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THE FORMATION OF THE NUCLEI OF GALAXIES. I. M31

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ABSTRACT

Globular clusters passing near the center of M31 interact with the background stars through dynamical friction and spiral in to the center of the galaxy, where they are tidally disrupted by interactions with the growing nucleus. By this process a distinct high-density central nucleus of mass $\sim 5 \times 10^7\ M_\odot$ will be formed in 10^{10} years. These predictions are consistent with recent Stratoscope balloon observations of the nucleus of M31.

Subject headings: galactic nuclei — globular clusters — stellar dynamics — M31

I. INTRODUCTION

Several of the galaxies in the Local Group have distinct central nuclei of diameter $\lesssim 10$ pc, and similar structures presumably exist in more distant galaxies. We propose that such nuclei are naturally and inevitably formed as globular clusters spiral in to the center of their galaxy in the process of attempting to reach equipartition of energy with the field stars of the galaxy. Here we show that this hypothesis predicts many of the observed properties of the nucleus of M31, reserving for a subsequent paper details and applications to other galaxies.

II. THEORY

We consider a globular cluster of mass m and core radius $a_{\rm cl}$ moving with speed v in a galaxy represented locally by a Maxwellian distribution of stars of mass m_f, density ρ, and mean square velocity $\langle v^2 \rangle \equiv 3/2j^2$. Its motion is dominated by the large-scale, time-independent potential field of the galaxy, but it also experiences random perturbations in its velocity due to encounters with individual field stars, which cause it to evolve toward equipartition of energy with the stars. For $jv \sim 1$ and $m \gg m_f$, only the perturbations of the velocity vector parallel to itself (dynamical friction) are important (Spitzer 1962), and the diffusion coefficient is

$$\left(\frac{dv_\parallel}{dt}\right)_{\rm fr} \equiv \langle \Delta v_\parallel \rangle$$

$$= -4\pi G^2 m\rho \ln \Lambda \frac{[\varphi(jv) - jv\varphi'(jv)]}{v^2}\,, \quad (1)$$

where φ is the error function. The exact diffusion coefficient for impact parameters between $r_{\rm min}$ and $r_{\rm max}$ (Chandrasekhar 1943) can be evaluated numerically; we find that the approximation $\Lambda = r_{\rm max}/r_{\rm min}$ is adequate for the conditions occurring in the model described below. We take $r_{\rm min} = 1$ pc $(\sim a_{\rm cl})$ and $r_{\rm max} = \max(a, r)$, where a is the core radius of the galaxy (see below) and r is the distance of the cluster from the center of the galaxy. The results depend only weakly on the choices for $r_{\rm max}$ and $r_{\rm min}$.

We show below that the spheroidal component of M31 may be represented adequately by an isothermal sphere (Zwicky 1957), characterized by a central density ρ_0 and a structural length α, or core radius $a = 3\alpha$. The velocity distribution is Maxwellian, so that equation (1) applies with line-of-sight dispersion,

$$\sigma^2 = \tfrac{1}{3}\langle v^2 \rangle = \frac{1}{2j^2} = 4\pi G\rho_0\alpha^2\,. \quad (2)$$

As a globular cluster orbits in the galactic potential field $U(r)$ [where $U(0) \equiv 0$], dynamical friction produces relatively slow changes in its orbital parameters; i.e., the energy and angular momentum per unit mass (E, J) are approximate integrals of the motion, which decrease slowly as the cluster settles into the center over many orbits.

With these approximations, the evolution of the cluster orbit is represented by a track on a Lindblad (1933) diagram—the (E, J)-plane, shown in figure 1. A cluster moves along a track in a direction determined only by the instantaneous values of (E, J), at a rate proportional to its mass. We have calculated vectors $[\dot{E}(E, J), \dot{J}(E, J)]$ for some standard mass cluster at grid points in the allowed region of the Lindblad diagram $(E \geq 0, E - J^2/2r^2 - U(r) \geq 0)$. Then the orbital evolution of any cluster at any point (E, J) can be found by interpolating among calculated values of $\dot{E}$ and $\dot{J}$. We assume that initially the globular clusters had the same, isothermal distribution in phase space as the field stars. This is not an entirely ad hoc assumption since some processes of galaxy formation such as violent relaxation (Lynden-Bell 1967) could produce a phase-space distribution independent of mass. Since most of the clusters in the galaxy have not yet entered the nucleus, we may assume that the initial distribution of cluster masses is similar to the mass distribution seen presently in M31.

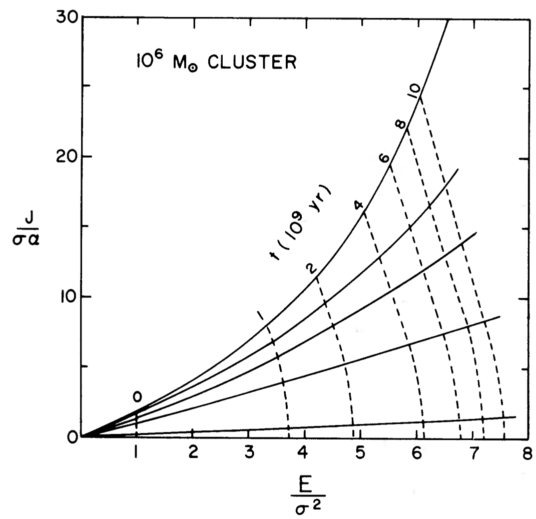

FIG. 1.—Lindblad diagram for an isothermal sphere. The abscissa is energy per unit mass, and the ordinate is angular momentum per unit mass, both in units of the line-of-sight velocity dispersion σ and the structural length α. Typical evolutionary tracks have been drawn. Dashed lines mark points from which a given time is required for a $10^6\ M_\odot$ cluster to reach $E/\sigma^2 = 1$; this is very nearly the time required for it to enter the nucleus. To scale the times to other cases, note that the time unit $\tau \equiv \alpha^2/\sigma G m$ has the value $\tau = 1.70 \times 10^8$ yr in the figure.

We then choose clusters both from the isothermal phase-space distribution and from an assumed mass distribution with maximum cluster mass $m_{\max}$ by a Monte Carlo technique. More massive clusters can be dragged in from a greater distance and from more energetic initial orbits. There is thus an energy per unit mass $E_{\max}$ such that clusters with $E \geq E_{\max}$ will not enter the nucleus in 10^{10} years if $m < m_{\max}$. In estimating the nuclear mass we thus need choose clusters only from the finite region of phase space with $E < E_{\max}$, rather than from the entire infinite isothermal distribution. Values of $E_{\max}$ are listed in table 1. The clusters are then placed on the Lindblad diagram and their evolution followed numerically. A cluster is considered to have entered the nucleus when $E < \sigma^2$, since beyond this point the cluster is in the core of the galaxy and orbits very rapidly toward the center.

TABLE 1

RESULTS OF CALCULATIONS

$m_{\max}$	$5.2 \times 10^6\ M_\odot$	$1 \times 10^8\ M_\odot$
$L_{\max}$	$2.6 \times 10^6\ L_\odot$	$5 \times 10^7\ L_\odot$
$\langle r\rangle$* (kpc)	0.8	1.1
$\langle mr\rangle/\langle m\rangle$* (kpc)	1.3	3.8
$E_{\max}/\sigma^2$†	10.1	15.0
Nuclear mass ($M_\odot$)	$(2.4 \pm 0.7) \times 10^7$	$(8 \pm 4) \times 10^7$
Mean number of clusters composing nucleus	24	26

* $\langle r\rangle$ is the mean initial radius of the clusters which enter the nucleus. $\langle mr\rangle/\langle m\rangle$ is the mass-weighted mean initial radius of the clusters entering the nucleus.

† See § II.

III. SCALING ARGUMENTS

Before presenting our numerical calculations, we show how the results may be scaled to other cases of interest.

For most galaxies, the mass of the nucleus will be dominated by clusters which were initially in orbits far outside the core of the galaxy ($r \gg \alpha$) where most of the evolutionary time will be spent. In this region $\rho(r) = 2\rho_0\alpha^2/r^2$ in an isothermal sphere, and the mass interior to r is thus $M(r) = 8\pi\rho_0\alpha^2 r$. It is easy to show that the velocity v_c in a circular orbit remains constant as the orbit decays inward:

$$v_c^2 = \frac{GM(r)}{r} = 8\pi G\rho_0\alpha^2 = 2\sigma^2 . \qquad (3)$$

Then, if $E(r)$ is the energy per unit mass on a circular orbit of radius r, we have

$$\frac{dE(r)}{dr} = \frac{dU(r)}{dr} = \frac{GM(r)}{r^2} = \frac{8\pi G\rho_0\alpha^2}{r} = \frac{2\sigma^2}{r} . \qquad (4)$$

But from equation (1), neglecting changes in ln Λ,

$$\frac{dE}{dt} = v_c\langle\Delta v_\parallel\rangle$$

$$\propto -\frac{G^2 m\rho(r)}{v_c}[\varphi(jv_c) - jv_c\varphi'(jv_c)] . \qquad (5)$$

Since jv_c remains a constant ($=1$), the function in square brackets is constant and

$$\frac{dE}{dt} \propto -\frac{G^2 m\rho(r)}{\sigma} \propto \frac{Gm\sigma}{r^2} . \qquad (6)$$

Dividing equation (6) by equation (4) yields

$$dr/dt \propto Gm/\sigma r \quad \text{or} \quad r^2(t) \propto Gmt/\sigma . \qquad (7)$$

Clusters initially inside of $r(t)$ reach the center in time $\leqslant t$.

The mass of the nucleus $M_n(t)$ at time t equals the mass of all the clusters initially within $r(t)$ given by equation (7). If the number density of field stars (of mass m_f) at any radius is $1/f$ times the initial number density of clusters at that radius, then we obtain

$$M_n(t) \propto (m_f^{-1} f G^{-1/2})\sigma^{3/2} m^{3/2} t^{1/2} . \qquad (8)$$

Note that the mass of the nucleus does not depend on the "size" α of the parent galaxy. If there is a distribution in cluster masses, the mean value of $m^{3/2}$ must be used in equation (8). In the more realistic case examined in the numerical calculations, the clusters do not all follow circular orbits; rather, their initial velocity distribution is isothermal and Maxwellian. It can be shown that equation (8) is valid in this case as well.

IV. APPLICATION TO M31

The distribution of light in M31 may be represented as the sum of a nucleus, a disk, and a spheroidal component, or bulge. We model the nuclear component (for all r) as an isothermal sphere whose central 1″ has the properties observed by Stratoscope II (Light, Danielson, and Schwarzschild 1975): central blue surface brightness of $B = (13.7 \pm 0.3 \text{ mag}) \sec^{-2}$ and structural lengths along the major and minor axes of (0″.16, 0″.12). De Vaucouleurs (1958) has shown that the disk has a constant surface brightness $B = 22.8 \text{ mag sec}^{-2}$ inside 1800″ from the center. The distribution of light in the inner 1000″ of the major axis of M31 is then reproduced well by assuming that the bulge contribution is an isothermal sphere with central surface brightness $B = 18.3 \text{ mag sec}^{-2}$ and structural length 23″ (see fig. 2). We neglect the fact that the isophotes are ellipses of axial ratio ~ 0.7 and assume a spherical bulge. At a distance of 0.69 Mpc (van den Bergh 1968), 23″ is 77 pc.

The line-of-sight velocity dispersion in the bulge has been measured to be 225 km s^{-1} 10″ from the nucleus (Minkowski 1962) and 120 ± 30 km s^{-1} 13″ from the nucleus (Morton and Thuan 1973). We will use Morton and Thuan's value since it is roughly consistent with the rotational velocities observed by Rubin and Ford (1970) within 1 kpc of the nucleus, and also with the dispersion of 104 km s^{-1} found for 20 globular clusters in M31 within 20′ of the nucleus whose velocities have been measured by van den Bergh (1969). We adopt $\rho_0 = 50\, M_\odot\, \text{pc}^{-3}$, $\alpha = 77$ pc, which from equation (2) gives $\sigma = 127$ km s^{-1}.

Between ~ 100 pc and ~ 1 kpc most of the mass lies in the bulge, and we therefore neglect the effect of the disk on the relaxation process. Between ~ 1 kpc and 30 kpc a large fraction of the mass of M31 lies in the disk. However, we believe that the isothermal sphere used to represent the bulge still provides a rough approximation to the relaxation rate in this region, since the rotation curve in this region resembles the flat rotation curve of an isothermal sphere (Roberts and Rots 1973), and neither the rotation curve nor the relaxation rate is strongly dependent on whether the mass lies in a disk or a sphere.

For the visual mass-to-light ratio of the globular clusters in M31 we adopt $(M/L)_v = 2$ in solar units. Illingworth and Freeman (1974) obtained a ratio of 1.7 ± 0.4 for the globular cluster NGC 6388 in our Galaxy. Scaling to other values of M/L is possible using equation (8).

The luminosity distribution $n(L)$ of the globular clusters in M31 has been determined by van den Bergh (1969). It may be represented by

$$\begin{aligned} \frac{dn(L)}{dL} &\propto \frac{(L/L_0)^a}{[1 + (L/L_0)^{a+1}]^b}\,, \quad && L < L_{\max}\,, \\ &= 0\,, && L > L_{\max}\,, \end{aligned} \qquad (9)$$

where $a = 1.2$, $b = 1.6$, and L_0 corresponds to $M_v = -8.3$ (see fig. 3). The brightest observed cluster in M31 has $M_v = -11.2$ (van den Bergh 1969), so $L_{\max} \geq 2.6 \times 10^6 L_\odot$; however, beyond this limit we have no information on $L_{\max}$. Yet the choice of $L_{\max}$ is critical, since for $L \gg L_0$,

$$dn/dL(L) \propto (L/L_0)^{a-(1+a)b} = (L/L_0)^{-2.3}\,,$$

so that as $L_{\max} \to \infty$, $\langle m^{3/2} \rangle$ diverges (assuming M/L is constant), and the nuclear mass predicted by equation (8) also diverges. The divergence is purely formal, since the mass of all of the clusters in the galaxy yields a finite upper bound on the nuclear mass, but it does indicate a dependence of the nuclear mass on $L_{\max}$. Accordingly, we will examine two models: with $L_{\max} = 2.6 \times 10^6 L_\odot$ and with $L_{\max} = 5 \times 10^7 L_\odot$.

Finally we must normalize the results of the Monte Carlo calculations to the actual number of clusters in M31. We estimate that the 108 clusters with $B-V > 0.5$ listed by Kinman (1963) within 20′ of the nucleus are a reasonably complete sample. We therefore choose the initial central number density of clusters in our model so that the number remaining within 20′ (projected distance) of the nucleus after 10^{10} years is 108.

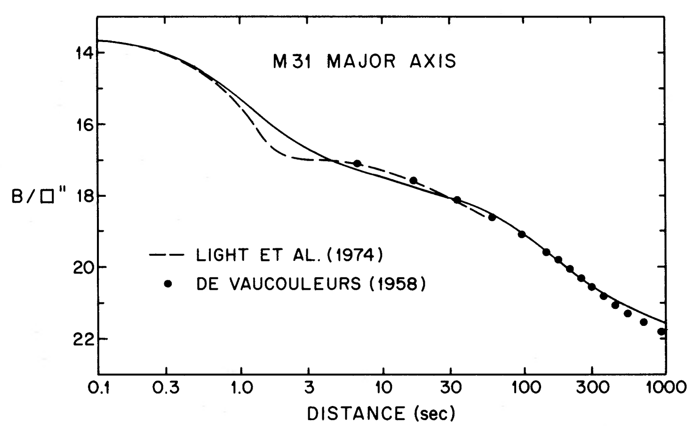

FIG. 2.—Surface brightness of the inner 1000″ of the major axis of M31. The solid curve is calculated from the model discussed in the text.

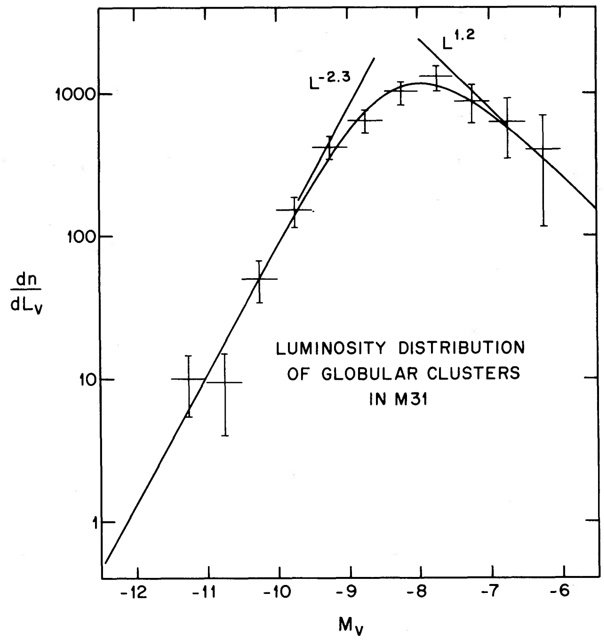

FIG. 3.—The luminosity of the globular clusters in M31 (van den Bergh 1969). The vertical scale is arbitrary. For an interval containing n clusters the error bars represent populations of $n \pm \sqrt{n}$. The solid curve is a fit to the data described in § IV of the text.

The results of the calculations, which incidentally verify the $M_n \propto t^{1/2}$ dependence predicted by equation (8), appear in table 1. The quoted uncertainty in the nuclear masses is an intrinsic statistical uncertainty (determined from the Monte Carlo simulation) due to the small number of clusters which compose the nucleus and neglects the uncertainties in the parameters chosen to define the calculation.

As expected, the nuclear mass depends on whether or not the mass distribution function had a tail of massive globular clusters which are not now seen because most of them have been dragged into the nucleus. We may use equation (8) to estimate the effects of changes in the model parameters. If we had chosen $(M/L)_v = 1$, the nuclear masses would be reduced by $2^{3/2} = 2.83$. Choosing Minkowski's (1962) velocity dispersion of 225 km s^{-1} would increase the masses by $(225/127)^{3/2} = 2.36$.

We have glossed over the physical processes which occur as the clusters approach the growing nucleus. Dynamical friction tends to circularize the orbits so that most clusters enter the inner $\sim$100 pc on nearly circular orbits. The density distribution in this region will not be that of an isothermal sphere after the nucleus has begun to form, since its mass will distort the distribution, enhancing the density and speeding up the relaxation. Clusters will spiral in, gradually being stripped of their outer parts by the tidal forces from the nucleus. A cluster will completely dissociate at the Roche limit, when the mean density interior to it is roughly its own central density. Some clusters may dissociate before this point as a result of tidal heating by the bulge or by the growing nucleus. The component cluster stars will then be relatively unaffected by dynamical friction, although they will achieve some evolution toward equipartition of energy among themselves; using Light *et al.*'s (1975) estimates of the nuclear parameters and Spitzer's (1969) definition of the equipartition time t_{eq}, we find $t_{eq} = (1.8 - 3.1) \times 10^9$ yr $\times\, m_*^{-1}$, where m_* is a typical star mass in solar units. We do not have a detailed dynamical model capable of explaining the density in the nucleus of M31, although, as we show in the next section, our calculation of the nuclear mass is in reasonable agreement with observations.

V. COMPARISON WITH OBSERVATIONS

Light *et al.* (1975) find that M31 has a distinct nucleus with mass $M_n = (0.45–1.6) \times 10^8\, M_\odot$, consistent with the predictions made in table 1. They find $(M/L)_v$ between 8 and 30, assuming no absorption within M31. However, dust has been noted within 6″ of the nucleus (Johnson and Hanna 1972), and it is reasonable to assume that the high M/L is due to absorption. The lower values of mass and M/L follow from Morton and Thuan's (1973) measurement of the velocity dispersion of the nucleus, and the upper values from Minkowski's (1962) measurement. Although both authors assert that the velocity dispersion in the bulge is the same as the dispersion in the nucleus, we stress that there is no *a priori* reason why this should be so if the nucleus is indeed a distinct dynamical structure.

While the agreement in mass is the most striking evidence in favor of the model, there is other, indirect, evidence as well. Einasto (1972) has pointed out that the system of globular clusters in M31 has a much larger mean radius than the bulge. This result follows naturally from the much greater dynamical friction in the high-density central region, which will depopulate it of clusters without greatly affecting the outer regions.

Also, Spinrad, Smith, and Taylor (1972) have observed an increase in metallicity (as measured by the cyanogen band index) between 15 pc and 5 pc from the center of M31, followed by a dip in metallicity in the inner 5 pc, where the light is dominated by the nucleus. Such a dip is to be expected if the nucleus is composed of relatively metal-poor globular clusters.

VI. CONCLUSIONS

The action of dynamical friction on the globular clusters in M31 will cause the formation of a compact nucleus with mass $10^7–10^8\, M_\odot$. Further measurements of the velocity dispersion, color, metallicity, and stellar content of the nucleus would provide important tests of the theory proposed here, as would a measurement of the absorption within M31 at the nucleus. With more complete information on the distribution and kinematics of globular clusters in M31, it might be possible to see the effect of dynamical friction on the distribution of their orbits.

Scaling arguments indicate that galaxies similar to M31 should have distinct stellar nuclei with masses

$\sim 10^{7.5} \sigma_{100}{}^{3/2} M_{\odot}$, where σ_{100} is the line-of-sight velocity dispersion in the nuclear bulge, in units of 100 km s^{-1}. Thus "hot" enough galaxies or galaxies rich enough in globular clusters, should, by the mechanism outlined in this paper, develop massive nuclei within which stellar collisions and other exotic phenomena might possibly occur.

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