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ON THE SATELLITE CAPTURE PROBLEM

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SUMMARY

The possibility that the outer satellites of Jupiter were permanently captured is discussed. The various proposed capture mechanisms are examined briefly, and their shortcomings discussed. It is suggested that all the observed aspects of the outer satellites can be accounted for if the mass of Jupiter, at some time in the past, increased by about 130 earth masses, on a time scale of the order of one sidereal period of Jupiter.

Ever since their discovery, it has been tempting to postulate that the outer satellites of Jupiter are, in fact, captured asteroids rather than ' natural ' satellites. This is true especially for the retrograde satellites which, at first sight, appear to be only marginally stable. A number of theories have already been advanced to explain the capture procedure. These will be briefly examined here.

1. THE RESTRICTED THREE-BODY CASE

The possibility of satellite capture will be discussed first for the three-body case; the Sun, Jupiter and the satellite will be considered as point-masses. The satellite will be considered to have been captured by Jupiter if its Jovicentric osculating elements are elliptical and the satellite has completed at least one revolution about Jupiter. A permanent capture is said to occur if the satellite is captured by Jupiter and stays within the vicinity of the planet for ever after. Temporary captures are defined to be those where the satellite escapes at some later time.

It has been shown by Hopf (1930) that, except for a class of Lebesgue measure zero, either the evolution of a gravitating system is quasi-periodic or the system disintegrates. Since in the circular or elliptic case no particle escapes to infinity, these systems must be quasi-periodic. Each particle must therefore pass arbitrarily close to its initial position in phase space, ruling out the possibility of a satellite of one body becoming permanently captured by another body. This result is valid except for a class of zero measure (i.e. although some initial conditions may yield permanent captures, these conditions would have to be specified with infinite accuracy).

The Jacobi constant for the circular restricted three-body problem, C_J is given by:

$$C_J = (x^2+y^2)+\frac{2(1-\mu)}{r_1}+\frac{2\mu}{r_2}$$

where x, y are the coordinates of the infinitesimal body in Cartesian coordinates in which the x-axis remains in the line Sun–Jupiter; r_1, r_2 are the distances of the infinitesimal body from the Sun and from Jupiter respectively and μ is the ratio of the mass of Jupiter to the mass of the Sun. In this definition, the unit of length is taken to be the radius of the (assumed circular) heliocentric orbit of Jupiter (5·20 AU).

The Jacobi constant C_J has been used by Groves & Shaikh (1965), Bailey (1972) and others as a criterion for stability. Although in the circular restricted case it is a good indicator for direct satellite orbits with zero inclination, it leads to the conclusion that the effect of increasing the inclination is always to decrease the satellite's stability. In particular, direct orbits are found to be more stable than corresponding retrograde orbits. However, Hunter (1967a) and Henon (1970) have shown numerically that in fact the opposite is true. (In numerical calculations, a satellite orbit will be said to be stable if it completes at least 50 revolutions about Jupiter.) The limits of stability found by Henon for Hill's case (where the mass-ratio of Jupiter to the Sun, μ, approaches zero) are shown in Fig. 1. Although these results are for Hill's case, they correspond very closely to the results of Hunter and others who used more realistic values for the mass and orbit of Jupiter. It can be seen readily that the retrograde orbits are stable for much smaller values of C_J than the corresponding direct orbits. (It is interesting to note that for both satellite groups the average maximum distance from Jupiter has to be approximately doubled for instability to be reached along the $e = 0$ path. The possible significance of this will be discussed later.) In general, therefore, the Jacobi constant is not a useful stability criterion.

In a recent paper, Bailey (1972) claims that the direct and retrograde satellites of Jupiter correspond to captures at perihelion and aphelion respectively. He finds capture to be possible only (i) with Jupiter at perihelion or aphelion, and (ii) for mass-ratios μ greater than $\sim 3 \times 10^{-6}$ in the circular case (hence capture in a circular orbit is possible only for the four most massive planets), and (iii) through the Lagrange point L_2. Since Bailey introduces no dissipative mechanism, it is difficult to see how permanent capture could occur. It is therefore assumed that these conditions are meant for temporary captures only.

A number of configurations were integrated numerically to test Bailey's assumptions. This was done with the help of a computer program developed at the Department of Aerospace Engineering and Engineering Mechanics at the University of Texas at Austin. It uses a seventh-order Runge–Kutta–Fehlberg algorithm. All integrations were for the coplanar case. The satellite was initially at L_2 with the same angular velocity as Jupiter, but with a small velocity du towards the planet. The following results were obtained.

(1) All capture orbits were direct, the duration of capture increasing with decreasing du.

(2) For $\mu = 10^{-7}$, a temporary capture took place lasting for at least 33 revolutions about Jupiter (~ 100 years).

(3) For Jupiter's orbit having an eccentricity of 0·05, temporary captures with Jupiter at perihelion, 90°, aphelion and 270° (true anomaly) lasted for 36, 15, 56 and > 100 years respectively.

(4) For Jupiter's orbit having an eccentricity of 0·33 a capture lasting 12 years took place when Jupiter was initially at a true anomaly of 90°.

It was found also that the retrograde orbits were stable to greater distances from Jupiter than their corresponding direct orbits. Although it may be true that direct satellites enter through L_2, this does not in general appear to be the case for retrograde satellites. Our numerical results confirm those of Henon and of Hunter, and do not support Bailey's postulates.

It should be mentioned that Bailey arrives at a limiting mass-ratio (below which capture is impossible) as a result of his adoption 'for simplicity' of twice the radius of Jupiter's sphere of influence as 'a reasonable limiting value for the semi-major axis of a stable satellite orbit'. This seems unrealistic because the sphere of influence does not take into account Jupiter's angular motion. It would appear to be more realistic to choose as limiting radius some fraction of the distance from Jupiter to the inner Lagrange point L_2. If this is done, capture would be possible for all mass ratios.

In conclusion, Hopf's results prove that permanent captures are not possible in the elliptic restricted problem, although temporary captures lasting for very long times may occur. Since at present Jupiter's satellites are far from instability, some other mechanism is required to make temporary captures permanent. Some possible candidates will now be considered.

2. CAPTURE WITH DISSIPATIVE MECHANISMS

Various capture theories invoking dissipative mechanisms will now be examined. For a theory to be acceptable it should be able to explain (i) how the orbits became stable, (ii) why the direct and retrograde satellites form two distinct groups, (iii) why the range of major semi-axes in each group is small, and (iv) why for both groups the ratio of the present maximum distances to the escape distance is the same (approximately).

(a) *Tidal friction.* Although the secular accelerations of the Moon and perhaps some other satellites may be attributed to tidal friction caused by tides raised on the primaries by the satellites, the small masses and large distances from Jupiter of the outer satellites would cause tides far too small to affect their motions appreciably. Even if the effect of tidal dissipation were not negligible, the fact that the orbital periods of the satellites are greater than the rotational period of Jupiter would cause the retrograde satellites to approach Jupiter, but the direct satellites to recede from Jupiter. In addition, since the satellites within each group have different masses, tidal dissipation would also spread out the orbits of the satellites within a group. Tidal friction does not, therefore, provide a stabilizing influence.

(b) *A resisting medium.* The effect of a resisting medium (either a Jovian atmosphere or an interplanetary medium) appears also to be negligible. Kuiper (1950) supposed that, in the past, the atmosphere of Jupiter may have been much denser. If the medium were sufficiently dense to affect the motions of the satellites, its force would be proportional to the cross-sectional area/mass ratios of the satellites; hence, all else being equal, inversely proportional to their diameters. Since the satellites vary considerably in size (by a factor of 5 or 6) the result of travelling through a resisting medium would be to spread out the satellite orbits over a wide range. The satellites would then not form two distinct, compact groups.

(c) *The effect of other bodies.* The influence of Saturn is too small to perturb the satellite orbits appreciably. It would also be difficult to explain satellite groupings in this manner. Near collisions between two unstable satellites could leave one stable, and near collisions between one stable and one unstable satellite could leave both stable. However, the satellite grouping would be still unexplained.

According to Kuiper, Jupiter's mass decreased in the past. As a result, some natural satellites escaped. Later, some of these were recaptured, being braked by Jupiter's atmosphere. The two satellite groups could be due to only two captures, with each mass breaking up into pieces as it entered the atmosphere. However, since the atmosphere would have had to be dense in order to cause the ingressing satellites to disrupt, its effect would have been to spread out the satellite groups very rapidly, as discussed previously.

Columbo & Franklin (1971) investigated the possibility that the two satellite groups are due to only one collision between a satellite and an asteroid, resulting in the direct and retrograde groups respectively. This theory avoids some of the difficulties already discussed. However, whereas criteria (ii) and (iii) impose already stringent conditions on the collision parameters, condition (iv) imposes an even more stringent condition, unless condition (iv) is taken to be purely accidental.

3. INCREASE OF JUPITER'S MASS ON A SHORT TIME SCALE

We now examine a possibility which would automatically satisfy conditions (i)–(iv). The postulate is that the mass of Jupiter increased, at some time in the past, on a time scale of the order of the sidereal period of Jupiter. Such a mass increase would cause a hitherto temporary satellite to be pulled in and made stable. Now the critical distance from Jupiter for which a temporary satellite would make many revolutions of Jupiter is different for retrograde and direct satellites. Thus, as a result of an increase of Jupiter's mass, the direct and retrograde satellites would form two compact groups, for which the ratio of the orbital radii would be the same as the ratio of the radii of the original critical orbits. Provided that the rate of increase of Jupiter's mass was not too great (i.e. on a time scale long compared with the Jovicentric periods of the satellites), no great changes in eccentricity would result (since eccentricity is an adiabatic invariant).

From Fig. 1 it is found that the ratio α of the present average maximum distance from Jupiter to the distance r_E at which instability occurs along the path $e = 0$ is about $0{\cdot}5 \pm 0{\cdot}05$ *for both groups*. For small mass-ratio μ the instability distance r_E is related to μ by

$$r_E \propto \mu^{1/3}.$$

It follows that

$$\frac{r_E'}{r_E} = \left\{\frac{m'}{m}\right\}^{1/3} \qquad (1)$$

where m' and r_E' correspond to the initial mass of Jupiter and initial instability distance, while m and r_E correspond to the present values.

If the mass increase is slow compared to the orbital period of the satellite, then the effect may be averaged over an orbit, and we find

$$\frac{r_E'}{r} = \frac{m}{m'} \qquad (2)$$

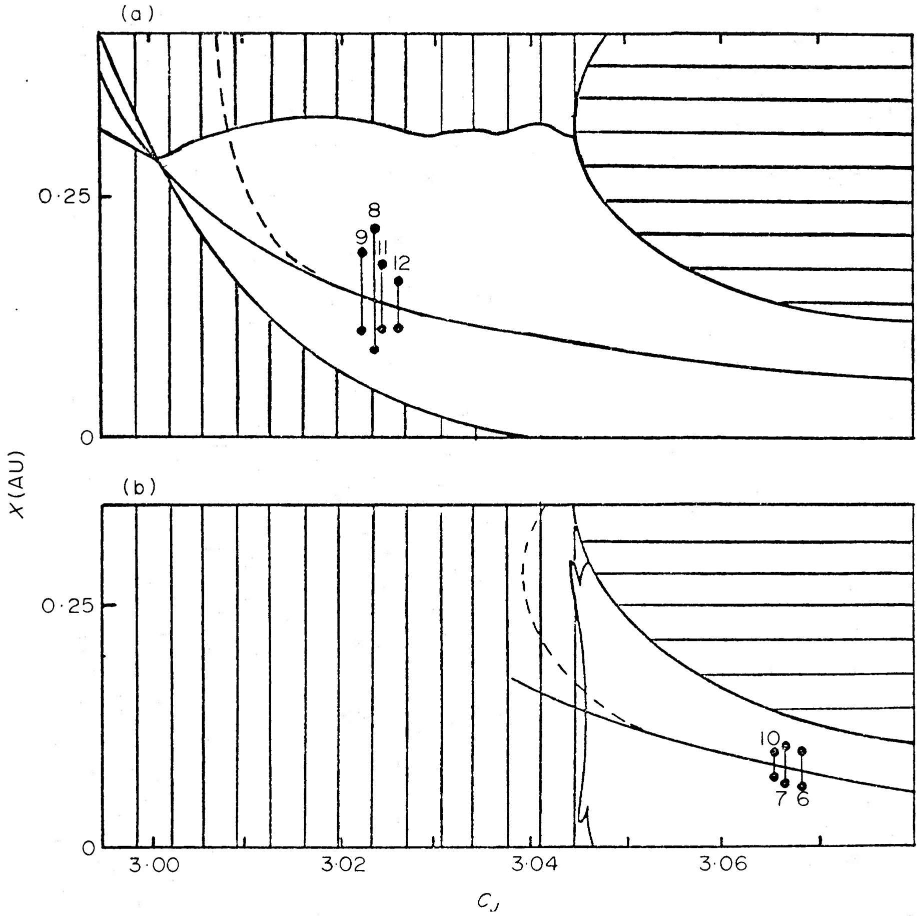

FIG. 1. (*Based on Fig.* 15 *of Henon.*) *a and b represent retrograde and direct satellites respectively. Horizontal hatching: forbidden regions; vertical hatching: unstable regions; dashed line: circular orbits; solid line: periodic orbits; X = distance from Jupiter; numbers refer to respective Jovian satellites.*

since angular momentum is conserved, and e is assumed invariant. From (1) and (2) it is then found that

$$m' = m\alpha^{3/4}. \tag{3}$$

Since α is about 0·5, it follows that $m' \doteqdot 0{\cdot}6m$. Hence an increase of about 0·4 times the present mass of Jupiter (i.e. an increment ~130 earth masses) would be required if permanent captures occurred in this manner.

The above result is independent of any postulated cause of Jupiter's mass increase. The close similarity of inclinations within the two satellite groups cannot be deduced from the mass increase hypothesis. Such a similarity would arise, however, if the satellites' pre-capture orbits were similar for each group. A possible explanation of this is that the satellites originated from a single planetary body that disintegrated. The stringent conditions for temporary capture would then cause the particles captured in the first approach to Jupiter to be limited to a very small range of pre-capture orbital elements. It is therefore of interest to note that the required mass increase is of the same order of magnitude as that derived by Ovenden (1973), from dynamical considerations, for a planet postulated to have once existed between Mars and Jupiter.

4. SUMMARY

No previous theory of satellite capture appears capable of giving a satisfactory account of the observed orbital characteristics of the outer satellites of Jupiter. It has been shown that a satisfactory explanation can be made, in general terms, if the mass of Jupiter increased (at some time in the past) by an amount ~ 130 earth masses, on a short time scale.

It is, of course, possible that the outer satellites are 'natural' satellites which have been always in stable orbits, although it is difficult to understand how retrograde 'natural' satellites could have been formed. It would be interesting to compare the light variations (if any) of the outer satellites and the inner satellites, as suggested by Kuiper. 'Natural' satellites tend to have more or less spherical shapes, with small light variations, whereas asteroids tend to have irregular shapes and significant light variations.

It should be noted that any change in the constant of gravitation G will affect the sizes of the satellite orbits and the planetary orbits. Changes of G will change the distance- and time-scales, but will have no effect on the topology of the system as regards capture or non-capture.

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