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# RADIATION PRESSURE ON GRAINS AS A MECHANISM FOR MASS LOSS IN RED GIANTS

SUN KWOK
School of Physics and Astronomy, University of Minnesota, Minneapolis
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## ABSTRACT

A quantitative model is constructed for the process of radiation pressure on dust to explain the mass loss observed in cool giants. Results are given for six sample stars, demonstrating the effects of stellar luminosity, mass, and effective temperature on the mass loss rate. The envelope expansion velocity and average grain size related to the fraction of grain material condensed and the gas density is found to be the key factor in determining the feasibility of mass ejection under this mechanism.

*Subject headings:* late-type stars — mass loss — radiative transfer

## I. INTRODUCTION

It has been known for nearly 40 years that in most M giants, deep, narrow absorption lines are found displaced toward the violet from the normal broad absorption lines by about 2–25 km s$^{-1}$. In 1956, Deutsch interpreted this displaced component as an indication of mass loss from the star after his analysis of the spectrum of the visual companion of $\alpha$ Her (Deutsch 1956).

Since that time, many physical mechanisms have been proposed to explain this phenomenon. These include actions of turbulent motions, vaporization, shock waves, and electromagnetic acceleration. The turbulent velocities implied by the curve of growth are large compared with thermal motions in some cases, but fall far short of the escape velocities needed (Deutsch 1960). Rubbra and Cowling (1959) also demonstrated the insufficiency of other mechanisms. Radiation pressure has always been a popular source of power because of the abundance of energy available. However, Weymann (1962*a*) has shown that radiation pressure on atoms and molecules is probably inadequate.

Hoyle and Wickramasinghe (1962) suggested that radiation pressure acting on dust may drive the gas to escape velocity. Recent infrared observations have indicated the presence of silicate material in the Trapezium region of M42 (Ney and Allen 1969) and in clouds surrounding certain cool stars (Woolf and Ney 1969). Gilman (1969) has shown that graphite and silicon carbide could arise from cool carbon stars, and iron and silicate particles may arise from cool, oxygen-rich stars. Hackwell (1972) found that carbon stars have a common emission feature at 10.8 $\mu$ which could possibly be due to silicon carbide.

After the existence of circumstellar dust had been established, the mechanism of radiation pressure acting on grains became more plausible. So far this process has been only theoretically studied by analytical approximations (Gehrz and Woolf 1971; Gilman 1972). In this paper we shall seek numerical solutions to the equation of motion by including the effects of (*a*) radiation pressure on grains; (*b*) gravitational attraction by the star; (*c*) growth of grains; (*d*) momentum transfer from the grain to the gas; and (*e*) sputtering of the grains.

## II. METHOD OF CALCULATION

We shall assume that the flow is steady and spherically symmetric. The evidence that the flow is steady comes from the fact that no significant variations in the velocity of the flow have been observed, and at least in the case of $\alpha$ Ori, these observations cover a span of 25 years (Weymann 1962*b*). As for spherical symmetry, the star's magnetic field and the interstellar medium will probably cause the flow, particularly at great distances from the star, to deviate from spherical symmetry. However, these are details that go well beyond current observations, and at present there seems to be little justification to abandon the simplicity of spherical symmetry. Under these assumptions we have the following equations:

conservation of mass:
$$\frac{d}{dr}(\rho v r^2) = 0\,; \tag{1}$$

conservation of momentum:

$$\frac{d}{dr}\left(\tfrac{1}{2}v^2\right) = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM_*}{r^2}\left[1 - \mathscr{F}(r)\right], \tag{2}$$

where $v$, $P$, and $\rho$ are the velocity, pressure, and density of the gas, respectively. The function $\mathscr{F}(r)$ is the ratio of the strength of the radiation pressure (via the grains) to the gravitational attraction. $M_*$ is the mass of the star.

The energy, if defined in the usual sense as

$$E = m\left(\tfrac{1}{2}v^2 + \frac{\gamma}{\gamma - 1}\frac{P}{\rho} - \frac{GM_*}{r}\right),$$

is not conserved because energy is being transferred from the radiation field to the gas.

Following the treatments of Bondi (1952) and Holzer and Axford (1970), we choose the Mach number as the dependent variable. The Mach number $M$ is defined by

$$M = v/u ,$$

where $u$ is the speed of sound given by

$$u^2 = \gamma P/\rho .$$

Weymann's work (1960) showed that for temperatures in the range considered here (less than $10^4$ K), radiative heating and cooling can be ignored in the first approximation; thus the only input from the radiation field that needs to be considered is the mechanical energy of the radiation pressure on grains. In this calculation we shall make the assumption that the flow is adiabatic. Making use of the adiabatic relation

$$\frac{d}{dr}(P\rho^{-\gamma}) = 0 , \qquad (3)$$

we arrive at the following equation:

$$\frac{dM^2}{d\xi} = \frac{2M^2}{\xi(M^2 - 1)} \times \left[M^2(\gamma - 1) + 2 - \frac{(\gamma + 1)(1 - \mathscr{F})GM_*}{2r_0\xi u^2}\right] , \qquad (4)$$

where the dimensionless independent variable is chosen to be $\xi = r/r_0$. The quantity $r_0$ is the point where $M = 1$, and will be referred to as the sonic point from now on.

Assuming that the ideal-gas law is valid, we have

$$\frac{P}{\rho} = \frac{kT}{\mu m_{\rm H}} , \qquad (5)$$

where $\mu$ is the mean molecular weight of the gas and $m_{\rm H}$ is the mass of the hydrogen atom. As a result we have

$$u^2 = \frac{\gamma kT}{\mu m_{\rm H}} ,$$

and

$$T = T_0(\rho/\rho_0)^{\gamma - 1} , \qquad (6)$$

where the subscript zero refers to the sonic point. From equation (1) we have

$$\rho v r^2 = \rho_0 v_0 r_0^2 ,$$

or

$$\rho = \rho_0(M^2\xi^4)^{-1/(\gamma+1)} . \qquad (7)$$

The velocity of the gas is then given by

$$v = Mu = \frac{\rho_0 v_0^2}{\rho\xi^2} . \qquad (8)$$

The working of the radiation pressure on grain mechanism relies on the effectiveness of the transfer of momentum from the grains to the gas. In his discussion of mass loss mechanisms, Weymann (1962*a*) rejected this mechanism because of the unreasonable long mean free path of gas between collisions by grains. His results were derived from the relation

$$\frac{\bar{Q}\pi a^2 L}{4\pi c} = GM_* m_{\rm H} ,$$

where $a$ is the size of the grain, $m_{\rm H}$ is the mass of hydrogen atoms, and $\bar{Q}$ is the ratio of the radiation pressure cross section to the geometric cross section. This relation, in fact, assumes that gas molecules can gain momentum only through direct collisions with the grains. It was later pointed out by Gilman (1972) that the mean free path of gas molecules within the gas is very small ($\sim 10^7$ cm), and the majority of the gas molecules gain momentum through collisions with other gas molecules. As a result, the momentum of the grain is diffused throughout the gas.

In addition to the requirement that momentum be effectively distributed, we must also know how much of the momentum received from the radiation field is actually transferred to the gas and how much is retained by the grains themselves. For this reason we shall now consider the equation of motion of the grains,

$$F_{\rm rad} = (\tfrac{4}{3}\pi a^3\rho_s)\frac{dv_{\rm gr}}{dt} + \frac{GM_*(\tfrac{4}{3}\pi a^3\rho_s)}{r^2} + F_{\rm drag} ,$$

where $v_{\rm gr}$ is the velocity of the grain, $\rho_s$ is the density of the grain, and $F_{\rm drag}$ the drag force produced by collision with gas molecules. For late-type giants, it can be shown that for grain sizes of the order of 0.1 $\mu$ the gravitational force on grains is about 1000 times smaller than the radiation force and can therefore be safely neglected. In an approximate solution to the above equation, Gilman (1972) found that for $\alpha$ Ori, the grains will approach terminal velocity over a distance of $\sim 10^{11}$ cm, which is small compared with the characteristic dimensions of the envelope. For the purpose of this calculation, which concerns the dynamics of the circumstellar envelope, the following approximation should be a good one:

$$F_{\rm rad} = F_{\rm drag} .$$

The drag force can be estimated by adopting the following simplified picture: A perfect sphere is moving at a velocity $v_d$ in a field of gas particles with Maxwellian velocities of temperature $T$. After taking the average of encounters from all possible directions, the rate of momentum transfer takes the form

$$F_{\rm drag} = \pi a^2 \rho v_d v_T$$

in the limit of $v_d \ll v_T$, where $v_T = 3/4(3kT/\mu m_{\rm H})^{1/2}$.

In the case of $v_d \gg v_T$, the drag force can be approximated by

$$F_{\rm drag} = \pi a^2 \rho v_d^2 .$$

The following expression gives a convenient way of connecting these two regions:

$$F_{\rm drag} = \alpha\pi a^2 \rho v_d(v_T^2 + v_d^2)^{1/2} .$$

We have also introduced a factor $\alpha$, whose value depends on the elasticity nature of the collisions. In the calculation we shall assume $\alpha = 3/4$.

The radiation force is given by

$$F_{\mathrm{rad}} = \frac{\pi a^2 \bar{Q} L}{4\pi c r^2} .$$

Equating $F_{\mathrm{rad}}$ to the drag force and letting $K = \bar{Q}L/4\pi c\alpha\rho r^2$, we have

$$v_d = \{\tfrac{1}{2}[(4K^2 + v_T^{\,4})^{1/2} - v_T^{\,2}]\}^{1/2} . \tag{9}$$

The dimensionless function $\mathscr{F}(r)$ can be written as

$$\mathscr{F}(r) = \left(\frac{\pi a^2 \bar{Q} L n_{\mathrm{gr}}}{4\pi c r^2}\right) \Big/ \left(\frac{G M_* \rho}{r^2}\right) ,$$

where $n_{\mathrm{gr}}$ is the number density of the grains. In a steady flow, we have from equation (1)

$$\frac{d\mathfrak{M}}{dt} = 4\pi\rho v r^2 ,$$

the integration constant $d\mathfrak{M}/dt$ being the mass loss rate of the star. At each point in the envelope, the grain flux can be related to the gas flux via the following expression:

$$4\pi(\tfrac{4}{3}\pi a^3 \rho s) n_{\mathrm{gr}}(v + v_d) r^2 = \frac{Ayf}{\mu}\frac{d\mathfrak{M}}{dt} ,$$

where $A$ = molecular weight of the grain material, $y$ = relative number abundance of the grain material, $f$ = fraction of grain material condensed, and $\mu$ = mean molecular weight of the gas.

Taking the ratio of the above two equations, we have

$$\frac{n_{\mathrm{gr}}}{\rho} = \frac{3Ayf}{4\pi a^3 \mu \rho_s}\left(\frac{v}{v + v_d}\right) .$$

Substituting into the expression for $\mathscr{F}(r)$, we have

$$\mathscr{F}(r) = \frac{3\bar{Q}LAyf}{16\pi c G M_* \mu a \rho_s}\left(\frac{v}{v + v_d}\right) . \tag{10}$$

Assuming that all grains are formed at the base of the flow and that no grain is totally destroyed in its passage through the envelope, we have

$$f = f_0(a/a_0)^3 . \tag{11}$$

## III. GRAIN GROWTH AND SPUTTERING

As the grains travel through the gas, collisions between the grains and the gas will cause some of the atoms to be adsorbed onto the grain surface. As a result, the grains will grow in size. The growth rate due to this process is

$$\frac{da}{d\xi} = \frac{\beta r_0 y(1 - f)\rho A}{4\mu\rho_s(v + v_d)}(v_T^{\,2} + v_d^{\,2})^{1/2} , \tag{12}$$

where $\rho$ is the probability that an atom of the appropriate grain material will condense on the grain during collision.

When the drift velocity of the grains with respect to the gas is large, the kinetic energy of the gas may exceed the surface potential energy of the grain, and atoms may be knocked out from the grain. The energy dependence of the sputtering yield can be divided into four regions (Wickramasinghe 1972):

$$\begin{aligned} S(E) &= 0, & E &< E_T, \\ &= S_0(E - E_T)/(E_A - E_T), & E_T &< E < E_A, \\ &= S_0, & E_A &< E < 10E_B, \\ &= S_0(10E_B/E), & E &> 10E_B . \end{aligned}$$

$E_T$, $E_B$, $E_A$, and $S_0$ are sputtering parameters depending on both the target material and the impinging atom:

$$E_T \propto (M_t + m)^2/4M_t m , \tag{13a}$$

$$E_A \propto (M_t + m)/M_t , \tag{13b}$$

$$E_B \propto m/M_t , \tag{13c}$$

$$S_0 \propto m(M_t + 1)/(M_t + m) , \tag{13d}$$

where $m$ and $M_t$ are atomic weights of the impinging and target atoms, respectively.

If silicon is assumed to be the target atom, then the parameters for sputtering by hydrogen on silicon are (Wickramasinghe 1972; Kaminsky 1965): $E_T = 25$ eV, $E_A = 849$ eV, $E_B = 1109$ eV, and $S_0 = 0.05$. The corresponding parameters for sputtering of other atoms on silicon are obtained by equations (13a) to (13d).

If $Z_i$ are the relative cosmic abundances of elements, we have the following sputtering rate:

$$\frac{da}{d\xi} = \frac{Ar_0\rho}{4\mu\rho_s}\left(\frac{v_d}{v + v_d}\right)\sum_i Z_i S_i .$$

Combining growth and sputtering, we have the following equation:

$$\frac{da}{d\xi} = \frac{Ar_0\rho}{4\mu\rho_s}\left(\frac{1}{v + v_d}\right) \times \left[\beta y(1 - f)(v_d^{\,2} + v_T^{\,2})^{1/2} - \sum_i Z_i S_i\right] . \tag{14}$$

We can easily see that the relative importance of sputtering and growth by collisions depends mainly on the sputtering energy threshold—in other words, on the magnitude of the drift velocity. As long as the grain material is not completely condensed, sputtering becomes dominating at a drift velocity of about 20 km s$^{-1}$.

## IV. INPUT PARAMETERS

There are seven variables of interest: $v$, $v_d$, $T$, $\rho$, $a$, $f$, and $P$. We also have seven coupled equations (1), (2), (3), (5), (9), (11), and (14). Since four of them are first-order differential equations, four initial conditions are required. The obvious initial point to choose is the sonic point. The four possible parameters are $T_0$, $\rho_0$, $a_0$, and $f_0$.

The flow in the envelope must be supersonic, since a subsonic flow would require too high a density in the envelope. Parker (1960) has shown that for $5/3 > \gamma > 1$, there is a unique point where the solutions can pass from subsonic to supersonic, and only one solution passes through this point.

From equation (4) we can observe that a singularity occurs at $M = 1$. For a steady flow this singularity must vanish, and, in consequence, the following relation must be satisfied:

$$r_0 = \frac{(1 - \mathscr{F}_0)GM_*}{2u_0^2} . \tag{15}$$

This condition further restricts our freedom in choosing the initial conditions. Since $\mathscr{F}(r_0)$ is a function of $a_0$ and $f_0$, therefore only one of these two is a truly free parameter.

In order to have a physically acceptable solution, there are other limitations on the remaining parameters. Assuming that only a small fraction of grain material is condensed at the condensation point, there will be a lot of uncondensed material left in the gas, and the condensed grains will be able to grow rapidly and reach a large size. This will result in a large drift velocity and in turn lead to a large sputtering rate. If sputtering dominates over growth too early, then the gas may never reach the sonic speed. On the other hand, if most of the grain material has already condensed at the base of the flow, the grains will have little chance of growing further and will remain small. In other words, there will be a lot of small grains. However, the grain size cannot be smaller than a certain minimum below which the radiation pressure on the grain is inadequate to overcome the gravitational force. Since the nucleation process is unknown, and may well be different for different stars, it is very difficult for us to estimate the fraction of grain material condensed at the base of the flow. Therefore we shall leave $a_0$ (or equivalently $f_0$) as a free parameter, and calculate all the possible cases.

Before we actually go into the numerical calculations, we shall now try to roughly estimate the mass loss rate under this mechanism. At the sonic point, $\mathscr{F}(r)$ must have attained a value close to unity. From equation (5), and using the relation

$$\frac{v_d}{v} = \left[\frac{\bar{Q}L}{\alpha c v_0 (d\mathfrak{M}/dt)}\right]^{1/2} ,$$

we have

$$\frac{d\mathfrak{M}}{dt} \approx \left(\frac{\bar{Q}L}{\alpha c v_0}\right)\left[\frac{3Ayf}{16\pi c G \mu \rho_s}\left(\frac{\bar{Q}}{a}\right)_{r=r_0}\left(\frac{L}{M_*}\right) - 1\right]^{-2} .$$

Recognizing that the Eddington limit luminosity is given by

$$L_{\mathrm{Ed}} = \frac{16\pi c G \rho_s}{3\bar{Q}_0} M_* ,$$

the mass loss rate can be expressed as

$$\frac{d\mathfrak{M}}{dt} \approx \left(\frac{\bar{Q}L}{\alpha c v_0}\right)\left[\left(\frac{Ayf}{\mu}\right)\frac{L}{L_{\mathrm{Ed}}} - 1\right]^{-2} . \tag{16}$$

Since $f < 1$, we have

$$\frac{d\mathfrak{M}}{dt} > 3.16 \times 10^{17} \bar{Q}_0 \left(\frac{L}{L_\odot}\right) \times \left[7.3 \times 10^{-8}\left(\frac{\bar{Q}}{a}\right)_{r=r_0}\left(\frac{L}{L_\odot}\right)\left(\frac{M_\odot}{M_*}\right) - 1\right]^{-2} , \tag{17}$$

where we have assumed the sonic-point temperature to be 3000 K.

If we take into account the effects of sputtering by excluding cases that have $v_d > 20\ \mathrm{km\ s^{-1}}$, we can estimate the minimum mass loss rate for any star, given its luminosity, mass, and effective temperature. Table 1 lists the minimum rates for six sample stars.

## V. RESULTS

Calculations have been performed for the circumstellar envelopes of six different stars. The grain material is assumed to be magnesium silicate ($Mg_2SiO_4$), appropriate for oxygen-rich stars (Gilman 1969). We have used the minimum condensation radius calculated by Gilman (Gilman and Woolf 1974) as the base of the flow. The values of the mean radiative pressure cross section $\bar{Q}(a, T_e)$ are taken from Gilman (1974*b*). The value of $\gamma$ is chosen to be 5/3.

The velocity gradient $dM/d\xi$ at the sonic point is first derived using the initial conditions. Next, a

TABLE 1

MINIMUM MASS LOSS RATES FOR SIX SAMPLE STARS

| | $L = 10^4 L_\odot$ $M_* = 1 M_\odot$ | | | $L = 10^5 L_\odot$ $M_* = 5 M_\odot$ | | | $L = 10^6 L_\odot$ $M_* = 25 M_\odot$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $T_e$ (K) | $\frac{d\mathscr{M}}{dt}$ ($10^{20}$ g s$^{-1}$) | $\frac{d\mathscr{M}}{dt}$ ($10^{-6}$ $M_\odot$ yr$^{-1}$) | $a_0$ ($\mu$) | $\frac{d\mathscr{M}}{dt}$ ($10^{20}$ g s$^{-1}$) | $\frac{d\mathscr{M}}{dt}$ ($10^{-6}$ $M_\odot$ yr$^{-1}$) | $a_0$ ($\mu$) | $\frac{d\mathscr{M}}{dt}$ ($10^{20}$ g s$^{-1}$) | $\frac{d\mathscr{M}}{dt}$ ($10^{-6}$ $M_\odot$ yr$^{-1}$) | $a_0$ ($\mu$) |
| 2000....... | 0.35 | 0.55 | 0.15 | 1.1 | 1.8 | 0.11 | 3.8 | 6.0 | 0.08 |
| 3000....... | ... | ... | ... | 0.62 | 0.98 | 0.06 | 1.9 | 3.1 | 0.045 |
| 5000....... | ... | ... | ... | ... | ... | ... | 1.0 | 1.6 | 0.02 |

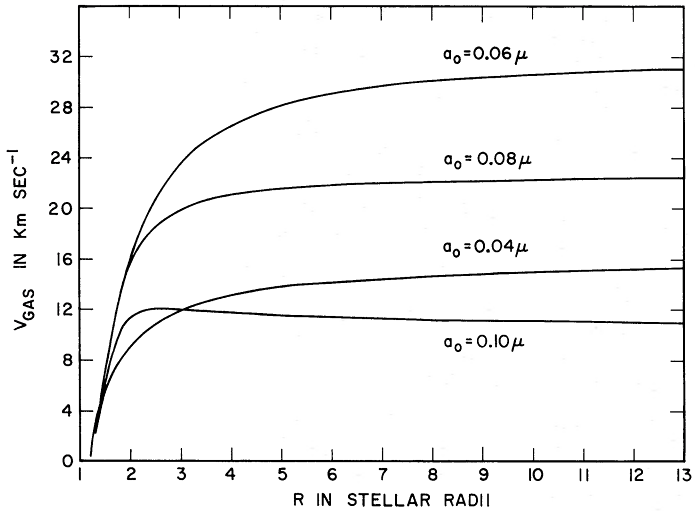


FIG. 1.—The gas velocity as a function of distance for a star of $L = 10^5 L_\odot$, $M_* = 5M_\odot$, $T_e = 3000$ K, $d\mathscr{M}/dt = 2 \times 10^{20}$ g s$^{-1}$. The number labeling each curve is the dust grain size at the sonic point.

series of iterations is performed to determine the position of the sonic point. The solution is then obtained by integrating both inward and outward from the sonic point.

Figure 1 shows the gas velocity as a function of distance for a star of luminosity $10^5$ $L_\odot$, mass 5 $M_\odot$, and effective temperature 3000 K. The sonic-point temperature is chosen to be 3000 K, and the mass loss rate $2 \times 10^{20}$ g s$^{-1}$. Four solutions corresponding to different values of sonic-point grain sizes are shown. We can see that between the extremes of many small grains and a few large grains lies the optimal expansion velocity. This result is expected from our discussion in the last section.

All these velocity curves show a characteristic sharp rise and then stabilize at a constant velocity over a large part of the circumstellar envelope. This is in agreement with the established narrowness of the circumstellar lines.

Figure 2 shows the corresponding absorption-line profile of the $a_0 = 0.08\ \mu$ case for an atom whose state is independent of position in the envelope and for which the line is produced by pure absorption. A violet-displaced circumstellar line can clearly be seen.

Table 2 shows the results for six sample stars. The first mass loss rate for each star corresponds to the lowest possible. In all cases, the mass loss rate was chosen such that the total mass column density of the grains will not exceed $10^{-3}$ g cm$^{-2}$. A higher mass loss rate (and, therefore, a higher mass column density) will mean that the envelope is optically thick. Since the effects of radiative transfer are not included in this calculation, we did not consider such cases.

Note that the agreement between Tables 1 and 2 is better for hotter stars. This is because results of Table 1 are derived under the assumption of complete condensation. In hotter stars, the allowed grain sizes are smaller and grain condensation is more complete; therefore, there is better agreement.

When the fraction of grain material condensed is small, the grains tend to grow to a large size, and as a result they receive a greater initial push from the star's radiation field. In cases of complete or near complete condensation, not only are the grains smaller and have

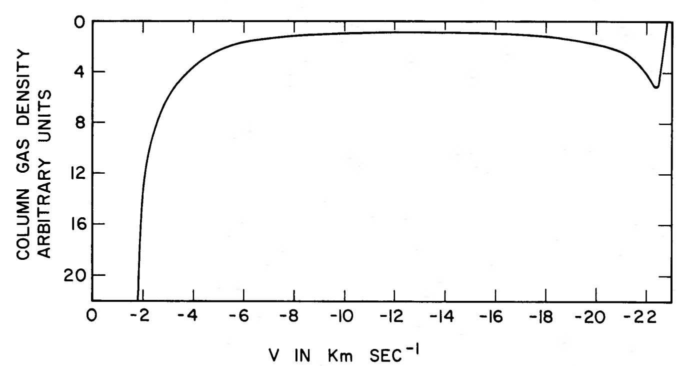


FIG. 2.—The stellar absorption line profile produced by pure absorption of a material whose state does not depend on temperature and density.

TABLE 2

NUMERICAL RESULTS OF THE MODEL FOR SIX SAMPLE STARS

| Star | $\frac{d\mathcal{M}}{dt}$ ($10^{20}$ g s$^{-1}$) | $a_0$ ($\mu$) | Gas Terminal Velocity (km s$^{-1}$) | Mass Column Density of the grains ($10^{-4}$ g cm$^{-1}$) | $f_0$ |
|---|---|---|---|---|---|
| $L = 10^4 L_\odot$, $M = 1 M_\odot$, $T_e = 2000$ K...... | 1.8 | 0.08 | 8.1 | 7.6 | 0.67 |
| | | 0.10 | 12.3 | 4.7 | 0.48 |
| | | 0.16 | 16.8 | 1.7 | 0.26 |
| | 2.0 | 0.08 | 8.4 | 8.7 | 0.68 |
| | | 0.12 | 15.8 | 3.7 | 0.38 |
| | | 0.15 | 17.9 | 2.3 | 0.29 |
| | | 0.22 | 14.6 | 0.8 | 0.16 |
| | 2.2 | 0.08 | 8.7 | 9.7 | 0.69 |
| | | 0.20 | 18.9 | 1.2 | 0.20 |
| | | 0.25 | 13.1 | 0.7 | 0.14 |
| $L = 10^5 L_\odot$, $M = 5 M_\odot$, $T_e = 2000$ K...... | 3.0 | 0.06 | 5.2 | 4.2 | 0.85 |
| | | 0.12 | 15.6 | 1.0 | 0.44 |
| | | 0.19 | 6.4 | 0.3 | 0.26 |
| | 5.0 | 0.06 | 6.6 | 8.1 | 0.85 |
| | | 0.12 | 24.5 | 1.7 | 0.39 |
| | | 0.20 | 10.4 | 0.6 | 0.24 |
| | 7.0 | 0.06 | 8.9 | 10.0 | 0.83 |
| | | 0.10 | 25.7 | 3.1 | 0.43 |
| | | 0.20 | 14.9 | 0.7 | 0.22 |
| | | 0.22 | 10.4 | 0.6 | 0.18 |
| $L = 10^6 L_\odot$, $M = 25 M_\odot$, $T_e = 2000$ K..... | 5.0 | 0.06 | 7.1 | 1.4 | 0.87 |
| | | 0.08 | 12.4 | 0.7 | 0.71 |
| | | 0.10 | 5.5 | 0.5 | 0.58 |
| | 7.0 | 0.06 | 12.8 | 1.9 | 0.84 |
| | | 0.08 | 18.5 | 1.0 | 0.65 |
| | | 0.11 | 7.5 | 0.6 | 0.49 |
| | 10.0 | 0.05 | 7.7 | 4.2 | 0.91 |
| | | 0.08 | 24.4 | 1.3 | 0.56 |
| | | 0.12 | 9.9 | 0.6 | 0.40 |
| $L = 10^5 L_\odot$, $M = 5 M_\odot$, $T_e = 3000$ K...... | 1.0 | 0.05 | 15.7 | 1.3 | 0.84 |
| | | 0.07 | 15.1 | 0.6 | 0.62 |
| | | 0.08 | 9.0 | 0.5 | 0.55 |
| | 2.0 | 0.04 | 15.3 | 3.7 | 0.85 |
| | | 0.06 | 31.0 | 1.4 | 0.56 |
| | | 0.08 | 22.6 | 0.8 | 0.43 |
| | | 0.10 | 10.7 | 0.6 | 0.36 |
| | 3.0 | 0.04 | 20.7 | 4.7 | 0.76 |
| | | 0.09 | 24.6 | 0.9 | 0.33 |
| | | 0.12 | 11.4 | 0.7 | 0.28 |
| $L = 10^6 L_\odot$, $M = 25 M_\odot$, $T_e = 3000$ K..... | 4.0 | 0.04 | 27.8 | 1.1 | 0.77 |
| | | 0.05 | 17.6 | 0.7 | 0.63 |
| | | 0.06 | 8.3 | 0.6 | 0.54 |
| | 6.0 | 0.03 | 22.3 | 3.0 | 0.88 |
| | | 0.05 | 27.9 | 0.9 | 0.54 |
| | | 0.07 | 8.7 | 0.7 | 0.40 |
| | 10.0 | 0.03 | 29.8 | 3.6 | 0.76 |
| | | 0.06 | 28.8 | 0.8 | 0.38 |
| | | 0.09 | 7.0 | 0.7 | 0.27 |
| $L = 10^6 L_\odot$, $M = 25 M_\odot$, $T_e = 5000$ K..... | 1.0 | 0.018 | 8.8 | 0.4 | 0.91 |
| | | 0.020 | 10.5 | 0.3 | 0.85 |
| | | 0.022 | 5.4 | 0.2 | 0.79 |
| | 2.0 | 0.015 | 18.4 | 0.7 | 0.87 |
| | | 0.020 | 25.0 | 0.4 | 0.68 |
| | | 0.025 | 15.5 | 0.3 | 0.56 |
| | 4.0 | 0.015 | 30.7 | 1.1 | 0.70 |
| | | 0.020 | 39.0 | 0.6 | 0.53 |
| | | 0.030 | 20.3 | 0.3 | 0.37 |

a slower acceleration, but also the sputtering rate can dominate over growth at moderate velocities such as 10 km s$^{-1}$ (see eq. [14]). This greatly limits the value of the terminal drift velocity. Figure 3 shows the drift velocity as a function of distance in three different degrees of condensation.

From Table 2 we observe that the maximum allowed grain size decreases with increasing $L/M_*$ and increasing effective temperature. The main limiting factor here is sputtering. A large grain for stars with high $L/M_*$ is not permissible because this will lead to a high drift velocity and a large sputtering rate.

## VI. DOMAIN OF MASS LOSS

One very important question remains: Within what limits will this mechanism be applicable? From our

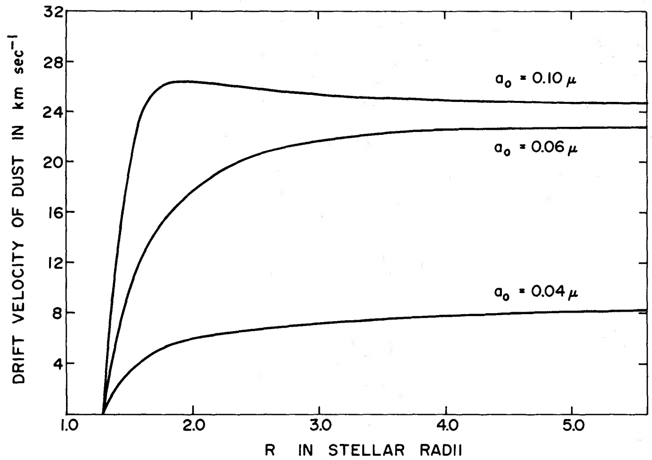


FIG. 3.—The drift velocity as a function of distance

past discussions we can see that the success of this mechanism relies on four physical parameters: the stellar luminosity $L$, stellar mass $M_*$, condensation-point gas density $\rho_c$, and the effective temperature $T_e$ (through its influence on the efficiency factor $\bar{Q}$). For a star of mass 1 $M_\odot$, the Eddington limit luminosity is

$$L_{\mathrm{Ed}} = \frac{16\pi c G \rho_s}{3} M_*(a/\bar{Q})$$

$$\approx 5 \times 10^4 (a/\bar{Q}) L_\odot .$$

For a star of 3000 K, $L_{\mathrm{Ed}}$ varies from $\sim 500\, L_\odot$ (if $a = 10^{-7}$ cm) to $5\, L_\odot$ (if $a = 10^{-5}$ cm). Therefore, the Eddington limit can easily be satisfied in the case of late-type giants. This implies that in the absence of gas the loss of dust particles will probably be occurring (Wickramasinghe 1972; Gilman 1973). The question now is whether the gas drag is important, and if so, whether the gas can be ejected. As we have shown, among the stars that we are considering, the $L$, $M_*$, $T_e$ combination is generally sufficient to drive the grains; therefore the most critical factor is $\rho_c$. Not only has $\rho_c$ to be large enough to ensure momentum coupling, it has also to be large enough to maintain a low drift velocity in order to avoid an unacceptable sputtering rate. The lower $L/M_*$ is, the larger $\rho_c$ has to be.

Having recognized the importance of $\rho_c$ in determining the occurrence of mass loss, we may ask what controls $\rho_c$. Gilman has shown that the significant determiner of the grain temperature is the radiation field and not the gas temperature (Gilman and Woolf 1975). Therefore, the condensation-point distance will be a function of the effective temperature. Using his results, we shall now attempt to define a domain where mass loss is theoretically possible.

Assuming that the effective temperature represents the kinetic temperature of the stellar atmosphere, the gas density at the condensation point is

$$\rho_c' = \frac{x}{h} \exp(-r_c/h) ,$$

where $r_c$ is the distance of the condensation point from the surface of the star and $x$ is the mass column density of the gas in the photosphere. A value of $x = 300$ gm cm$^{-2}$ inferred from the opacity is used in this calculation. Let us further assume that stars of interest obey a mass-luminosity law in the form of

$$\frac{L}{L_\odot} = 10^4 \left(\frac{M}{M_\odot}\right)^{0.82} .$$

TABLE 3

RATIO OF GAS DENSITY AVAILABLE AT THE CONDENSATION POINT TO MINIMUM REQUIRED GAS DENSITY FOR MASS EJECTION

| | | $T_e$ (K) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Log $(L/L_\odot)$ | $M/M_\odot$ | 3600 | 3400 | 3200 | 3000 | 2800 | 2600 | 2400 | 2200 | 2000 |
| 5.4 ........ | 50 | 8.3(−53) | 5.5(−37) | 4.9(−26) | 8.8(−18) | 6.4(−12) | 4.9(−8) | 1.3(−4) | 2.7(−2) | 1.2(+1) |
| 5.2 ........ | 27 | 5.1(−37) | 1.4(−25) | 8.8(−18) | 7.3(−12) | 1.1(−7) | 6.7(−5) | 1.9(−2) | 1.0(+0) | 6.9(+1) |
| 5.0 ........ | 16 | 1.2(−25) | 2.0(−17) | 7.6(−12) | 1.3(−7) | 1.4(−4) | 1.2(−2) | 6.8(−1) | 1.2(+1) | 2.5(+2) |
| 4.8 ........ | 9.4 | 2.1(−17) | 1.4(−11) | 1.4(−7) | 1.6(−4) | 2.3(−2) | 6.1(−1) | 9.3(+0) | 7.4(+1) | 6.3(+2) |
| 4.6 ........ | 5.3 | 1.6(−11) | 2.4(−7) | 2.0(−4) | 2.6(−2) | 9.0(−1) | 9.7(+0) | 7.4(+1) | 2.7(+2) | 1.2(+3) |
| 4.4 ........ | 3.1 | 2.7(−7) | 2.6(−4) | 3.3(−2) | 1.0(+0) | 1.3(+1) | 7.2(+1) | 3.0(+2) | 7.8(+2) | 2.0(+3) |
| 4.2 ........ | 1.7 | 3.6(−4) | 4.7(−2) | 1.3(+0) | 1.7(+1) | 8.8(+1) | 3.1(+2) | 7.7(+2) | 1.6(+3) | 3.3(+3) |
| 4.0 ........ | 1.0 | 5.5(−2) | 1.7(+0) | 1.8(+1) | 1.1(+2) | 4.1(+2) | 8.9(+2) | 1.6(+3) | 2.6(+3) | 4.3(+3) |



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Then the scale height is given by

$$\frac{h}{R_*} = \frac{1}{2G(4\pi\sigma)^{1/2}} \left(\frac{3kT}{\mu M_{\rm H}}\right) \frac{L}{T_e^2} \left(\frac{L_\odot^{1/2}}{M_\odot}\right) \times (10^4)^{0.82} \left(\frac{L}{L_\odot}\right)^{-0.71} . \quad (18)$$

Table 3 gives the ratio of the gas density available at the base of the flow ($\rho_c'$) to the required gas density for mass ejection ($\rho_c$), the latter being calculated from the minimum mass loss rate given by equation (17). The gas velocity at the condensation point is taken to be 1 km s$^{-1}$. Mass loss is theoretically possible for those stars with this ratio greater than unity.

Results shown in Table 3 seem to preclude any early M luminous stars from ejecting mass. Grains can only form at a large distance from a hot star, and at such a point the gas density may be too low. However, some early M supergiants are known to be losing mass; $\alpha$ Ori is a good example (Weymann 1962*b*). Gilman and Woolf (1975) have suggested that the large macroscopic motion in the atmosphere may increase the scale height and bring enough gas to the condensation point. They found that the required turbulent velocities for $\alpha$ Ori, $\alpha$ Her, $\mu$ Cep, and HR 5171 are in good agreement with their observed values. Another piece of evidence comes from the observation of mid-infrared excesses in some low-surface-gravity stars, which according to Gilman (1974*a*) may also imply the existence of photospheric turbulence.

## VII. COMPARISON WITH OBSERVATIONS

Two methods have been devised to measure the rate of mass ejection from cool stars. Deutsch (1956) and Weymann (1962*b*) have employed an optical method which used the curve of growth to measure the surface density of Ca II ions. This density is related to the total number of Ca atoms and therefore to the total number of hydrogen atoms. Assuming a size and expansion velocity of the envelope, the mass loss rate can be obtained. Another way of estimating this rate is by means of infrared observation. Gehrz and Woolf (1971) have measured the [3.5 $\mu$] − [8.4 $\mu$] and [8.4 $\mu$] − [11 $\mu$] color excesses for M stars. From these measurements it is possible to obtain the optical depth of the dust and its amount. Assuming a complete condensation of silicates, the mass loss rate can then be determined.

Although the absolute values of the mass loss rates estimated by the above methods differ from each other, they nevertheless show similar trends of behavior, which can be summarized by the following rules: (1) The mass loss rate is greater for more luminous stars (Deutsch 1960). (2) The mass loss rate increases for decreasing effective temperature. The rates probably more than double for each change of spectral class (Gehrz and Woolf 1971; Weymann 1963). (3) Compared with the mass loss rates for similar stars, there seems to be a large spread for individual cases (Gehrz and Woolf 1971).

Results of our calculations as shown in Table 1 and 2 are in agreement with the above rules. For a star with larger luminosity, more momentum is fed into the outflowing motion, and mass loss is possible even if the drift velocity is large. On the other hand, a star with larger mass will have a larger gravitational attraction, and cannot afford to have a large drift velocity. Since the drift velocity is inversely proportional to the square root of density (and the mass loss rate), therefore mass ejection cannot occur unless a higher gas density at the base of the flow inhibits the growth of drift velocity. Equation (17) shows that the mass loss rate is roughly proportional to $M_*^2/L$. For stars with a mass-luminosity relation $L = aM_*^b$ and $b < 2$ (which is appropriate for late-type giants), a more luminous star will always be observed to have a higher mass loss rate.

The fact that stars in the later spectral classes have a higher mass loss rate can also be understood in this model. Since $\bar{Q}(T_e, a)$ increases with $T_e$ if $a > 10^{-6}$ cm and $T_e > 2000$ K (Gilman 1974*b*), a star with a higher surface temperature can drive grains more efficiently and, therefore, by the same argument above, is able to eject mass on a smaller scale. The more important factor, however, is the fact that the condensation-point gas density is higher for cool stars than for warm stars. As a result, more mass will be ejected.

As for the third empirical relation, the mass loss process can take place, according to this model, whenever the condensation-point gas density exceeds a certain minimum value. This implies that mass loss can occur on various scales, depending on the actual scale height of the stellar atmosphere.

## VIII. SUMMARY AND CONCLUSION

A quantitative model of the circumstellar envelope of cool stars has been constructed. It is found that once dust condensation takes place, radiation pressure on grains can drive the gas to the observed expansion velocities, under the condition that the gas density at the condensation point is high enough. The range of grain sizes can be determined if the properties of the star are known. Admittedly, the upper bound of grain size is less certain, due to its dependence on the sputtering parameters. A correlation between grain sizes and $L/M_*$, $T_e$ is also predicted. Although we did not discuss the nucleation of grains, this model nevertheless provides a link between the observed expansion velocity and the nucleation process, due to the fact that the expansion velocity is a function of the fraction of silicates condensed at the base of the flow. It was also pointed out that in the case of red giants, the condensation-point gas density is the main factor in determining whether mass loss is possible. The theoretical relation between $L$, $M_*$, $T_e$ and the mass loss rate is also found to be consistent with the observations. The mass loss rates calculated by this model are possibly more reliable than those determined by observational means because of fewer assumptions involved.

OH and $H_2O$ maser emissions have been observed in the circumstellar envelopes of many late-type stars (Hyland *et al.* 1972; Wilson *et al.* 1972; Dickinson *et al.* 1973). The results of the present dynamic model, when combined with a particular pumping mechanism, can give prediction to the spectra of maser emissions. A calculation under the simplifying assumption of total saturation has already been performed (Kwok, Gilman, and Woolf 1975), but a more detailed calculation remains to be done.

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SUN KWOK: Department of Physics, University of British Columbia, Vancouver, B.C., Canada V6T 1W5