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TEST FOR THE EXISTENCE OF GRAVITATIONAL RADIATION*

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ABSTRACT

Predictions are presented for the gravitational-radiation–induced decrease of the orbital period and eccentricity of the recently discovered radio pulsar in a binary system, in terms of the mass ratio and angle of inclination of the orbit. It is shown how measurements of the rotation of the periastron and the term of order $(v/c)^2$ in the pulsar period variation would allow these two unknowns to be determined.

Subject headings: binaries — gravitation — pulsars

The discovery by Hulse and Taylor (1975) of a radio pulsar (PSR 1913+16) in a binary system of short period (7.75 hours) appears to provide a new testing ground for gravitation theory. Indeed, it may be the first system to be found which has observable relativistic effects much larger than those in the solar system and which is at the same time relatively free of complicating astrophysical effects. In this *Letter* we will not discuss in detail the many tests that may be possible, but instead will present definite predictions of the observable consequences of gravitational radiation and other general-relativistic effects on such binaries.

The essence of the test is the fact that the loss of energy and angular momentum due to gravitational radiation results in a decrease in the orbital period and eccentricity of the binary, both of which are obtainable from the observed time variation of the pulsar period. A related test has been proposed by Faulkner (1971), who found that the reduction in the separation of U Geminorum cataclysmic variable binaries due to gravitational radiation could induce the observed rate of mass transfer in such systems. Unfortunately, theoretical and observational uncertainties preclude an accurate quantitative test at this time. However, it should be noted that Faulkner did mention the possibility of looking for a decrease in orbital periods.

Let us consider instead a binary consisting of well-separated masses m_1 and m_2 whose relative orbit is characterized by period P, eccentricity ϵ, semimajor axis a, inclination angle i, and angle ω between the ascending node and periastron point, as usually defined. In evaluating the secular effects of gravitational radiation on nonrelativistic systems, one can neglect all other post-Newtonian effects on the orbits. The energy E and angular momentum L (normal to the orbital plane) of the binary system are then given by $E = -\frac{1}{2} Gm_1m_2a^{-1}$ and $L^2 = Gm_1{}^2m_2{}^2M^{-1}a(1-\epsilon^2)$, while their slow changes due to gravitational radiation loss are (when time-averaged)

$$\frac{dE}{dt} = -\frac{32}{5}\frac{G^4m_1{}^2m_2{}^2M}{a^5}F(\epsilon)\ , \tag{1}$$

$$\frac{dL}{dt} = -\frac{32}{5}\frac{G^{7/2}m_1{}^2m_2{}^2M^{1/2}}{a^{7/2}}G(\epsilon) \tag{2}$$

(Peters and Mathews 1963; Peters 1964), where $M = m_1 + m_2$, $c = 1$, and

$$F(\epsilon) = (1-\epsilon^2)^{-7/2}(1 + \frac{73}{24}\epsilon^2 + \frac{37}{96}\epsilon^4)\ , \tag{3}$$

$$G(\epsilon) = (1-\epsilon^2)^{-2}(1 + \tfrac{7}{8}\epsilon^2)\ . \tag{4}$$

Also using Kepler's law $P = 2\pi a^{3/2}(GM)^{-1/2}$, one finds that the evolution of the period and eccentricity are given by

$$\frac{1}{P}\frac{dP}{dt} = -\frac{3}{2E}\frac{dE}{dt} = -\frac{96}{5}\frac{G^3m_1m_2M}{a^4}F(\epsilon) \tag{5}$$

$$\frac{P}{\epsilon}\frac{d\epsilon}{dP} = \frac{19}{18}H(\epsilon)\ , \tag{6}$$

$$H(\epsilon) = (1-\epsilon^2)(1+\frac{121}{304}\epsilon^2)(1+\frac{73}{24}\epsilon^2+\frac{37}{96}\epsilon^4)^{-1}\ . \tag{7}$$

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Now in the present case one is observing the variation with time of the pulse period Δt_r of one member of the binary (designated by the subscript 1) due to its orbital motion. This can be shown to be related to its "center-of-mass pulse period" $\Delta t_{\rm cm}$ by

$$\Delta t_r = \Delta t_{\rm cm}[1 + \boldsymbol{n}\cdot\boldsymbol{v}_1 + \tfrac{1}{2}v_1^2 + Gm_2/r + O(\zeta^3)]$$
$$= \Delta t_{\rm cm}\{C_{(0)} + C_{(1)}(1 - \epsilon\cos u)^{-1}[(1-\epsilon^2)^{1/2}\cos\omega\cos u - \sin\omega\sin u] + C_{(2)}(1-\epsilon\cos u)^{-1} + O(\zeta^3)\}\ , \quad (8)$$

where $\boldsymbol{v}_1$ is the velocity of body 1 with respect to the center of mass of the binary, which moves with velocity $\boldsymbol{v}_{\rm cm}$ with respect to the observer, taken to move with the center of mass of the solar system. The unit vector $\boldsymbol{n}$ points from the observer to the source, as measured in the frame moving with the center of mass of the binary. The center-of-mass period is related to the emitted period Δt_e by the unknown Lorentz transformation $\Delta t_{\rm cm} = \Delta t_e[1 + \boldsymbol{n}\cdot\boldsymbol{v}_{\rm cm} + O(v_{\rm cm}{}^2)]$.

We have introduced the dimensionless relativity parameter

$$\zeta \equiv \left(\frac{2\pi GM_\odot}{P}\right)^{1/3} \sim v\ , \quad (9)$$

so that the constants $C_{(n)} \sim \zeta^n$. The mean eccentric anomaly u is related to the time of pulse reception t_r by

$$t_r = \text{const.} + (P_{\rm cm}/2\pi)\{u - \epsilon\sin u + C_{(1)}\,[(1-\epsilon^2)^{1/2}\cos\omega\sin u + \sin\omega(\cos u - \epsilon)] + O(\zeta^2)\}\ ,$$

in terms of which the separation of the masses is $r = a(1 - \epsilon\cos u)$. Note that $C_{(0)} = 1 + O(\zeta^2)$, while the second term on the right-hand side of equation (8) represents the usual "radial velocity," and the third term includes the effects of "transverse Doppler shift" and gravitational redshift.

In the usual analysis of the radial-velocity curve $\Delta t_r(t_r)$, one can obtain the constants P, ϵ, ω, and

$$C_{(1)} = \zeta(f_1/M_\odot)^{1/3}\ , \quad (10)$$

where the mass function $f_1 = M^{-2}(m_2\sin i)^3$. In terms of these observables, equation (5) becomes

$$\frac{1}{P}\frac{dP}{dt} = -2.50\times 10^{-9}\,\frac{F(\epsilon)(f_1/M_\odot)^{5/3}X(1+X)^3}{\sin^5 i(P/10^4\ \text{s})^{8/3}}\ \text{year}^{-1}\ , \quad (11)$$

where the mass ratio $X = m_1/m_2$. Using the data $P = 2.791\times 10^4$ s, $\epsilon = 0.615$, and $f_1 = 0.13\ M_\odot$ for this object (Hulse and Taylor 1975) gives

$$\frac{1}{P}\frac{dP}{dt} = -6.3\times 10^{-11}\,\frac{X(1+X)^3}{\sin^5 i}\ \text{year}^{-1}\ . \quad (12)$$

Note that although the present lack of information about X and i precludes a definite prediction for the rate of decrease of orbital period, the relation between the decrease in the eccentricity and period of this system is found from equation (6) to have the definite value

$$\frac{P}{\epsilon}\frac{d\epsilon}{dP} = 0.342\ . \quad (13)$$

However, it would appear difficult to measure ϵ to the accuracy required to test this prediction. On the other hand, values of $|P^{-1}dP/dt| \gtrsim (10^9\ \text{years})^{-1}$ (which would be predicted if $m_1 \geq M_\odot$) may be observable within a few years.

From the value of P quoted above, the relativity parameter for this system is $\zeta = 1.035\times 10^{-3}$. Thus when one measures the variation of Δt_r to a relative accuracy of $\zeta^2 \sim 10^{-6}$, it is seen from equation (8) that one can also determine the constant

$$C_{(2)} = \zeta^2\left(1 + \frac{m_2}{M}\right)\left(\frac{m_2}{M}\right)^{1/3}\left(\frac{m_2}{M_\odot}\right)^{2/3} = \zeta^2(2+X)(1+X)^{-2}(M/M_\odot)^{2/3}\ . \quad (14)$$

Unfortunately, at present $\omega = 179°$ (Hulse and Taylor 1975), so an accurate measurement of $C_{(2)}$ can only be made when ω changes sufficiently, as can be seen from equation (8). In any case, a measurement of $C_{(2)}$ requires a time span long enough to allow the time variation of ω to be significant in equation (8). The third equation necessary to determine m_1, m_2, and i is that for the periastron shift.

$$\frac{d\omega}{dt} = \frac{6\pi GM}{aP(1-\epsilon^2)} = \frac{6\pi\zeta^2}{(1-\epsilon^2)P}\left(\frac{M}{M_\odot}\right)^{2/3}\ . \quad (15)$$

For this pulsar, we obtain $d\omega/dt = 2.10\,(M/M_\odot)^{2/3}$ degrees per year. Although the effect of this rotation of the orbit on the observed pulsar period variation is formally of order ζ^3, in reality it is larger than the neglected terms in equation (8) because it accumulates with time. This is also the reason why the gravitational radiation effects may be observable.

Of course, all these predictions are valid only if the effects of tidal interactions and mass loss are negligible. There is a fair chance that this will be the case, for the following reasons. The minimum separation of the masses is

$$a(1-\epsilon) = GM_\odot\,\zeta^{-2}(M/M_\odot)^{1/3}\,(1-\epsilon) = 5.31 \times 10^{10}(M/M_\odot)^{1/3}\,\text{cm} = 0.763(M/M_\odot)^{1/3}R_\odot\,.$$

If the companion of the pulsar were an ordinary star, then the proximity of its surface to the neutron star should have produced observable effects of mass transfer on the pulsar, such as are seen in compact X-ray sources. In addition, Hulse and Taylor (1975) have shown that the absence of eclipses rules out the possibility that the companion is a main-sequence star. If the companion is a white dwarf, neutron star, or black hole, tidal interactions probably have negligible effects on the orbit (Will 1975).

On the other hand, mass loss from the binary will cause the orbital period P to increase. For instance, an energy output from the pulsar of 10^{37} ergs s^{-1} in the form of relativistic particles corresponds to a mass loss of $M^{-1}|dM/dt| \sim P^{-1}dP/dt \sim 10^{-10}$ year^{-1}. Any nonrelativistic matter lost by the pulsar would increase this number.

In any case, if X and i can be determined and the resulting change in the orbital period predicted by equation (12) is observed, it certainly will provide strong evidence for the existence of gravitational radiation.

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Note added in proof.—Equation (12) may be put in the form

$$\frac{1}{P}\frac{dP}{dt} = -0.96 \times 10^{-9}\,\frac{(M/M_\odot)^{4/3}}{\sin i}\left[1 - 0.507\,\frac{(M/M_\odot)^{-1/3}}{\sin i}\right]\text{yr}^{-1}\,,$$

which is more useful when the periastron shift has been measured.

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