-2/3}s\left[\left(\frac{t}{t_0}+\frac{r^3x^3}{M_*^3\omega_0^3}\right)^{1/3}\right], \tag{8}$$

where we have introduced $r \equiv R/R_0 = (1+Z)^{-1}$, a function of t.

One can see from this formula that if $\omega_0 \gg (t_0/t)^{1/3}rx/M_*$ and if $e^{-\tau}$ is insensitive to ω_0 over the dominant part of the integral (generally $t \sim t_0$, $r \sim 1$, $x \sim 0.2$, and $e^{-\tau} \sim 1$, so $\omega_0 \gg 0.2M_*^{-1} \approx 120$ MeV), then the integral has no dependence upon ω_0 and $dJ/d\omega_0$ is proportional to ω_0^{-3}, independent of the form of $s(y)$, $R(t)$, and $\tau(\omega_0, t)$ except for pathological cases. For small values of ω_0 the integral is cut off by redshift and opacity factors. This means that the function $s(y)$, which determines the shape of the initial number spectrum of PBHs, is important only in the region near $y = 1$. We shall assume that in this region it has a power-law form:

$$s(y) = y^{-\beta}\,. \tag{9}$$

Such a form for $s(y)$ is supported by the work of Carr (1975), who finds that a certain reasonable class of density fluctuations in the early universe favors PBH formation with a power-law spectrum where the exponent β is related to the ratio γ of pressure to energy density in the early universe by

$$\beta = \frac{2+4\gamma}{1+\gamma}\,. \tag{10}$$

(A very soft equation of state with $\gamma = 0$ gives $\beta = 2$; a noninteracting relativistic gas with $\gamma = \frac{1}{3}$ gives $\beta = 2.5$; and a stiff equation of state with $\gamma = 1$ gives $\beta = 3$.)

As a model for $R(t)$, we shall take a standard Friedmann model with noninteracting dust and radiation obeying Einstein's field equations with cosmological constant $\Lambda = 0$. Such a model may be labeled by the Hubble constant H_0 to set the scale and by two dimensionless parameters to determine the matter and radiation content:

$$\Omega_m = \frac{8\pi\rho_{\text{matter}}}{3H_0^2}\,, \qquad \Omega_r = \frac{8\pi\rho_{\text{radiation}}}{3H_0^2}\,, \tag{11}$$

where the densities are measured at the present epoch. We take H_0 to be 60 km s^{-1} Mpc^{-1}. We take Ω_r to be 0.0001 on the basis of observations of the microwave background and the assumption that nondegenerate electron and muon neutrinos were in thermal equilibrium with photons in the early stages of the universe. The value of Ω_m is not well known. A lower limit seems to be 0.0013, and Gott *et al.* (1974) suggest that 0.06 is the most probable value but the observations do not completely rule out $\Omega_m \geq 1$. In fact, the value of Ω_m makes very little difference to the predicted γ-ray spectrum except below about 10 MeV where it is strongly influenced by the opacity in the universe at redshifts $Z \gtrsim 100$. This opacity arises mainly from pair production caused by high-energy γ-rays striking neutral hydrogen or helium atoms. We have used the cross sections derived by Bethe and Heitler (1934) with corrections by Wheeler and Lamb (1939) for hydrogen and by Knasel (1968) for helium. They find that the total cross section for pair production in hydrogen is 0.0124 cm^2 g^{-1} and in helium is 0.0083 cm^2 g^{-1} independent of the γ-ray energy provided it is above about 100 MeV. A primordial abundance of 70 percent hydrogen and 30 percent helium by mass was assumed (cf. Danziger 1970), making the opacity 0.0112 cm^2 g^{-1} at high energies, and a crude correction for lower energies was made.

Figure 1 shows the predicted background γ-ray spectrum from primordial black holes for $\Omega_r = 0.0001$ and $\Omega_m = 0.06$ and for various values of the exponent β in the initial number spectrum of the primordial black holes. Figure 2 shows the spectra with $\Omega_m = 1$ (approximately a $k = 0$ cosmology). As expected, the curves all agree more or less above 120 MeV and have a logarithmic slope of -3. Below 120 MeV the curves differ for different values of β, but they all flatten and turn over at about 10 MeV. All the curves can be moved up or down by

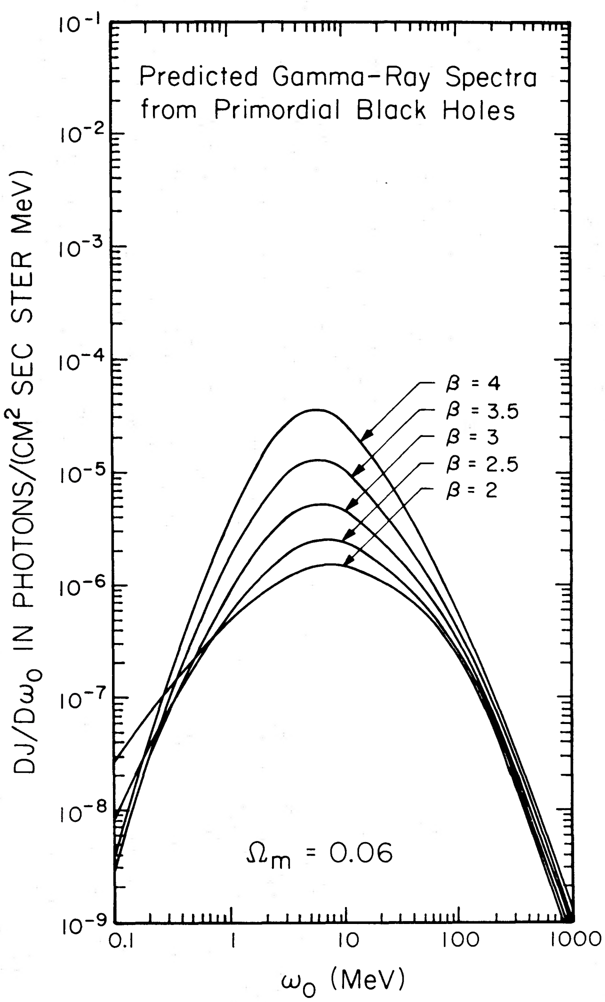

FIG. 1

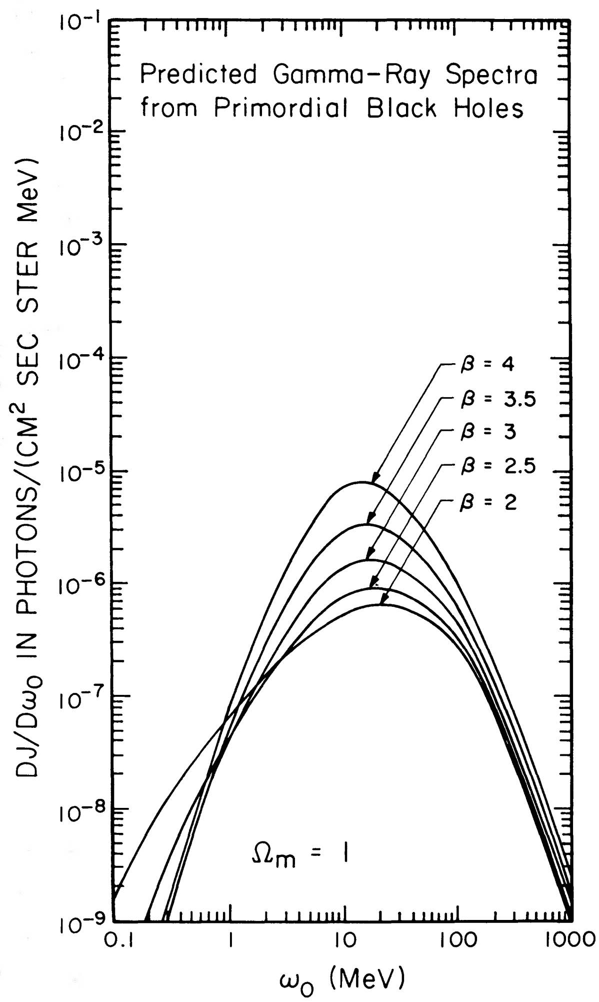

FIG. 2

FIGS. 1 and 2.—Predicted number spectra $dJ/d\omega_0$ of γ-rays from primordial black holes having initial number spectra $dn/d(M_i/M_*) = 10^4\ pc^{-3}(M_i/M_*)^{-\beta}$ for initial masses M_i around $M_* \simeq 5 \times 10^{14}$ g, where $\boldsymbol{\beta}$ is given values from 2 (*bottom curve*) to 4 (*top curve*) in steps of $\frac{1}{2}$. Fig. 1 assumes the present matter density is 0.06 of the critical value for closure of the universe; Fig. 2 assumes it is at the critical value.

adopting different values of the constant $\mathcal{N}$ that multiplies the factor $s(y)$ in the initial PBH number spectrum $M_* dn/dM$. In Figures 1 and 2 the value of $\mathcal{N}$ was chosen as 1×10^4 pc^{-3} to be consistent with the upper limit set by the observations which are shown in Figure 3. These seem to fit roughly a power-law spectrum with exponent -2.4 from about 0.3 MeV to 200 MeV. There is no evidence of a break in the spectrum at 120 MeV but the observations in this region, which were by Fichtel *et al.* (1975), were statistical in nature and were fitted to an assumed power-law spectrum. Nevertheless, it is clear that $dn/d \ln M$ at $M = M_*$ cannot be greater than 10^4 pc^{-3} and that this is only an upper limit.

The considerations above have been based on the assumption of a uniform distribution of primordial black holes throughout the universe. The observed matter in the universe, however, is strongly concentrated in galaxies and possibly in halos. Any initial velocity with which primordial black holes were created would have been reduced almost to zero by the expansion of the universe. Thus one might expect that they would be concentrated in the gravitational potential wells of galaxies. Unlike the gas, they would encounter very little friction in passing through the plane of the galaxy and so they would be distributed throughout the halo. If we assume that the primordial black holes are concentrated in halos of the order of 40 kpc around each galaxy with an r^{-2} density distribution instead of being spread uniformly, the upper limit on the number density of PBHs averaged over the whole universe would be about the same but the local density would be about a factor of 10^6 greater.

III. BURSTS

In the calculations of Page (1976) it was assumed that the emitted particles interacted only with the gravitational field and not with each other. This should be a good approximation for the emission of gravitons, photons, and leptons. It will break down when the mass of the black hole falls below about 2×10^{14} g corresponding to a

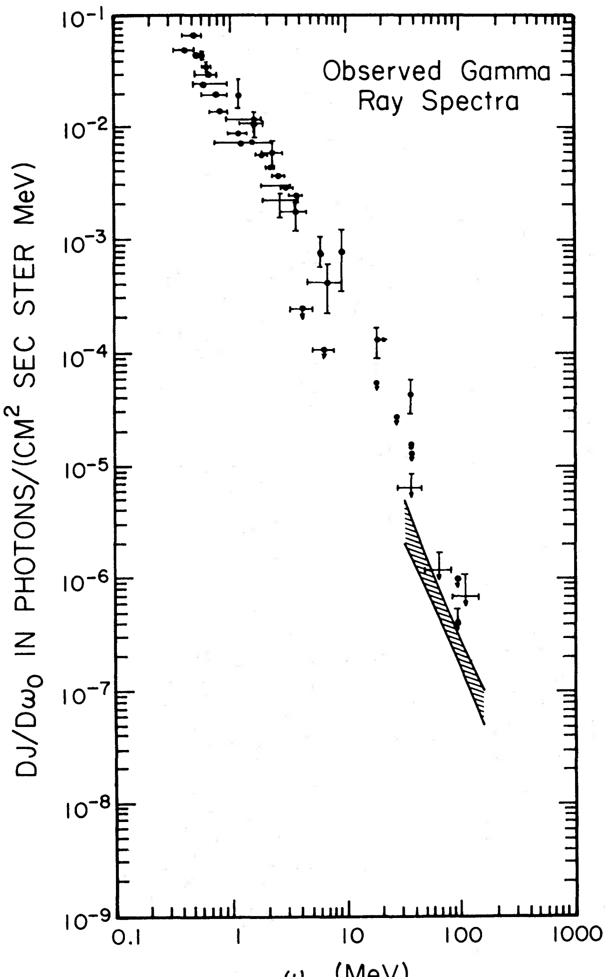

FIG. 3.—Observed diffuse γ-ray spectrum as reported in Fichtel *et al.* (1975). The shaded region represents their SAS-2 measurements and uncertainties; the other points represent previous measurements they enumerate and reference.

temperature of about 50 MeV at which pions, the lightest hadrons, would begin to be emitted in significant numbers. Although the present field theory derivations of particle creation by black holes break down when strong interactions become important, there are thermodynamic and statistical arguments which indicate that a black hole would still emit thermal radiation with a temperature related to the mass in the same way as before (Hawking 1975*c*). The problem is to calculate what thermal radiation consists of in the presence of strong interactions and how it decays as it moves away from the black hole. At present there is little experimental knowledge on either of these questions, so one has to resort to theoretical models. Possibly the simplest of such models is the statistical bootstrap theory (Hagedorn 1965; Frautschi 1971; Hagedorn 1973). In this approach one considers the eigenstates of the strongly interacting fields contained within some box of volume V. Let $\sigma(E, V)$ be the number of eigenstates with energy between E and $E + dE$. One can define a quantity $\rho(m, V)$ such that $\sigma(E, V)$ is equal to the number of eigenstates of a system of noninteracting particles with mass spectrum $\rho(m, V)$ and total energy between E and $E + dE$ contained in a box of volume V. One regards $\rho(m, V)$ as representing the spectrum of resonances in the strongly interacting fields. If one neglects long-range gravitational and electromagnetic fields, one might expect that $\rho(m, V)$ would be independent of V for V greater than a hadron volume $V_h \approx 10^{-39}$ cm^3 because the strong interactions have a range of order 10^{-13} cm. One then makes the bootstrap assumption that the density of energy levels in a volume V_h is just given by this mass spectrum, i.e.,

$$\rho(E) = \sigma(E, V_h)\,. \tag{12}$$

This gives an effective mass spectrum of the form

$$\rho(m) = am^{-b} \exp{(m/c)} \tag{13}$$

where $5/2 \leq b \leq 7/2$ and $c \approx 160$ MeV.

Similar mass spectra are obtained from dual-resonance models of strong interactions (Fubini and Veneziano 1969; Huang and Weinberg 1970).

If one regards this mass spectrum as representing different species of noninteracting particles all of which a black hole would emit thermally like point particles, the rate of energy emission would become infinite when the black hole got down to a mass $M_{\rm H}$ of about 7×10^{13} g corresponding to a temperature of about 160 MeV because the Boltzmann factor $\exp{(-E/T)}$ would be canceled out by the exponential in the mass spectrum. The black hole would convert itself into a fireball of very heavy hadrons. In the conventional statistical bootstrap theory, which neglects gravitational interactions, these heavy hadrons would decay slowly with lifetimes of the order of 10^{13} s (Carlitz, Frautschi, and Nahm 1973). However, gravitational interactions between the hadrons would be significant compared to thermal energies for particle masses above 10^{-5} g. They would increase the rate of collisions between such heavy hadrons and hence, by detailed balance, the rate at which they can emit lighter hadrons and decay. Thus the fireball could probably be treated as a pressureless fluid which maintained itself in thermal equilibrium at a temperature of about 160 MeV as it expanded with parabolic velocity (cf. Carter *et al.* 1975). One would expect the fireball to radiate electrons, positrons, muons, photons, and perhaps neutrinos thermally from its surface. It would radiate away all its energy in a time of about 10^{-7} s, giving a burst of γ-rays peaked around 250 MeV with total energy about 10^{34} ergs.

This picture can be criticized on the ground that, even if there were in some sense an exponential mass spectrum of hadrons, they would be of the same size as the black hole or larger and thus would not be emitted as point particles. One might regard hadrons as composite bodies made up from quarks and gluons which are point particles and which are asymptotically free at small distances but are strongly bound at separations greater than 10^{-13} cm (Gross and Wilczek 1973; Politzer 1973, 1974). In this case it might be that black holes smaller than 10^{-14} cm or 10^{14} g would emit individual quarks and gluons as noninteracting point particles. When they had traveled a distance of about 10^{-13} cm from the black hole, they would feel the interaction with other quarks and gluons and would join up with them to form hadrons which would then decay into lighter particles. The rate of energy emission would be about $10^{46}\eta(M/g)^{-2}$ ergs s^{-1}, where η is number of species of quarks, gluons, leptons, photons, and gravitons with rest mass less than the black-hole temperature. In the original quark-gluon theory (Fritzch and Gell-Mann 1972) there were 18 species of quarks (three flavors, three colors, and their antiparticles) and eight species of gluons. Thus η would be 36. About 1.5 percent of the rest-mass energy of the black hole would be emitted directly in high-energy photons, and further photons would arise from the decay of highly relativistic hadrons. One might therefore expect that between 10 and 30 percent of the rest-mass energy of the black hole would emerge as photons at around $500\,(M/10^{14}\text{ g})^{-1}$ MeV. The emission would become very rapid when the mass of the black hole got down to 10^{10} g, giving a burst of about 10^{30} photons at around 5×10^6 MeV.

The recent discovery of the J or Ψ particles (Aubert *et al.* 1974; Augustin *et al.* 1974) suggests that there may be a fourth flavor of quark with a rather higher mass. It also seems that it may be necessary to postulate a fifth and a sixth flavor to explain the electron-positron annihilation cross section into hadrons. It is therefore possible that there is an infinite sequence of quarks with higher masses. These higher-mass quarks would increase the rate of energy loss of a black hole hot enough to emit them. The final burst of very rapid emission could therefore come at some mass between the Hagedorn mass $M_{\rm H}$ and 10^{10} g. In this case one would expect the number of photons to be q^{-2} times the number in the statistical bootstrap and picture the energy of each photon to be q times greater, where q is the ratio of the Hagedorn mass to the mass at which the burst occurs.

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