THE ASTROPHYSICAL JOURNAL, **205**: 346–353, 1976 April 15

RADIAL VELOCITIES AND MASSES OF GALAXIES IN GROUPS FROM 21-CENTIMETER LINE OBSERVATIONS

HERBERT J. ROOD
Department of Astronomy and Astrophysics, Michigan State University
AND
JOHN R. DICKEL
Department of Astronomy, University of Illinois

Received 1975 August 4; revised 1975 September 22

ABSTRACT

New 21-cm radial velocities are given for 50 galaxies. The mean error of a determination is typically 9 km s^{-1}. Conventional Bottlinger-Lohman total masses are derived for these galaxies from the width of 21-cm line profiles.

Comparison of new 21-cm radial velocities with values tabulated in the Reference Catalogue of Bright Galaxies (BGC) indicates that, except for an occasional anomalous value, the BGC velocities have accuracies well described by their listed mean errors.

The 21-cm velocities and galactic masses are used to estimate directly $M_{\rm vt}$ (virial mass) and M (sum of the conventional masses of component galaxies) for six de Vaucouleurs groups. Groups 5, 6, 9, 12, 33, and 51 have $M_{\rm vt}/M$ = 15, 44, 3, 28, 75, and 590, respectively. The M101 and M51 groups, which together comprise group 5, each have $M_{\rm vt}/M = 1$.

Subject headings: galaxies: clusters of — radio sources: 21-cm radiation

I. INTRODUCTION

If a group of galaxies contains only a small amount of intergalactic matter, then its mass M is the sum of the masses of its component galaxies,

$$M = \sum_i m_i \,. \tag{1}$$

If the group is (i) isolated in space and (ii) violently relaxed with (iii) negative total energy, if (iv) its galaxies are small compared to their average separation, (v) its instantaneous total kinetic and gravitational potential energies are equal to the values averaged over a characteristic free-fall time, and (vi) the position and velocity vectors relative to the center of mass are randomly oriented, then the mass of the group can be estimated through the virial theorem by

$$M_{\rm vt} = V^2 R/G \,, \tag{2}$$

$$V^2 = 3 \sum_i m_i \Delta v_i^2/M \,, \tag{3}$$

$$R = \pi M^2 \Big/ \left(\sum_i \sum_j m_i m_j / r_{ij} \right) , \tag{4}$$

where r_{ij} is the distance between two galaxies projected onto the celestial sphere, and Δv_i is the line-of-sight (radial) velocity of a galaxy relative to the center of mass of the group,

$$\Delta v_i = v_i - \sum_j m_j v_j / M \,, \tag{5}$$

where v_i is the radial velocity relative to the Local Group.

When the above equations are evaluated with available observational data for nearby loose groups, compiled primarily by de Vaucouleurs (1968), it is found that $M_{\rm vt}$ is often larger than M by factors which frequently exceed 10 (Rood, Rothman, and Turnrose 1970, herein called RRT). This suggests that one or more of the above assumptions and/or unstated assumptions are not valid, which indicates the need for further investigation.

RRT did not correct Δv_i^2 for uncertainties in the observed radial velocities, which they believed to be of order 50 km s^{-1}. Moreover, they were forced to estimate the mass m_i of each galaxy indirectly from its absolute luminosity and the average ratios of mass to luminosity for spiral and elliptical galaxies. The latter values were based on direct mass estimates from rotation-curve and other position-velocity data for a small number of very nearby galaxies.

Two improvements to the above analysis were forthcoming. For de Vaucouleurs's group 7, Materne (1974) applied relations which rigorously correct Δv_i^2 for the mean error in a radial velocity. For the M101 group, Sandage and Tammann (1974*b*) used direct determinations of the masses m_i to evaluate $M_{\rm vt}$. The velocity correction procedure was extended to 13 more groups and the direct mass measurements were extended to two more groups by Materne and Tammann (1975) (also see Chincarini, Rood, and Welch 1975).

The generally large values for the ratio $M_{\rm vt}/M$ were substantiated by Geller and Peebles (1973), who developed a new statistical method of evaluating the virial theorem for a collection of groups to give the average value $\langle M_{\rm vt}\rangle/\langle M\rangle$ for the groups without requiring a knowledge of whether a particular galaxy belongs to a particular group. Moreover, their method automatically discards any galaxy with a radial velocity discordantly different from the values of group members because (i) the galaxy is superposed on a group or (ii) the cataloged value for the radial velocity is in error. The result, $\langle M_{\rm vt}\rangle/\langle M\rangle \approx 30$, is sensitive to the assumption that the mean errors of radial velocity determinations listed in the literature are approximately correct. If the mean errors were greatly underestimated, then the derived value of $\langle M_{\rm vt}\rangle/\langle M\rangle$ would be much too large.

Observations of the 21-cm neutral hydrogen line of galaxies in groups are uniquely suited for further exploration of the above problems. The center of the line profile can provide an accurate radial velocity of each galaxy. The results can be used to check the accuracy of estimates of mean errors in the literature (Ford, Rubin, and Roberts 1971). Moreover, they can also yield accurate estimates of Δv_i. Finally, the full width of the line profile provides an estimate of the line-of-sight component of the maximum rotational velocity of a galaxy, which can be used to estimate its mass directly (see Roberts 1969 and references therein). Hence, accurate estimates of M and $M_{\rm vt}$ for groups are feasible.

For the present study, we have searched for 21-cm emission in 65 galaxies located primarily in de Vaucouleurs (1968) groups; 50 galaxies were detected. In § II, the new 21-cm radial velocities are compared with optical values in the literature. Conventional Bottlinger-Lohman masses of galaxies are estimated in § III. The new velocities and masses of the galaxies are then used to derive M and $M_{\rm vt}$ for six groups (§ IV). The results are discussed in § V. It is found that, except for a small fraction of cases, the error estimates for radial velocities given in the literature are reliable. The de Vaucouleurs groups are generally found to have large values of $M_{\rm vt}/M$, in qualitative agreement with the results of RRT and Geller and Peebles (1973).

II. RADIAL VELOCITIES

The 21-cm data were obtained with the 300 foot (91 meter) radio telescope at the National Radio Astronomy Observatory. Each galaxy was observed during its transit of the meridian at the position given in the *Reference Catalogue of Bright Galaxies* (BGC, de Vaucouleurs and de Vaucouleurs 1964). Galaxies with angular sizes significantly larger than the beamwidth (10′ at half-power) were also observed at positions north and south of the BGC position. The normal observing procedure was to use an autocorrelation spectrometer of 192 channels each with a width of 6.6 km s^{-1} but separated by 5.5 km s^{-1} to cover a velocity range of 1964 km s^{-1} centered on the known or estimated (from the average velocity of group members) optical radial velocity of each galaxy. A small number of observations were obtained with a channel width of 13.2 km s^{-1} and a range of 2128 km s^{-1}. Further details of the observational procedure are given by Dickel and Rood (1975).

The results are presented in Table 1. Column (1) gives the group designation by de Vaucouleurs (1968). Group 5 is considered by Sandage and Tammann (1974*b*, 1975) to actually constitute two groups: the M101 group and the M51 group.

Columns (2), (3), and (4) contain NGC number, morphological type, and $B(0)$ magnitude from the BGC. A colon following a magnitude means that $B(0)$ has been estimated from a Harvard magnitude.

Column (5) contains the observed 21-cm radial velocity of the galaxy relative to the Sun, defined according to the practice in optical astronomy,

$$V_{21} = c\frac{\lambda - \lambda_0}{\lambda_0} = c\frac{\nu_0 - \nu}{\nu}\,. \qquad (6)$$

This velocity is measured at the midpoint between one-fourth peak-intensity points of the 21-cm temperature versus radial velocity profile. With the resolution available, the major uncertainty in all the velocity measurements discussed herein is created by noise on the records which has three distinct causes: (1) *Receiver noise*—we summed the signals from two identical receivers each with a noise temperature of 50 K and an integration time of $4^{\rm m}$ sec δ, where δ is the declination of the galaxy, which then gave an rms noise on the output record of less than 0.02 K antenna temperature. (2) *Spurious reflections* between various members of the telescope which can often cause ripples in the baseline whose amplitude depends upon the observed central frequency. (3) *Confusing sources* within the 10′ beam of the telescope. Repeated observations of the weak source NGC 779 indicate that the mean error of a radial velocity determination is typically 9 km s^{-1} (see Dickel and Rood 1975 for further details). Radial velocities which are less certain than the typical 9 km s^{-1} are indicated by a colon and explained in the notes in column (13). For the six galaxies common to our study and a recent investigation by Shostak (1975), the mean absolute difference $\langle |V_{21}(\text{Shostak}) - V_{21}(\text{here})|\rangle$ is 8 km s^{-1}.

Column (6) contains the 21-cm radial velocity relative to the Local Group. The conversion from column (5) to column (6) was performed following the BGC.

Column (7) contains the difference between a radial velocity listed in the BGC or de Vaucouleurs and de Vaucouleurs (1967) and the 21-cm value.

Column (9) contains the antenna temperature of a profile averaged over all radial velocities. This parameter is a measure of the noisiness of a profile.

Column (10) lists the half-width of the observed velocity profile corrected for the resolution (usually 6.6 km s^{-1}) which thus is the line-of-sight rotational velocity of the galaxy.

Column (11) contains csc^2 i, where i is the orbital inclination of a galaxy estimated according to the procedures by Roberts (1969). The csc^2 i factor is a factor which enters into the mass determination of a galaxy (§ III).

TABLE 1

New Radial Velocities and Masses of Galaxies

Group	NGC	Type	$B(0)$ (mag)	V_{21} (km s^{-1})	V_{021} (km s^{-1})	$V_{BGC} - V_{21}$ (km s^{-1})	$V_{ST} - V_{21}$ (km s^{-1})	$\langle \Delta T_A \rangle$ (0.01 K)	$\csc^2 i$	V_m (km s^{-1})	M_T ($10^{10} M_\odot$)	Notes
40..........	470	Sb	12.75	2555:	2654	2	...	2.0	1.77	111	2.4	*a*, *n*
40..........	474	S0^0	12.51	2385:	2484	...	...	2.7	5.70	188	26.5:	*a*
40..........	488	Sb	11.41	2268	2372	−88	...	3.0	2.42	231	27.2	*b*
40..........	520	P	12.75	2162	2258	61	...	10.7	1.06	114	2.5	
40..........	521	Sbc	12.75	...	...	...	...	...	...	...	...	*c*
33..........	584	E4	11.71	...	...	...	...	...	...	...	...	
33..........	596	E0	12.31	...	...	...	...	...	...	...	...	
33..........	615	Sb	12.51:	1857	1902	89	...	3.3	1.14	239	8.5	
33..........	720	E5	11.47	...	...	...	...	...	...	...	...	
33..........	779	Sb	12.20	1386	1416	37	...	4.0	1.06	192	5.2	
7..........	891	Sb:/	11.24	531	702	−459	−11	28.5	1.00	250	10.0	
7..........	925	Sd	10.96	551	701	17	19	81.7	1.43	126	3.4	
6..........	2500	SBd	12.39	515	558	−45	−19	23.4	5.70	61	0.85	
6..........	2537	Imp	12.55	443	460	−39	−39	16.4	3.48	59	0.34	
6..........	2541	Scd	12.14	555	588	46	20	45.0	1.43	110	1.5	
....	2545	SBab	13.64	3381:	3273	...	...	1.8	1.71	229	12.8	*n*
6..........	2552	Sm?	12.54:	517	554	...	−6	14.5	1.93	77	0.56	
6..........	2681	S0/a	11.34	...	...	...	...	...	...	...	...	
6..........	2841	Sb	10.27	636	677	−1	4	17.5	1.17	311	13.0	
12..........	3184	Scd	10.59	594	593	−176	−11	53.7	21.3	78	20.1	
47..........	3190	Sap	12.20	...	...	...	...	...	...	...	...	*d*
47..........	3193	E2	12.37	...	...	...	...	...	...	...	...	*d*
12..........	3198	SBc	11.09	663	684	−14	13	44.7	1.19	162	6.0	
47..........	3226	E$^+$2p	12.77	Note e	...	...	...	...	...	...	...	*e*
47..........	3227	Sa	11.75	1284:	1178	...	...	6.7	1.64	120	3.3	*e*
11..........	3239	SBm	12.23	751	632	129	...	33.4	1.43	112	1.7	*f*
12..........	3319	SBcd	11.95	743	749	83	11	26.4	1.40	120	2.9	
11..........	3351	SBd	10.75	776	640	4	19	13.8	1.93	150	5.7	
11..........	3368	Sab	10.32	891	756	44	36	16.7	1.93	185	10.0	
11..........	3377	E5-6	11.75	...	...	...	...	...	...	...	...	*g*
11..........	3379	E+1	10.83	...	...	...	...	...	...	...	...	*h*
43..........	3395	Scdp	12.46	1621:	1587	...	...	15.7	1.60	128	2.7:	*i*
43..........	3430	Sc	12.39	1594:	1561	...	...	12.6	1.43	178	6.9:	*j*
12..........	3432	SBmsp	11.94	615	600	−6	31	38.9	1.00	141	2.6	
11..........	3447	Sdp	...	1071	961	−106	...	8.5	1.43	92	0.89	
9..........	3489	S0+	11.24	...	...	...	...	...	...	...	...	
9..........	3593	S0/a	11.91	627	509	−80	0	3.0	1.09	150	2.2	
49..........	3607	S0^0	11.42	...	...	...	...	...	...	...	...	*k*
49..........	3608	E2	12.31	...	...	...	...	...	...	...	...	*k*
9..........	3623	Sa	10.51	818	703	−63	−63	3.5	1.06	258	10.8	

TABLE 1—*Continued*

Group	NGC	Type	$B(0)$ (mag)	V_{21} (km s^{-1})	V_{021} (km s^{-1})	$V_{BGC} - V_{21}$ (km s^{-1})	$V_{ST} - V_{21}$ (km s^{-1})	$\langle\Delta T_A\rangle$ (0.01 K)	csc$^2 i$	V_m (km s^{-1})	M_T ($10^{10} M_\odot$)	Notes
49..........	3626	S0+	12.11	1537:	1447	−85	...	3.6	1.71	119	2.2	*n*
9..........	3627	Sb	9.89	721	607	−15	−7	9.5	1.23	201	8.8	
9..........	3628	Sbp	10.43	847	735	−5	−5	37.2	1.00	248	14.2	
49..........	3686	Scd	12.24	1157	1065	−135	...	3.1	2.25	98	3.0	*b*
17..........	3813	Sb	12.88:	1464	1469	...	...	9.5	1.24	163	2.5	
....	4062	Sc	12.17	774	769	...	...	5.0	1.17	161	2.1	
17..........	4145	Sd	11.68:	1019	1053	...	...	21.7	2.25	117	4.9	
17..........	4369	Sa	12.84:	1052	1091	...	...	4.4	7.44	104	4.2	
13..........	4670	SBap	13.44	1071	1070	139	39	4.6	1.77	109	0.81	*l*
....	4712	Sbc	13.73	4379	4374	...	...	2.2	1.26	242	20.1	
13..........	4725	Sabp	10.21	1215	1210	−101	−101	17.7	1.93	215	17.6	
....	4747	SBc?	13.22	1197	1194	...	...	11.0	1.06	104	0.83	
Coma Cl....	4966	S	...	...	...	...	...	...	...	...	...	*n*
(M51) 5.....	5055	Sbc	9.52	509	589	11	4	44.1	1.40	208	7.6	
(M51) 5.....	5194	Sbcp	9.03	471	578	...	...	59.8	1.64	110	2.2	*m*
(M51) 5.....	5195	I0p	10.94	465:	573	...	...	32.7	2.58	116	2.4:	*m*
(M101)5.....	5204	Sm	12.00	212	357	...	...	41.3	1.55	74	0.52	*b*
(M101)5.....	5457	Scd	8.58	255	404	11	...	121.1	10.9	127	38.2	
(M101)5.....	5474	Scdp	11.74	289	437	−42	...	79.3	7.43	50	1.05	
(M101)5.....	5585	Sd	11.66	319	482	−15	...	45.8	1.71	92	0.97	
51..........	6217	Sbc	12.17	1370	1600	15	−39	17.9	2.42	127	4.3	
51..........	6340	S0/a	12.21	...	...	...	...	...	...	...	...	
51..........	6412	Sc	12.62	1335	1578	173	73	9.8	3.48	81	2.4	
51..........	6643	Sc	11.97	1501	1754	37	−1	7.8	1.30	174	5.6	
51..........	6654	S0/a	12.80	1821:	2077	103	...	0.6	2.13	168	6.7	*n*

NOTES TO TABLE 1

a) N470 and N474 separated by 6′.

b) A galaxy with $m_p = 15.1$–15.7 mag less than 10′ away.

c) I1694 ($m_p = 14.9$ mag) within 10′ of N521.

d) N3190, N3193, and N3187 (SBpc, $m_p = 13.8$ mag) within 10′ of each other.

e) N3226 and N3227 separated by 2′.

f) Six $m_p = 15.1$–15.7 mag galaxies within 10′ of N3239.

g) Very diffuse $m_p = 15.0$ mag galaxy within 10′ of N3377.

h) N3379, N3384 (S0B, $m_p = 10.0$ mag) and N3389 (Sc, $m_p = 12.0$ mag) within 10′ of each other.

i) N3396 (IBm, $m_p = 12.6$ mag) within 10′ of N3395.

j) N3424 (SBb, $m_p = 13.2$ mag) within 10′ of N3430.

k) N3607, N3608, and N3605 (E, $m_p = 12.7$ mag) within 10′ of each other.

l) N4673 (E, $m_p = 13.7$ mag) within 5′ of N4670.

m) N5194 and N5195 separated by 4′; the galaxies are extended.

n) Strong baseline fluctuations making measurements of the profile uncertain.

1976ApJ...205..346R

Column (12) contains the mass of the galaxy derived in § III. An uncertain value is indicated by a colon.

Column (13) contains notes. The catalog by Zwicky *et al.* (1960–1968) was consulted to detect galaxies with apparent magnitude m_p equal to or less than 15.7 mag located within 10′ of each program galaxy.

From Table 1, we see that not one elliptical galaxy was detected in our survey. Four of eight S0 or S0/a galaxies were detected, although NGC 6654 was only marginally found. Three spirals were not detected: NGC 521 has an unknown velocity; but if it is really in de Vaucouleurs group 40, its radial velocity should have been within 500 km s^{-1} of the predicted value; the baselines were too uncertain for NGC 4966 to put a good limit on its antenna temperature; the record for NGC 3190 is good, and apparently either the 21-cm emission is just too weak for detection or the optical radial velocity is in error. In addition, NGC 470 was only marginally found.

In general, the 21-cm velocities agree well with the optical values tabulated in the BGC or by de Vaucouleurs and de Vaucouleurs (1967) (Table 2). The difference is greater than 200 km s^{-1} for only one galaxy, NGC 891. For this galaxy, the BGC velocity is 72 km s^{-1} and the 21-cm velocity is 531 km s^{-1}, in good agreement with a previous measurement by Materne (1974). The difference is nearly 5 times the estimated optical mean error of 100 km s^{-1}. Discarding NGC 891, the remaining 32 galaxies with accurate 21-cm velocities and optical values given by the BGC or de Vaucouleurs and de Vaucouleurs (1967) show a mean difference, $\langle V_{\rm BGC} - V_{21} \rangle = -0.7$ km s^{-1} (essentially zero) and a mean absolute difference, $\langle |V_{\rm BGC} - V_{21}| \rangle = 59$ km s^{-1}. The comparison of 21-cm velocities with values compiled by Sandage and Tammann (1975) shows similar good agreement (Table 2).

Studies by Roberts (1972) and Lewis (1974) demonstrate that the Lick Observatory radial velocities in the range ~1200 to ~2400 km s^{-1} listed by Humason, Mayall, and Sandage (1956), Mayall *et al.* (1961), and Mayall and de Vaucouleurs (1962) are on the average about 100 km s^{-1} larger than 21-cm radial velocities. Three of our galaxies (NGC 520, NGC 615, and NGC 6412) are in this velocity range and have optical velocities derived from Lick data exclusively. These galaxies have $(V_{\rm BGC} - V_{21}) = 18$, 78, and 180 km s^{-1}, respectively, which are consistent with the Roberts-Lewis result.

III. MASSES OF THE GALAXIES

Brandt (1960) gives a general expression for rotation curves of galaxies defined by the maximum rotational velocity, the radius corresponding to this velocity, and a shape parameter, *n*. A Bottlinger-Lohman rotation curve corresponds to the Brandt curve with $n = 3$ (which reduces to a quasi-Keplerian rotation in the outer parts of a galaxy). This relation well describes our own Galaxy and has been extensively applied in the literature to estimate masses of other galaxies (e.g., Roberts 1969). Using this relation, we find that the total mass of a galaxy is given by

$$M_T = \tfrac{3}{2}\csc^2 i V_m{}^2 R_m d/G\,, \qquad (7)$$

where i is the orbital inclination of the galaxy, V_m is the line-of-sight component of the maximum rotational velocity, R_m is the angular radius of the galaxy corresponding to the location of V_m, d is the distance to the galaxy, and G is the gravitational constant.

By adopting these conventional Bottlinger-Lohman masses, we can readily compare our values with results of previous studies. However, it should be noted that recent results for several galaxies suggest that a Brandt curve with $n = 1$ (flat rotation curve) would give a better fit to some high-resolution data (e.g., see Rogstad and Shostak 1972; Roberts and Rots 1973). Several other galaxies show a descending rotation curve in closer agreement with $n = 3$ (Huchtmeier 1975). Shostak (1975) estimates that the rotation curves found by Rogstad and Shostak (1972) yield masses which are 4 times the Bottlinger-Lohman masses. Hence, there could be large systematic uncertainties in masses derived by the Bottlinger-Lohman relation, and these masses should be regarded as highly provisional. To evaluate equation (7), the necessary parameters have been estimated as follows:

i) The inclination i of each galaxy is derived from the axial ratio given in the BGC by the use of Roberts's (1969) formula (see col. [11] of Table 1).

ii) $2V_m$ is estimated from the full width of the line profile (see col. 10 of Table 1) and is the distance between the two points on the profile which intersect the baseline. No correction has been applied for random motions. If they were 10 km s^{-1}, then our values for V_m would be too large by typically 5–10 percent. If one were to allow for a random motion effect of this size, then our masses would not differ

TABLE 2

COMPARISON OF NEW 21-CENTIMETER RADIAL VELOCITIES WITH RADIAL VELOCITIES IN PUBLISHED CATALOGS

Sample	N	$\langle V_{\rm cat} - V_{21} \rangle$	$\langle \lvert V_{\rm cat} - V_{21} \rvert \rangle$	Derived ⟨mean error⟩	Literature ⟨mean error⟩
A.........	34	−0.7	58.8	58.1	57.1
B..........	23	−1.8	24.3	22.6	24.0

NOTE.—Sample A: BGC + de Vaucouleurs and de Vaucouleurs 1967. Sample B: Sandage and Tammann 1975. Radial velocities in units of km s^{-1}.

systematically from those recently derived by Shostak (1975).

iii) R_m is estimated statistically to be one-sixth of the Holmberg (1958) maximum angular diameter (Roberts 1962; Epstein 1964). When the diameter is not given by Holmberg (1958), it is estimated from the major diameter D in the BGC through a conversion relation by Roberts (1969). For our sample of galaxies, one-sixth of the Holmberg radius is always less than 5′ and usually considerably less, so the maximum of the rotation curve lies within the antenna beam. Although large H I holes are known to exist in the centers of several spiral galaxies, R_m always appears to lie outside the hole in a region where there is sufficient hydrogen to evaluate V_m (e.g., see Roberts and Rots 1973; Huchtmeier 1973).

iv) The distance d of a galaxy is estimated from the distance to its group (de Vaucouleurs 1968) when available, and otherwise from the measured radial velocities and a Hubble constant $H = 100\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$. Group distances were given preference over redshift distances because the latter could be affected by virial motions and other deviations from the Hubble flow. The adopted distances in megaparsecs to groups 5, 6, 7, 9, 11, 12, 13, 17, 33, 40, 43, 47, 49, and 51 are 4.6, 6.0, 6.3, 7.6, 8.3, 9.6, 11.5, 14.5, 16, 16, 16.5, 16.5, and 16.5, respectively. The resultant masses should be divided by 0.55 to convert them to values based upon $H = 55\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ (Sandage and Tammann 1974*a*), but in any case the desired ratio $M_{\rm vt}/M$ is independent of distance.

Sixteen of the galaxies in our sample with newly derived masses have previous mass estimates by Roberts (1969). The average difference is $\langle \log M_T(\text{here}) - \log M_T(\text{Roberts 1969})\rangle = -0.043$, and the average absolute difference is $\langle |\log M_T(\text{here}) - \log M_T(\text{Roberts 1969})|\rangle = 0.281$. Hence, there is no systematic difference, and the average mean error corresponds to a factor of 2 in the ratio $M_T(\text{here})/M_T(\text{Roberts 1969})$. This mean error is comparable to that between previous 21-cm masses, and between previous optical and 21-cm masses (Roberts 1969). The reduction procedures and estimates of i, R_m, and d (eq. [7]) for our masses and for Roberts's (1969) masses are identical, so the remaining uncertainty is in the value for V_m which rests on the determination of the full profile width. As stated above, for the present data, this measurement is affected primarily by noise on the records and by baseline uncertainties. For a typical evaluation of the noise contribution, our determination of the profile width of the weak source NGC 779 from four independent records has a fractional mean error about the average value of 3 percent, which corresponds to a fractional mean error of 6 percent in the mass estimate. Therefore, we believe that previous profile widths, which are a basis for Roberts's (1969) mass estimates, are uncertain by a factor of about $\sqrt{2}$. The accuracy of these earlier data was probably seriously affected by the less sensitive equipment and smaller number of channels in the filter spectrometers available at that time.

We have used the masses based on equation (7) for all the observed galaxies in this paper. For the irregular galaxies, this method can be compared with a virial theorem method which assumes that the motions are random (Volders and Högbom 1961). For three galaxies in our sample, NGC 2537 (Imp), NGC 5195 (I0p), and NGC 520 (P), this method yields values of 0.41, 3.0, and $4.9 \times 10^{10}\ M_\odot$, respectively; these masses are within a factor of 2 of corresponding masses based on equation (7).

IV. MASSES OF LOOSE GROUPS

The observations for six de Vaucouleurs (1968) groups in our sample are complete in that all definite spiral and irregular members listed by de Vaucouleurs have been detected and their radial velocities and individual masses measured. Estimates of M, $M_{\rm vt}/M$, V, and R have been derived for the six groups by use of equations (1)–(5) (Table 3). Following Materne (1974), we have corrected the relative radial velocities, Δv_i, for observational error by means of the following relations:

$$\Delta v_i^2 = \Delta v_{i\,\rm obs}^2 - \sigma^2(\Delta v_i)\,, \tag{8}$$

$$\sigma^2(\Delta v_i) = \left(1 - \frac{2m_i}{M}\right)\sigma^2(v_i) + \sum_j \left(\frac{m_j}{M}\right)^2 \sigma^2(v_j)\,, \tag{9}$$

where $\Delta v_{i\,\rm obs}$ is the observed relative radial velocity and $\sigma(v_i)$ is the mean error in the determination of a radial velocity v_i. For cases where $\sigma(\Delta v_i)$ is greater than $\Delta v_{i\,\rm obs}$, Δv_i is set equal to zero. A misprint in Materne's (1974) paper has been corrected in equation (9).

To evaluate the parameters for each group, radial velocities v_i, and mean errors $\sigma(v_i)$ were taken in order of priority from Table 1 [$\sigma(v_i) = 9\ \mathrm{km\ s^{-1}}$], Sandage and Tammann (1975), and the BGC or de Vaucouleurs and de Vaucouleurs (1967). Masses of galaxies, $m_i = M_T$, were taken from Table 1. Direct masses are not available for seven elliptical and S0 galaxies in the six groups. For these galaxies, masses have been estimated indirectly from absolute blue luminosities, L_B (corrected for Galactic absorption following Sandage 1973), multiplied by a ratio of mass to blue luminosity, m_i/L_{Bi}. The calculations were performed twice, once for $m_i/L_{Bi} = 20$ solar units and again for $m_i/L_{Bi} = 300$ solar units. The first value is suggested by measurements of velocity dispersions in galactic cores (King and Minkowski 1972; Morton and Chevalier 1972, 1973). The second value is suggested by application of the virial theorem and other dynamical procedures to the Coma cluster (Rood *et al.* 1972) and other clusters (Oemler 1974).

The mean error in V, $\sigma(V)$, and the fractional mean error in $M_{\rm vt}$ caused by uncertainties in radial velocity determinations, $\sigma(V^2)/V^2$ (Table 3), are derived from

$$\sigma^2(V^2) = 9\sum_i \left(\frac{m_i}{M}\right)^2 \sigma^2(\Delta v_i^2)\,, \tag{10}$$

$$\sigma(V) = \frac{1}{2V}\sigma(V^2)\,, \tag{11}$$

TABLE 3

Masses and Other Dynamical Parameters of de Vaucouleurs (1968) Groups

Group	M (10^{10} $M_\odot$)	M_{vt} (10^{10} $M_\odot$)	M_{vt}/M	R (Mpc)	V (km s^{-1})	$\sigma(V)$ (km s^{-1})	$\sigma(V^2)/V^2$
5...........	53.3	795	15	1.64	144	5.6	0.08
6...........	19.6–65.2	862–665	44–10	3.80–4.08	98.8–83.8	11–10	0.22–0.25
9...........	41.4–116	140–454	3.4–3.9	0.421–1.99	119–99.1	20–29	0.34–0.59
12...........	31.6	890	28	4.67	90.5	5.8	0.13
33...........	49.6–553	3710–1370	75–2.5	1.89–1.88	291–177	36–68	0.25–0.77
51...........	30.5–192	18100–24900	590–130	4.99–25.2	395–206	29–12	0.15–0.12
(M101)5......	41.1	31.8	0.77	2.36	24.1	3.3	0.28
(M51)5.......	12.2	14.2	1.2	0.22	52.7	9.8	0.37

and

$$\sigma^2(\Delta v_i^2) = 2\sigma^2(\Delta v_i)\cdot[2\Delta v_{i\,\mathrm{obs}}{}^2 + \sigma^2(\Delta v_i)]\,. \quad (12)$$

Equation (12) is from Materne (1974).

From Table 3, we see that the fractional mean error in M_{vt} is equal to or less than 37 percent for all groups, except 9 and 33 in the case where the ellipticals and S0's, which are undetected in 21-cm emission, are assumed to have a ratio of mass to blue luminosity of 300 solar units.

Values of M_{vt}/M for the six de Vaucouleurs groups range from 2.5 to 590. Group 5 has $M_{\mathrm{vt}}/M = 15$, but this group is actually the superposition of the M101 and M51 groups (Sandage and Tammann 1975; Materne and Tammann 1975); these groups individually have $M_{\mathrm{vt}}/M = 1$. Group 9 may be part of the Leo group (Materne and Tammann 1975; Sandage and Tammann 1975). Group 12 is composed of four bright spiral galaxies with mass determinations from the 21-cm observations; it has $M_{\mathrm{vt}}/M = 28$. Application of $m_i/L_{Bi} = 300$ solar units for ellipticals and S0's undetected in the 21-cm line reduces M_{vt}/M to values between 2.5 and 130, which are still significantly larger than unity.

V. DISCUSSION

Our results indicate that, except for an occasional anomalous value, the optical radial velocities given in the BGC have uncertainties which are well described by the listed mean errors. Thus, the principal uncertainty in the analysis by Geller and Peebles (1973) is removed, and their result, that loose groups have $\langle M_{\mathrm{vt}}\rangle/\langle M\rangle \approx 30$, is supported.

Materne (1974) found two anomalous optical radial velocities in de Vaucouleurs (1968) group 7. We have not uncovered a single additional anomalous velocity in our survey which includes complete data for six groups.

Our determinations of M_{vt}/M are more certain than those of RRT for two reasons: (i) We have corrected for uncertainties in radial velocities. (ii) We have used directly determined masses for spiral and irregular galaxies. Moreover, for groups 6, 12, 33, and 51 (which have M_{vt}/M = 44–10, 28, 75–2.5, and 590–130, respectively), there appears to be no controversy whatsoever concerning group membership. All of the galaxies listed as definite members by de Vaucouleurs (1968) are also listed as members by Sandage and Tammann (1975).

As discussed in § III, the conventional Bottlinger-Lohman masses are probably very uncertain. In most of the groups analyzed, a very large fraction ($\geqslant 0.5$) of the conventional mass is attributed to a single member of the group. Thus, the value of M_{vt}/M is often dependent on essentially a single conventional mass. It is conceivable that M_{vt}/M may in fact represent the correction factor which must be applied to the Bottlinger-Lohman mass to give the actual mass of a galaxy; that is, the mass of a group could be contained in its galaxies but the galaxies would have large heavy halos (Ostriker, Peebles, and Yahil 1974). The large mass ratios for groups 6, 12, 33, and 51 found in the present study are consistent with that hypothesis; however, the mass ratio $M_{\mathrm{vt}}/M = 1$ found for the M101 and M51 groups may conflict with the hypothesis.

Turner and Sargent (1974) interpret a two-peaked frequency distribution of crossing times for de Vaucouleurs groups, with the second peak occurring at a crossing time roughly equal to the Hubble time, to mean that some groups are bound and the others are expanding apart in the Hubble flow (Gott, Wrixon, and Wannier 1973). Among the groups in our sample, only group 5 is considered by Turner and Sargent to be bound, and this "group" is actually the M101 and M51 groups, each with $M_{\mathrm{vt}}/M = 1$. The other groups in our sample are considered by Turner and Sargent to be unbound, and these groups indeed have large mass ratios. Thus, our values of M_{vt}/M are consistent with the hypothesis of Turner and Sargent. However, this agreement may be fortuitous, because Jackson (1975) has shown that when an *accurate definition* of crossing time is used, the frequency distribution of crossing times becomes single-peaked, and nearly all of the crossing times of the groups are significantly shorter than the Hubble time.

The six groups of the present study are too few to firmly test the validity of the correlations between M_{vt}/M, V, and R suggested by the work of RRT. However, the results for these groups indicate that the presence of correlations may be sensitive to the adopted ratio of mass to blue luminosity for elliptical galaxies (20 or 300 solar units).

We thank J. R. Fisher for information on the equipment and M. S. Roberts for valuable comments on the program. We thank G. Chincarini, H. Corwin, P. Crane, P. J. E. Peebles, and G. de Vaucouleurs for remarks on the M101 group. The NRAO is operated by Associated Universities, Inc., under contract with the National Science Foundation. This research was supported in part by the National Science Foundation Research grants GP-36167A#1 to Michigan State University and GP-41560 to the University of Illinois.

REFERENCES

Brandt, J. C. 1960, *Ap. J.*, **131**, 293.
Chincarini, G., Rood, H. J., and Welch, G. A. 1975, *M.N.R.A.S.*, **170**, 441.
de Vaucouleurs, G. 1968, in *Stars and Stellar Systems*, Vol. **9**, ed. A. and M. Sandage and J. Kristian (Chicago: University of Chicago Press) (in press).
de Vaucouleurs, G., and de Vaucouleurs, A. 1964, *Reference Catalogue of Bright Galaxies* (Austin: University of Texas Press) (BGC).
———. 1967, *A.J.*, **72**, 730.
Dickel, J. R., and Rood, H. J. 1975, *A.J.*, **80**, 584.
Epstein, E. E. 1964, *A.J.*, **69**, 490.
Ford, W. K., Jr., Rubin, V. C., and Roberts, M. S. 1971, *A.J.*, **76**, 22.
Geller, M. J., and Peebles, P. J. E. 1973, *Ap. J.*, **184**, 329.
Gott, J. R., Wrixon, G. T., and Wannier, P. 1973, *Ap. J.*, **186**, 777.
Holmberg, E. 1958, *Medd. Lund*, Ser. 2, No. 136.
Huchtmeier, W. 1973, *Astr. and Ap.*, **22**, 91.
———. 1975, preprint.
Humason, M. L., Mayall, N. U., and Sandage, A. 1956, *A.J.*, **61**, 97.
Jackson, J. C. 1975, *M.N.R.A.S.*, **173**, 41P.
King, I. R., and Minkowski, R. 1972, *External Galaxies and Quasi-stellar Objects*, ed. D. S. Evans (New York: Springer-Verlag), p. 87.
Lewis, B. M. 1974, *Observatory*, **94**, 9.
Materne, J. 1974, *Astr. and Ap.*, **33**, 451.
Materne, J., and Tammann, G. A. 1975, *Astr. and Ap.*, in press.
Mayall, N. U., and de Vaucouleurs, A. 1962, *A.J.*, **67**, 360.
Mayall, N. U., Lovasich, J. L., Neymann, J., and Scott, E. L. 1961, in *Proc. Fourth Berkeley Symposium on Mathematical Statistics and Probability*, **3**, 187.
Morton, D. C., and Chevalier, R. A. 1972, *Ap. J.*, **174**, 489.
———. 1973, *Ap. J.*, **179**, 55.
Oemler, A. 1974, Ph.D. thesis, California Institute of Technology.
Ostriker, J. P., Peebles, P. J. E., and Yahil, A. 1974, *Ap. J. (Letters)*, **193**, L1.
Roberts, M. S. 1962, *A.J.*, **67**, 437.
———. 1969, *ibid.*, **74**, 859.
———. 1972, in *External Galaxies and Quasi-stellar Objects*, ed. D. S. Evans (New York: Springer-Verlag), p. 12.
Roberts, M. S., and Rots, A. H. 1973, *Astr. and Ap.*, **26**, 483.
Rogstad, D. H., and Shostak, G. S. 1972, *Ap. J.*, **176**, 315.
Rood, H. J., Page, T. L., Kintner, E. C., and King, I. R. 1972, *Ap. J.*, **175**, 627.
Rood, H. J., Rothman, V. C. A., and Turnrose, B. E. 1970, *Ap. J.*, **162**, 411 (RRT).
Sandage, A. 1973, *Ap. J.*, **183**, 711.
Sandage, A., and Tammann, G. A. 1974*a*, *Ap. J.*, **190**, 525.
———. 1974*b*, *ibid.*, **194**, 559.
———. 1975, *ibid.*, **196**, 313.
Shostak, G. S. 1975, *Ap. J.*, **198**, 527.
Turner, E. L., and Sargent, W. L. W. 1974, *Ap. J.*, **194**, 587.
Volders, L., and Högbom, J. A. 1961, *B.A.N.*, **15**, 307.
Zwicky, F., Herzog, E., Karpowicz, M., Kowal, C. T., and Wild, P. 1960–1968, *Catalogue of Galaxies and of Clusters of Galaxies*, Vols. **1–6** (Pasadena: California Institute of Technology).

JOHN R. DICKEL: University of Illinois Observatory, Urbana, IL 61801
HERBERT J. ROOD: 52 Elizabeth St., S. Bound Brook, NJ 08880

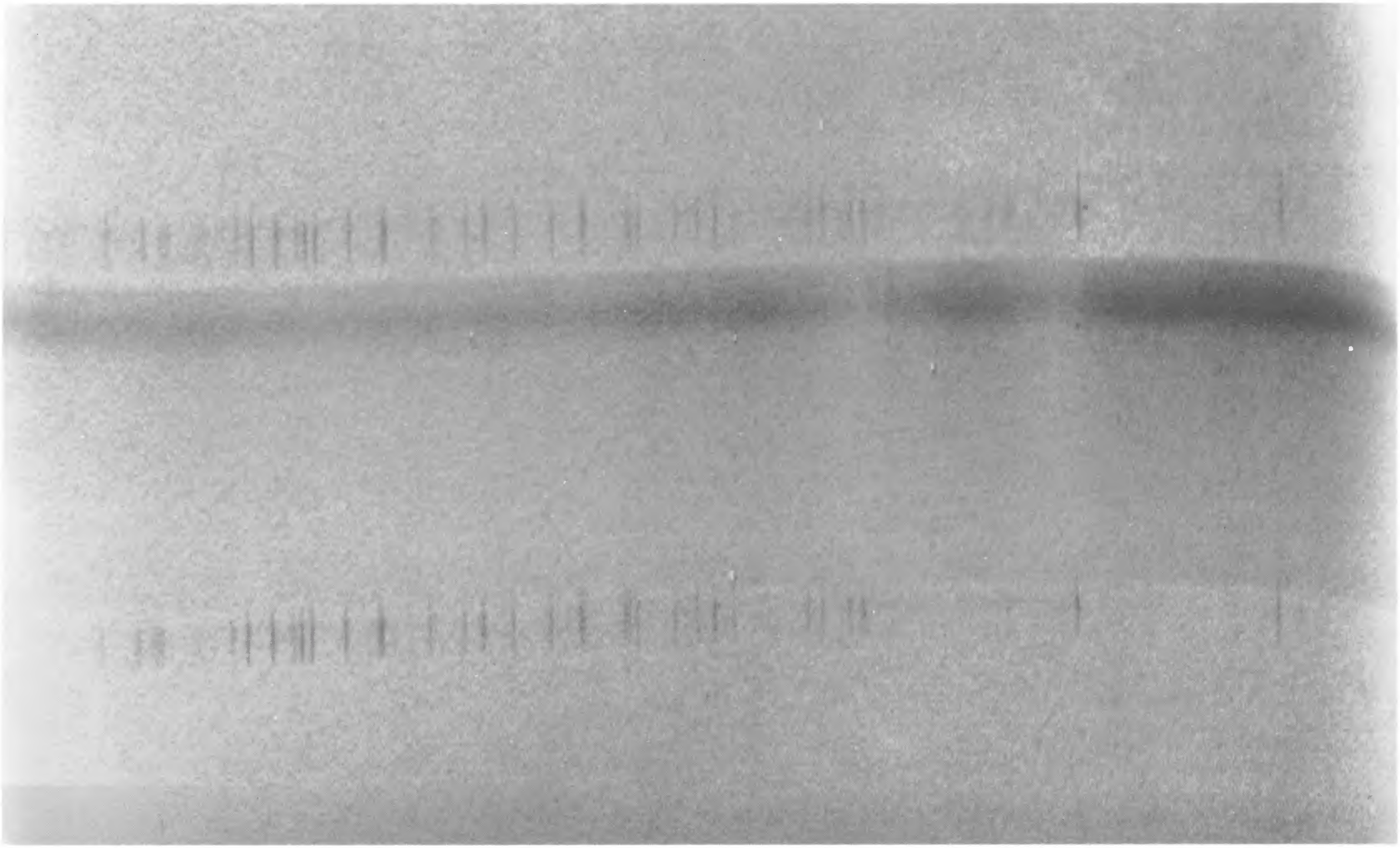

FIG. 1.—The spectrum of the nucleus and adjacent bulge of M31 reproduced from the digitized TV readout. Wavelengths increase from λ3770 on the left to λ4030 on the right. The spectrum covers a width of 34″ at a position angle of 52°, with the lower edge north and east of the nucleus. Four narrow absorptions can be seen superposed on the broad K line, which is closer to the center of the figure than the H line.

MORTON AND ANDERECK (*see* page 356)