

## TIME VARIATIONS IN THE OH MICROWAVE AND INFRARED EMISSION FROM LATE-TYPE STARS

PAUL M. HARVEY,\* KENNETH P. BECHIS,† WILLIAM J. WILSON,‡ AND JOHN A. BALL§

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### ABSTRACT

The OH microwave and infrared variability of 14 OH/IR stars were studied. Many of the sources show periodic flux variations between 1 and 10  $\mu$  and in the 1612-MHz OH lines with no appreciable phase difference between the infrared and microwave. The best explanation for this correlation seems to be that the OH masers are radiatively pumped. In at least two sources the 1665- and 1667-MHz variations are significantly greater and more random than in the 1612-MHz sources. These data are not of sufficient quality to determine if the 1665/1667-MHz and infrared variations are significantly correlated. A correlation does exist between the period of variation, the amplitude of variation at 2.2  $\mu$ , the [3.5  $\mu$ ] - [10  $\mu$ ] color index, the separation in velocity of the two OH emission features, and the existence of 1.35-cm H<sub>2</sub>O emission in the Mira type stars observed. No evidence is seen in the infrared for periodic variations of the color temperatures of the circumstellar shells which are believed to surround many of these stars.

*Subject headings:* circumstellar shells — infrared sources — late-type stars — luminous stars — long-period variables — masers

### I. INTRODUCTION

The class of OH microwave emission sources associated with infrared stars was first studied in detail by Wilson, Barrett, and Moran (1970). Since then, there have been several microwave and infrared studies of these and other similar objects (Hyland *et al.* 1972; Wilson and Barrett 1972; and Wilson *et al.* 1972). The properties of these sources can be summarized as follows. The stars with which the OH sources are associated are either irregular supergiant variables or Mira-type long-period variables with periods from one to two years. All appear to be oxygen-rich stars with surface temperatures of 1800°–2800° K. Most are believed to be evolved stars undergoing mass loss and surrounded by 600°–800° K dust shells. These shells presumably absorb near-infrared stellar radiation and emit strongly between 3 and 20  $\mu$ . The OH emission typically occurs in two velocity ranges separated by 5–50 km s<sup>-1</sup>. There is a tendency for those sources with the thickest dust shells to have 1612-MHz OH emission stronger than any 1665- or 1667-MHz OH emission, and conversely. The OH emission is typically weakly polarized if at all, and there is no detectable radio continuum or 1720-MHz OH emission.

The above studies included some mention of the

variability of the sources, and, on the basis of four OH microwave measurements, Wilson (1970) suggested a possible correlation between the 1612-MHz OH and infrared emission variations of IRC+10011. However, no detailed investigation of the variability has been reported. Such a study is especially important for the stellar OH/IR sources because the OH masers are apparently coincident with known variable stars which are probably the best understood of any type of object with which OH masers are associated.

In this paper we report results of a study initiated in the fall of 1969. The aim of this program was to measure monthly the microwave spectra at 1612 MHz and the broadband infrared fluxes at six wavelengths for 10 OH/IR sources. These observations represent the first conclusive evidence for well-defined, periodic variations in the flux from any type of OH microwave emitter. Following the discovery of OH/IR stars that emit mainly at 1665 and/or 1667 MHz (see, e.g., Wilson *et al.* 1972), the program was expanded to include four of these sources, as well as the 1665/1667-MHz emission of three of the 1612-MHz sources. These data are not nearly as extensive as those on the 1612-MHz sources, but they are interesting enough to warrant inclusion in this study.

### II. INSTRUMENTATION

#### a) Microwave Equipment and Techniques

Nearly all the OH spectral-line measurements were made by K.P.B. with the Harvard College Observatory-Smithsonian Astrophysical Observatory 84-foot (26-m) radio telescope at Agassiz Station, in Harvard, Massachusetts. In a Cassegrainian configuration the half-power beamwidth of the antenna at 18-cm wavelength is approximately 30', and the pointing accuracy is approximately  $\pm 3'$ . The receiver contained an uncooled

\* George W. Downs Laboratory of Physics, California Institute of Technology, Pasadena, California (Present address: Steward Observatory, University of Arizona, Tucson, Arizona).

† Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts.

‡ Electronics Research Laboratory, The Aerospace Corporation, El Segundo, California.

§ Center for Astrophysics, Harvard College Observatory and Smithsonian Astrophysical Observatory, Cambridge, Massachusetts.

nondegenerate parametric amplifier. Signal processing was done with a 50-filter spectral-line radiometer, and the spectral resolution was usually 1.5 or 5 KHz (0.3 or 1.0 km s<sup>-1</sup>). Over the three years of the observations, the total system-noise temperature of the receiver varied from about 150° K to about 250° K. This slow change has been taken into account in all the measurements and calibrations. The two senses of circular polarization were observed separately and later summed to give Stokes parameter  $S_0$ , the total flux.

Flux calibrations were done using mainly 3C 274 (Virgo A) and occasionally Cas A. The calibration sources and the OH emission sources were assumed to be small compared to the beamwidth. The flux density of 3C 274 was taken to be  $S_0 = 180.5$  f.u. at 1612 MHz (1 f.u. = 10<sup>-26</sup> W m<sup>-2</sup> Hz<sup>-1</sup>). The Cas A flux density was calculated from a formula suggested by Allen (1967):  $S_0 = 6500(\nu/0.4)^{-0.785}$  f.u. for  $\nu$  in GHz for the epoch 1964.4, and a secular decrease of 1.1 percent per year. This gives, for example, 2015 f.u. at 1612 MHz for the epoch 1971.4, which is approximately halfway through the observing program. The ratio of flux to antenna temperature (both  $S_0$ ) for a small diameter source is approximately 6.5 f.u. °K<sup>-1</sup>.

Five times during the monitoring program additional OH observations were made with the 140-foot (43-m) telescope of the National Radio Astronomy Observatory (NRAO).<sup>1</sup> These observations were made during 1969 August/September, 1970 February, 1970 September/October, 1971 October/November, and 1972 April by W. J. W., K. P. B., and J. A. B. This antenna was illuminated at the prime focus with a scalar feed that gave a half-power beamwidth of 18' at a wavelength of 18 cm. The feed provided two orthogonally polarized ports, and a cooled parametric amplifier receiver was connected to each port. The system-noise temperature of each receiver was about 75° K. Spectral-line processing was done with the NRAO 384-channel autocorrelation system, which was used either as one 384-channel autocorrelator or as two 192-channel autocorrelators. For all observations at least two orthogonal senses of linear polarization were observed and later summed to give  $S_0$ . In some cases left- and right-circular polarizations were measured and summed to provide an independent measurement of  $S_0$ .

Flux calibrations at the NRAO were made using the unpolarized continuum source 3C 218, which was assumed to have a flux density of 37.5 f.u. at 1612 MHz and 36.4 f.u. at 1666 MHz (Kellermann, Pauliny-Toth, and Williams 1969). The ratio of flux to antenna temperature (both  $S_0$ ) for a small-diameter source was about 1.98 f.u. °K<sup>-1</sup>.

#### b) Infrared Equipment and Techniques

The broad-band infrared fluxes between 1 and 10  $\mu$  were measured primarily by P. M. H. with the 60-inch

(1.5-m) telescope of the Hale Observatories on Mount Wilson, using the photometer described by Becklin and Neugebauer (1968). Measurements were made relative to five or six standard stars on each night. These were used to determine a sensitivity for the night at each wavelength, and, for all but the faintest stars, the relative agreement of the standards determined the errors assigned to the measurements. Nominal air-mass corrections were applied to all the data, and, whenever possible, measurements were made at zenith angles less than 60°. The wavelengths observed were 1.25  $\mu$  (1.15–1.35  $\mu$ ), 1.65  $\mu$  (1.50–1.75  $\mu$ ), 2.2  $\mu$  (2.0–2.4  $\mu$ ), 3.5  $\mu$  (3.2–4.1  $\mu$ ), 4.8  $\mu$  (4.6–5.1  $\mu$ ), and 10  $\mu$  (8–13  $\mu$ ). The absolute calibration has been described by Wilson *et al.* (1972).

### III. OBSERVATIONS AND DATA ANALYSIS

#### a) The Data

The microwave and infrared observations of all the observed stars are displayed as light curves in the Appendix. We plotted the following five parameters of the 1612-MHz OH data: the two peak flux densities within the low- and high-velocity features, the integrated flux in each group of features, and the total integrated flux. These five parameters generally specify all the 1612-MHz variations because the velocities and shapes of the OH features did not vary significantly in most cases. Except for some of the spectra of R Aql, VY CMa, and NML Cyg, the 1665- and 1667-MHz data were characterized only by the peak and integrated flux densities of all the features in each spectrum.

Errors in the microwave and infrared observations arise both from random noise in the detectors and from changes in the calibration of the detecting system. For many of the microwave data and for almost all of the infrared data, the principal contribution to the errors is from the calibration changes.

The error limits on the microwave observations were estimated from observed changes in system gain and from the agreement of measurements made less than a few days apart. These limits are about  $\pm 15$  percent. On the 84-foot telescope the integration times on the weaker sources were usually chosen to give a statistical error of about  $\pm 2$  f.u. (2  $\sigma$ ), and this was added in quadrature to the 15 percent absolute calibration error for all measurements. These error estimates are believed to represent 90–95 percent confidence limits. One confirmation of the estimated size of the errors is that the 1612-MHz flux of VY CMa has remained constant within the errors for the entire period of the observations. This seems unlikely unless the source had remained nearly constant and the errors were estimated correctly.

The error limits on the infrared observations were determined from the agreement of the standard sources measured along with the OH/IR stars. The agreement is usually within about  $\pm 10$  percent; the corresponding

<sup>1</sup> The National Radio Astronomy Observatory, Green Bank, West Virginia, is operated by Associated Universities, Inc., under contract with the National Science Foundation.

error limits on the magnitudes are  $\pm 0.15$  to  $\pm 0.10$  magnitude.

A  $\chi^2$  test for the degree of variability, several sine-wave fits, and a correlation coefficient test were performed on the infrared and microwave data. These are described in detail in the Appendix. In the following sections we discuss the observed infrared and microwave variability and the results of these statistical tests.

### b) Infrared Variations

Except in NML Cyg and VY CMa, the values of  $\chi^2$  given in table A1 of the Appendix indicate significant variation of all the stars at all infrared wavelengths. Both the appearance of the light curves and the values of  $\chi^2$  suggest that NML Cyg has probably shown small long-term variations while VY CMa has probably remained constant within the errors.

For the stars for which both visual and infrared periods were determined, the periods agree well (Appendix, table A2). In addition, the phases of the infrared variations are equal at all wavelengths except that the long-wavelength flux sometimes peaks slightly ahead of the shorter-wavelength fluxes (Appendix, table A3). Where available, the times of visual maxima and minima obtained from light curves of the AAVSO (Mayall, private communication) are indicated on the infrared light curves in the Appendix. The infrared light maximum lags the visual typically by 0.1 to 0.2 period, in agreement with the previous observations of Pettit and Nicholson (1933), Mendoza (1967), Lockwood and Wing (1971) and Frogel (1971). The amplitudes of variation of the broad-band  $2.2\text{-}\mu$  fluxes are somewhat smaller than the  $1.04\text{-}\mu$  amplitudes of Lockwood and Wing, and marginally larger than the narrow-band  $2.25\text{-}\mu$  amplitudes of Frogel.

For the Mira variables observed, we found a definite correlation among the four quantities plotted in figure 1—the period, the  $2.2\text{-}\mu$  amplitude, the  $[3.5\text{ }\mu] - [10\text{ }\mu]$  color index, and the separation in velocity between the two groups of OH emission features. Several aspects of this correlation were seen previously. In particular, Merrill (1960) noted a possible correlation between the

periods of variation and the visual amplitudes of Mira stars, and Lockwood and Wing (1971) found a period-amplitude relationship at  $1.04\text{ }\mu$ , especially for a sample restricted to pure M stars. Furthermore, Hyland *et al.* (1972) noted a tendency for longer-period stars to have larger amplitudes and larger  $[0.8\text{ }\mu] - [2.2\text{ }\mu]$  color indices, and Wilson *et al.* (1972) found that the stars with larger  $[3.5\text{ }\mu] - [10\text{ }\mu]$  color indices have larger separations in velocity between the OH microwave emission features. In addition to the above correlations, the six shorter-period stars in figure 1 all show  $1.35\text{-cm}$   $\text{H}_2\text{O}$  emission while none of the five longer-period stars are known to have such emission.

Most of the stars observed show variations in color during their cycles of light variation. The amplitudes of variation at each wavelength shown in figure 2 for four of the stars observed in this study are illustrative of the general behavior of all the stars (Appendix, table A6). The amplitudes generally decrease from  $1.2$  to  $3.5\text{ }\mu$  and are roughly constant between  $3.5$  and  $10\text{ }\mu$ . The phase of the color variations is the same as that of the flux variations, that is, the stars appear reddest at minimum light. This is consistent with Frogel's (1971) broad-band observations between  $1.2$  and  $3.5\text{ }\mu$ . The stars with the least color variation are those with the shortest periods and smallest  $[3.5\text{ }\mu] - [10\text{ }\mu]$  color indices; these stars also tend to have their  $1665/1667\text{-MHz}$  OH emission stronger than any  $1612\text{-MHz}$  OH emission that may be present.

### c) OH Emission Variations

The significance of the observed OH microwave variability was investigated by performing a  $\chi^2$  test on all the data (Appendix, table A1). Except for NML Cyg and VY CMa, the value of  $\chi^2$  is greater than about 2 for the variations of virtually all the peak and integrated  $1612\text{-MHz}$  flux densities of all the sources. In the three cases where the value of  $\chi^2$  is less than 2 at  $1612\text{ MHz}$ , the average value of the peak flux density is only about 5 f.u. In these cases the errors are apparently masking the variations which were observed in the data with a higher signal-to-noise ratio taken

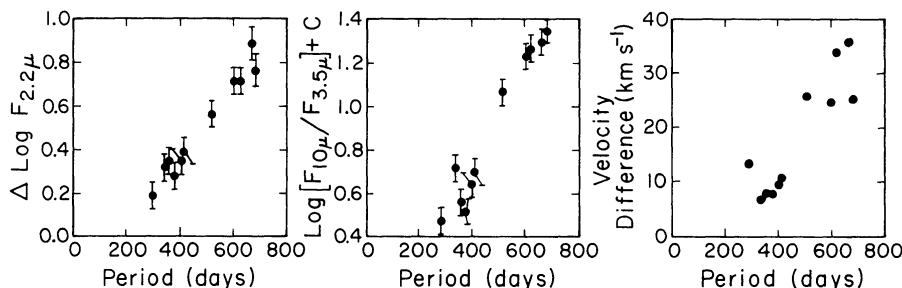


FIG. 1.—The log of the amplitude of variation at  $2.2\text{ }\mu$ , the  $[3.5\text{ }\mu] - [10\text{ }\mu]$  color index, and the separation in velocity between the two OH emission features are shown as functions of the period of variation for the 11 Mira type variables observed, including both  $1612\text{-}$  and  $1665/1667\text{-MHz}$  sources. Velocity differences are measured between the two strongest OH emission features in each spectrum. Uncertainties in the periods and velocities are on the order of the size of the points plotted. The constant,  $C$ , equals  $0.88$  (Wilson *et al.* 1972).

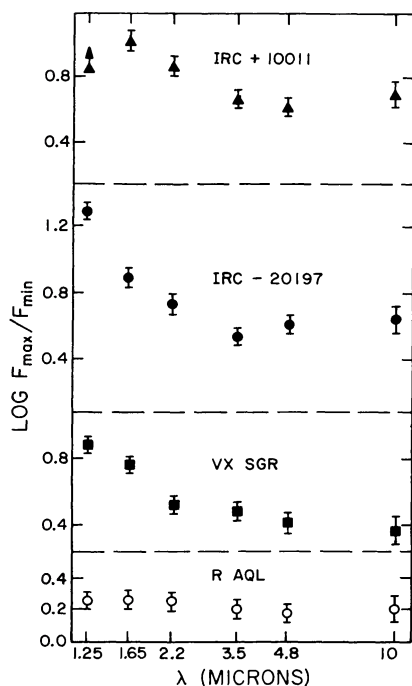


FIG. 2.—The log of the amplitude of variation at 1.2, 1.6, 2.2, 3.5, 4.8, and 10  $\mu$  is shown for four representative stars.

with the 140-foot antenna. The values of  $\chi^2$  for the 1612-MHz fluxes of VY CMa and NML Cyg indicate that VY CMa remained constant within the errors but that the integrated flux of NML Cyg probably varied with a larger amplitude than that of the peak flux density.

Except for some of the parameters of the 1665/1667-MHz fluxes of VY CMa, the only cases where the values of  $\chi^2$  indicate no significant variability at 1665 and 1667 MHz are those for which the peak flux densities are  $\leq 5$  f.u. Therefore, all these sources may be variable. Several of the parameters of the 1665/1667-MHz fluxes of VY CMa appear to be nearly constant while others show large variations. Robinson, Caswell, and Goss (1970) found large changes in the intensity and polarization of the 1665/1667-MHz spectra of VY CMa. Since we did not measure all the Stokes parameters of the emission, we can only conclude from our data that the 1665/1667-MHz spectra of VY CMa vary considerably more than the 1612-MHz spectra. The variations in the 1667-MHz and perhaps the 1665-MHz flux of NML Cyg are also larger than the 1612-MHz variations, and in general, the variations of the 1665/1667-MHz fluxes of all the sources seem to be greater than the typical 1612-MHz variations.

The periods of the 1612-MHz variations determined by sine-wave fitting are given in table A2 of the Appendix. The 1665/1667-MHz data are not of sufficient quality or quantity to obtain meaningful determinations of the periods, if periods exist.

#### d) Correlations between Microwave and Infrared Variations

The most important aspect of this study is the indication of a correlation between the infrared and 1612-MHz OH variability of the sources observed. This is illustrated in figure 3 for four of the stronger sources and was confirmed for all the sources in several ways.

It is clear from table A2 of the Appendix that for virtually all the sources the periods determined by the sine-wave fitting for the 1612-MHz and infrared fluxes are roughly equal. The only exceptions are IRC-20540 which is a relatively weak microwave source and VX Sgr which has large differences in amplitude from cycle to cycle. The phases of the infrared and microwave variations in table A3 of the Appendix for the 1612-MHz sources are also equal within the confidence limits for virtually all the sources. Finally, correlation coefficients between the microwave and infrared variations were computed as described in the Appendix and listed there in table A4 for the 1612-MHz sources. These coefficients show that the microwave and infrared variations are clearly correlated in the stronger sources, although less obviously correlated in several of the weaker sources. In the weaker microwave sources, however, the error limits are comparable to the probable size of the variations and a smaller correlation would be expected. Correlation coefficients were also computed for NML Cyg because it has probably shown small but significant variations. The variations in the integrated flux of NML Cyg are well correlated with the infrared, but the variations in the peak flux densities do not show any definite correlation. It should be noted that the variations of NML Cyg may be periodic but of such a long period and small amplitude that the time scale of these observations is too short to establish the periodicity.

The uncertainties in the OH microwave amplitudes are too large to determine if a correlation exists between the infrared and 1612-MHz OH amplitudes. The data in table A6 of the Appendix, however, suggest that, except for R Aql, the amplitudes of the 1612-MHz variations are significantly less than, or at most equal to, the infrared amplitudes.

For the 1665/1667-MHz sources, the phases derived by sine-wave fitting with the period fixed to that of the visual variations, and the correlation coefficients for the variations of these sources, are given in table A5 of the Appendix. The short time coverage and weaker fluxes of the 1665/1667-MHz sources make the detection of a correlation with the infrared difficult, if such exists. The results in table A5 are not conclusive; within the errors they possibly indicate a correlation between the infrared and 1665/1667-MHz variability of most sources.

#### e) Summary of Observational Results

The most important observational results of this study are:

- 1) The infrared and 1612-MHz OH variations of VX

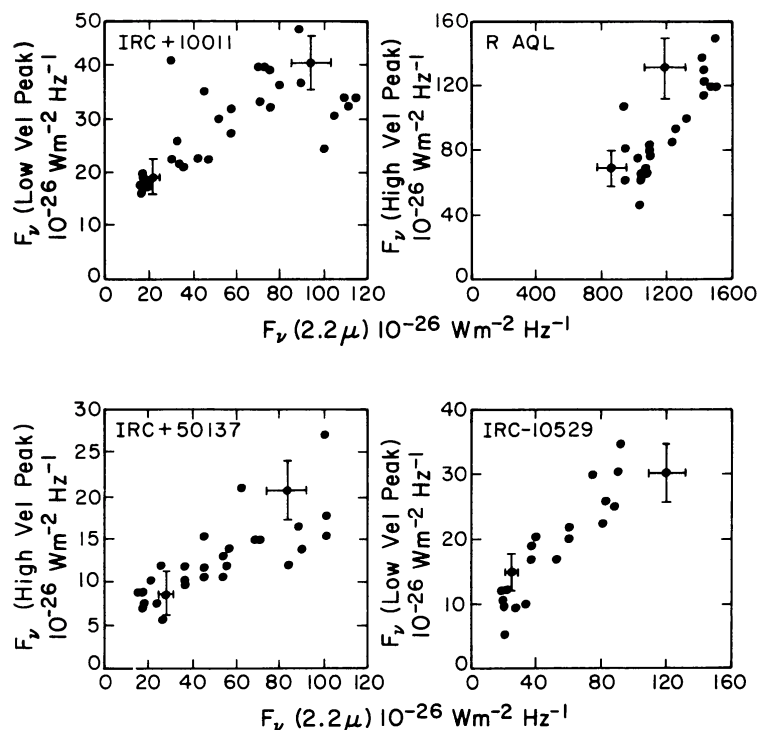


FIG. 3.—The flux density of the stronger of the two OH emission features is shown as a function of the flux density at  $2.2\ \mu$  at various times in the light cycles for four of the stronger 1612-MHz sources. The fluxes at  $2.2\ \mu$  were interpolated between nearby observations. Typical error limits are shown.

Sgr and of all the observed Mira type OH/IR stars are roughly periodic. The periods in the infrared and microwave are equal and range from about 300 to 700 days (Appendix, table A2). The maxima and minima of the infrared and 1612-MHz fluxes of the periodic sources roughly repeat from cycle to cycle. The infrared and 1612-MHz fluxes of VY CMa remained constant while NML Cyg probably showed small long-term variations at both wavelengths.

2) The amplitudes of variation in the infrared generally decrease with increasing wavelength from  $1.2$  to  $3.5\ \mu$  and are roughly constant between  $3.5$  and  $10\ \mu$  (fig. 2; Appendix, table A6). The ratios of maximum to minimum flux range from less than 2 to greater than 10 for different stars at different wavelengths.

3) The amplitude of variation in the OH microwave flux is usually on the order of or smaller than the amplitude at all observed infrared wavelengths (Appendix, table A6). The ratio of maximum to minimum 1612-MHz flux density is typically between 2 and 4.

4) The phases of the variations at all infrared wavelengths and at 1612 MHz for the periodic variables are equal within the confidence limits of the observations, generally about 60 days, or 0.1 to 0.2 period (Appendix, table A3). The 1612-MHz and infrared variations of the periodic variables are correlated (fig. 3; Appendix, table A4) as are the small long-term variations of NML Cyg.

5) There are no velocity variations of the 1612-MHz OH lines within the observational uncertainty—about  $0.3\ \text{km s}^{-1}$  for strong features.

6) There are no systematic or periodic changes in the 1612-MHz feature widths, separations, or the number of features of the spectra as the total flux varies, but there are occasional random variations in feature strength.

7) The 1665/1667-MHz flux variations are not definitely correlated with the infrared variations, and in at least two sources, NML Cyg and VY CMa, they are significantly greater and more random than the 1612-MHz variations.

8) A close correlation exists between the period of variation, the amplitude of variation, the  $[3.5\ \mu] - [10\ \mu]$  color index, and the separation in velocity between the OH emission features for the Mira type stars as shown in figure 1. Some correlation between these four quantities and the occurrence of 1.35-cm  $\text{H}_2\text{O}$  emission is also present.

#### IV. INTERPRETATION AND SPECULATION

##### a) Radiative Coupling

The main conclusion of this study is that the correlation between the infrared and 1612-MHz OH variations is due to some radiative coupling mechanism between the stars and the masering OH clouds. This conclusion

is based on the premise that in all the 1612-MHz sources the distance between the OH maser and the star is too large for any mass motion or pressure wave of reasonable velocity to travel from the star to the maser without an observable phase lag. This premise will now be discussed.

Three considerations suggest that the OH is at least  $3 \times 10^{14}$  cm away from the stars. Assuming a distance to NML Cyg of 500 pc, which is probably accurate to within a factor of 3, Davies, Masheder, and Booth (1972) showed from VLBI measurements that the typical separation of OH maser emitting points in NML Cyg is about  $10^{16}$  cm. Therefore, the masering OH clouds must be located at least  $10^{16}$  cm away from the star. Second, as Wilson (1970) noted, because no variation is seen in the velocity of the OH maser lines, although variations are seen in the velocity of optical spectral lines, the OH is probably mechanically decoupled from the stars. Therefore, the OH is probably at least several stellar radii or a few times  $10^{14}$  cm distant from the stars. Third, Litvak (1969) noted that far-infrared resonance fluorescence, which is probably necessary in the maser pump to select particular hyperfine states, is quenched at densities greater than about  $10^8 \text{ cm}^{-3}$ . These densities are at least  $10^3$  times lower than typical stellar photospheric densities, also suggesting that the OH is at least several stellar radii away from the stars.

An upper limit to the velocity of mass motion is about  $50 \text{ km s}^{-1}$ . This is the separation in velocity of the microwave or optical spectral lines. The velocity of sound in the circumstellar medium is even smaller, probably only a few  $\text{km s}^{-1}$ . Therefore, the minimum phase lag for mass motion or pressure waves to travel from the star to the OH cloud is  $(3 \times 10^9 \text{ km}) / (50 \text{ km s}^{-1})$ , or about two years.

There are several radiative coupling mechanisms which might cause the maser output to follow the stellar variations: (1) the masers may be radiatively pumped by the infrared radiation from the star and/or dust shell; (2) the stellar light may be able to affect the creation or destruction of OH molecules in the maser region; (3) the maser may be amplifying variable 1612-MHz continuum radiation directly from the star; (4) the varying stellar radiation may be able to affect the collision rate populating or depopulating the inversion by changing the temperature in the masering region.

The first suggestion, radiative pumping of the maser, seems to be the most likely and will be discussed in detail below. The second suggestion seems unlikely since the minimum energy necessary to affect the electronic configuration of OH corresponds to  $0.4 \mu$  radiation. Assuming equal fractional linewidths in the visual and microwave, the photon emission rate at  $0.4 \mu$  for the late-type stars studied by the OAO (Doherty 1972) is typically less than  $10^{-2}$  times the 1612-MHz photon emission rate of the OH sources.

There is no reason to believe that the OH sources would have an ultraviolet excess relative to the late-type stars studied by Doherty. Amplification of the stellar 1612-MHz continuum background is also unlikely for two reasons. If the maser were amplifying the stellar background, emission would only be seen in the line of sight to the star and, at least in the case of NML Cyg, this is contrary to the findings of Davies, Masheder, and Booth (1972). Secondly, if the masers are saturated, as seems likely, then the maser output is almost independent of the brightness of the source being amplified unless the beaming of the maser were variable. It is also unlikely that the microwave variations are due to a varying temperature in the maser medium, although this possibility cannot definitely be ruled out. Between 1 and  $10 \mu$ , where most of the stellar energy is emitted, the principal sources of opacity are the fundamental vibrational bands of  $\text{H}_2\text{O}$ , CO, and OH. Using some typically assumed values for the parameters of the circumstellar cloud (see, e.g., Litvak and Dickinson 1972)— $r = 10^{15}$  cm,  $n(\text{H}) = 10^6 \text{ cm}^{-3}$ ,  $n(\text{H}_2\text{O}) \simeq n(\text{CO}) \simeq n(\text{OH}) \simeq 10^{-4} n(\text{H})$ ,  $L_* = 10^4 L_\odot$ —the energy absorbed by the circumstellar gas in a few times  $10^6$  s (the upper limit to the phase difference between the microwave and infrared) can only change the gas temperature by 0.1–1 percent. Furthermore, if the masers were sensitive to such small changes in the kinetic temperature of the gas, then random variations in the gas flow and turbulence would probably lead to large random variations in the maser emission which are not seen. The Reynolds number for the probable outward flow of the gas in which the maser is situated is on the order of  $10^6$  for the observed flow velocities and reasonable densities and temperatures. Therefore, the flow may be highly turbulent. In fact, such turbulence could be the prime factor determining the temperature in the maser medium.

#### *b) Radiative Pumping*

Radiative pumping of the maser population inversion seems to be the most likely coupling mechanism because it is the mechanism most consistent with all the observations to date including our measurements of the variability.

Hyland *et al.* (1972) showed that, if one assumes isotropic microwave and infrared emission with equal fractional line widths, the number of photons emitted from most of the OH/IR stars at 2.8 and  $35 \mu$  is comparable to or exceeds the number of 1612-MHz photons. This conclusion is consistent with our observations; the ratios of infrared to 1612-MHz photons do not vary by more than a factor of 2 to 3 during the cycles of variation. Since a radiative pumping mechanism that yields more maser photons than pump photons is possible but unlikely, the most likely infrared pump transitions are at those wavelengths where the ratio of infrared to 1612-MHz photons is greater than 1. Assuming a reasonable extrapolation of

the energy distributions, this occurs for most of the sources between about 2 and 40  $\mu$ . Since the only allowed transitions from the ground state of the OH molecule in this range are at 2.8 or 35  $\mu$ , these seem to be the most likely pump wavelengths.

The functional relationship between the 1612-MHz and infrared fluxes is not inconsistent with the conclusion that the masers are radiatively pumped, despite the fact that the infrared and microwave amplitudes are not equal. Regardless of the degree of saturation, almost any radiative pumping model would predict an increase in the maser output with an increase in the pump flux, and this is observed in all the 1612-MHz sources. Without a detailed theory for the pumping, however, it is impossible to predict the exact dependence of the maser flux on the pump flux. In fact, one might expect different relationships under the different physical conditions in different sources. For example, in a model proposed by Litvak (1969) wherein collisions are considered negligible, the unsaturated population inversion depends only on the density of OH and to a small extent on the spectral distribution of the incident radiation, but not on the intensity of the infrared radiation. In that case, if the maser were not fully saturated, then the periodic microwave variations would be smaller than the infrared variations, although random variations would probably be larger.

It seems more likely, however, that the masers are saturated. In a saturated maser the output should be proportional to the pump rate, but in the OH/IR stars there are at least three ways in which the pump rate may not be exactly proportional to the infrared photon rate. Some other inverting or anti-inverting mechanism may exist such as the collisional pumping suggested by Litvak and Dickinson (1972). Alternatively, nonlinearities in the pumping may result from stimulated emission in some of the far-infrared transitions; or the maser intensity may depend not only on the intensity of the infrared radiation but on its spectral distribution. In summary, because of the many ways in which the maser and infrared amplitudes can differ, one cannot determine any details of the pumping process from the relationships between the pump and maser fluxes.

Our observation of a correlation between the infrared and microwave variability is not only consistent with general considerations of radiative pumping, but also with the particular model of Litvak and Dickinson (1972) as they noted. This fact does not necessarily confirm their model, but it does support the conclusion that the masers are radiatively pumped.

#### c) Saturation

The output of a fully saturated maser is proportional to the pump rate and independent of the input signal, whereas in an unsaturated maser the output is proportional to the input signal being amplified and exponentially dependent on the population inversion. Therefore, in an unsaturated maser relatively large random

variations are possible because of random variations in the population inversion caused by changes in the collision rate, density, etc. The relatively small and periodic variations of most of the sources and the constancy of one source, VY CMa, thus suggest that the masers associated with infrared stars are saturated. This is in agreement with the conclusions of Wilson *et al.* (1970) based on polarization measurements.

#### d) Circumstellar Dust Shells and Stellar Properties

The interpretation generally given to the low color temperatures at long wavelengths of many late-type stars is that these stars are surrounded by dust envelopes which absorb the visible and near-infrared radiation of the star, are heated to about 600° K, and reradiate the energy between 3 and 20  $\mu$ . Recently, however, Humphreys (1974) and Gilman (1974) have suggested that in at least some of these stars the long-wavelength radiation excesses may be explained by free-free emission from stellar chromospheres. In this section we first consider the color variations of the three stars—IRC+10011, IRC+50137, and IRC—10529—that have the largest radiation excesses at 10  $\mu$  and thus presumably the densest dust or gas shells. We then discuss possible interpretations of the period-amplitude-color-velocity separation relationship for all the Mira sources.

Frogel (1971) established a relationship between the variation of the  $[1.2 \mu] - [3.5 \mu]$  color index and the amplitude of variation at 2.25  $\mu$  for stars with little excess 10- $\mu$  radiation. The color variations of the three OH/IR stars with large excess 10- $\mu$  radiation are consistent with the variations predicted from their 2.2- $\mu$  amplitudes and Frogel's relationship. Therefore, the difference in amplitude of variability between the short and long wavelengths in the OH/IR stars is probably related to the central star rather than to the nature of the shell. The amplitude difference can probably be explained by variations of both the effective temperature of the stars and of the molecular band strengths at the shorter wavelengths.

Table A6 of the Appendix shows that the color temperatures of the above three stars, as well as all the OH/IR stars studied here, do not vary within the observational limits at wavelengths longer than 3  $\mu$ . An upper limit to the variability of the  $[4.8 \mu] - [10 \mu]$  color for all the stars in table A6 is  $\pm 0.2$  mag. If the long-wavelength emission arises from a dust shell at a temperature of  $\sim 600^\circ$  K, this implies a maximum color-temperature variation of  $\pm 40^\circ$  K. For the three stars—IRC+10011, IRC+50137, and IRC—10529—the 4.8- and 10- $\mu$  emission arises almost entirely from the circumstellar shells. Therefore, if the emission is from dust, the physical temperature of the dust-shell particles which are seen probably does not vary much more than  $\pm 40^\circ$  K. On the other hand, if the long-wavelength emission is due to the mechanisms suggested by Gilman (1974), the lack of color variation is probably

due to the insensitivity of the free-free spectral shape to temperature variations.

The correlation between the four quantities—period, amplitude, color, and velocity—implies an interdependence among all of them. Clearly, the longer-period stars have larger amplitudes of variation, but the interpretation of the other relationships in terms of cause and effect is unclear. Larger amplitudes may be accompanied by greater mechanical activity leading to more mass loss and higher velocities of expulsion of the gas, or the longer-period stars may have intrinsically greater luminosities which drive greater mass loss and larger velocities of expulsion by means of radiation pressure acting on the dust grains (Gilman 1972).

The existence of 1.35-cm  $\text{H}_2\text{O}$  emission from the six shorter-period stars in figure 1, but not from the longer-period stars, may be due to an additional correlation with one or more of the other quantities above. It may also be due to a correlation with the temperature of the stars which in turn may be correlated with one of the above quantities, or it could be a distance or luminosity effect. For example, with the assumption that all these stars have equal luminosities of  $10^4 L_\odot$ , Hyland *et al.* (1972) and Wilson *et al.* (1972) found that the distances of the six shorter-period stars are less than about 500 pc while the distances of the five longer-period stars are greater than about 500 pc. If all the sources have comparable  $\text{H}_2\text{O}$  luminosities, this could lead to the observed absence of 1.35-cm emission from the more distant stars.

#### V. SUMMARY

The principal conclusions of this study are:

- 1) The observed correlation between the infrared and 1612-MHz variability of the OH/IR stars is probably due to a radiative coupling mechanism between the stars and the OH clouds.
- 2) The coupling mechanism most consistent with these and other observations is radiative pumping of the masers, possibly at 2.8 or 35  $\mu$ .
- 3) The masers are probably saturated.

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#### APPENDIX

On the following pages we show the infrared and OH microwave observations of the variability of all the sources considered in this study. The infrared data are plotted in magnitudes,  $-2.5 \log F_\nu + C_\nu$ . The absolute calibration of  $C_\nu$  is described by Wilson *et al.* (1972). The microwave observations are plotted linearly in terms of flux density and integrated flux.

Several tables containing the statistical parameters of the microwave and infrared variability follow the observations (fig. 4-35). Table A1 lists the reduced  $\chi^2$ , that is, the ratio of the rms deviation from the mean to the rms error. Table A2 lists the periods for the OH microwave and infrared variations of the 1612-MHz sources. These periods were determined by fitting the data with sine waves characterized by four parameters—average level above zero, amplitude, period, and phase—all of which were determined from the fit. When known, the visual period is also listed for comparison. Table A3 lists the phases of the 1612-MHz and infrared variations for the 1612-MHz OH sources. These phases were determined by fitting sine waves to the variations with the period fixed to either the visual or 2.2- $\mu$  period, whichever was best determined. The sign of the phase is defined such that a numerically smaller value of the phase implies a time lag. Table A4 lists the values of the correlation coefficients between the 2.2- $\mu$  and 1612-MHz variations and between the 10- $\mu$  and 1612-MHz variations for the nine variable 1612-MHz sources. These correlation coefficients were computed from the infrared fluxes estimated with an interpolation procedure for the dates of the microwave observations. Table A5 contains the phases and correlation coefficients for the infrared and microwave variations of the 1665/1667-MHz sources, determined as for the 1612-MHz sources. Table A6 lists the amplitudes of variation of all the sources in the microwave and infrared. The amplitudes of the 1612-MHz sources are those determined by the sine-wave fitting program. Since the microwave variations of the 1665/1667-MHz sources are not clearly periodic, their amplitudes were estimated directly from the observations.

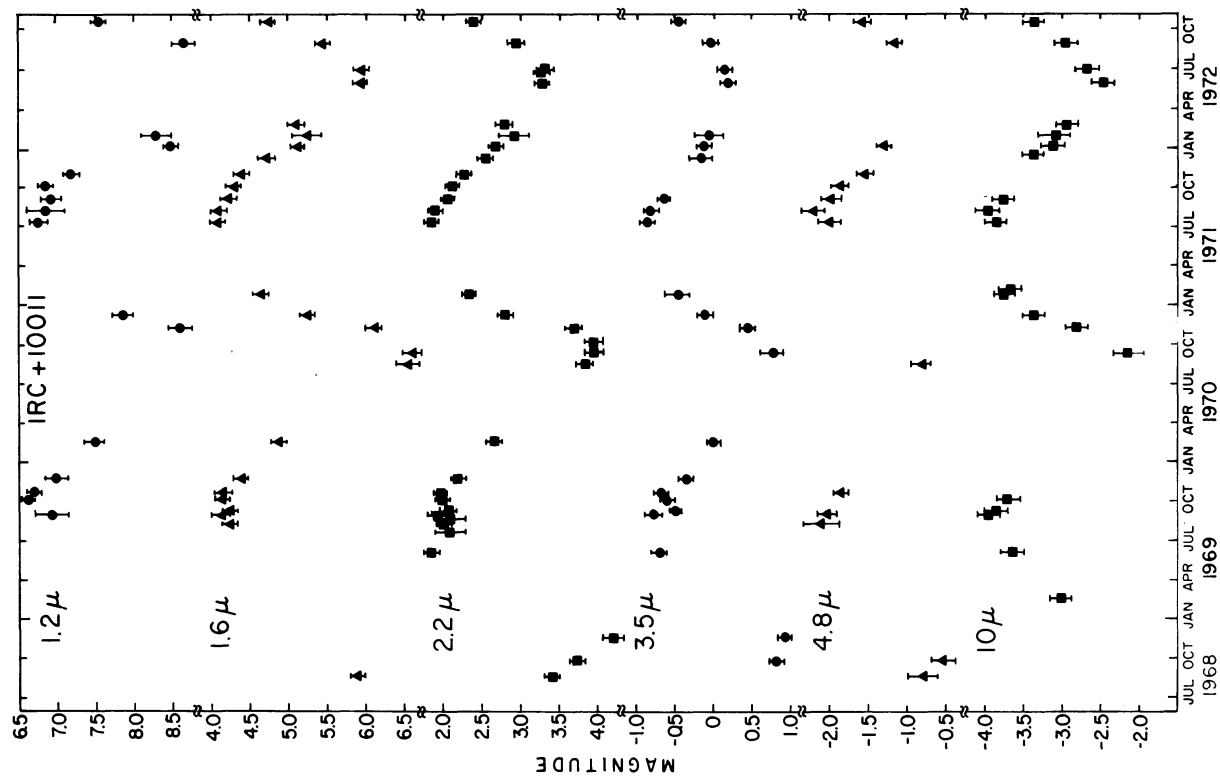


Fig. 4.—The infrared observations of IRC+10011

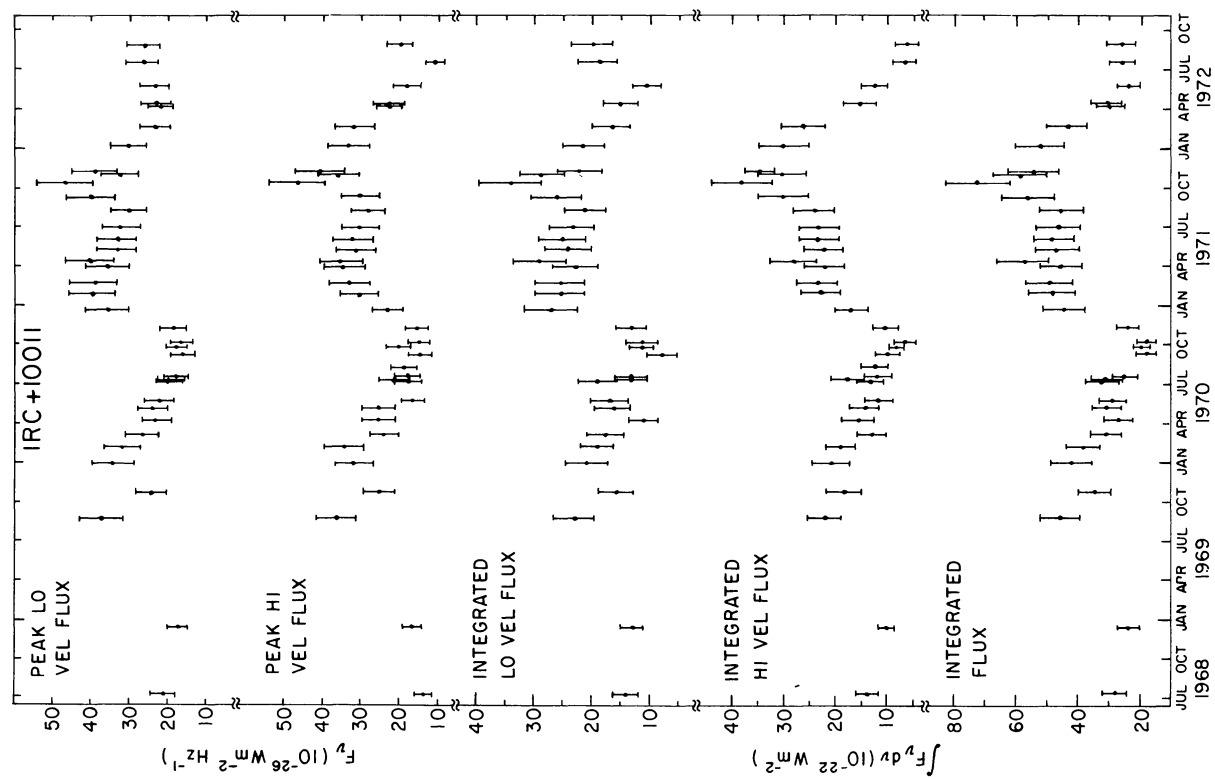


Fig. 5.—The 1612-MHz observations of IRC+10011

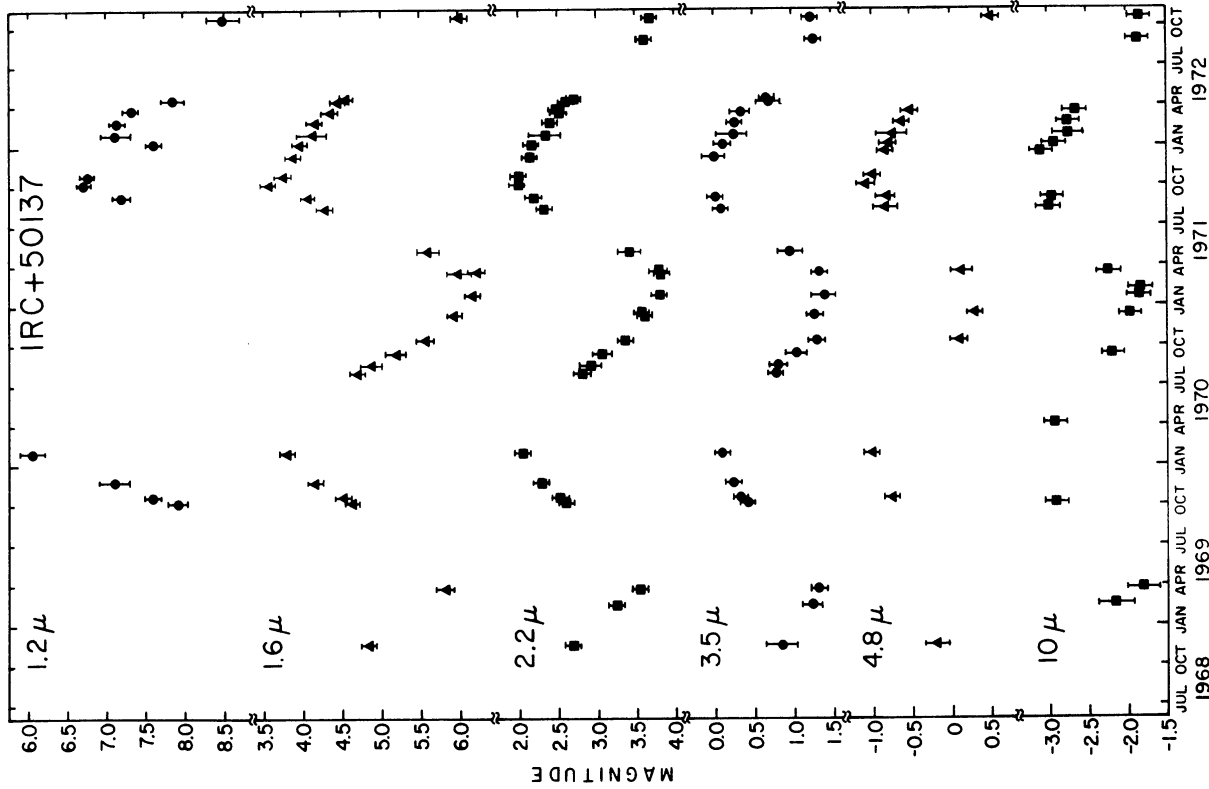


FIG. 6.—The infrared observations of IRC+50137

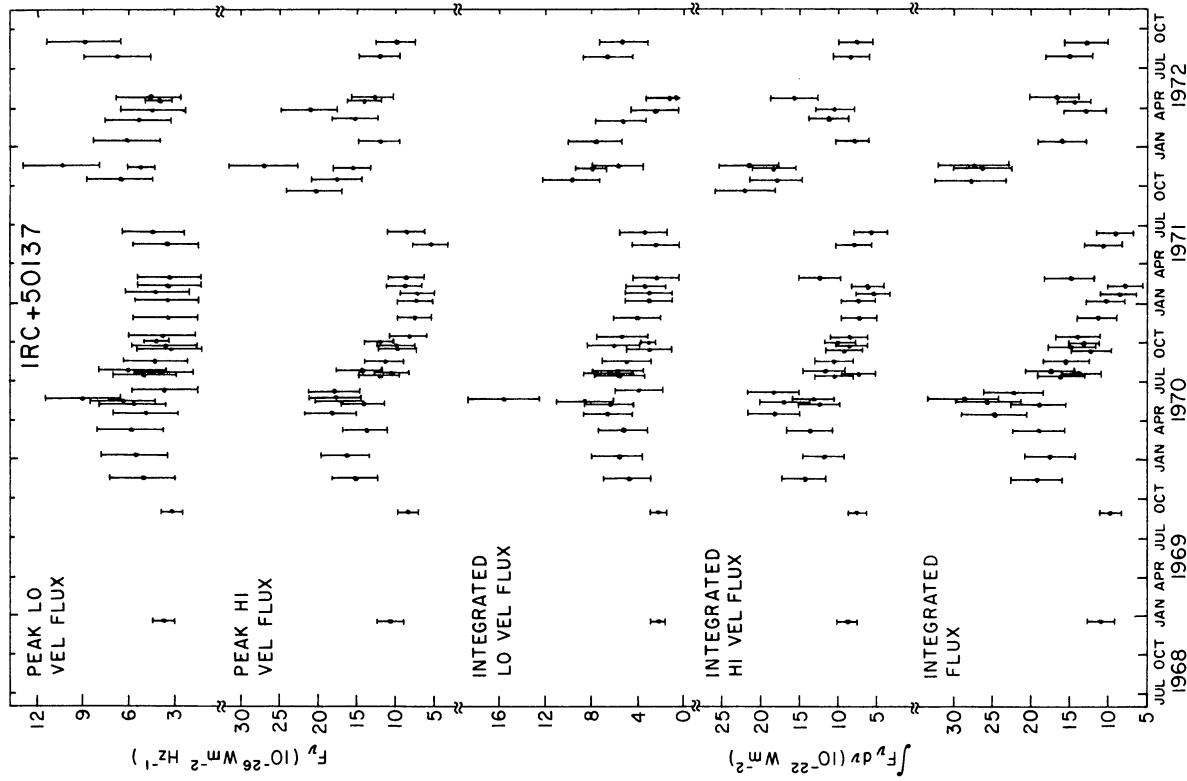


FIG. 7.—The 1612-MHz observations of IRC+50137

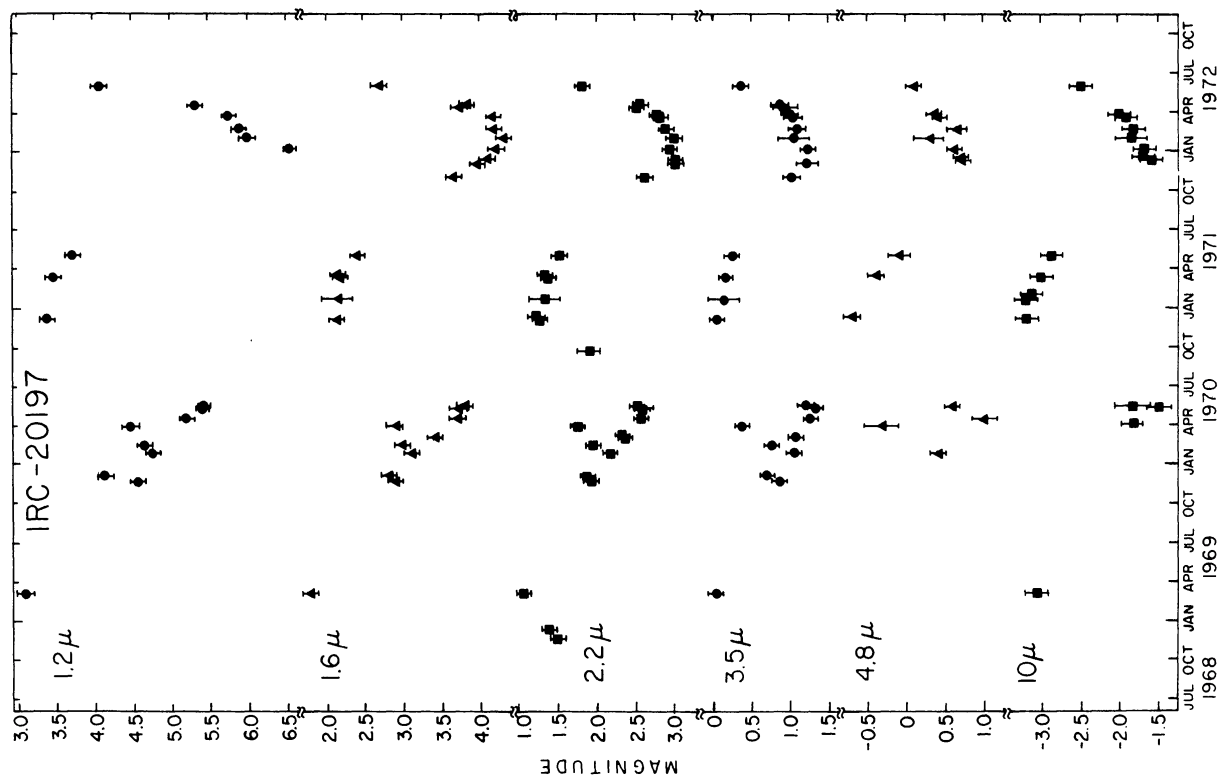


Fig. 8.—The infrared observations of IRC—20197

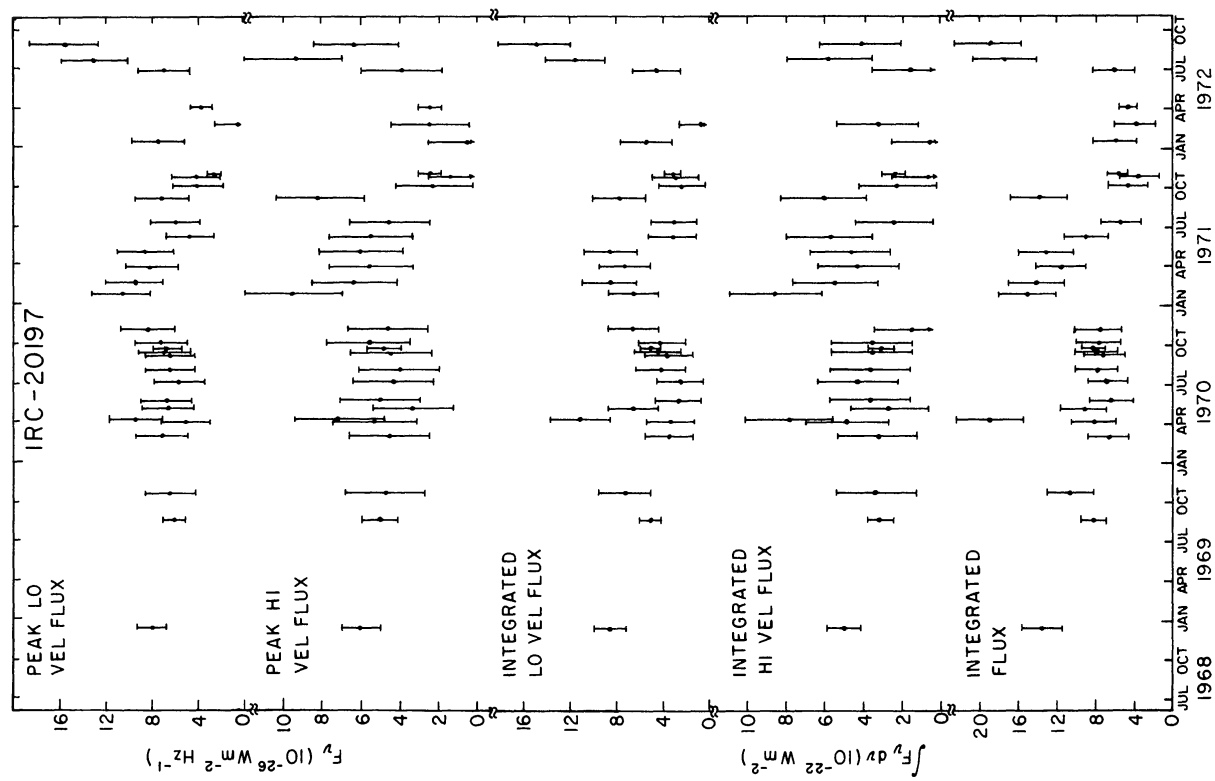


Fig. 9.—The 1612-MHz observations of IRC—20197

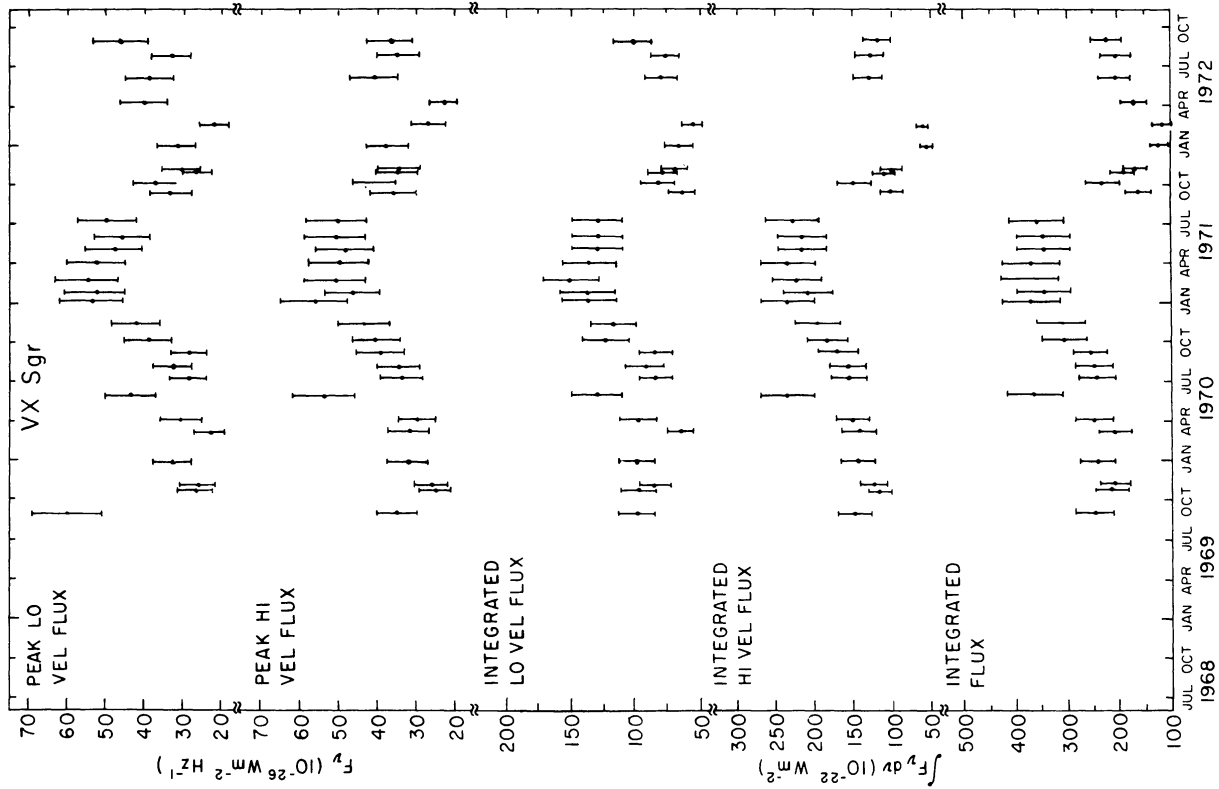


Fig. 11.—The 1612-MHz observations of VX Sgr

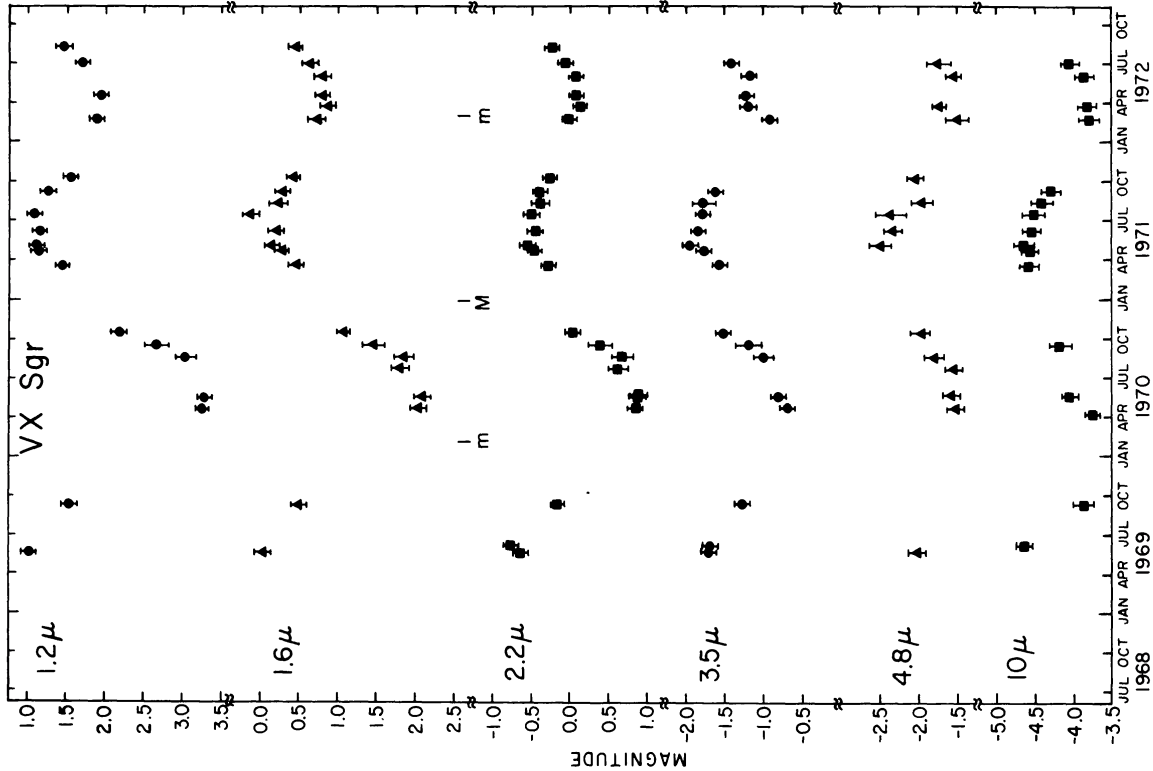


Fig. 10.—The infrared observations of VX Sgr

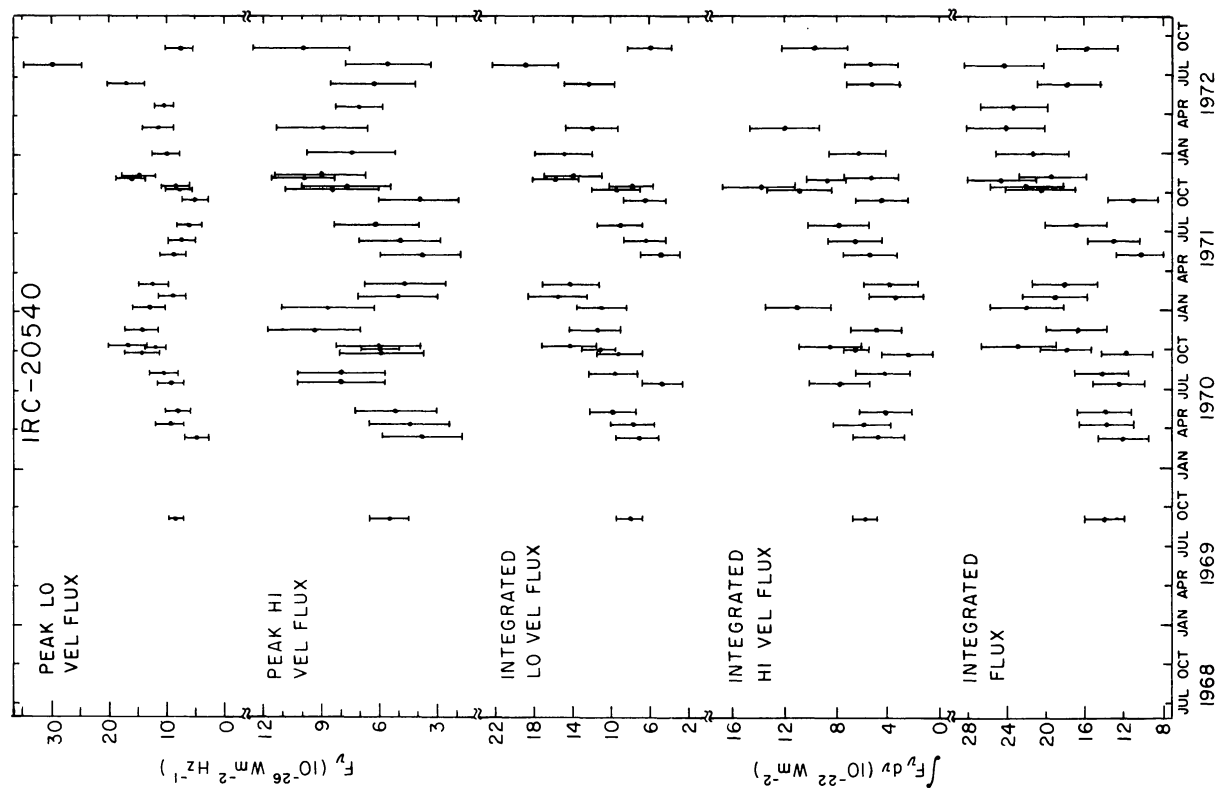


FIG. 13.—The 1612-MHz observations of IRC-20540

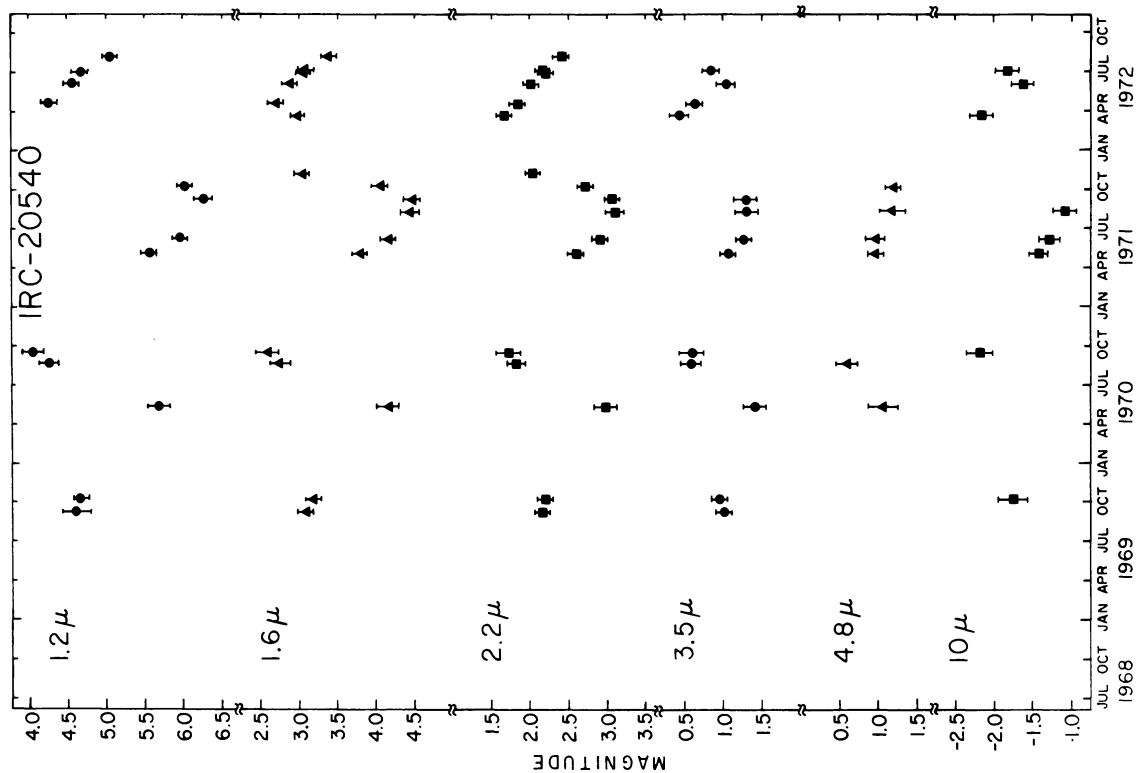


FIG. 12.—The infrared observations of IRC-20540

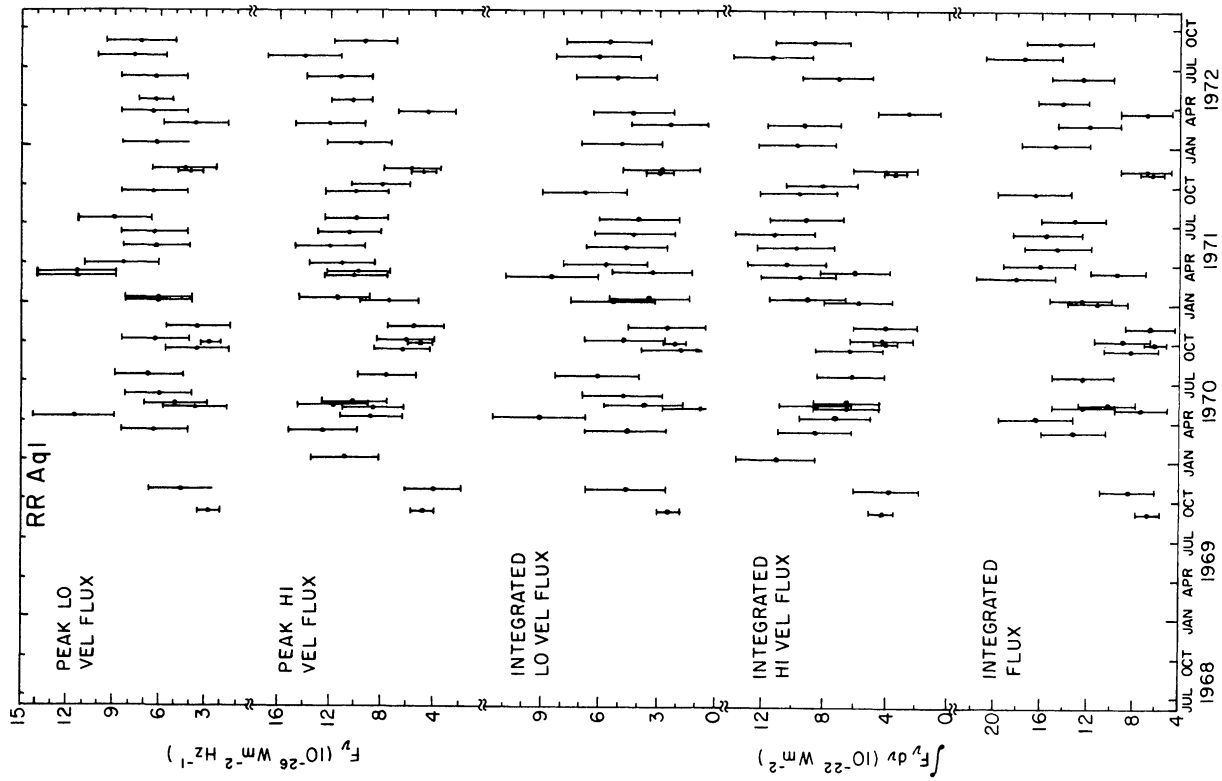


Fig. 15.—The 1612-MHz observations of RR Aql

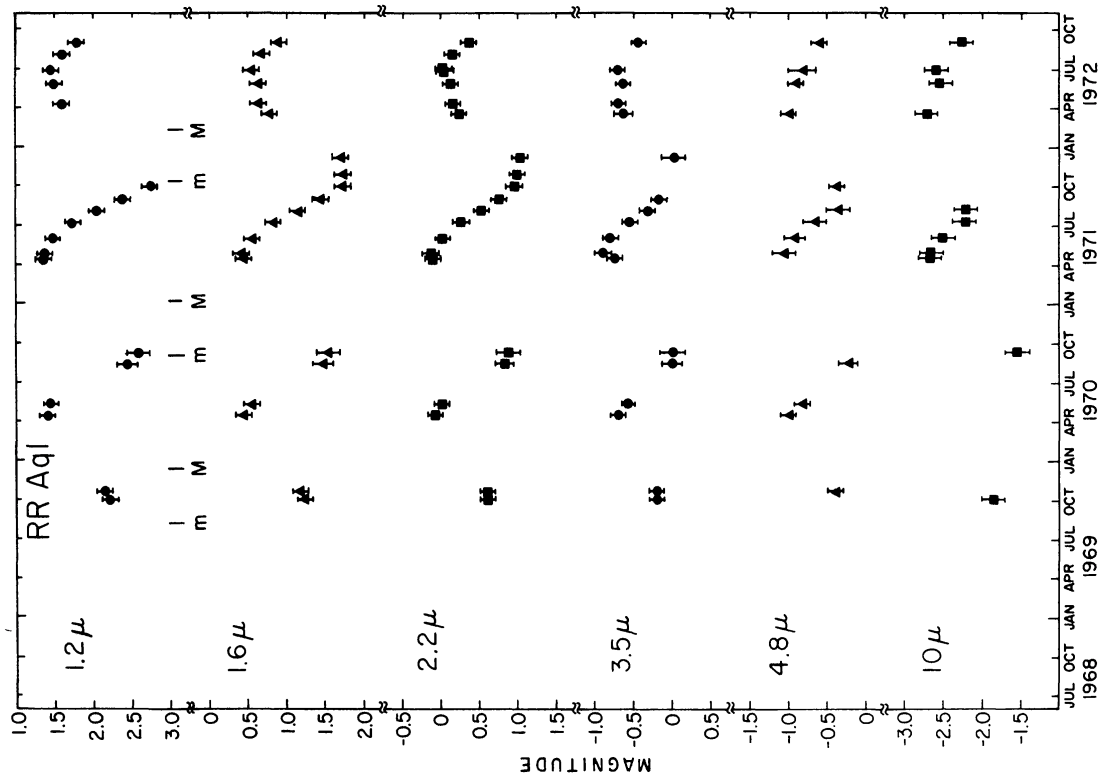


Fig. 14.—The infrared observations of RR Aql

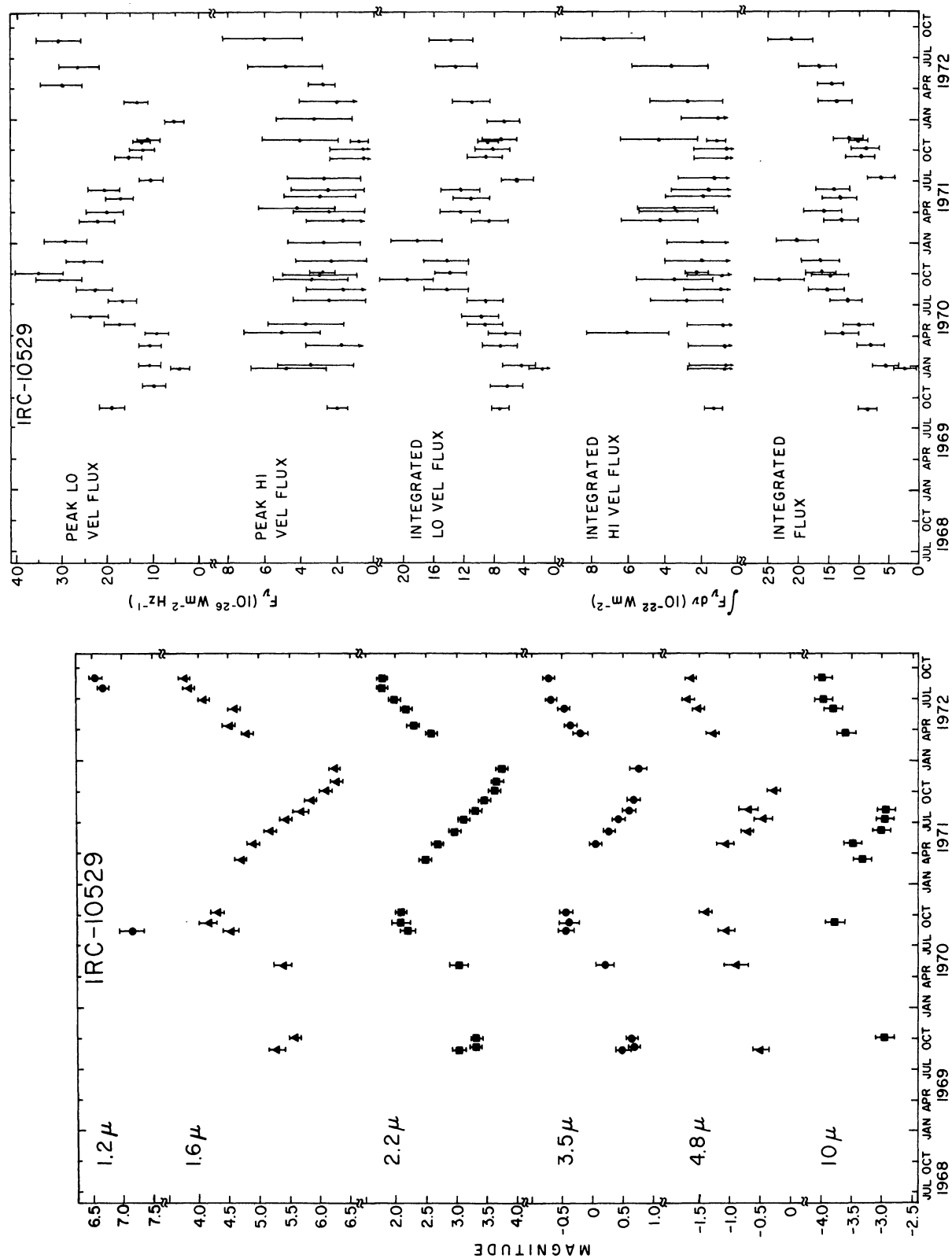


Fig. 16.—The infrared observations of IRC—10529

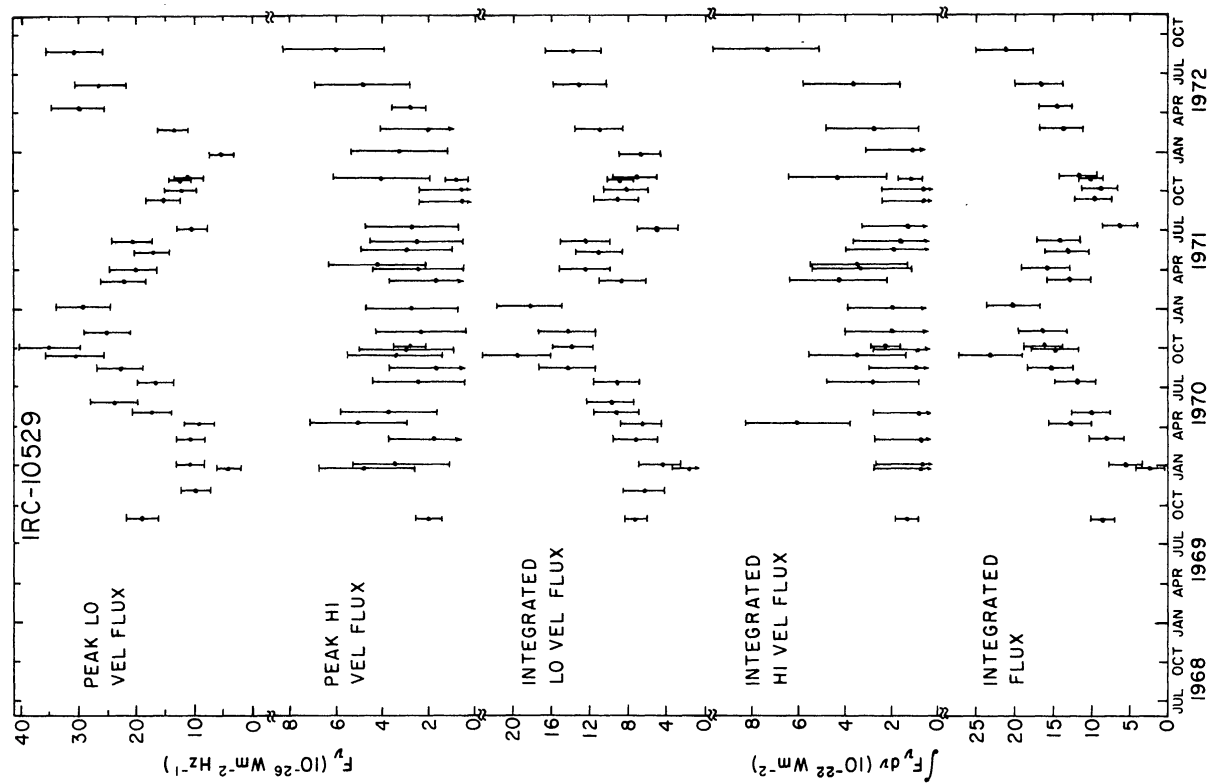
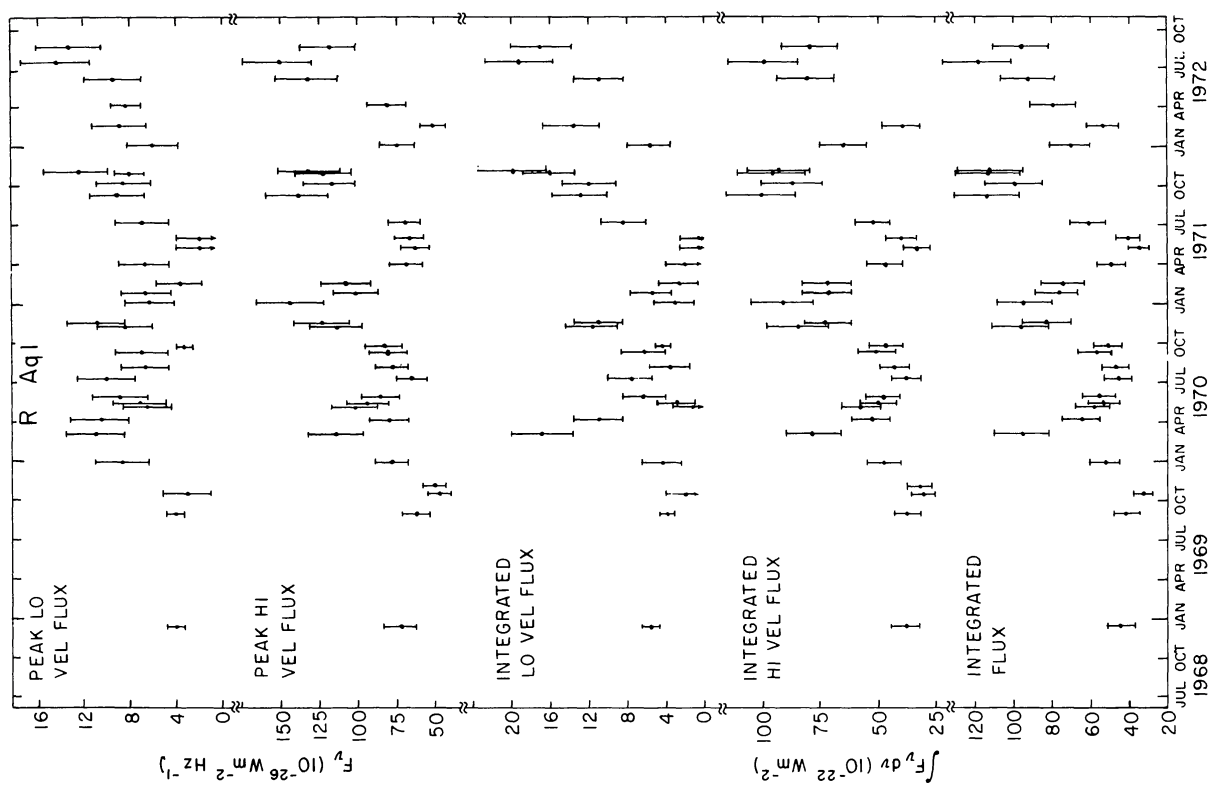
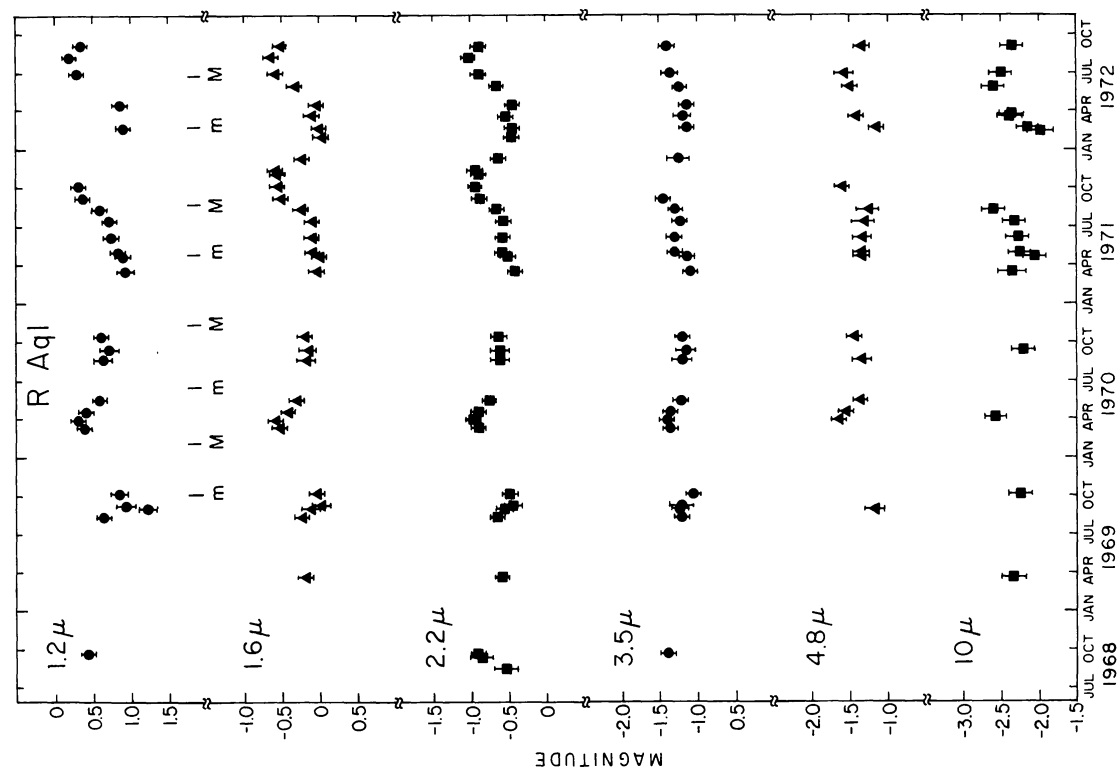


Fig. 17.—The 1612-MHz observations of IRC—10529



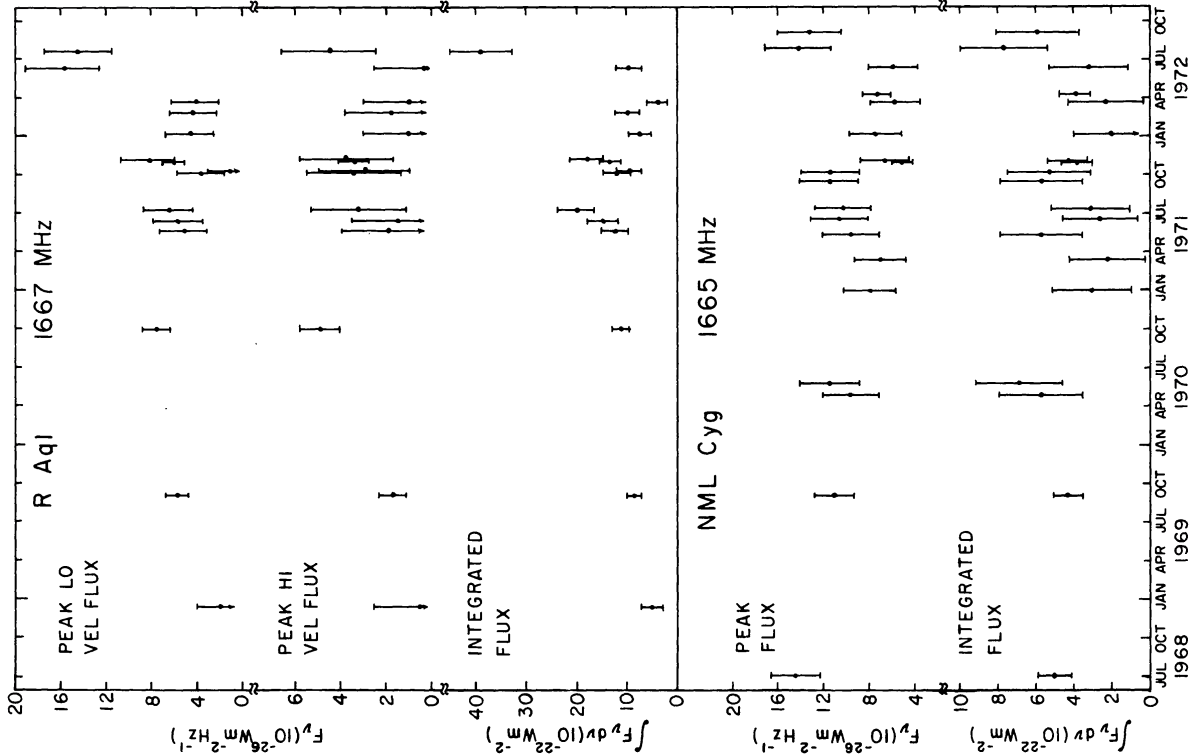


Fig. 20.—The 1667-MHz observations of R Aql and the 1665-MHz observations of NML Cyg.

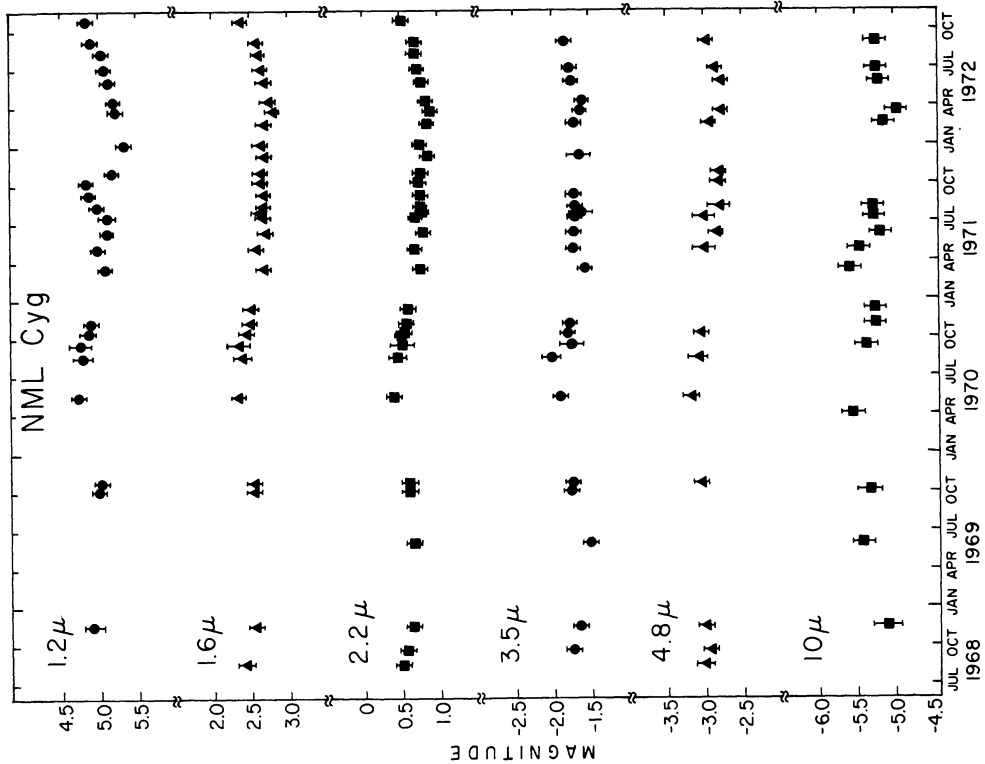


Fig. 21.—The infrared observations of NML Cyg

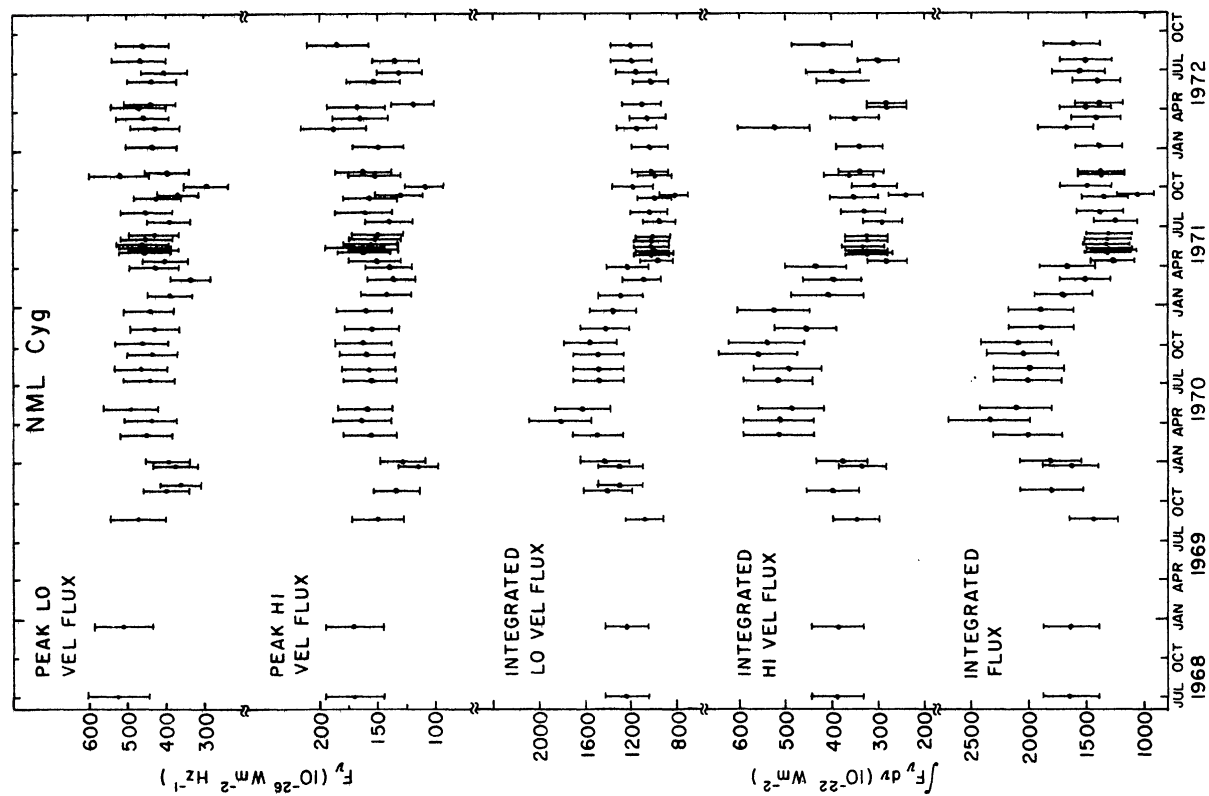


Fig. 22.—The 1612-MHz observations of NML Cyg

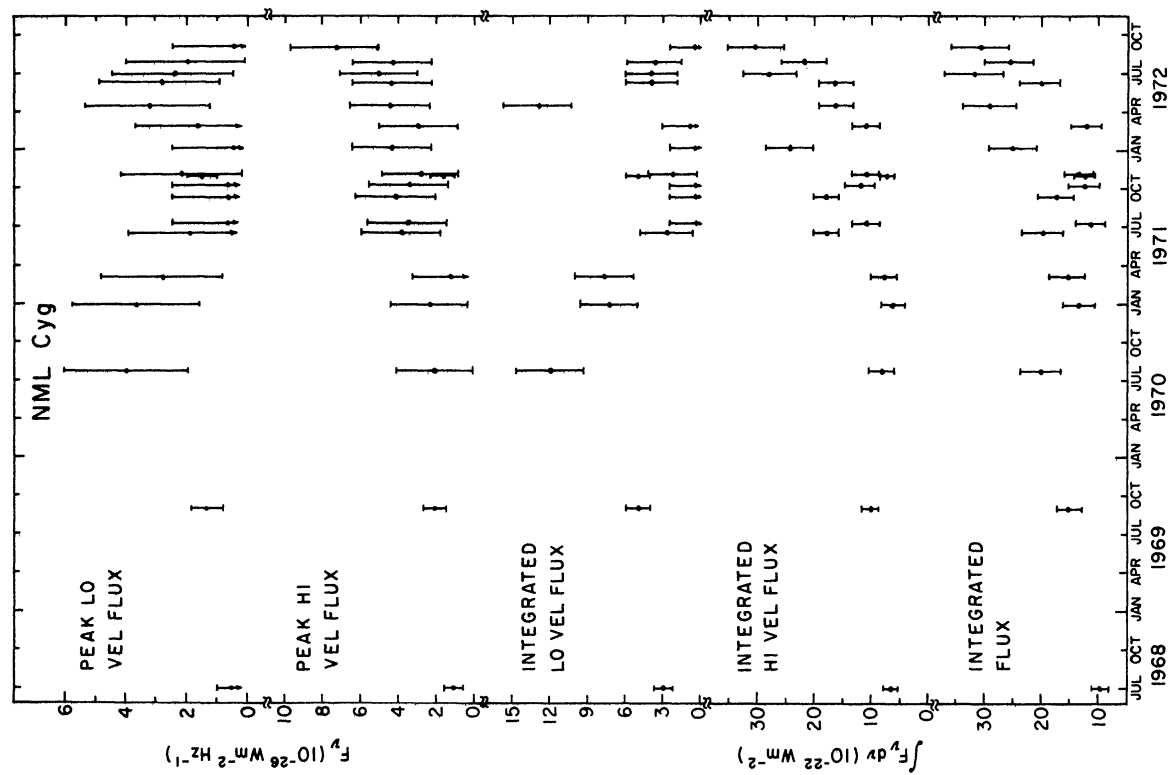


Fig. 23.—The 1667-MHz observations of NML Cyg

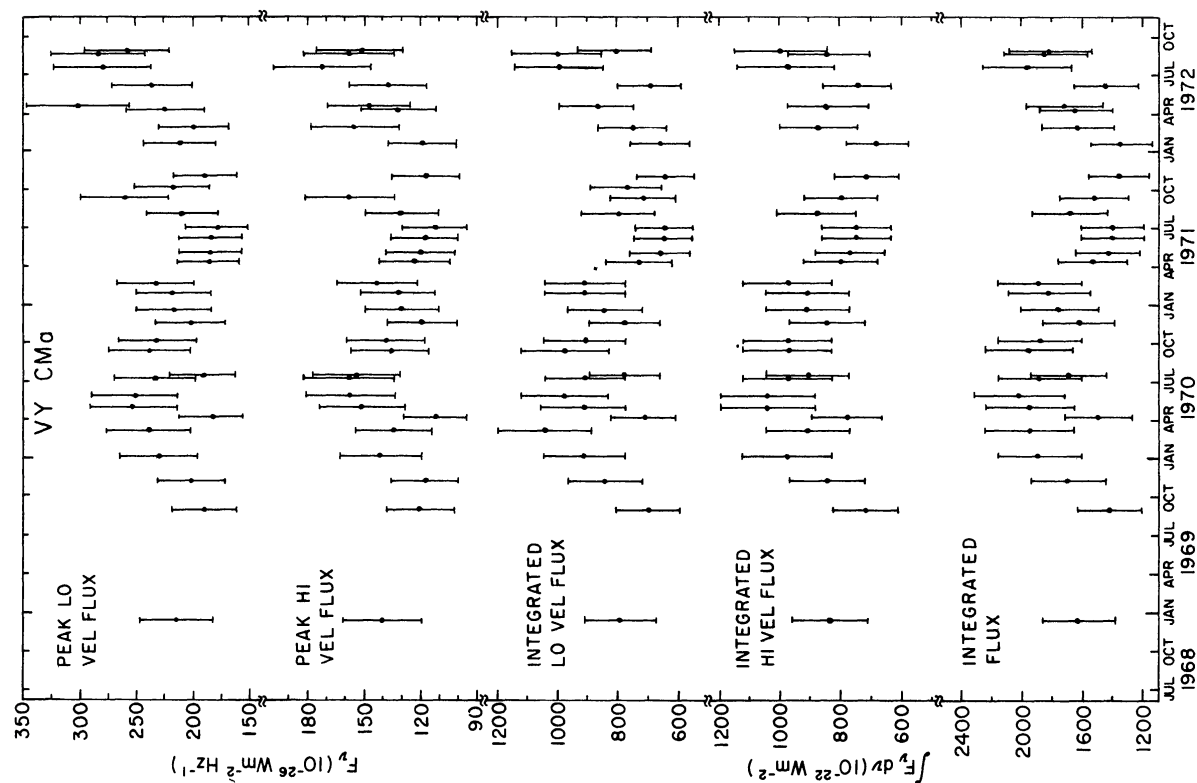


FIG. 25.—The 1612-MHz observations of VY CMa

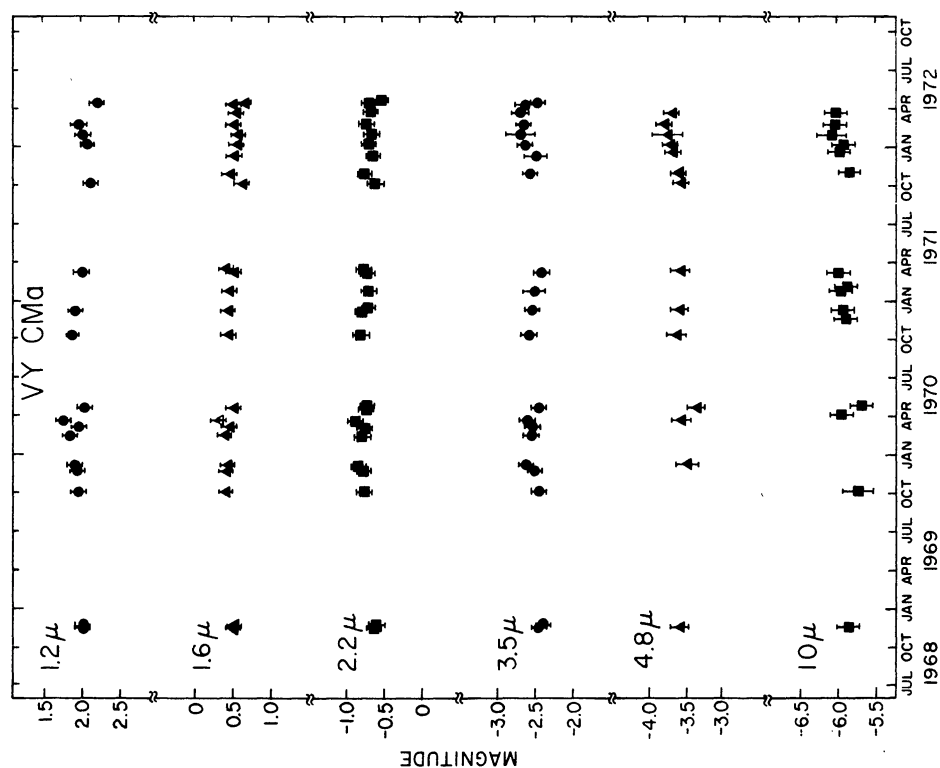


FIG. 24.—The infrared observations of VY CMa

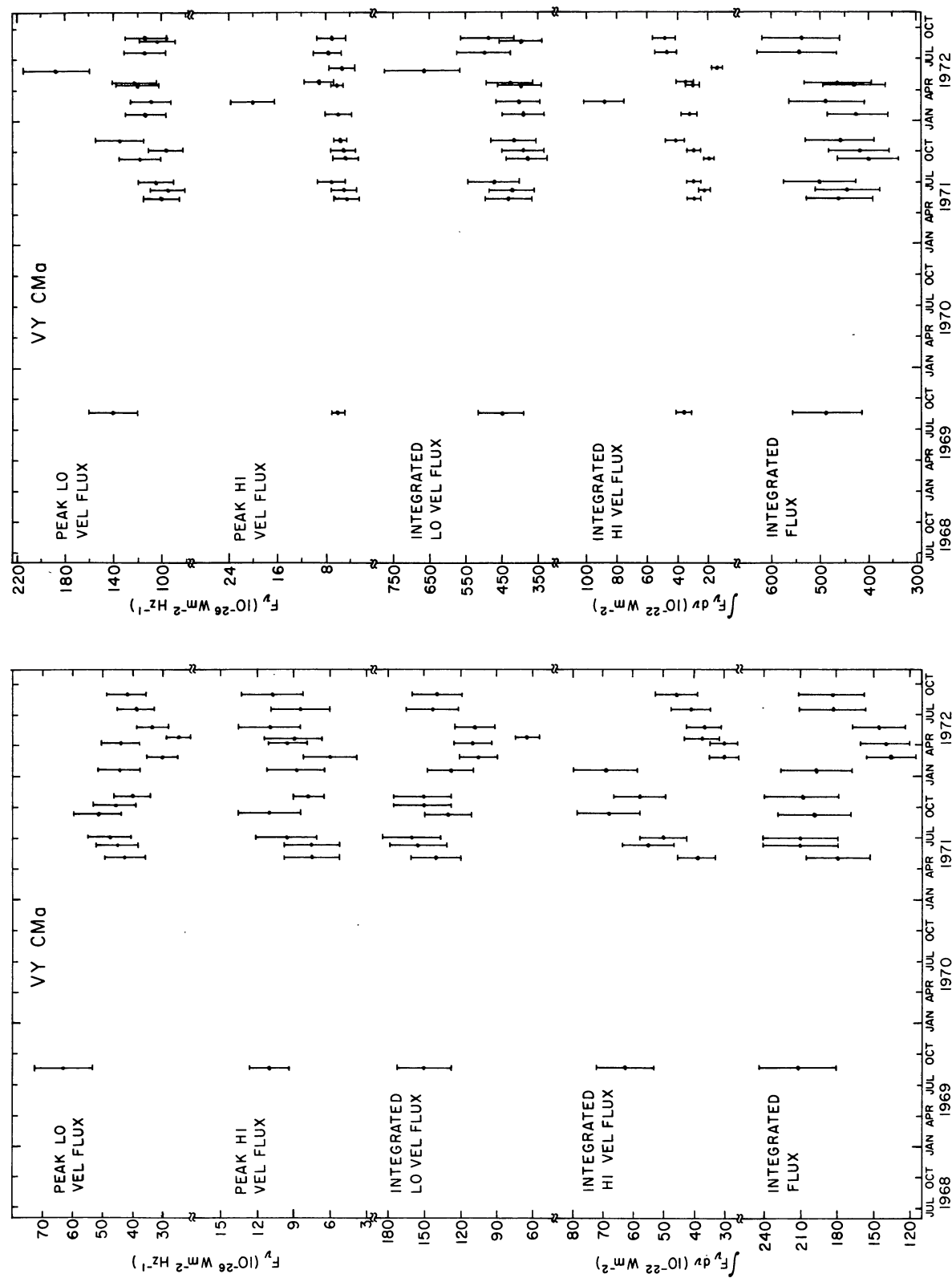


FIG. 26.—The 1665-MHz observations of VY CMa

FIG. 27.—The 1667-MHz observations of VY CMa

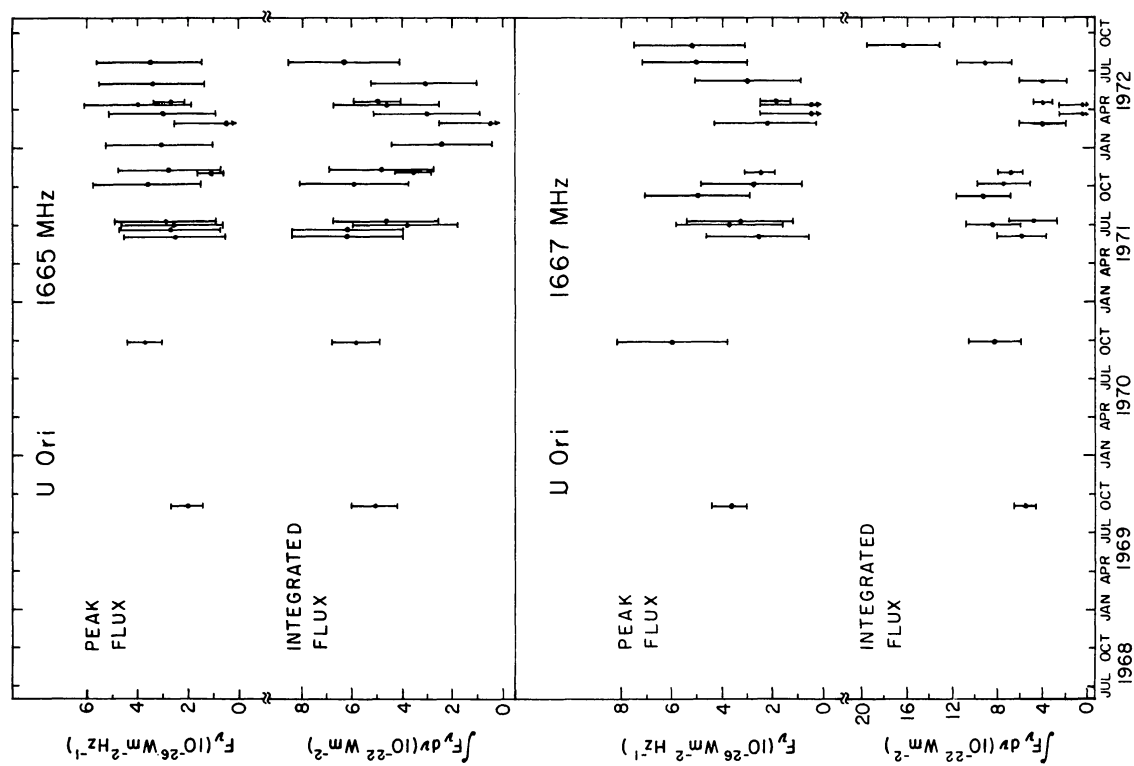


FIG. 29.—The 1665- and 1667-MHz observations of U Ori

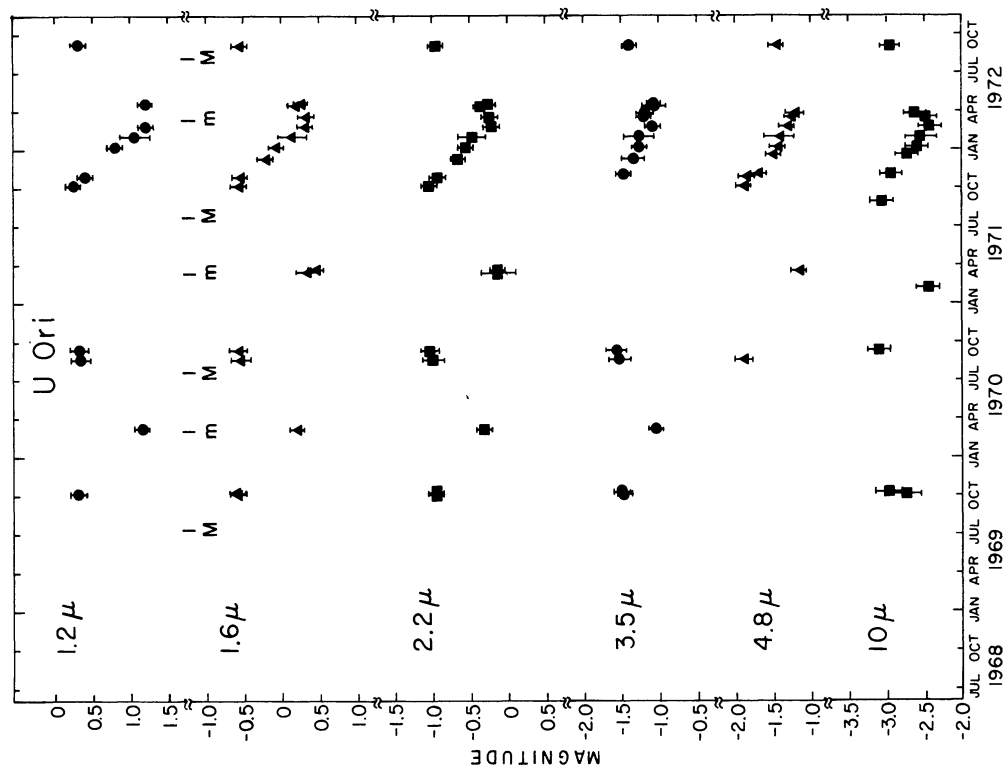


FIG. 28.—The infrared observations of U Ori

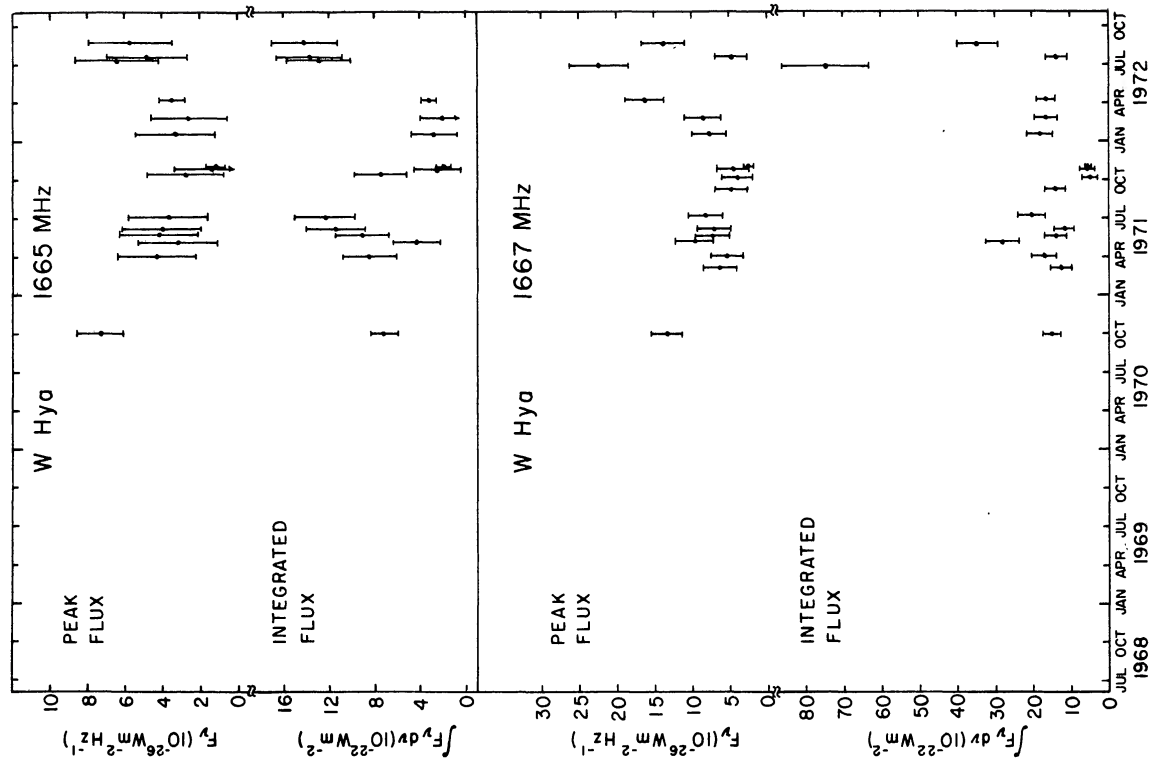


FIG. 31.—The 1665- and 1667-MHz observations of W Hya

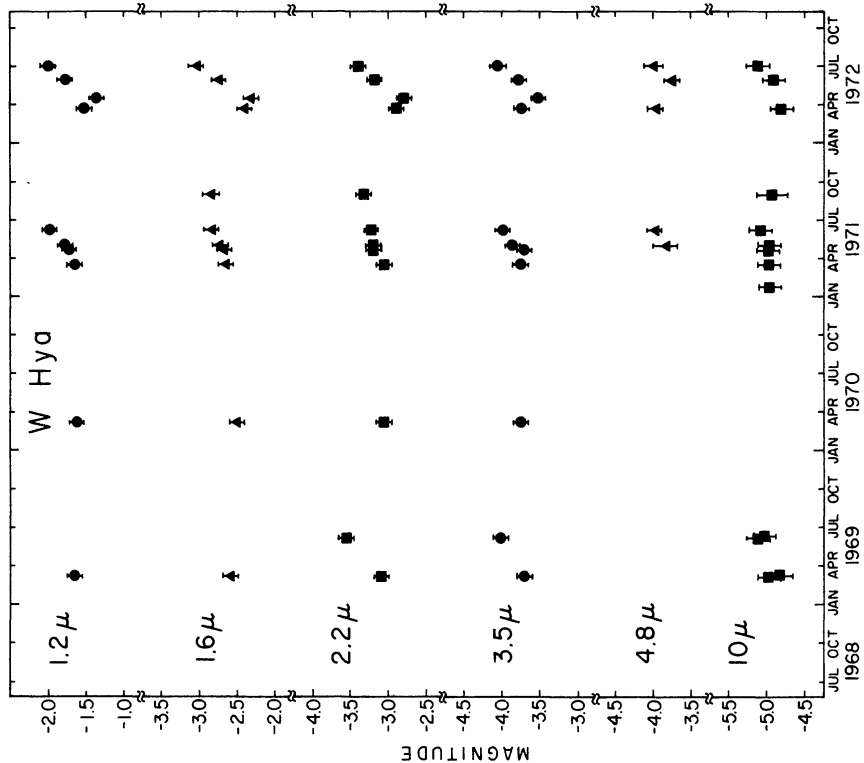


FIG. 30.—The infrared observations of W Hya

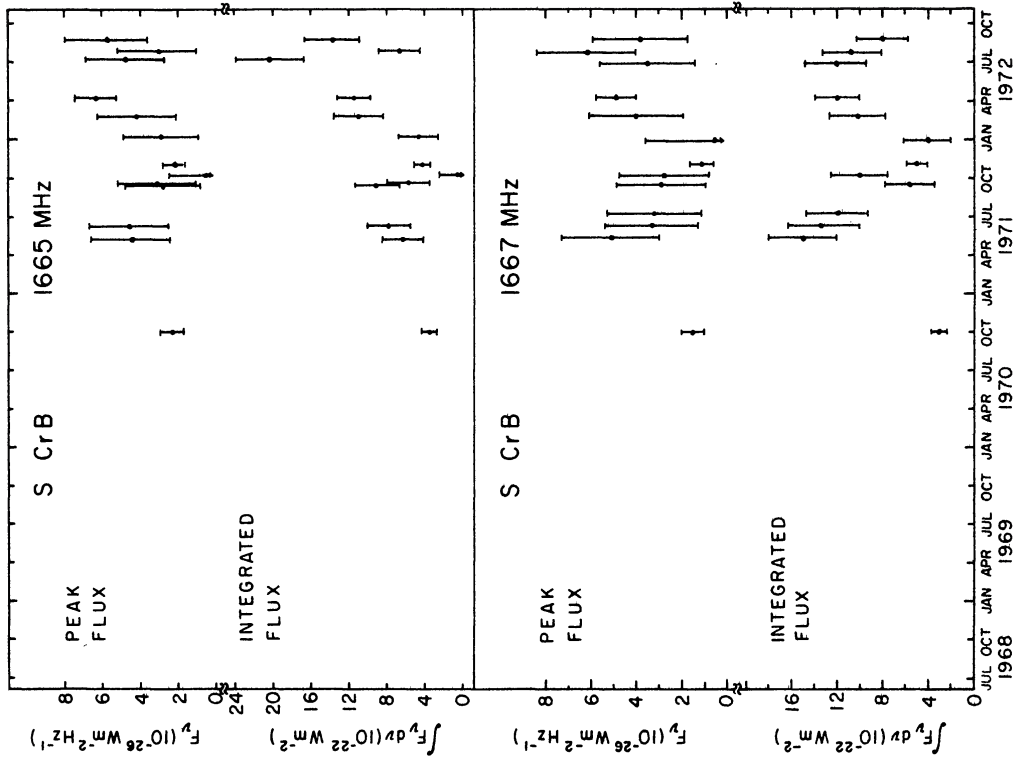


Fig. 33.—The 1665- and 1667-MHz observations of S CrB

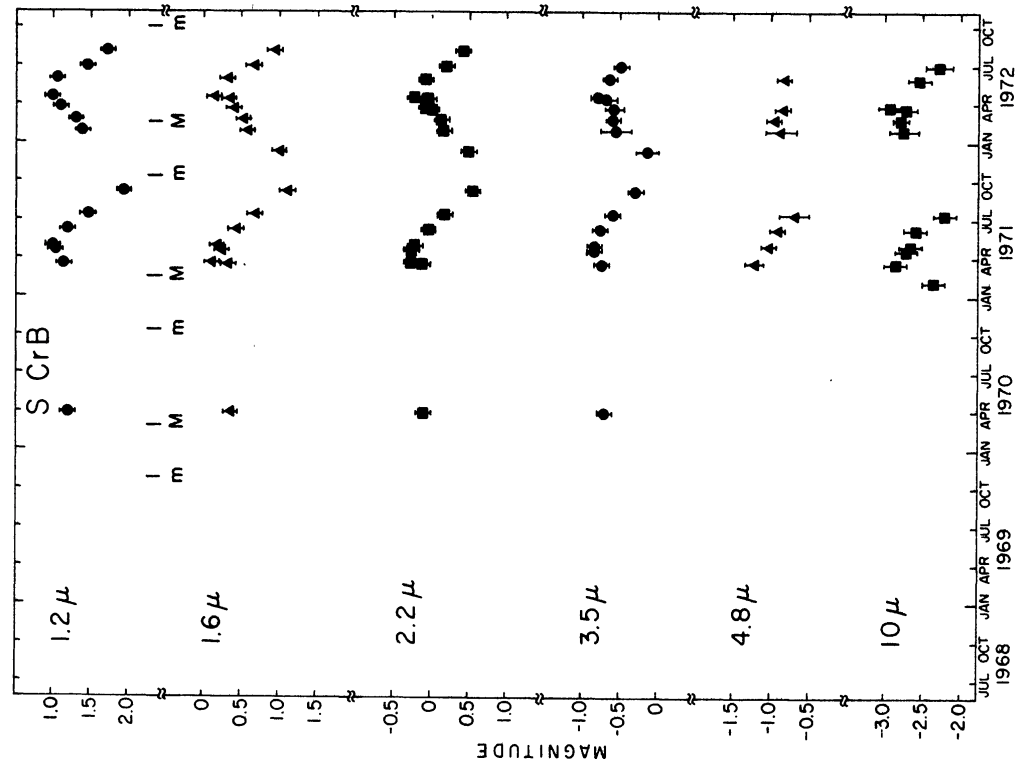


Fig. 32.—The infrared observations of S CrB

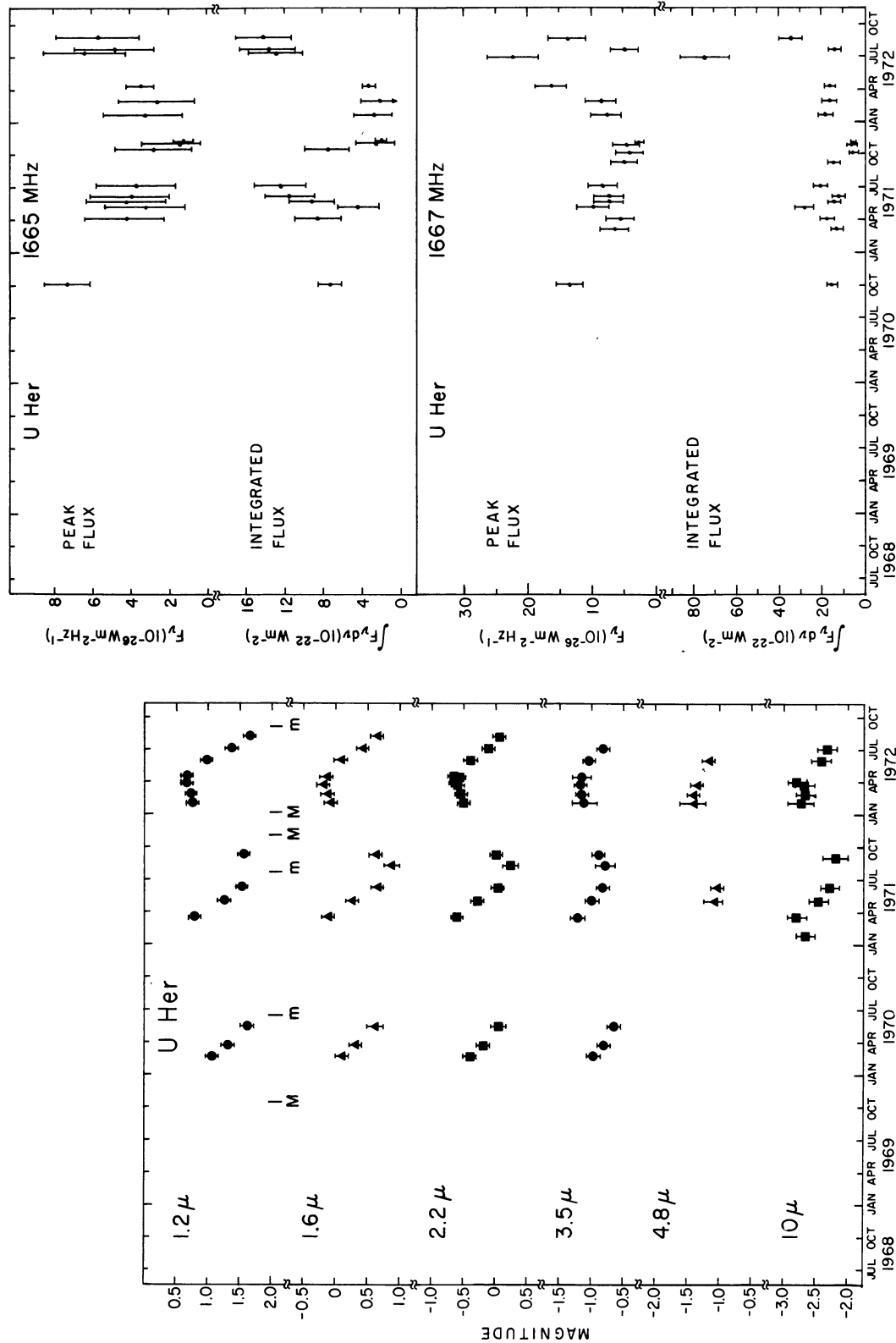


FIG. 35.—The 1665- and 1667-MHz observations of U Her

FIG. 34.—The infrared observations of U Her

TABLE A1  
VALUES OF  $\chi^2$

| STAR        | $\nu$ -OH<br>(MHz) | OH MICROWAVE            |                          |                                    |                                     |                                     | INFRARED  |           |           |           |           |          |
|-------------|--------------------|-------------------------|--------------------------|------------------------------------|-------------------------------------|-------------------------------------|-----------|-----------|-----------|-----------|-----------|----------|
|             |                    | Peak<br>Low<br>Velocity | Peak<br>High<br>Velocity | Inte-<br>grated<br>Low<br>Velocity | Inte-<br>grated<br>High<br>Velocity | Inte-<br>grated<br>Both<br>Velocity | 1.2 $\mu$ | 1.6 $\mu$ | 2.2 $\mu$ | 3.5 $\mu$ | 4.8 $\mu$ | 10 $\mu$ |
| +10011..... | 1612               | 4.4                     | 6.3                      | 4.8                                | 10                                  | 7.6                                 | 94        | 189       | 136       | 53        | 25        | 22       |
| +50137..... | 1612               | 1.1                     | 3.7                      | 6.0                                | 4.0                                 | 5.0                                 | 27        | 232       | 74        | 29        | 48        | 20       |
| VY CMa..... | 1612               | 0.9                     | 0.7                      | 1.0                                | 0.7                                 | 0.8                                 | 1.0       | 0.5       | 0.6       | 0.5       | 0.8       | 0.5      |
| VY CMa..... | 1665               | 2.1                     | 0.5                      | 4.5                                | 4.1                                 | 1.5                                 | ...       | ...       | ...       | ...       | ...       | ...      |
| VY CMa..... | 1667               | 1.1                     | 3.8                      | 0.8                                | 12                                  | 0.4                                 | ...       | ...       | ...       | ...       | ...       | ...      |
| -20197..... | 1612               | 4.7                     | 2.8                      | 5.3                                | 1.8                                 | 5.7                                 | 512       | 132       | 58        | 15        | 18        | 50       |
| VX Sgr..... | 1612               | 3.6                     | 2.7                      | 4.0                                | 14                                  | 7.0                                 | 216       | 116       | 42        | 16        | 5.7       | 9.1      |
| R Aql.....  | 1612               | 5.6                     | 6.2                      | 16                                 | 7.9                                 | 8.6                                 | 6.4       | 4.2       | 3.2       | 1.0       | 1.6       | 1.8      |
| R Aql.....  | 1667               | 6.3                     | 0.7                      | ...                                | ...                                 | 10.5                                | ...       | ...       | ...       | ...       | ...       | ...      |
| -20540..... | 1612               | 4.1                     | 1.1                      | 3.1                                | 2.5                                 | 2.5                                 | 107       | 79        | 28        | 6.8       | 2.1       | 9.6      |
| RR Aql..... | 1612               | 3.9                     | 3.5                      | 2.4                                | 3.8                                 | 4.8                                 | 30        | 32        | 19        | 7.6       | 6.5       | 10       |
| -10529..... | 1612               | 12                      | 1.5                      | 4.0                                | 2.6                                 | 4.5                                 | 4.0       | 160       | 73        | 33        | 32        | 13       |
| NML Cyg...  | 1612               | 0.4                     | 0.7                      | 1.4                                | 2.1                                 | 1.5                                 | 2.0       | 1.2       | 1.4       | 0.8       | 1.2       | 1.3      |
| NML Cyg...  | 1665               | ...                     | 2.6                      | ...                                | ...                                 | 1.0                                 | ...       | ...       | ...       | ...       | ...       | ...      |
| NML Cyg...  | 1667               | 0.7                     | 1.9                      | 8.8                                | 12                                  | 5.9                                 | ...       | ...       | ...       | ...       | ...       | ...      |
| U Ori.....  | 1665               | ...                     | 1.0                      | ...                                | ...                                 | 1.0                                 | 20        | 18        | 11        | 2.5       | 5.8       | 2.9      |
| U Ori.....  | 1667               | ...                     | 1.2                      | ...                                | ...                                 | 6.7                                 | ...       | ...       | ...       | ...       | ...       | ...      |
| W Hya.....  | 1665               | ...                     | 2.6                      | ...                                | ...                                 | 26                                  | 3.6       | 4.1       | 4.2       | 2.2       | 0.9       | 0.5      |
| W Hya.....  | 1667               | ...                     | 20                       | ...                                | ...                                 | 44                                  | ...       | ...       | ...       | ...       | ...       | ...      |
| S CrB.....  | 1665               | ...                     | 2.8                      | ...                                | ...                                 | 11                                  | 10        | 11        | 7.2       | 3.1       | 1.1       | 2.5      |
| S CrB.....  | 1667               | ...                     | 4.0                      | ...                                | ...                                 | 5.5                                 | ...       | ...       | ...       | ...       | ...       | ...      |
| U Her.....  | 1665               | ...                     | 1.1                      | ...                                | ...                                 | 2.1                                 | 16        | 14        | 8.7       | 2.6       | 2.0       | 2.0      |
| U Her.....  | 1667               | ...                     | 1.5                      | ...                                | ...                                 | 2.1                                 | ...       | ...       | ...       | ...       | ...       | ...      |

TABLE A2  
PERIODS OF 1612-MHz OH SOURCES (in days)

| Star        | Microwave<br>Period* | Infrared<br>Period* | Visual<br>Period† |
|-------------|----------------------|---------------------|-------------------|
| +10011..... | 700 $\pm$ 30         | 660 $\pm$ 15        | ...               |
| +50137..... | 635 $\pm$ 30         | 635 $\pm$ 10        | ...               |
| -20197..... | 680 $\pm$ 40         | 650 $\pm$ 20        | ...               |
| VX Sgr..... | 740 $\pm$ 40         | 770 $\pm$ 150       | 732               |
| R Aql.....  | 285 $\pm$ 10         | 285 $\pm$ 10        | 293               |
| -20540..... | 520 $\pm$ 80         | 510 $\pm$ 15        | ...               |
| RR Aql..... | 425 $\pm$ 20         | 410 $\pm$ 10        | 394               |
| -10529..... | 650 $\pm$ 50         | 670 $\pm$ 30        | ...               |

\* Confidence limits determined from statistics of fit and agreement of fits to different microwave parameters and infrared wavelengths.

† Kukarin *et al.* (1969).

TABLE A3  
PHASE OF VARIATIONS OF 1612-MHz OH SOURCES\*

| STAR      | AS-SUMED PERIOD (days) | RELATIVE PHASE: 1612 MHz (days) |                    |                         |                          |                    | RELATIVE PHASE: INFRARED (days) |              |              |              |              |              |
|-----------|------------------------|---------------------------------|--------------------|-------------------------|--------------------------|--------------------|---------------------------------|--------------|--------------|--------------|--------------|--------------|
|           |                        | Peak Low Velocity               | Peak High Velocity | Integrated Low Velocity | Integrated High Velocity | Integrated Flux    | 1.2 $\mu$                       | 1.6 $\mu$    | 2.2 $\mu$    | 3.5 $\mu$    | 4.8 $\mu$    | 10 $\mu$     |
| +10011..  | 650                    | 128 $\pm$ 50                    | 115 $\pm$ 50       | 136 $\pm$ 50            | 88 $\pm$ 50              | 108 $\pm$ 50       | 143 $\pm$ 10                    | 135 $\pm$ 20 | 104 $\pm$ 20 | 148 $\pm$ 30 | 144 $\pm$ 20 | 180 $\pm$ 30 |
| +50137..  | 630                    | 578 $\pm$ 75                    | 567 $\pm$ 40       | 586 $\pm$ 50            | 585 $\pm$ 50             | 572 $\pm$ 40       | 585 $\pm$ 80                    | 572 $\pm$ 15 | 574 $\pm$ 15 | 601 $\pm$ 20 | 601 $\pm$ 20 | 604 $\pm$ 20 |
| -20197..  | 640                    | 282 $\pm$ 50                    | 260 $\pm$ 50       | 256 $\pm$ 65            | 259 $\pm$ 100            | 252 $\pm$ 65       | 217 $\pm$ 25                    | 205 $\pm$ 25 | 222 $\pm$ 20 | 249 $\pm$ 25 | 240 $\pm$ 35 | 229 $\pm$ 25 |
| VX Sgr..  | 730                    | 345 $\pm$ 55                    | 357 $\pm$ 50       | 370 $\pm$ 50            | 370 $\pm$ 45             | 373 $\pm$ 45       | 235 $\pm$ 60                    | 233 $\pm$ 70 | 251 $\pm$ 65 | 284 $\pm$ 70 | 254 $\pm$ 70 | 315 $\pm$ 60 |
| R Aql.... | 290                    | 155 $\pm$ 35                    | 163 $\pm$ 15       | 170 $\pm$ 45            | 164 $\pm$ 20             | 166 $\pm$ 20       | 169 $\pm$ 20                    | 159 $\pm$ 15 | 157 $\pm$ 15 | 163 $\pm$ 25 | 172 $\pm$ 40 | 180 $\pm$ 30 |
| -20540..  | 510                    | 212 $\pm$ $\infty$              | 272 $\pm$ 100      | 213 $\pm$ $\infty$      | 292 $\pm$ $\infty$       | 207 $\pm$ $\infty$ | 183 $\pm$ 15                    | 189 $\pm$ 15 | 196 $\pm$ 20 | 203 $\pm$ 45 | 169 $\pm$ 40 | 204 $\pm$ 60 |
| RR Aql..  | 410                    | 303 $\pm$ 50                    | 297 $\pm$ 35       | 304 $\pm$ 60            | 299 $\pm$ 45             | 298 $\pm$ 50       | 318 $\pm$ 10                    | 317 $\pm$ 10 | 314 $\pm$ 10 | 315 $\pm$ 10 | 319 $\pm$ 25 | 324 $\pm$ 20 |
| -10529..  | 670                    | 364 $\pm$ 45                    | 390 $\pm$ 120      | 386 $\pm$ 60            | 392 $\pm$ 140            | 388 $\pm$ 60       | †                               | 374 $\pm$ 20 | 383 $\pm$ 20 | 398 $\pm$ 20 | 408 $\pm$ 50 | 437 $\pm$ 45 |

\* Phase is defined as the time of maximum emission measured from 1969 August 15.

† Insufficient data.

TABLE A4  
CORRELATION COEFFICIENTS\* BETWEEN 1612 MHz AND INFRARED VARIATIONS

| STAR        | 1612 MHz-2.2 $\mu$ |                    |                         |                          |                 | 1612 MHz-10 $\mu$ |                    |                         |                          |                 |
|-------------|--------------------|--------------------|-------------------------|--------------------------|-----------------|-------------------|--------------------|-------------------------|--------------------------|-----------------|
|             | Peak Low Velocity  | Peak High Velocity | Integrated Low Velocity | Integrated High Velocity | Integrated Flux | Peak Low Velocity | Peak High Velocity | Integrated Low Velocity | Integrated High Velocity | Integrated Flux |
| +10011..... | 0.73               | 0.71               | 0.69                    | 0.68                     | 0.74            | +0.79             | +0.72              | +0.70                   | 0.61                     | 0.71            |
| +50137..... | 0.37               | 0.75               | 0.69                    | 0.77                     | 0.85            | +0.31             | +0.72              | +0.44                   | 0.75                     | 0.79            |
| -20197..... | 0.75               | 0.78               | 0.82                    | 0.74                     | 0.87            | +0.79             | +0.78              | +0.74                   | 0.80                     | 0.80            |
| VX Sgr..... | 0.53               | 0.44               | 0.30                    | 0.18                     | 0.22            | +0.67             | +0.80              | +0.74                   | 0.73                     | 0.76            |
| R Aql.....  | 0.68               | 0.77               | 0.71                    | 0.69                     | 0.72            | +0.37             | +0.66              | +0.04                   | 0.65                     | 0.62            |
| -20540..... | 0.17               | 0.20               | 0.34                    | 0.01                     | 0.37            | +0.05             | +0.74              | +0.30                   | 0.75                     | 0.55            |
| RR Aql..... | 0.67               | 0.64               | 0.52                    | 0.45                     | 0.46            | +0.17             | +0.31              | -0.32                   | 0.13                     | 0.00            |
| -10529..... | 0.87               | 0.36               | 0.79                    | 0.35                     | 0.79            | +0.81             | +0.57              | +0.65                   | 0.53                     | 0.60            |
| NML Cyg.... | 0.11               | 0.12               | 0.85                    | 0.68                     | 0.84            | -0.20             | -0.02              | +0.42                   | 0.32                     | 0.40            |

\* Correlation coefficients greater than 0.4-0.5 imply greater than 95 percent probability that the data are correlated.

TABLE A5  
PHASES AND CORRELATION COEFFICIENTS FOR VARIATIONS OF 1665/1667 MHz SOURCES

| STAR       | $\nu$ -OH (MHz) | VELOCITY FEATURE | ASSUMED PERIOD* (days) | RELATIVE 2.2 $\mu$ PHASE (days) | PEAK MICROWAVE FLUX   |                          | INTEGRATED MICROWAVE FLUX |                          |
|------------|-----------------|------------------|------------------------|---------------------------------|-----------------------|--------------------------|---------------------------|--------------------------|
|            |                 |                  |                        |                                 | Relative Phase (days) | Correlation Coefficient† | Relative Phase (days)     | Correlation Coefficient† |
| U Ori..... | 1665            | Both             | 370                    | 75 $\pm$ 15                     | 180 $\pm$ $\infty$    | -0.28                    | 140 $\pm$ $\infty$        | -0.20                    |
| U Ori..... | 1667            | Both             | 370                    | 75 $\pm$ 15                     | 115 $\pm$ 30          | +0.64                    | 90 $\pm$ 50               | +0.59                    |
| W Hya..... | 1665            | Both             | 380                    | 140 $\pm$ 30                    | 200 $\pm$ 35          | +0.78                    | 165 $\pm$ 20              | +0.94                    |
| W Hya..... | 1667            | Both             | 380                    | 140 $\pm$ 30                    | 235 $\pm$ 40          | +0.05                    | 225 $\pm$ 45              | +0.45                    |
| S CrB..... | 1665            | Both             | 360                    | 200 $\pm$ 15                    | 185 $\pm$ 50          | +0.48                    | 165 $\pm$ 90              | -0.07                    |
| S CrB..... | 1667            | Both             | 360                    | 200 $\pm$ 15                    | 165 $\pm$ 40          | +0.33                    | 165 $\pm$ 30              | +0.79                    |
| U Her..... | 1665            | Both             | 405                    | 395 $\pm$ 15                    | 355 $\pm$ $\infty$    | +0.57                    | 385 $\pm$ $\infty$        | +0.43                    |
| U Her..... | 1667            | Both             | 405                    | 395 $\pm$ 15                    | 240 $\pm$ $\infty$    | -0.46                    | 130 $\pm$ $\infty$        | +0.22                    |
| NML Cyg..  | 1665            | Both             | ...                    | ...                             | ...                   | +0.36                    | ...                       | +0.56                    |
| NML Cyg..  | 1667            | Low              | ...                    | ...                             | ...                   | +0.69                    | ...                       | +0.31                    |
| NML Cyg..  | 1667            | High             | ...                    | ...                             | ...                   | +0.22                    | ...                       | +0.02                    |
| R Aql..... | 1667            | Low              | 290                    | 155 $\pm$ 15                    | 210 $\pm$ $\infty$    | +0.37                    | 180 $\pm$ $\infty$        | +0.40                    |
| R Aql..... | 1667            | High             | 290                    | 155 $\pm$ 15                    | 150 $\pm$ 70          | +0.70                    | ...                       | ...                      |

\* Kukarkin *et al.* 1969.

† Greater than 0.6 to 0.7 implies greater than 95 percent probability that data are correlated.

TABLE A6  
AMPLITUDES OF VARIATION— $F_{\max}/F_{\min}$

| STAR         | $\nu$ -OH<br>(MHz) | MICROWAVE                    |                                |                                         |                                          |                         | INFRARED  |           |           |           |           |          |
|--------------|--------------------|------------------------------|--------------------------------|-----------------------------------------|------------------------------------------|-------------------------|-----------|-----------|-----------|-----------|-----------|----------|
|              |                    | Peak<br>Low<br>Ve-<br>locity | Peak<br>High<br>Ve-<br>locity* | Inte-<br>grated<br>Low<br>Ve-<br>locity | Inte-<br>grated<br>High<br>Ve-<br>locity | Inte-<br>grated<br>Flux | 1.2 $\mu$ | 1.6 $\mu$ | 2.2 $\mu$ | 3.5 $\mu$ | 4.8 $\mu$ | 10 $\mu$ |
| +10011.....  | 1612               | 2.0                          | 2.7                            | 2.1                                     | 3.0                                      | 2.8                     | >7.0      | 10.0      | 7.0       | 4.4       | 4.0       | 4.8      |
| +50137.....  | 1612               | 2.3                          | 2.6                            | 2.5                                     | 2.4                                      | 2.7                     | >9.0      | 9.1       | 5.2       | 3.3       | 3.3       | 3.0      |
| VY CMa.....  | 1612               | <1.3                         | <1.3                           | <1.3                                    | <1.3                                     | <1.3                    | <1.3      | <1.3      | <1.3      | <1.3      | <1.3      | <1.3     |
| VY CMa.....  | 1665               | 1.5                          | <1.5                           | 1.5                                     | 2.0                                      | 1.5                     | ...       | ...       | ...       | ...       | ...       | ...      |
| VY CMa.....  | 1667               | <1.3                         | <1.5                           | <1.3                                    | 2.5                                      | 1.4                     | ...       | ...       | ...       | ...       | ...       | ...      |
| -20197.....  | 1612               | 2.4                          | 1.8                            | 2.5                                     | 2.0                                      | 2.3                     | 19.0      | 7.6       | 5.2       | 3.3       | 4.0       | 4.4      |
| VX Sgr.....  | 1612               | 1.8                          | 1.8                            | 1.9                                     | 2.9                                      | 2.2                     | 7.6       | 5.8       | 3.3       | 3.0       | 2.5       | 2.3      |
| R Aql.....   | 1612               | 1.7                          | 2.0                            | 2.7                                     | 2.6                                      | 2.5                     | 1.7       | 1.7       | 1.7       | 1.6       | 1.5       | 1.6      |
| R Aql.....   | 1667               | 3.0                          | <2.0                           | ...                                     | ...                                      | 4.0                     | ...       | ...       | ...       | ...       | ...       | ...      |
| -20540.....  | 1612               | 2.0                          | 1.8                            | 2.2                                     | 2.2                                      | 2.0                     | 7.6       | 5.8       | 3.3       | 2.3       | >1.7      | 2.7      |
| RR Aql.....  | 1612               | 2.5                          | 2.0                            | 2.0                                     | 2.5                                      | 2.0                     | 3.6       | 3.3       | 2.8       | 2.3       | 2.3       | 2.7      |
| -10529.....  | 1612               | 4.5                          | †                              | 2.5                                     | †                                        | 2.5                     | †         | 6.3       | 4.8       | 3.3       | >2.8      | >2.3     |
| NML Cyg..... | 1612               | <1.3                         | <1.3                           | 1.4                                     | 1.5                                      | 1.4                     | 1.6       | 1.6       | 1.6       | 1.5       | 1.5       | 1.6      |
| NML Cyg..... | 1665               | ...                          | 1.7                            | ...                                     | <2.0                                     | ...                     | ...       | ...       | ...       | ...       | ...       | ...      |
| NML Cyg..... | 1667               | <2.0                         | 1.5                            | 3.0                                     | 3.0                                      | 2.5                     | ...       | ...       | ...       | ...       | ...       | ...      |
| U Ori.....   | 1665               | ...                          | <2.0                           | ...                                     | ...                                      | <2.0                    | 2.3       | 2.3       | 2.3       | 1.9       | 1.9       | 1.7      |
| U Ori.....   | 1667               | ...                          | <2.0                           | ...                                     | ...                                      | 2.5                     | ...       | ...       | ...       | ...       | ...       | ...      |
| W Hya.....   | 1665               | ...                          | <2.0                           | ...                                     | ...                                      | 2.0                     | 1.6       | 1.6       | 1.6       | 1.3       | 1.4       | 1.3      |
| W Hya.....   | 1667               | ...                          | 3.0                            | ...                                     | ...                                      | >2.5                    | ...       | ...       | ...       | ...       | ...       | ...      |
| S CrB.....   | 1665               | ...                          | <2.0                           | ...                                     | ...                                      | 2.5                     | 2.3       | 2.3       | 2.1       | 1.9       | >1.6      | 1.7      |
| S CrB.....   | 1667               | ...                          | <2.0                           | ...                                     | ...                                      | 2.5                     | ...       | ...       | ...       | ...       | ...       | ...      |
| U Her.....   | 1665               | ...                          | <2.0                           | ...                                     | ...                                      | 1.7                     | 2.3       | 2.3       | 2.1       | 1.7       | >1.4      | 1.6      |
| U Her.....   | 1667               | ...                          | <2.0                           | ...                                     | ...                                      | 1.8                     | ...       | ...       | ...       | ...       | ...       | ...      |

\* Or both features.

† Insufficient data.

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