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MASSES AND LUMINOSITIES OF POPULATION II CEPHEIDS

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ABSTRACT

Mean luminosities, effective temperatures, and radii of 11 Population II Cepheids are derived from three color photometry and, in some cases, radial velocities. From a comparison of the observed and the theoretical period-radius relation masses of $0.55 \pm 0.05\ \mathfrak{M}_\odot$ are derived for all Population II Cepheids except the variables No. 1 and 2 in M13, which appear to be less massive and possibly helium-rich. The period-luminosity relation is also discussed.

Subject headings: Cepheids and W Virginis stars — globular clusters — luminosities — mass loss

I. INTRODUCTION

Cepheid variables can provide critical tests of the theory of stellar evolution and pulsation. For a star of known composition, effective temperature, luminosity, and pulsation period, the results of pulsation theory may be used to estimate the mass. While we expect that the masses of Population II Cepheids must be less than masses at main-sequence turnoff points of globular clusters, there is no *a priori* way to establish the degree of mass loss during evolution through the giant branch, horizontal branch, and up to the Cepheid region. However, by determining the masses of Population II Cepheids and comparing them with those of red giants we can learn how much mass has been lost during this evolution.

In order to determine Cepheid masses we have to find the "equilibrium" values of their luminosities and effective temperatures.

Since pulsation is not expected to affect the energy generation in the stars, we can determine the equilibrium luminosity from

$$L = \frac{1}{P}\int_0^P L dt = \int_0^1 L dp, \qquad (1)$$

t being the time, P the period, and p the phase. $L(t)$ can be determined from M_v once the bolometric correction is known. This correction will depend on $T_{\rm eff}$ and on the metal abundance Z. $T_{\rm eff}$ can be determined from the $B - V$ colors once the relation between $B - V$ and $T_{\rm eff}$ is established, depending on M_v and Z.

These relations between $B - V$, M_v and $T_{\rm eff}$, L and Z have been determined in four papers (Böhm-Vitense 1970, 1972, 1973 and Böhm-Vitense and Szkody 1974) for different metal abundances. With an approximate temperature $T_{\rm eff}$ (estimated from $B - V$ and an approximate Z), the bolometric correction can be determined to yield the luminosity. Once the luminosity is known, $B - V$ can serve to determine $T_{\rm eff}$ with the estimated Z. Since

$$R^2 = L/4\pi\sigma T_{\rm eff}{}^4\,, \qquad (2)$$

L and $T_{\rm eff}$ determine R^2.

Our aim is to determine the equilibrium value of $T_{\rm eff}$. We have the choice of determining $T_{\rm eff} = \int_0^1 T_{\rm eff}\, dp$ or of computing

$$\langle R\rangle^2 \int_0^1 R^2\, dp \qquad (3)$$

and then

$$\langle T_{\rm eff}\rangle^4 = L/4\pi\sigma R^2\,. \qquad (4)$$

We have selected the second approach since L and R^2 are more fundamental characteristics than is $T_{\rm eff}$. In any case, *the difference between the two estimates of* $\langle T_{\rm eff}\rangle$ *is always less than* 50° K.

For our investigation we have selected 11 Population II Cepheids for which color and light curves are available and for which the metal abundance Z as well as the reddening correction $E(B - V)$ can be estimated with reasonable accuracy. We have listed these objects in table 1, where we also give their periods, the reddening corrections, the distance moduli, and the estimated metal abundances. The corresponding references are given in the footnotes.

The colors $B - V$ and $U - B$ depend on the gravity $g(\mathfrak{M}, R)$ rather than on L. On the other hand, M_v determines L and thereby only R, so we still have to assume a mass $\mathfrak{M}$ for our study. We have usually assumed $\mathfrak{M} = 1\ \mathfrak{M}_\odot$, but for some stars we find better agreement in $U - B$ if $\mathfrak{M} = 0.5\ \mathfrak{M}_\odot$.

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TABLE 1

MEAN LUMINOSITIES $\bar{L}$, EFFECTIVE TEMPERATURES $\langle T_{eff} \rangle$, AND RADII $\bar{R}$ OF POPULATION II CEPHEIDS

Star	$V - M$*	$E(B - V)$	$Z/Z_\odot$	log $P[d]$†	log $L/L_\odot$	log $R/R_\odot$	log T_{eff} K
M5 No. 42	14.39‡	0.02§	0.05§	1.410	3.20	1.656	3.739
W Vir	12.3‡‖	0.12‖	0.1#	1.238	2.89	1.504	3.736
	12.4††	0.12‖	0.1	...	2.93	1.524	...
TW Cap	14.3‡	0.04	0.1**	1.455	3.28	1.682	3.745
UY Eri	11.3‖	0.05	0.02	0.342	1.91	0.929	3.780
	11.86††	0.05	0.02	...	2.14	1.044	3.780
M13 No. 1	14.59‡‡	0.05§§	0.05‖‖	0.176	2.15	0.841	3.884
	14.62	0.00‖	0.05	...	2.12	0.904	3.854
	14.52	0.00	0.05	...	2.08	0.884	3.854
	14.02##	0.00	0.02	...	1.89	0.764	3.858
M13 No. 2	14.59	0.05	0.05	0.708	2.56	1.143	3.833
	14.62	0.00	0.05	...	2.56	1.202	3.817
	14.52	0.00	0.05	...	2.52	1.182	3.817
M13 No. 6	14.59	0.05§§	0.05	0.322	2.16	1.012	3.800
	14.62	0.00	0.05	...	2.16	1.033	3.790
	14.52††	0.00	0.05	...	2.12	1.013	3.790
M10 No. 2	14.50	0.25§§	0.05	1.272	3.01	1.580	3.729
M10 No. 3	14.50	0.25	0.05	0.899	2.62	1.387	3.728
M2 No. 1	15.80††***	0.05	0.01	1.193	2.92	1.498	3.745
M2 No. 11	15.80	0.05	0.01	1.525	3.39	1.727	3.748

* $V - M$ is meant to be the difference between apparent visual magnitude (uncorrected for reddening) and absolute visual magnitude. † Sawyer 1955. ‡ Böhm-Vitense 1974*a*. § Arp 1962. ‖ Kwee 1968. # Barker *et al.* 1971.
** Anderson and Kraft 1971. †† This study. ‡‡ Sandage 1970. §§ Fitzgerald 1968. *** Demers 1969.
Trial value in order to check possible changes.

In the following we shall discuss the Cepheids in groups.

II. CEPHEID DATA

a) *The Field Population II Cepheids*

The stars M5 No. 42, W Vir, and TW Cap were discussed by one of us (Böhm-Vitense 1974*a*) where distances were checked or determined by a new version of the Baade-Wesselink method, requiring that the derived dR/dt and the measured radial velocity curve $v_r(t)$ should have the same amplitude. In table 1 we give the distance moduli $m - M$ derived in this way. In this paper $L(t)$, $T_{eff}(t)$, and $R(t)$ are inserted in equations (1), (3), and (4) to determine $\bar{L}$, $\bar{R} = \langle R^2 \rangle^{1/2}$ and $T_{eff} = \langle T_{eff}{}^4 \rangle^{1/4}$. Results are given in table 1. [For W Vir an increase in R by 10 percent leading to $\Delta(m - M) = 0.1$ would give even better agreement in the velocity amplitude; however, this appears to be within the limits of uncertainty.]

Since no measured radial velocities are available, the distance to UY Eri cannot be determined by the same method. Assuming with Kwee (1968) that $V - M = 11.3$, a mean absolute visual magnitude $\langle M_v \rangle = 0$ is obtained for UY Eri. However, the other short-period Population II Cepheids studied by Kwee have an average $\langle M_v \rangle \simeq -0.56$, so UY Eri would be exceptionally faint if $V - M = 11.3$. We have therefore also carried out the calculation for $V - M = 11.86$, which was used in figure 11. (V = apparent visual magnitude, uncorrected for interstellar absorption). A reddening of $E(B - V) = 0.05$ was assumed by Kwee and by us. The star appears to have rather weak metal lines; we therefore assumed log $Z/Z_\odot = -1.66$.

In figure 1 we reproduce the $B - V$ and $U - B$

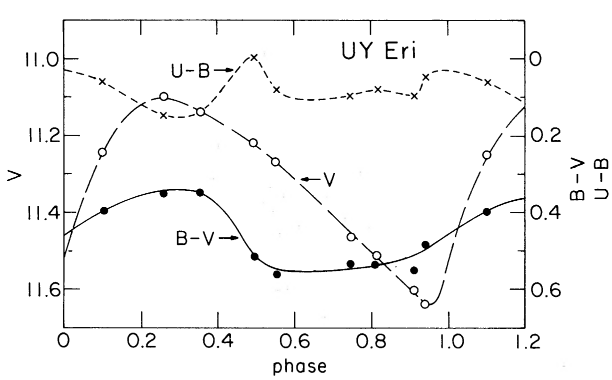

FIG. 1.—Measured visual magnitude V, and color $U - B$, $B - V$ for UY Eri according to Kwee and Braun 1967. Measured points are given. Also shown are the interpolating curves which were used for this investigation.

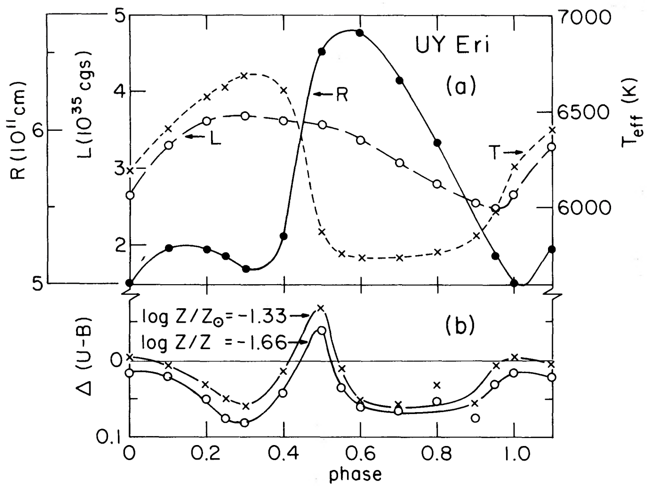

FIG. 2*a*.—Derived $T_{\rm eff}$, radii R, and luminosities L as a function of phase for UY Eri, assuming a distance modulus $V - M = 11.3$. (We actually favor 11.86.) A metal abundance $\log Z/Z_\odot = -1.66$ was used.

FIG. 2*b*.—Residuals $\Delta(U - B)$, measured minus theoretical values, for UY Eri as a function of phase assuming metal abundances $\log Z/Z_\odot = -1.66$ and $\log Z/Z_\odot = -1.33$. The slightly higher metal abundance gives better average agreement. For phase of increasing R the observed colors are bluer than expected, indicating a larger pressure in the atmosphere than expected from the gravitational acceleration. At phases of decreasing radius the opposite is true.

measurements of Kwee and Braun (1967), and the somewhat smoothed values which we actually used. [The strongly smoothed curves given by Kwee and Braun do not give a reasonable $R(t)$.] In figure 2*a* we show L, $T_{\rm eff}$, and R as a function of phase obtained assuming $\log Z/Z_\odot = -1.66$. From $T_{\rm eff}$ and L the theoretical values of $U - B$ can be determined and compared with the observed values to provide a check on the assumed metal abundance, the distance modulus, and possibly the mass.

In figure 2*b* we show the difference $\Delta(U - B) = (U - B)_{\rm observed} - (U - B)_{\rm computed}$. During outward acceleration the observed $U - B$ are more blue than the computed ones. We expect bluer colors for higher pressures. $\Delta(U - B)$ decreases when the *acceleration* is directed outward, indicating that this leads to an increasing pressure. For inward acceleration the opposite effect is observed (elevator effect). This effect seems to be present also for the phase $p \simeq 0.1$ when we see a small hump in the radius curve, indicating that

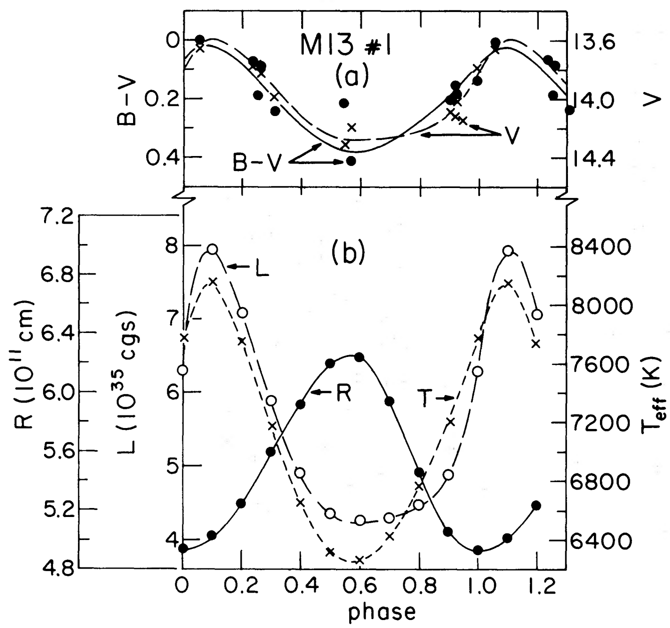

FIG. 3.—Top graph shows the measured $B - V$ colors (*dots*) as a function of phase for the variable No. 1 in M13 with the interpolating curve (*solid line*), and the measured V-magnitudes (*crosses*) (Demers 1971) with the interpolating curve (*dashed line*). For discussion see text.

this hump may be real. The same phase relations for $\Delta(U-B)$ were also found for M5 No. 42, W Vir, and TW Cap (Böhm-Vitense 1974*a*). The measured $U-B$ colors of UY Eri tend to be somewhat redder than the ones computed for $\log Z/Z_\odot = -1.66$ and $V-M = 11.3$, indicating that Z may be larger by about a factor of 2. Better agreement is also achieved by choosing a mass smaller than 1 $\mathfrak{M}_\odot$, as indicated by the analysis in the last section of this paper.

b) *The Variables in M13*

For M13 the distance modulus and the degree of reddening are not very well known. Sandage (1970) suggests $m-M = 14.42$ and $E(B-V) = 0.05$. For variables No. 2, Joy (1949) gives quite a number of velocity measurements for which the points show substantial scatter so the velocity amplitude cannot be well determined.

Because of these uncertainties we have studied the variables for different distance moduli and reddening corrections. All variables were studied for $E(B-V) = 0.05$ with $V-M = 14.59$ [corresponding to $m-M = 14.42$ and $E(B-V) = 0.05$] and for $E(B-V) = 0$ with $V-M = 14.62$. For variable No. 1 also $V-M = 14.02$, $E(B-V) = 0$ was used. Since the variables No. 1 and 2 have unexpectedly blue colors the assumption $E(B-V) = 0$ seemed to fit the theory best (as discussed in § III).

Colors and magnitudes were measured by Demers (1971), whose measurements show large scatter, and the cycles are not covered very well so there is much room for arbitary interpolation.

The mean values obtained for different assumptions are given in table 1. For $V-M = 14.59$ and $E(B-V) = 0.05$ the color and magnitude interpolation curves given by Demers were used. Since the theory (see § III) requires larger radii and lower temperatures for the stars No. 1 and 2 the second set of reductions, i.e., for $V-M = 14.62$ and $E(B-V) = 0$, was calculated with the interpolation curves shown in figures 3, 4, and 5, which are biased purposely to give temperatures as low as possible and visual magnitudes as bright as possible in order to obtain large radii to see whether the theoretical expectations can possibly be matched. A metal abundance $Z/Z_\odot = 0.05$ was used in the reduction, though a somewhat lower metal abundance seems possible. The results for the variable No. 1 for $V-M = 14.62$ and $E(B-V) = 0$ are shown in figure 3.

Figure 4 shows the results for M13 No. 6. The scatter of the observed points especially around phase 0.2 is very large leaving room for arbitrary interpolation. For this star the mean radius and temperature fits rather well with the theoretical expectations.

In figure 5 we show the results for M13 No. 2, which again shows too high temperatures and too small radii to match the theory. Therefore the observations were purposely interpolated in such a way as to give as low temperatures and as large radii as possible. In figure 5*c* we show the velocities dR/dt, determined from the radius of the continuum-forming layers, i.e., from differentiation of the R curve shown in figure 5*b*, and we compare these values with the measured radial velocities (Joy 1949, adjusted to match the phases of our curves). The values obtained from dR/dt are much larger, especially during decreasing radius, than the measured radial velocities. Decreasing the distance modulus would reduce dR/dt and R; however, R is too small anyway. The only possibility would be to de-

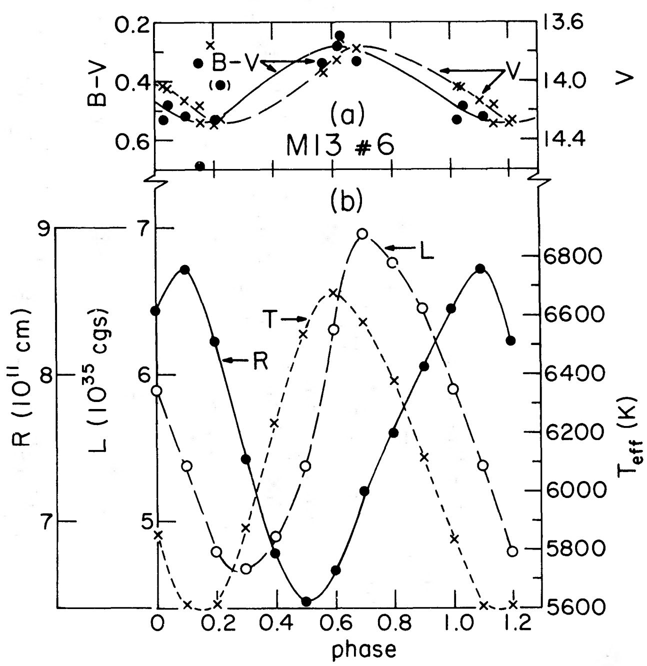

FIG. 4.—Same as fig. 3, but for the variable No. 6 in M13

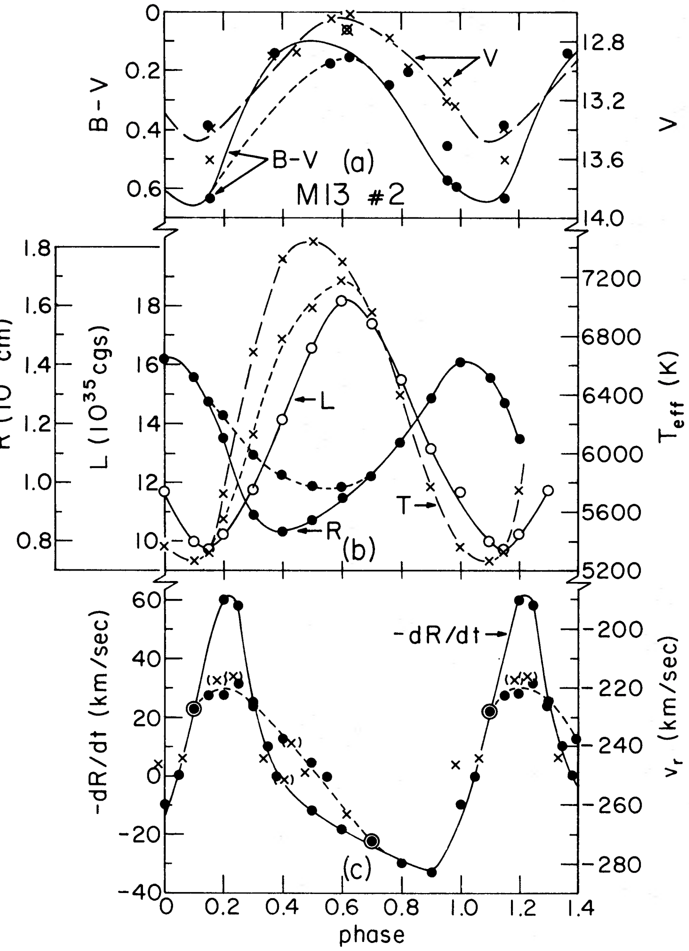

FIG. 5.—Same as fig. 3, but for the variable No. 2 in M13. In addition we compare the measured radial velocities v_r (Joy 1949) with $-dR/dt$ (i.e., the time change of the radius of the continuum-forming layer). The amplitudes of the two velocity curves are expected to be the same (Böhm-Vitense 1974a). The difference probably indicates either wrong temperatures, because of wrong $B - V$ values (see text) leading to too large R variations, or too large a distance modulus. The dotted $B - V$, T, and R curves would resolve the problems for this variable as discussed in the text.

crease the radius variation by increasing the minimum radius. This would mean an even more biased interpolation of the $B - V$ colors, possibly as shown by the dotted line in figure 5*a*. With these colors the discrepancies could be resolved for this star.

Could the color measurements be wrong? Variable No. 6 is in the least crowded region of M13 and for this star we find "reasonable" T and R. The stars No. 1 and 2 are in fairly crowded fields..

In figure 6 we have plotted the theoretical two-color diagram for stars with a metal abundance $Z/Z_\odot = 0.025$ and different luminosities. In the same graph we show the two-color measurements by Demers (1971) for all three variables. Most of the measured $U - B$ colors are discordant with any computed colors for any plane-parallel static atmosphere. Those of the variable No. 6 fit best. We suspect that the color measurements, especially in the U, are very uncertain due to the crowding in the field. Rather large uncertainties are also possible in the $B - V$ colors for star No. 1 and 2. New measurements are urgently needed for the M13 variables as well as *UBV* observations of M15 No. 1, whose colors and period correspond very closely to M13 No. 1 (Arp 1955).

c) *The Variables in M10*

For the variable No. 2 radial velocities were measured by Joy (1949). The largest difference between any two values is 84 km s^{-1}. The scatter of the measured points is ± 20 km s^{-1}.

With the help of this radial-velocity curve we can attempt to determine the distance to M10 by means of the velocity-amplitude method. The $B - V$ colors and the visual magnitudes are given by Arp (1957). The cluster is heavily reddened. We have adopted a reddening correction $E(B - V) = 0.25$ as given by Fitzgerald (1968). The uncertainty in $E(B - V)$ is at least 0.03.

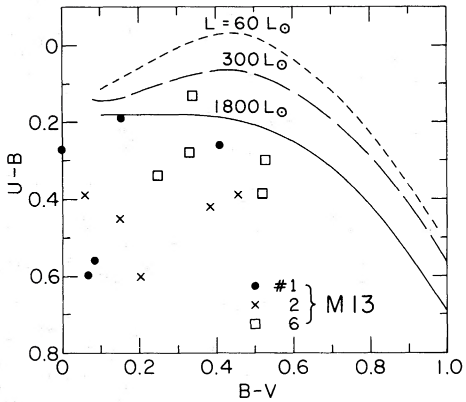

FIG. 6.—Theoretical two-color diagram for stars with log $Z/Z_\odot = -1.66$. The different curves are for stars with different luminosities. We show the measured values for the variables in M13, *Dots*, for variable No 1; *crosses*, for variable No. 2, and *squares* for variable No. 6 (Demers 1971). Measured values are in disagreement with theoretical expectation. We suspect errors in the measurements due to crowding in the field.

In figure 7*a* we give the values of $B - V$ and V which we used. With a distance modulus $V - M = 14.5$, we find the luminosities, temperatures, and radii shown in figure 7*b*. The values of $\bar{L}$, $\bar{R}$, and $\langle T_{\rm eff}\rangle$ are given in table 1.

The gradient dR/dt is not very well determined but it appears that we get a maximum differerence in dR/dt of 76 km s^{-1} with a rather large uncertainty. This is in reasonable agreement with the observed amplitude in radial velocity, showing that $V - M = 14.5$ is an acceptable choice.

The residuals $\Delta(U - B) = (U - B)_{\rm obs} - (U - B)_{\rm comp}$ are given in figure 7*c*. They are equally distributed over positive and negative values, and show the effects of increasing and decreasing pressure during outward and inward acceleration.

In figure 8 we show the results for the variable No. 3 in M10. The reductions were made assuming $\mathfrak{M} = \mathfrak{M}_\odot$. For smaller masses only a negligible change in the temperature would be obtained.

d) The Variables in M2

Joy (1949) measured radial velocities for the variable No. 11 in M2, which is really an RV Tauri (Arp and Wallerstein 1956). The light curves do not repeat and we may expect that the velocity curves will not repeat either. The visual light curve and the $B - V$ color curve were measured by Demers (1969). We interpolated in such a way as to give a reasonable R (phase) curve, shown in figures 9*a* and *b*. For the derivation of temperature, luminosity, and radius we assumed $Z = 0.01\, Z_\odot$. The distance modulus $m - M = 15.63$ was adopted by Demers. A reddening of $E(B - V) = 0.05$ leads to $V - M = 15.80$. We find somewhat better agreement in the velocity amplitude for $V - M = 16.00$ which we use in figure 9*c* to give T, L, and R. Figure 9*d* shows the values of dR/dt and compares them with the velocities measured by Joy (1949). At phases around $p \sim 0.1$ the radius curve gives rather large velocities of the continuum-forming layers, due to the large measured visual brightness. In Joy's velocity curve—shifted by $\Delta p = -0.3$—we find a rather large scatter in the measured velocities but the amplitude is not comparable to the values derived for dR/dt. We have, however, to keep in mind that Joy's velocities were measured in 1949 while the light curve was measured in 1969. We therefore cannot necessarily expect agreement. Better agreement can perhaps be expected for phases during which the radius is shrinking smoothly, i.e., for phases between 0.4 and 0.8. Here the amplitude in $v_{\rm abs}$ appears to be somewhat larger than that of dR/dt. For this reason we have finally adopted the value $V - M = 15.8$, keeping in mind that it may have to be increased. This distance modulus will lead to values of dR/dt smaller by 10 percent than those shown in figure 9*c*.

The results for the variable No. 1 in M2 are given in figure 10. The light and color of Demers (1969) were used. For the reduction a value $V - M = 15.9$ was used. However, the values for $\bar{R}$ and $\bar{L}$ given in table 1 refer to $V - M = 15.8$, the distance modulus adopted above.

e) Summary of Results in the Period-Luminosity Diagram

In figure 11 we show the period-luminosity relation for the stars discussed here. (We omitted the variables in M13 when drawing the estimated mean relation.) The relation appears to be much narrower than obtained for Population I Cepheids (Sandage and Tammann 1971). The data given were obtained by assuming the stars to have $\mathfrak{M} = 1\, \mathfrak{M}_\odot$, except for M13 where $\mathfrak{M} = 0.5\, \mathfrak{M}_\odot$ was used. For slightly smaller masses we would have obtained only minor decreases in temperatures, thanks to lower gravities. Concomitantly, we would have found only minute changes in luminosities since, for a given obseved M_v, only the change in the bolometric correction enters.

III. COMPARISON WITH PULSATION THEORY AND ESTIMATES OF MASS

Pulsation theory establishes a relationship between M, R, and P that is essentially independent of composition, but is slightly dependent on the choice of opacity. In figure 12 are shown lines of constant mass in the (P, R)-plane for pulsation in the fundamental mode. Lines for $\mathfrak{M}/\mathfrak{M}_\odot = 0.4$, 0.6, and 0.8 are based on the calculations of Tuggle and Iben (1972) who used Cox and Stewart (1971) opacities interpolated by means of cubic splines. The line for $0.5\, \mathfrak{M}_\odot$ is based on the calculations of Iben (1971), who used Cox-Stewart opacities as approximated by Christy (1966*a*). For a given choice of opacity, the positions of these lines are nearly independent of Z and Y in the ranges $Z \leq 0.004$ and $0.2 \leq Y \leq 0.3$.

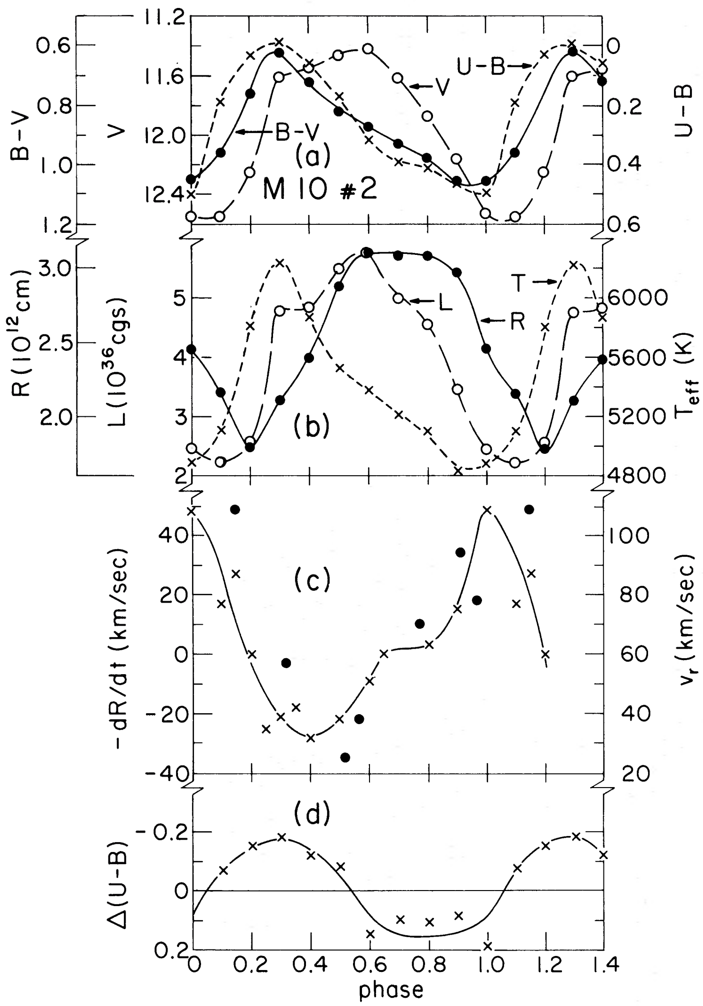

FIG. 7*a*.—Measured visual magnitudes and $U - B$, $B - V$ colors for the variable No. 2 in M10 according to Arp 1957.

FIG. 7*b*.—T_{eff}, R, and L for the variable No. 2 in M10 derived for $V - M = 14.5$, $E(B - V) = 0.25$, and $\log Z/Z_{\odot} = -1.33$.

FIG. 7*c*.—Velocities dR/dt obtained from the radius curve in fig. 5*b* (xxx) are compared with V_r (absorption lines) measured by Joy 1949 (●●●). The amplitudes agree within the limits of error.

FIG. 7*d*.—Residuals $\Delta(U - B) = (U - B)_{\text{measured}} - (U - B)_{\text{theoretical}}$ as a function of phase. The star is bluer (higher pressure) during phases of outward acceleration and redder (lower pressure) during inward acceleration.

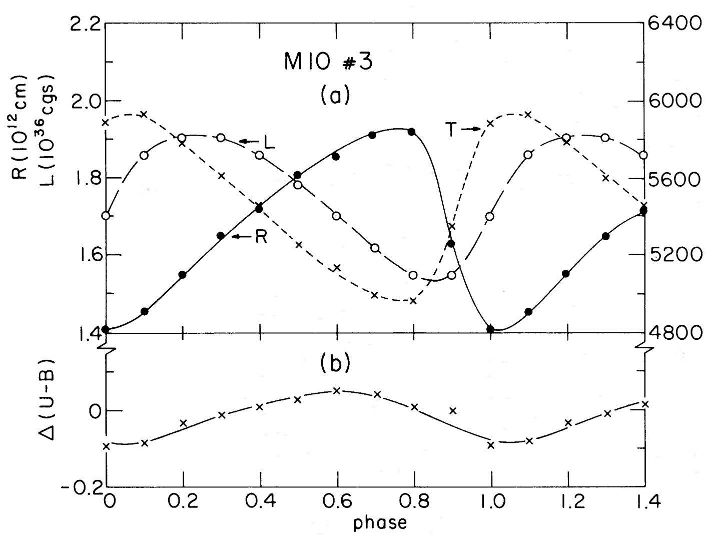

FIG. 8*a*.—T_{eff}, R, and L for the variable No. 3 in M10, assuming $V - M = 14.50$ and $E(B - V) = 0.25$

FIG. 8*b*.—Residuals $\Delta(U - B)$ for the variable No. 3 in M10

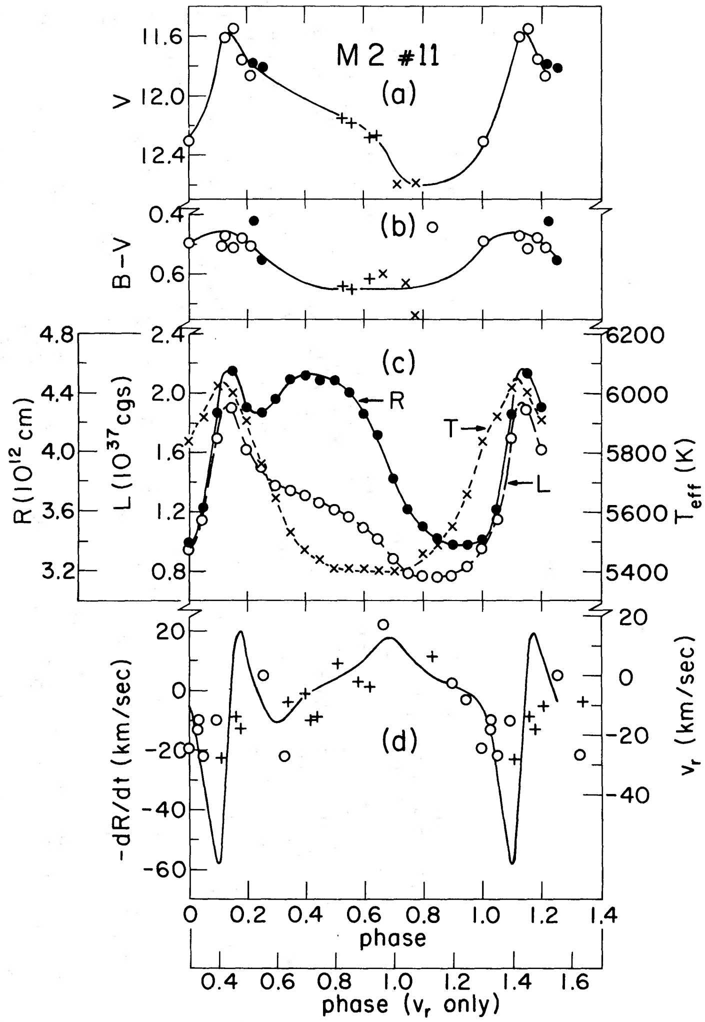

FIG. 9*a* and 9*b*.—Visual magnitudes and colors measured for variable No. 11 in M2 by Demers 1969. The interpolated curves used in this study are also shown.

FIG. 9*c*.—T_{eff}, *R*, and *L* for the variable No. 11 in M2, assuming $V - M = 16.00$, $E(B - V) = 0.05$, $\log Z/Z_{\odot} = -2$.

FIG. 9*d*.—A comparison between the velocities *dR/dt* (*solid curve*) derived from the radius curve shown in fig. 7*c* with the measured radial velocities v_r of the absorption lines given by Joy 1949. The crosses and circles correspond to the alternating cycles. The phases are shifted by 0.3 for the two sets of velocities. The amplitudes during the quiescent phases agree fairly well. There are large discrepancies during phases of maximum luminosity, which can possibly be attributed to time variations. The color measurements were made in 1969 while the v_r were measured in 1949.

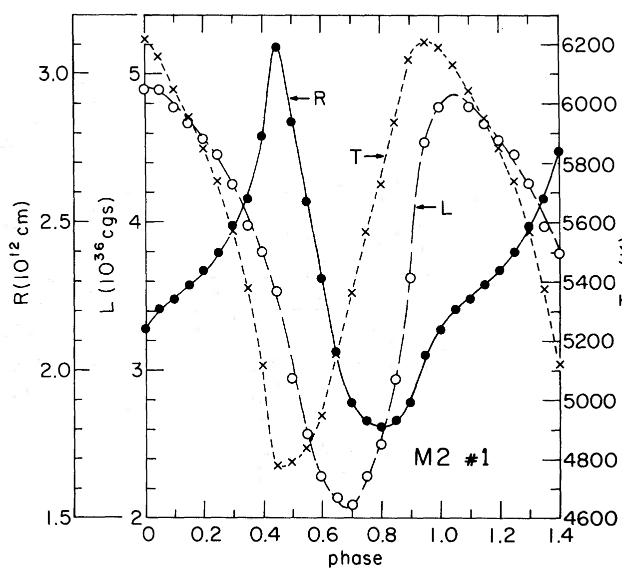

FIG. 10.—T_{eff}, *R*, and *L* for the variable No. 1 in M2, assuming $V - M = 15.90$, $E(B - V) = 0.05$, $\log Z/Z_{\odot} = -2$

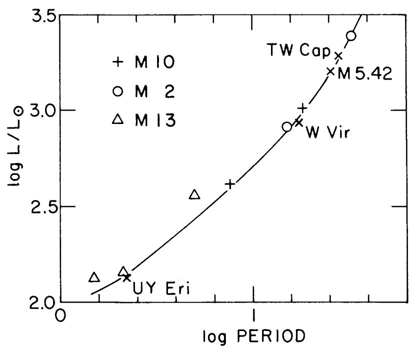

FIG. 11.—Period-luminosity relation for the variables studied here. The plusses give the position of the variables in M10, the circles those of the variables in M2, and the triangles those for the variables in M13. The relation may be much narrower than for Population I variables.

When splined opacities are chosen, the results may be represented by

$$\log P \simeq 1.72[-0.83 + \log (R/R_\odot) - 0.42 \log (\mathfrak{M}/0.5\,\mathfrak{M}_\odot)] \quad (5)$$

$$P^2 \propto \text{const}\,(R^3/\mathfrak{M})(R/\mathfrak{M})^{0.445}\,. \quad (5')$$

These equations are remarkably similar to one derived by Cogan (1970) who used similar Y's but much larger Z's and M's, as well as a different set of opacities.

Comparing the values of P and R given in table 1 with either equation (5) or with the lines in figure 12, one might conclude that masses for Population II

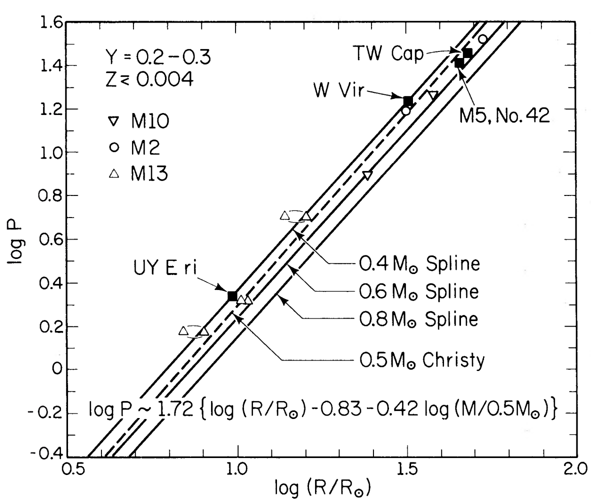

FIG. 12.—Lines of constant mass in the (period, radius)-plane. Lines for $\mathfrak{M}/\mathfrak{M}_\odot = 0.4$, 0.6, and 0.8 are constructed from data in Tuggle and Iben (1972), who used spline interpolation in Cox-Stewart opacities for $Y = 0.2$, 0.3 and $Z = 0.001$–0.004. The line for $\mathfrak{M} = 0.5\,\mathfrak{M}_\odot$ is from data in Iben (1971) who used a Christy (1966*a*) approximation to Cox-Stewart opacities. Estimates of R obtained in this paper for several Population II Cepheids are also shown. For the variables in M13, the smaller radii are appropriate if $E(B-V) = 0.05$; the larger radii are appropriate if $E(B-V) = 0.0$.

Cepheids are less than or equal to about 0.6 $\mathfrak{M}_\odot$. It is interesting that masses of order 0.5–0.65 $\mathfrak{M}_\odot$ are estimated for RR Lyrae stars in M5, M3, and M15 (e.g., Iben 1971).

The uncertainties in distance modulus, reddening corrections, and in the analysis presented here are sufficiently large so that the "pulsation" masses obtained by inspection of figure 12 cannot be accurate to better than $\pm 0.1\,\mathfrak{M}_\odot$. This is true also of estimates of the masses of RR Lyrae stars. Hence, we may argue that the observational data for most of the Cepheids discussed here and the theoretical data are consistent both (*a*) with no mass loss between the horizontal-branch phase and the asymptotic-branch phase and (*b*) with modest mass loss of about $0.1 - 0.2\,\mathfrak{M}_\odot$.

On the other hand, we can say with some certainty that the masses of Population II Cepheids are distinctly smaller than masses $\mathfrak{M} \simeq 0.8\,\mathfrak{M}_\odot$ currently assigned to stars at cluster turnoff in many globular clusters (e.g., Iben and Rood 1970).

Theoretical calculations for low-mass giants can in principle give a lower limit to the mass which a horizontal-branch star of a given initial composition may have. This lower limit is the mass in the helium core at the time of the helium flash. Calculations by Rood (1972) give this mass $\mathfrak{M}_c$ as

$$\mathfrak{M}_c/\mathfrak{M}_\odot \simeq 0.475 + 0.23(0.3 - Y) - 0.01(\log Z + 3) + 0.035(0.8 - \mathfrak{M}/\mathfrak{M}_\odot), \quad (6)$$

where $\mathfrak{M}$ is the mass of the star as it experiences the helium flash.

A substantial body of analysis (see Iben 1971, 1974) suggests that, for many globular-cluster stars on the horizontal branch, $Y \sim 0.2$–0.3. Therefore, one might expect the mass of a horizontal-branch precursor of a Population II Cepheid to have mass in excess of about 0.47 $\mathfrak{M}_\odot$. Once helium is exhausted at the center of a horizontal-branch stars, helium burns in a shell which progressively moves outward in mass until it almost touches the hydrogen-burning shell. At this point, a thermal instability sets in which causes the star to loop away from the asymptotic branch, across the Cepheid instability strip and back again to the asymptotic branch (Schwarzschild and Härm 1970). Thus, the mass of a Population II Cepheid is expected to be at least as large as the mass in the helium core of a horizontal-branch precursor.

Of the 11 Cepheids considered in this paper, all but two appear to have "pulsation" masses that are either larger than 0.47 $\mathfrak{M}_\odot$ or are close enough to 0.47 $\mathfrak{M}_\odot$ that one cannot argue that a discrepancy exists. Only the two Cepheids in M13 (No. 1 and No. 2) appear to be less massive [$\mathfrak{M} \simeq 0.3\,\mathfrak{M}_\odot$ if $E(B-V) = 0.05$ or $\mathfrak{M} \simeq 0.4\,\mathfrak{M}_\odot$ if $E(B-V) = 0$] than the minimum mass of a precursor characterized by $Y \simeq 0.2$–0.3. The third Cepheid in M13 No. 6) appears to be more massive by $\sim 0.2\,\mathfrak{M}_\odot$ than the other two. The trustworthiness of this remarkable result, however, depends on the reliability of the measured $B - V$ colors.

In order to achieve a precursor mass of order 0.4 $\mathfrak{M}_\odot$, the pulsation mass we find for variables No. 1

and 2 when $E(B - V) = 0$, we would have to assume $Y \gtrsim 0.6$. If $E(B - V) = 0.05$, then $Y \gtrsim 0.7$. Such large values of Y can be ruled out for the horizontal-branch stars in M13.

The reddest nonvariable horizontal-branch stars in this cluster may be assigned a surface temperature $T_e \simeq 7390°$ K. This temperature represents an upper limit to the temperature of the blue edge of the instability strip (at $V - M \simeq 14.62$). There are two RR Lyrae variables in M13 with periods $P = 0.38$ and 0.39 days (Sawyer Hogg 1974). These periods represent an upper limit to the period at the blue edge of the instability strip. From Tuggle and Iben (1972) we know that temperatute T_{BE} and period P_{BE} at the blue edge are related to Y by

$$Y \simeq 0.2 + 4.9 (\log T_{BE} - 3.859) + 0.35 (\log P_{BE} + 0.55). \quad (7)$$

Inserting our limits $\log T_{BE} < 3.869$ and $\log P_{BE} < -0.400$ in expression (7), we conclude that $Y < 0.3$. From equation (6) we infer that the mass of a horizontal-branch star in M13 is in excess of about $0.47\,\mathfrak{M}_\odot$.

If, then, the two Cepheids in M13 are really characterized by $Y \lessgtr 0.6$ and $\mathfrak{M} \gtrsim 0.4\,\mathfrak{M}_\odot$, we have learned that, following the horizontal-branch phase, at least some stars eject sufficient mass to leave a remnant less massive than the mass of the helium core at the time of the helium flash. In this event we would be forced to accept further that the entire hydrogen-rich envelope of these stars had been ejected and that the helium abundance at their surfaces is $Y \simeq 1$. The spectra of these Cepheids, however, show strong hydrogen lines and do not indicate a low opacity by the presence of enhanced metallic lines (Joy 1949).

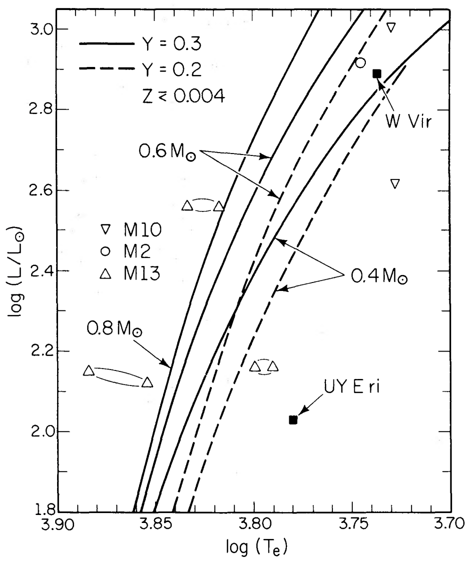

FIG. 13.—Blue edges in the H-R diagram for pulsation in the fundamental mode. These edges are constructed from data in Tuggle and Iben (1972). For the variables in M13, the larger temperatures are for $E(B - V) = 0.05$, and the smaller temperatures are for $E(B - V) = 0.0$.

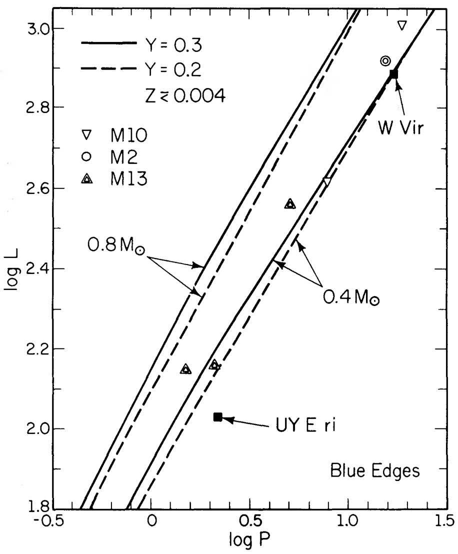

FIG. 14.—Blue edges in the *P-L* diagram for pulsation in the fundamental mode. Data from Tuggle and Iben (1972).

Variables No. 1 and 2 in M13 appear anomalous in several additional respects. Pulsation theory permits one to define "blue edges" for pulsation in the fundamental mode. When splined opacities are chosen, the blue edges in figure 13 (the *H-R* diagram) and figure 14 (the P-L plane) result. For a given mass and composition, a model is stable against pulsation in the fundamental mode if it lies in the region to the left of the appropriate blue edge. Hence, the location of a Cepheid variable in either of the two diagrams sets limits on the mass and composition of that variable: the approximate blue edge must lie to the left of the variable.

From their positions in the H-R diagram (fig. 13), it would appear that variables No. 1 and 2 in M13 must either be considerably more massive than $1\,\mathfrak{M}_\odot$ (if $Y \simeq 0.2$–0.3) or be characterized by an envelope Y in excess of 0.7 (if $\mathfrak{M} \simeq 0.4\,\mathfrak{M}_\odot$). Consistency in the *P-R* plane (fig. 12) would demand that the second alternative be chosen. However, we have to remember the uncertainties in the observed colors. On the other hand, Joy's (1949) spectral type A2 for M13 No. 1 at maximum T favours the high temperature.

Comparison between the observational data and the theoretical data in the *P-L* plane (fig. 14) suggests that either $\mathfrak{M} \simeq 0.6\,\mathfrak{M}_\odot$ (if $Y \simeq 0.2$–0.3), or that $Y > 0.5$ (if $\mathfrak{M} \simeq 0.4\,\mathfrak{M}_\odot$), or $Y > 0.6$ if $\mathfrak{M} \simeq 0.3\,\mathfrak{M}_\odot$. Again, consistency in the *P-R* plane can be achieved only if the second alternative is chosen.

In summary, if we accept the surface temperatures estimated in this paper, the variables No. 1 and 2 in M13 are of very low mass (~ 0.3–$0.4\,\mathfrak{M}_\odot$). Consistency with stellar evolution theory as well as consistency with blue edges in both the H-*R* and *P-L* diagrams would require that the helium abundance in the envelopes of these two stars must exceed $Y \simeq 0.6$–0.7.

The only way to avoid these rather startling conclusions is to admit that the surface temperatures derived for variables No. 1 and 2 [with $E(B - V) = 0.05$] could, on the average, be too large by $\Delta \log T_e \simeq 0.05$. Then one could achieve consistency with both pulsation theory and evolution theory with $Y \simeq 0.2$–0.3 and $\mathfrak{M} \simeq 0.5$–$0.6\, \mathfrak{M}_\odot$. In view of the large scatter of the measured $B - V$ for these variables and in view of the discrepant measured $U - B$ values we do not wish to exclude this possibility. New measurements of $B - V$ are required.

REFERENCES

Anderson, K. S., and Kraft, R. R. 1971. *Ap. J.*, **167**, 119.
Arp, H. C. 1955, *A.J.*, **60**, 1.
———. 1957, *ibid.*, **62**, 129.
———. 1962, *ibid.*, **135**, 311.
Arp, H. C., and Wallerstein, G. 1956, *A.J.*, **61**, 272.
Barker, T., Baumgart, L. D., Butler, D., Cudworth, K. M., Kemper, E., Kraft, R. P., Lorre, J., Rao, N. K., Reagan, G. H., and Soderblom, D. R. 1971, *Ap. J.*, **165**, 67.
Böhm-Vitense, E. 1970, *Astr. and Ap.*, **8**, 282.
———. 1972, *ibid.*, **17**, 335.
———. 1973, *ibid.*, **24**, 447.
———. 1974*a*, *Ap. J.*, **188**, 571.
———. 1974*b*, in preparation.
Böhm-Vitense, E., and Szkody, P. 1974, *Ap. J.*, **184**, 211.
Christy, K. F. 1966*a*, *Ap. J.*, **144**, 108.
———. 1966*b*, *ibid.*, **145**, 337.
Cogan, B. C. 1970, *Ap. J.*, **162**, 129.
Cox, A. N., and Stewart, J. 1971, private communication.
Demers, S. 1969, *A.J.*, **74**, 925.
———. 1971, *ibid.*, **76**, 445.
Fitzgerald, M. P. 1968, *A.J.*, **73**, 983.
Helfer, H. L., Wallerstein, G., and Greenstein, J. L. 1959, *Ap. J.*, **129**, 700.
Iben, I., Jr. 1971, *Pub. A.S.P.*, **83**, 697.
———. 1974, *Ann. Rev. of Astr. and Ap.*, **12**, in press.
Iben, I., Jr., and Rood, R. T. 1970, *Ap. J.*, **161**, 587.
Joy, A. H. 1949, *Ap. J.*, **110**, 105.
Kwee, K. K. 1968, *B.A.N.*, **19**, 374.
Kwee, K. K., and Braun, L. D. 1967, *B.A.N. Suppl.*, **2**, 77.
Rood, R. T. 1972, *Ap. J.*, **177**, 681.
Sandage, A. 1970, *Ap. J.*, **162**, 841.
Sandage, A., and Tammann, G. A. 1971, *Ap. J.*, **167**, 293.
Sawyer, H. B. 1955, *Pub. David Dunlap Obs.*, **2**, No. 2.
Sawyer Hogg, H. B. 1973, *Pub. David Dunlap Obs.*, **3**, No. 6.
Schwarzschild, M., and Härm, R. 1970, *Ap. J.*, **160**, 341.
Tuggle, R. S., and Iben, I., Jr. 1972, *Ap. J.*, **178**, 455.

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