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THE LOCATION OF THE HOT SPOT IN CATACLYSMIC VARIABLE STARS AS DETERMINED FROM PARTICLE TRAJECTORIES*

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SUMMARY

Warner & Peters using particle trajectories in the restricted three-body approximation, have previously published a set of models for the hot spot location in cataclysmic variable stars. This paper presents a new set of models, based on their assumptions, which corrects an error in their calculations. With the new models the spot radius is moved substantially closer to the blue star for most mass fractions, and other parameters are changed accordingly. It is pointed out that the well-defined picture for the hot spot geometry produced in this model is probably not directly applicable to real stars.

1. INTRODUCTION

At minimum light the novae, recurrent novae, dwarf novae, and nova-like variable stars, jointly referred to as cataclysmic variable (CV) stars in this paper, appear as similarly constituted, short period, binary stars. They consist of a Roche lobe filling, late type (red) star which transfers mass through its inner Lagrangian point, L_1, into a ring or disk surrounding a hot (blue) star, probably a white dwarf (Kraft 1963). Independently Warner & Nather (1971), and Krzeminski & Smak (1971) proposed an elaboration of this basic model which suggests that a shock front or hot spot at the intersection of the disk and stream produces an important fraction of the observed luminosity. The asymmetrically placed spot explains the characteristic hump concurrent with increased flickering which precedes eclipse, and eclipses which occur at variable phase but usually follow spectroscopic conjunction by several hundredths of an orbital period. Warner & Peters (1972) (designated as WP in this paper) published a series of models for the hot spot's size and location as a function of mass ratio based on particle trajectory calculations in the restricted three-body (RTB) approximation. In connection with an investigation of hydrodynamic gas flow in CV stars, to be reported elsewhere, the author found several discrepancies between the hydrodynamic and particle models which were traced to an error in the WP calculations. New particle models were calculated which agreed satisfactorily with the hydrodynamic results. Since these new particle models are substantially different from those of WP, the purpose of this paper is to communicate the corrected hot spot parameters derived from RTB trajectories and the stated assumptions of WP.

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2. METHOD AND RESULTS

The hot spot's location in this model is determined from two propositions: (1) an approximation that near the blue star motions are governed by one-body considerations; and (2) an assumption that at the spot's radius, r_d, from the blue star, both disk material in circular orbit and gas in the infalling stream from L_1 simultaneously have the same specific angular momentum, j_d, with respect to the blue star in a non-rotating frame of reference. To evaluate the hot spot's location RTB trajectories corresponding to thermal evaporation from L_1 in the binary orbital plane are numerically integrated using a predictor–corrector method which retains the Jacobi integral constant to at least one part in 10^6. After each integration step the quantities j_t, the particle's specific angular momentum as above, and r_t, the radius at which the gas would orbit in circular Keplerian motion about the blue star with angular momentum j_t, are both evaluated. The hot spot's location is defined as the point at which the trajectory first crosses inside the radius r_t appropriate to its instantaneous angular momentum.

The equations of motion are formulated in the conventional RTB problem frame of reference which corotates with the binary, and the physical variables are reduced to the standard dimensionless variables. Scale factors for relevant physical quantities are the following: length, the binary separation A; mass, the binary total mass M; time, the inverse of the orbital circular frequency ω^{-1}; velocity, ωA; energy, GM/A; and the angular momentum, ωA^2. The blue star is designated as star 1 and has mass fraction μ, i.e. $M_1 = \mu M$. With the X axis along the line of centres and the coordinate origin at the centre of mass, the equations of motion are

$$\ddot{x} = 2\dot{y}+x-\mu\frac{(x-x_1)}{r_1^3}-(1-\mu)\frac{(x-x_2)}{r_2^3}, \tag{1}$$

$$\ddot{y} = -2\dot{x}+y-\frac{\mu y}{r_1^3}-(1-\mu)\frac{y}{r_2^3}, \tag{2}$$

where $r_1(r_2)$ is the particle's distance from star 1(2), and dots represent time derivatives. The quantities j_t and r_t defined above are evaluated as

$$j_t = (x-x_1)\dot{y}-y\dot{x}+r_1^2, \tag{3}$$

$$r_t = j_t^2/\mu. \tag{4}$$

TABLE I

*Hot spot parameters derived from particle trajectories**

μ	q	r_d	α	β	j_d	ΔE	V_d
0·05	19·000	0·0402	74·4	2·2	0·0448	0·36	1·12
0·10	9·000	0·0475	74·1	2·7	0·0689	0·63	1·45
0·20	4·000	0·0575	73·4	3·2	0·1072	1·07	1·87
0·30	2·333	0·0665	72·6	3·7	0·1412	1·39	2·12
0·40	1·500	0·0762	71·6	4·2	0·1746	1·59	2·29
0·50	1·000	0·0878	70·4	4·9	0·2096	1·69	2·39
0·60	0·667	0·1028	68·8	5·7	0·2483	1·66	2·42
0·70	0·429	0·1237	66·6	6·8	0·2942	1·51	2·39
0·80	0·250	0·1567	63·3	8·6	0·3540	1·23	2·26
0·90	0·111	0·2227	57·2	12·0	0·4477	0·78	2·01
0·95	0·053	0·2984	50·7	15·9	0·5324	0·47	1·78

* Symbols are defined in the text. α and β are in degrees.

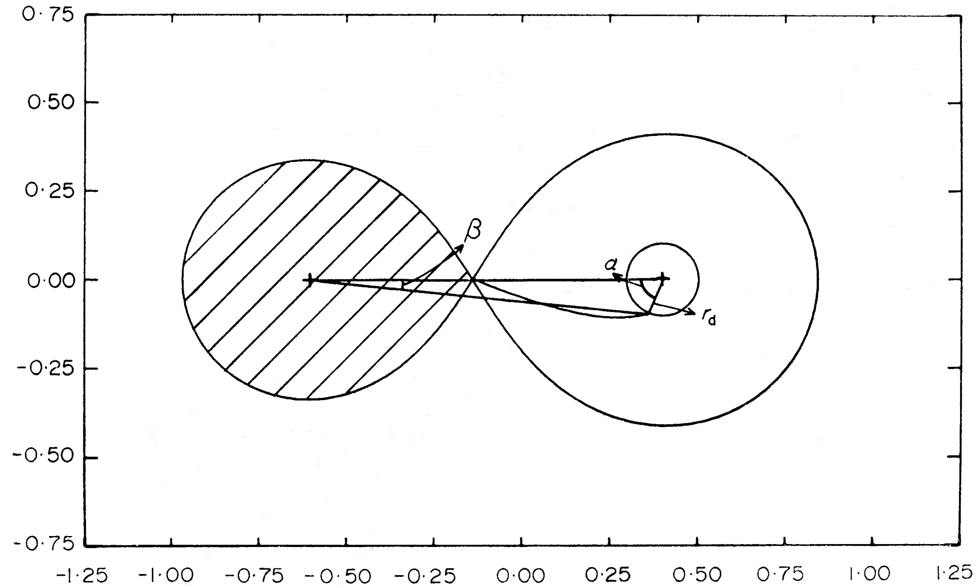

FIG. 1. *The particle trajectory hot spot model for a mass fraction of* 0·6. *The spot forms at the intersection of the trajectory from the inner Lagrangian point and the ring around the blue star at radius* r_d. *The bounding curve is the Roche equipotential through* L_1 *in the orbital plane.*

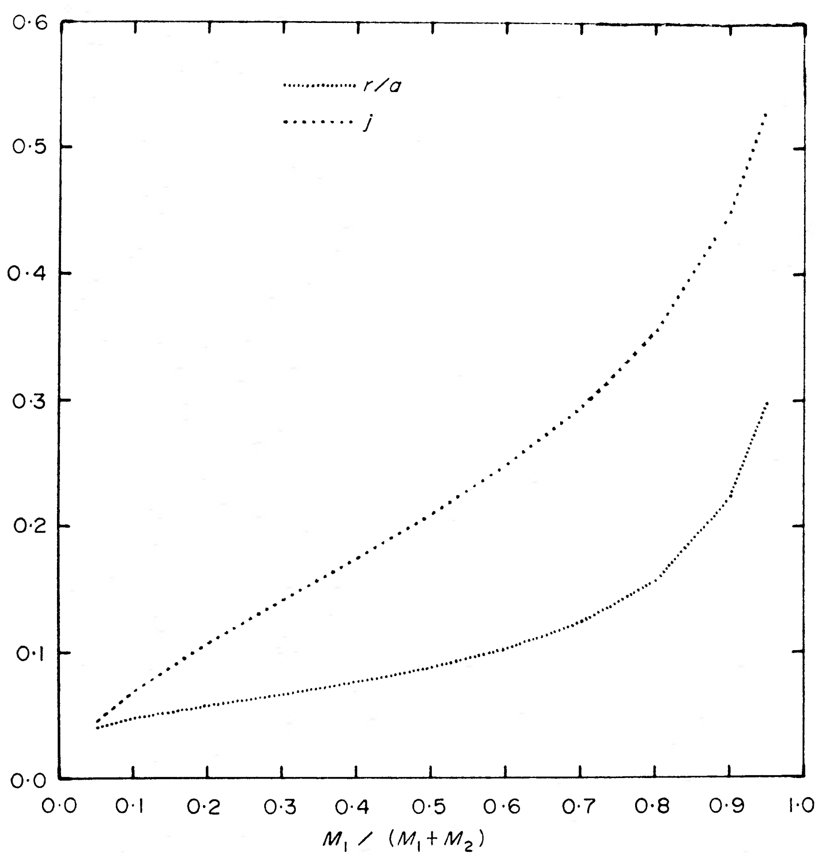

FIG. 2. *The specific angular momentum, j, of gas from the inner Lagrangian point as it reaches the hot spot, and the hot spot radius, r/a from the blue star are indicated as a function of mass fraction.*

For dimensionlessly small injection velocities from L_1 trajectories are nearly independent of the initial velocity, so orbits are only integrated for initial values of $\dot{x} = 0{\cdot}03, \dot{y} = 0, \pm 0{\cdot}03$. The orbit with $\dot{y} = 0$ is used to define the spot's location.

The resultant hot spot parameters, tabulated in Table I, are only a function of mass fraction. In Table I the quantities μ, r_d, j_d are as defined above; q is the mass ratio $(1-\mu)/\mu$, i.e. red to blue as in WP; the angles α and β are indicated in Fig. 1 for $\mu = 0{\cdot}6$; ΔE is the energy difference between particles in the stream and particles in the disk at the spot; and V_d is the circular velocity in a non-rotating frame of reference for particles in the disk at $r_d(V_d = j_d/r_d)$. Figs 2 and 3 display some of the parameters graphically.

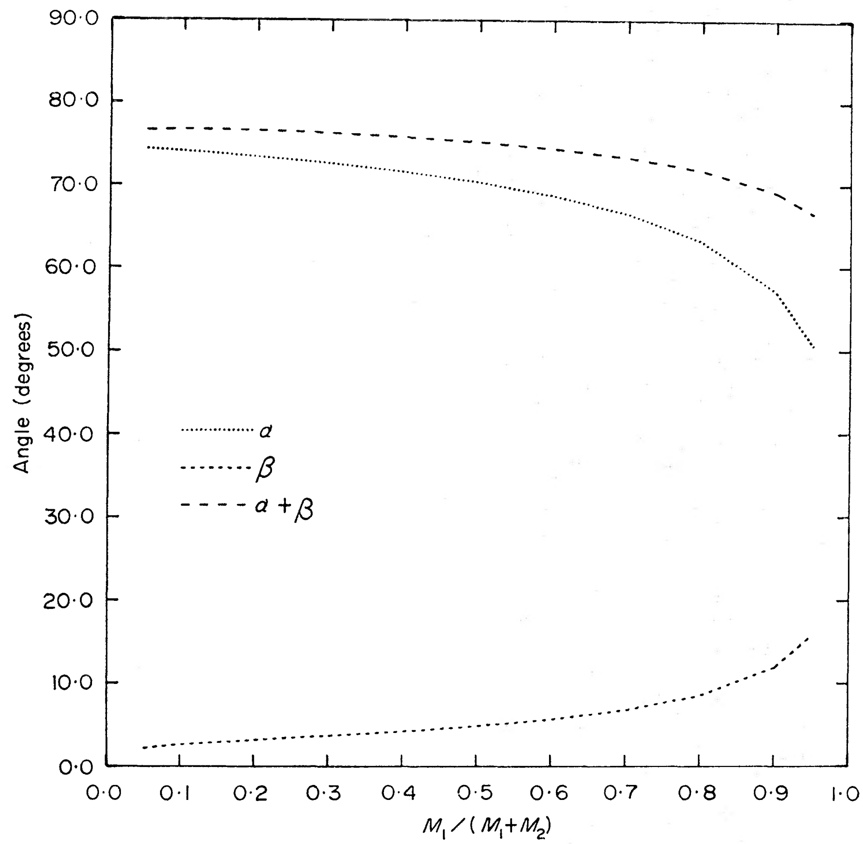

FIG. 3. *The angles α, β and their sum as a function of mass fraction. α and β are the angles separating the line of centres and the radius vector to the hot spot originating from the centre of mass of the blue and red star, respectively, as indicated in Fig.* 1.

3. DISCUSSION

(a) *Comparison with the results of Warner & Peters*

The substantially smaller radii and different values for the angles α and β found here apparently occur because WP incorrectly evaluated the quantity j_t. Because of the rotating frame of reference, particles with zero velocity at L_1 are endowed with an initial angular momentum $j_0 = \omega(L_1 - x_1)$. As a particle falls towards star 1, the retarding influence of star 2 causes j_t to decrease from j_0 until the particle is close enough to star 1 that j_t becomes essentially constant. Furthermore, as the mass fraction in star 1 increases, the trajectory should more closely approximate one-body motion; therefore, the ratio j_d/j_0 should tend towards unity,

TABLE II

A comparison with the Warner & Peters models

μ	L_1	j_0	j(WP)	j(WP)/j_0	j_d/j_0
0·05	0·7152	0·055	—	—	0·81
0·10	0·6090	0·085	0·085	1·00	0·81
0·20	0·4381	0·131	0·132	1·00	0·82
0·30	0·2861	0·171	0·173	1·01	0·83
0·40	0·1416	0·210	0·212	1·01	0·83
0·50	0·0	0·250	0·249	1·00	0·84
0·60	−0·1416	0·293	0·289	0·99	0·85
0·70	−0·2861	0·343	0·332	0·97	0·86
0·80	−0·4381	0·407	0·383	0·94	0·87
0·90	−0·6090	0·503	0·454	0·90	0·90
0·95	−0·7152	0·585	0·511	0·87	0·91

since angular momentum should more nearly be conserved. Table II is a comparison of the results of WP and those found here. In Table II L_1 is the x coordinate of L_1, j_0 is the initial angular momentum at L_1, j(WP) is the WP value for the angular momentum at the spot radius (interpolated from their results to conform with the mass fractions used in this paper), and J(WP)/j_0 and j_d/j_0 are the ratio of initial angular momentum to angular momentum at the spot according to WP and the author, respectively. The ratios found here seem in accordance with expected

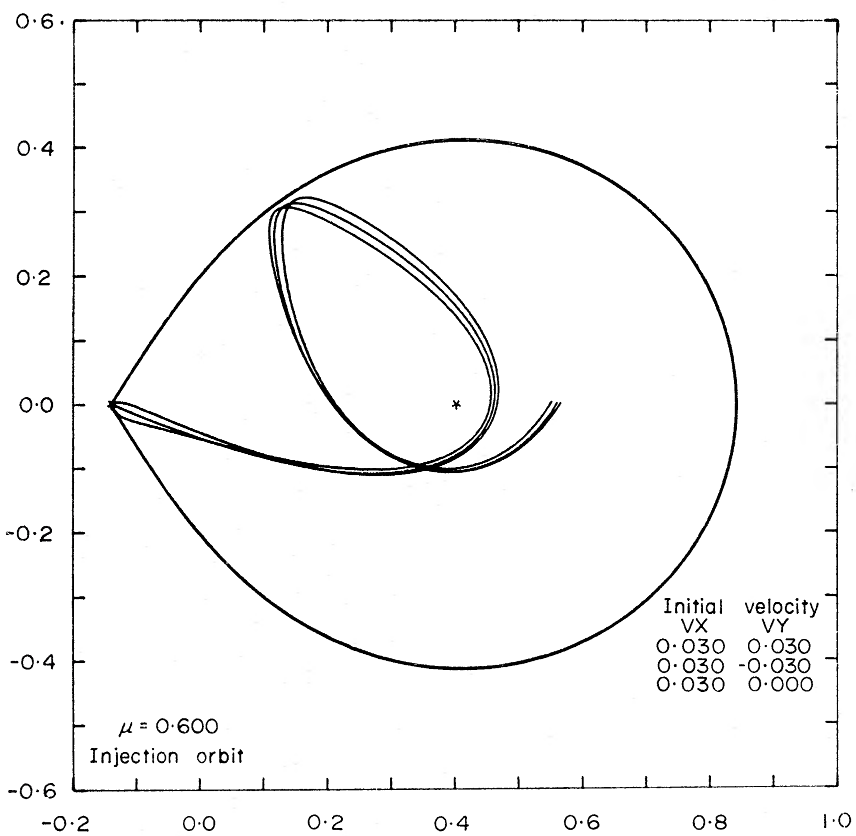

FIG. 4. *Particle trajectories in the orbital plane corresponding to thermal evaporation from the inner Lagrangian point for a mass fraction of* 0·6. *The bounding curve represents the Roche equipotential through* L_1. *Note that the trajectories cross before reaching the region of hot spot formation, which is* r_d = 0·10 *for this mass fraction.*

behaviour, while those of WP show far too little decrease in angular momentum along the stream for mass fractions less than 0·6, and the wrong trend as a function of increasing mass fraction.

WP also calculated an angle δ representing the hot spot's angular extent as viewed from the blue star. The width was derived by launching a series of trajectories from L_1 at a thermal speed but at various angles of ejection, and then noting the spread in positions at which these trajectories crossed the radius r_d. However, this procedure seems invalid. Fig. 4, displaying the trajectories from L_1 for $\mu = 0{\cdot}6$, clearly shows that the particles which define the spread at the spot have already crossed the median trajectory while in transit from L_1. Unless the stream is of extremely low density, collisions will prevent particles from proceeding undeviated along RTB trajectories where such crossing occur. While the mean stream path is probably satisfactorily represented by the mean particle trajectory, the spot's width must be determined from hydrodynamic considerations.

(b) *Applicability of the particle model*

Although the particle model provides a clear, well-defined picture for the various components of the hot spot model, its direct application to CV stars is suspect. Broadening of the gas flow into a disk through viscous dissipation would increase the disk's outer radius. Indeed, in WZ Sge Krzeminski & Smak find that the spot radius varies by as much as 25 per cent. Also, the broad, frequently double emission lines which dominate the spectra of CV stars display random variability in the separation of the line peaks. If one assumes that the emission lines originate in gas in circular orbit about the blue star, then the radius deduced from the peak separation often indicates that the gas orbits at a distance comparable with the blue star's Roche lobe. As examples the emission line disk radii (in dimensionless units) deduced for Z Cam (Robinson 1973), EM Cyg (Robinson 1974), and SS Cyg (Walker & Chincarini 1968) are 0·55, 0·40 and 0·66, respectively. The hot spot, however, seems to form at a radius much closer to the blue star. Presumably the stream from L_1 easily penetrates the low density outer disk before forming a hot spot by colliding with a much more dense inner disk. The characteristic hump in CV light curves, which is attributed to optically thick continuum emission from the spot, generally is visible for only one half the orbital period. This indicates that absorption by the ambient gas plays an important role in modulating the spot's visibility. In addition to these difficulties the veiling of the red star's spectrum in systems with periods less than 6 hr, and the masking effect of flickering all conspire to make a direct observational determination of the hot spot's location quite difficult.

The model meets with some success and some failure when applied to actual systems. For WZ Sge the radius of 0·27 found by Krzeminski & Smak is indicative of a high mass fraction, even if the radius varies by 25 per cent. The deduced mass fraction in the white dwarf of 0·95 is in accord with the model for WZ Sge proposed by Krzeminski & Smak. However, often the spot radius cannot be found, and only the relative phasing of eclipse and hump structure can be determined, as pointed out by WP. If mid-eclipse occurs along the ray defined by the angle β, and the hump first appears when the spot is viewed tangent to the disk, then the angle θ, between first appearance of the hump prior to eclipse and mid-eclipse, determines the angle $\alpha+\beta$ from

$$\alpha+\beta = \theta-90. \tag{5}$$

But the results derived here indicate that $\alpha+\beta$ is not very sensitive to mass fraction. For VV Pup, where $\alpha+\beta = 66°$, Fig. 2 still implies a high mass ratio in favour of the white dwarf as found by WP. However, for U Gem, where $\alpha+\beta = 54°$ according to WP, no reasonable solution seems possible. If, as suggested above, the spot radius exceeds that of the particle model, a solution becomes possible as the shock is displaced upstream along the trajectory from L_1. However, it is also likely that gas outside the spot radius modifies the phase of first appearance of the spot relative to the phase determined from the tangent to the disk through the spot. In summation the variability of observed gaseous features, and the presence of gas outside the predicted spot radius seriously complicate the direct application of these models to interpret CV stars.

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