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CONSTRAINTS ON MODELS FOR CHEMICAL EVOLUTION IN THE SOLAR NEIGHBORHOOD

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ABSTRACT

Chemical evolution in the local region of the Galaxy is described by analytical approximations to models based on a variety of hypotheses that have been proposed to account for the observed abundances in stars, interstellar gas, and the solar system. There are many alternative (although not always mutually exclusive), physically plausible models to be considered, and many parameters related to nucleosynthesis, star formation, gas flows, etc., which cannot be reliably determined *a priori*. In spite of these uncertainties, it is found that very few types of models are consistent with several important empirical constraints. These are the paucity of metal-poor stars, the small difference between the mean metal abundances of the oldest and youngest disk stars, and the lack of observable change in certain isotopic abundance ratios during the last 4.5 billion years. Unless there is an unknown Galactic site of deuterium production, the production rates in supernovae of other light elements relative to deuterium appear to restrict the class of viable models further to those in which a significant fraction of primordial deuterium has not been processed through stars. Either metal-enhanced star formation or the infall of primordial gas leads to evolutionary models consistent with all these constraints. It is found that the ratio between *s*-process abundances and stellar production factors is nearly model-independent, but that the relative importance of the big bang or red-giant stars as the source of ^{3}He depends on details of the evolutionary history.

Subject headings: abundances, stellar — nucleosynthesis — stellar evolution — stellar statistics

I. INTRODUCTION

Models for evolution of the solar neighborhood aim to explain the present chemical composition of the interstellar gas, its past history as recorded in the composition of stars of different ages (in particular, details of the composition 4.5×10^9 years ago, reflected in the solar system, and the present distribution of stellar ages and masses. Studies of such models by many authors have shown clearly that a unique, detailed model cannot be defined because there are more free parameters—related to the stellar birthrate as a function of mass and time, gas flows, and nucleosynthesis—than firm empirical or theoretical constraints.

The purpose of this paper is to discuss various specific models in a uniform, approximate, analytical formalism, in order to show how models of different types are affected by the known constraints. It will be found that, in spite of the indeterminacy, relatively few types of scenarios for evolution of the solar neighborhood are fully consistent, while on the other hand a number of interesting model-independent relations between theoretical production parameters and predicted abundances can be established. The remainder of this section will define what is meant by "the solar neighborhood" in the present context, and discuss two important empirical points that present nontrivial problems for evolutionary models. Section II describes alternative hypothesis made by a number of authors to explain these points. Section III contains the general equations for chemical evolution to be used, and discusses the approximations on which they are based. In § IV, solutions to the equations are obtained for models based on the alternative hypotheses. The consistent scenarios are summarized in § V, and the conclusions to be drawn from other predicted abundances and their ratios are used to limit further the possible types of model. The main results and conclusions are summarized in § VI.

The past history of the gas and stars now in the small "local swimming hole" (as Baade called it) around the Sun has been affected by the material in a much greater volume of the Galaxy. Our "nucleogenetic pool," as this volume might be called, includes at least the cylindrical shell defined by the orbital motion of the local material about the Galactic center, extending out of the plane as far as disk-population stars are carried by the vertical component of their random velcoities. A greater volume is involved if radial gas flows are important; but these can be allowed for, along with possible infall from above the Galactic plane, by terms for net inflow of gas in the relevant equations. Except when inhomogeneities are explicitly discussed, all properties of the models will refer to averages in space over this pool, and to averages in time over a Galactic orbital period at the solar radial distance. The estimates to be used for the stellar birthrate function and gas content include attempts to average over a fair sample in the pool.

Two empirical results provide powerful constraints on evolutionary models, because they contradict predictions based on the most straightforward assumptions. They will be referred to throughout this paper as "point A" and "point B."

A. The paucity of metal-poor stars. There are extremely few metal-poor stars in the local disk population. For example, Bond (1970) found that only one G–K dwarf in 10 or 100 has a metal abundance (Z) as small as $0.3Z_\odot$.

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It was first noted by van den Bergh (1962) and Schmidt (1963) that the rate of production of "metals" (which is usual in this field means most elements heavier than helium, or Z) is expected to be proportional to the stellar birthrate, so the fraction of stars with abundance less than Z' should vary roughly as Z'. Thus about one G–K dwarf in three should be an old star formed early in the lifetime of the Galaxy, with $Z \leqslant 0.3Z_{\odot}$.

B. The slow enrichment rate. Although disk-population stars of all ages have a considerable dispersion in Z, the mean value is only a very slowly increasing function of birth epoch. For example, Hearnshaw (1972) found an increase in the mean iron-to-hydrogen abundance of only about a factor 2 from the oldest to the youngest disk stars. Such a slow enrichment rate conflicts sharply with the simplest theoretical predictions: the present stellar birthrate is about 2 percent of the total mass (averaged throughout the pool) per 10^9 years, and about 2 percent of the matter formed into stars is expected to be returned promptly as new metals (Talbot and Arnett 1971, 1973*a*); since gas comprises about 5 percent of the mass of the pool, its metal abundance should be increasing by about 0.008 per 10^9 years, thus doubling the present value ($\sim$0.025) in only 3×10^9 years. Allowing for factor of 2 uncertainties in each of the three estimates, this discrepancy might not seem very serious; but it is enhanced by evidence from stellar age distributions that the birthrate was considerably greater in the past (Tinsley 1974).

Many simplifying assumptions have been implicit in the two discordant predictions just made. They include constancy of the initial mass function (IMF) for star formation, conservation of total mass, and homogeneity. These and other implicit assumptions will be explicitly altered in the models described in the next section.

II. DESCRIPTION OF MODELS

a) *Alternative Hypotheses*

The following list of hypotheses that have been proposed to account for points A and B is undoubtedly not exhaustive (although it includes all those known to the author), but it should be sufficiently representative that future hypotheses will lead to models with numerical and approximate evolutionary properties similar to one of these.

i) *Variable Initial Mass Function* ("*VIMF*" *Models*)

The problem arising from point A can be avoided if the yield of metals per unit mass of star formation was much greater at early epochs than subsequently. One possibility is that the IMF for massive stars was enhanced (Schmidt 1963). However, variations in the relative numbers of massive stars will alter the relative production rates of different elements, and according to Arnett and Schramm (1973) the excellent agreement between observed and theoretical abundances that arises using the present IMF precludes such variations. An alternative possibility that does not change any relative production rates is depletion of the *lower* IMF until $Z \sim Z_{\odot}$ in the interstellar gas. This postulates greatly increases the yield, so that Z increases rapidly, and it also accounts for point A by simply ensuring that long-lived metal-poor stars are not formed (Biermann and Tinsley 1974). The idea is not entirely ad hoc, since it is physically plausible that the lower rate of cooling in metal-poor gas would inhibit the formation of low-mass stars; it gains tentative observational support from Searle and Sargent's (1972) inference that only rather massive stars (say $\geqslant 2\ M_{\odot}$) are forming in two metal-poor galaxies.

ii) *Infall Models*

If the disk of the Galaxy has grown from a small initial mass by infall of metal-poor gas, forming stars efficiently so that the mass of gas has always been small, then Z increases at first much faster than the number of stars and rapidly achieves an equilibrium value (Larson 1972). This hypothesis accounts for both points A and B, with a constant IMF. Numerical models in which infall of extragalactic gas keeps Z nearly constant (after a rapid initial rise, produced in these models by VIMF), have been described by Quirk and Tinsley (1973) and Biermann and Tinsley (1974). The infall rate required is consistent with the infall interpretation of the high-velocity clouds (Oort 1970; Hulsbosch and Oort 1973; Gunn and Gott 1972; but see Verschuur 1973). Searle's (1972) hypothesis of inhomogeneous collapse of the Galaxy, in which stars formed only in small pockets that grew by accretion of unenriched gas, is mathematically equivalent to Larson's (1972) model. Another variant of the infall hypothesis is that the disk has grown from gas shed by stars in a massive halo (Ostriker and Thuan 1974). The first gas to reach the disk is from massive halo stars, so it is very enriched in newly synthesized metals, but later the infalling gas is from stars which shed only their very metal-poor envelopes; points A and B are thus both explained much as in the case of extragalactic infall.

iii) *Pregalactic Enrichment*

Truran and Cameron (1971) postulated that the initial enrichment required by point A was due to a pregalactic burst of very massive stars.

iv) *A Sink of Heavy Elements* ("*Comet Models*")

Point B can be explained in terms of any mechanism which provides a sink to use up metals (but not hydrogen or helium) as fast as they are formed. Tinsley and Cameron (1974) have discussed a model of this type, in which star formation is accompanied by condensation, as comets, of a mass of condensable elements approximately equal (on the average) to the mass of such elements in the star. The physical motivation for this hypothesis is Cameron's (1973) suggestion that the solar system originated with such a comet cloud.

v) *New Metals Not Ejected* ("*Black Hole Models*")

A rather similar hypothesis to account for point B is that massive stars swallow their newly synthesized metals into black-hole remnants, rather than ejecting them into the interstellar medium (Truran and Cameron 1971). Clearly an alternative site to ordinary massive stars, such as pregalactic very massive stars, must be postulated to account for the initial enrichment.

vi) *Metal-enhanced Star Formation* ("*MESF Models*")

Talbot and Arnett (1973*b*) have shown that points A and B can both be explained if stars form preferentially in regions of above-average Z, in a chemically inhomogeneous interstellar medium. This idea is physically plausible (Talbot 1974), and is conceptually similar to (although mathematically different from) the VIMF and inhomogeneous-collapse hypotheses, in that the birthrate of all or some stars is enhanced by a high metal abundance.

b) *Two Galactic Eras*

It is generally convenient to divide the lifetime of the nucleogenetic pool into two eras. In the first, which lasts for a brief interval from time $t = 0$ to t_1, the gas must be enriched to $Z \sim Z_{\odot}$ and very few long-lived metal-poor stars must be born. This era accounts for point A, and provides the initial condition for B to be satisfied. The second era must be characterized by a nearly constant or slowly increasing Z, and it lasts from time t_1 to the present, t_0. Of the above hypotheses, MESF and infall (starting from a small disk mass or with inhomogeneous collapse) can provide consistent accounts of both eras; VIMF (with depletion of the low-mass end) can give the first era; pregalactic enrichment replaces the first era; and infall (with any model for the first era), black holes, and comets give consistent models for the second era. In § IV, these results will be derived anew using the approximate equations of § III, and additional predictions of the various models will be used to select those which are consistent with other empirical constraints.

III. GENERAL EQUATIONS

a) *The Stellar Birthrate*

The stellar birthrate is given by the function $b(m, t)$, where $b(m, t)dmdt$ is the number of stars born of mass $(m, m + dm)$ in time $(t, t + dt)$. (This function differs from that of Talbot and Arnett [1971], because the distribution of stellar numbers, rather than mass fractions, in the IMF is needed for calculation of luminosities [Tinsley 1973].) The total birthrate, or simply "the birthrate" when there is no ambiguity, is defined as the mass of stars born per unit time, i.e.,

$$\psi(t) = \int_0^\infty mb(m, t)dm\,. \qquad (1)$$

If the IMF is constant, it can be denoted $\varphi(m)$, where

$$b(m, t) = \varphi(m)\psi(t) \qquad (2)$$

and

$$\int_0^\infty m\varphi(m)dm = 1\,. \qquad (3)$$

Equations (2) and (3) will be used in all the models discussed here, since VIMF models can be closely approximated by two constant IMFs, one during each era.

b) *The Mass of Gas*

The mass of gas in the pool changes because of star formation, gas loss from stars, and inflow:

$$dm_g/dt = -\psi(t) + R(t)\psi(t) + f(t)\,, \qquad (4)$$

where $f(t)$ is the net inflow rate, including infall and radial flows, and $R(t)\psi(t)$ is the rate of stellar mass loss. To evaluate the fraction $R(t)$, let τ_m be the lifetime of a star of mass m, which at death sheds (in a time $\ll \tau_m$) all but a remnant mass w_m. Let $m(t)$ be the stellar mass for which $\tau_m = t$. The rate of mass loss is given by

$$R(t)\psi(t) = \int_{m(t)}^\infty (m - w_m)\varphi(m)\psi(t - \tau_m)dm \approx \psi(t)\int_{m(t_0)}^\infty (m - w_m)\varphi(m)dm$$
$$\equiv R\psi(t)\,, \qquad (5)$$

where R is a constant fraction. The second integral with $\psi(t)$ outside makes use of the instantaneous recycling approximation, in which τ_m is neglected and the integral over the IMF is taken from the present turnoff mass ($\sim 1\ M_\odot$) to the upper limit for star formation. This approximation was investigated in some detail by Talbot and Arnett (1971), who showed that it is generally accurate enough to give useful quantitative predictions of the behavior of models in which $\psi(t)$ varies no faster than seems plausible for the solar neighborhood. Instantaneous recycling will be used throughout this paper. The value of R may be estimated to lie between 0.2 and 0.4 for the local IMF, with uncertainties arising in the values w_m and in the lower mass limit. Equation (4) now reduces to

$$dm_g/dt = -(1 - R)\psi + f\,, \tag{6}$$

where the functional dependence of ψ and f on t is to be understood.

c) Metals (*Z*)

The equations in this and the following subsections are mostly valid only for homogeneous models (including individual pockets in the model with inhomogeneous collapse [Searle 1972]), so cannot be used for MESF.

Let p_{Zm} be the mass fraction of a star of mass m that is ejected as newly synthesized metals, and let Z_f be a mean metal abundance in the inflow, defined so that $Z_f f$ is the net rate of inflow of metals. The mass of interstellar metals (Zm_g) depends on the rate at which they are used in star formation, the rate of ejection from stars of metals that had been there since birth and of new metals produced by the star, and inflow:

$$\begin{aligned} d(Zm_g)/dt &= -Z(t)\psi(t) + \int_{m(t)}^{\infty} [(m - w_m)Z(t - \tau_m) + p_{Zm}m]\varphi(m)\psi(t - \tau_m)dm + Z_f f(t) \\ &\approx -(1 - R)Z\psi + P_Z\psi + Z_f f\,, \end{aligned} \tag{7}$$

where the instantaneous recycling approximation has been used, and

$$P_Z \equiv \int_{m(t_0)}^{\infty} p_{Zm} m\varphi(m)dm\,. \tag{8}$$

This approximation is very good in the second era, since stars with finite p_{Zm} have lifetimes very short compared with the evolutionary time scales, and it is known empirically (point B) that the envelope abundances $Z(t - \tau_m)$ are not strongly time-dependent. Nucleosynthesis theory leads to the estimate $P_Z \sim 0.02$ for the local IMF (Talbot and Arnett 1971, 1973*a*), with an uncertainty on the order of a factor 2.

Equation (7) must be modified in so-called comet models (§ II*a*[iv]).

d) Astration

It is useful to define two astration parameters, describing the extent to which the interstellar matter has been processed through stars. These are α, the fraction of interstellar *gas* which has been in a star and ejected, and β, the fraction of interstellar *metals* that has been into a star *again* (after ejection from the star that produced them) and re-ejected. The first-order astration parameter, α, is closely related to the amount of primary nucleosynthesis and to the destruction of primordial deuterium (as well as other light elements), while the second-order parameter, β, is related to the amount of secondary nucleosynthesis and to the destruction of D and other light elements produced in supernovae.

The equations for the mass of astrated gas (αm_g) and mass of re-astrated metals in the gas ($\beta Z m_g$) can be written and then simplified in the instantaneous recycling approximation as follows:

$$\begin{aligned} d(\alpha m_g)/dt &= -\alpha(t)\psi(t) + R(t)\psi(t) + \alpha_f f(t) \\ &\approx (R - \alpha)\psi + \alpha_f f\,, \end{aligned} \tag{9}$$

$$\begin{aligned} d(\beta Z m_g)/dt &= -\beta(t)Z(t)\psi(t) + \int_{m(t)}^{\infty} (m - w_m)\varphi(m)Z(t - \tau_m)\psi(t - \tau_m)dm + \beta_f Z_f f(t) \\ &\approx (R - \beta)Z\psi + \beta_f Z_f f\,, \end{aligned} \tag{10}$$

where α_f and β_f are the mean inflow values defined in the appropriate way.

e) Deuterium and Helium-3

Deuterium and ^{3}He from the big bang (Wagoner 1973) may be present in the infalling gas, while D may also be produced in supernova shocks (Colgate 1973) and ^{3}He in red giants (Iben 1968). During astration, D is rapidly converted to ^{3}He, so no D is present in gas ejected from stars, while ^{3}He is partially destroyed (to ^{7}Li, which rapidly becomes a negligible addition to the abundance of ^{4}He). For the stellar contributions (including those from supernovae), production factors P_2 and P_3, for D and ^{3}He, respectively, can be defined analogously to P_Z (eq. [8]), using the instantaneous recycling approximation. Mean inflow abundances X_{2f} and X_{3f} can also be defined similarly to Z_f.

The mass of interstellar D ($X_2 m_g$) is given by

$$d(X_2 m_g)/dt = -X_2\psi + P_2\psi + X_{2f}f. \tag{11}$$

Now let q_{3m} be the fraction of the matter ejected from a star of mass m in which ^{3}He has *not* been destroyed, and, for instantaneous recycling, let

$$Q_3 R \equiv \int_{m(t_0)}^{\infty} q_{3m}(m - w_m)\varphi(m)dm\,. \tag{12}$$

The successive terms in the following equation for the mass of interstellar ^{3}He($X_3 m_g$) refer to star formation, ejection of the surviving fraction of original envelope ^{3}He from stars, ejection of the surviving fraction of ^{3}He made from D in the envelope, new production, and inflow:

$$\begin{aligned} d(X_3 m_g)/dt &= -X_3\psi(t) + \int_{m(t)}^{\infty} q_{3m}(m - w_m)\varphi(m)[X_3(t-\tau_m) + \tfrac{3}{2}X_2(t-\tau_m)]\psi(t-\tau_m)dm + P_3\psi(t) + X_{3f}f(t) \\ &\approx -(1 - Q_3R)X_3\psi + \tfrac{3}{2}Q_3RX_2\psi + P_3\psi + X_{3f}f\,. \end{aligned} \tag{13}$$

The values of q_{3m} suggested by Truran and Cameron (1971) lead to $Q_3 \sim 0.45$. Instantaneous recycling is not an accurate approximation for stellar production of ^{3}He, since it is thought to occur in stars with main-sequence lifetimes comparable to the age of the Galaxy (Iben 1968).

f) *Lithium, Beryllium, and Boron*

Stable isotopes of these elements (^{6}Li, ^{7}Li, ^{9}Be, ^{10}B, ^{11}B) can be produced in supernova shocks, in the interstellar medium by cosmic rays or suprathermal particles from supernovae (both of which will be referred to as "cosmic rays" since they contribute similar terms), and in the big bang (Audouze and Truran 1973; Colgate 1974; Epstein, Schramm, and Arnett 1974; Reeves *et al.* 1973; Wagoner 1973). Lithium is destroyed almost as rapidly as D by astration, Be more slowly, and B similarly to ^{3}He. The equation for the interstellar mass of light element L ($X_L m_g$) can be derived in the same way as that for ^{3}He, with the result:

$$d(X_L m_g)/dt = -(1 - Q_LR)X_L\psi + (\lambda_1 + \lambda_2 Z)\psi + (\gamma_1 + \gamma_2 Z)m_g\psi + X_{Lf}f\,. \tag{14}$$

The surviving fraction Q_L is defined analogously to Q_3 (eq. [12]), and it may be estimated that $Q_L \sim 0$ for Li, $\sim Q_3$ for *B*, and intermediate for Be. The term $\lambda_1\psi$ represents stellar production in reactions involving only H and He, while $\lambda_2 Z\psi$ refers to reactions involving heavier elements. (The latter should strictly allow for the fact that some production involves elements that are probably secondary, such as ^{14}N, but the additional complication would lead to no new insights here.) The term $\gamma_1 m_g\psi$ refers to spallation of interstellar H and He by cosmic rays, whose flux is assumed proportional to $\psi(t)$ (as would be the case if they arise in supernovae), while $\gamma_2 Z m_g\psi$ refers to cosmic-ray spallation of heavier elements in the interstellar gas. The last term is the inflow rate. Since the big-bang abundances of these elements are probably very much less than their observed abundances, the primordial contributions in infalling gas will be neglected.

g) *The s-Process*

The *s*-process will be considered as an example of secondary nucleosynthesis, and the results can easily be adapted for other types of secondary production. This process is thought to occur in red giants with two nuclear burning shells (e.g., Ulrich 1974), yielding a mass of new *s*-process elements proportional to the value of Z (the seed abundance) in the stellar envelope and the mass of envelope material exposed to slow neutrons in the intershell region. The exposure of envelope material also affects certain rare isotopes near the *r*-process peaks, whose observed abundances set stringent limits to the fraction of primary elements in the solar system that has been exposed to slow neutrons during re-astration (Peters, Fowler, and Clayton 1972; Blake and Schramm 1973). It is thus of interest to calculate an astration parameter σ, defined as the fraction of interstellar metals which has been re-astrated *and* exposed to *s*-processing. (The fraction of *r-process* elements so exposed will be given by σ if they are produced in proportion to Z, which requires only that they be made in stars with similar short lifetimes and that the upper IMF is invariant.)

Let s_m be the mass fraction of a star of mass m which is exposed to slow neutrons in the intershell region before ejection, and let $X_{Sm}{}^*$ be the mean abundance of new *s*-process material in the exposed fraction if its metal abundance is solar. Then the mass of metals exposed in the star before ejection is $s_m m Z(t - \tau_m)$ and the mass of new *s*-process material ejected is $s_m X_{Sm}{}^* m Z(t - \tau_m)/Z_\odot$. For the instantaneous recycling approximation, S is defined (analogously to R [eq. (5)]) as the average of s_m over the IMF above $m(t_0)$, and $X_S{}^*$ is defined so that $SX_S{}^*$ is the average of $s_m X_{Sm}{}^*$. The following equations then give the interstellar masses of exposed metals ($\sigma Z m_g$) and s-process elements ($X_S m_g$), in detail and then in the instantaneous recycling approximation.

$$\begin{aligned} d(\sigma Z m_g)/dt = & -\sigma Z\psi(t) + \int_{m(t)}^{\infty} (m - w_m - s_m m)\varphi(m)\sigma(t-\tau_m)Z(t-\tau_m)\psi(t-\tau_m)dm \\ & + \int_{m(t)}^{\infty} s_m m\varphi(m)Z(t-\tau_m)\psi(t-\tau_m)dm + \sigma_f Z_f f(t) \\ & \approx -(1 - R + S)\sigma Z\psi + SZ\psi + \sigma_f Z_f f , \end{aligned} \tag{15}$$

where the terms refer to stellar births, ejection of metals that had previously been exposed before going into that star, ejection of metals exposed in that star, and inflow, respectively. Equation (10) for β is of course the same as equation (15) if $S = R$.

$$\begin{aligned} d(X_s m_g)/dt = & -X_S\psi(t) + \int_{m(t)}^{\infty} (m - w_m)\varphi(m)X_S(t-\tau_m)\psi(t-\tau_m)dm \\ & + \int_{m(t)}^{\infty} s_m X_{Sm}{}^* m\varphi(m)[Z(t-\tau_m)/Z_\odot]\psi(t-\tau_m)dm + X_{Sf}f(t) \\ & \approx -(1 - R)X_S\psi + SX_S{}^*(Z/Z_\odot)\psi + X_{Sf}f , \end{aligned} \tag{16}$$

where the terms refer to stellar births, ejection of old and new s-process elements, and inflow, respectively. The stellar production factor analogous to P_Z is $SX_S{}^*Z/Z_\odot$, which differs significantly in having the time-dependent factor Z. As with ^{3}He, instantaneous recycling is not a good approximation for the s-process, and in some models may lead to quite misleading results, especially if most stellar production occurs in fairly low-mass stars, which may be indicated by FG Sagittae (Ulrich 1974).

IV. SPECIAL SOLUTIONS

a) The Simplest Model and its Failure

The above equations will be used first to show quantitatively the failure of the obvious, simple model to explain points A and B. With $f = 0$, and Z initially zero, equations (6) and (7) describe the evolution of Z in a homogeneous model with constant IMF and conservation of total mass. Eliminating ψ, and using as independent variable

$$g \equiv \ln(m_i/m_g) , \tag{17}$$

where m_i is the initial gas mass, we obtain from equations (6) and (7)

$$m_g dZ/dm_g = -P_Z/(1 - R) . \tag{18}$$

The solution is the well-known result,

$$Z = P_Z g/(1 - R) , \tag{19}$$

showing that $P_Z/(1 - R)$ is the quantity defined as the yield by Searle and Sargent (1972). Equation (19) predicts that Z should increase steadily to a present value (when $g \sim 3$) between about 0.05 and 0.3. Both the steady enrichment and the large final value disagree with point B. (It is not possible to argue that Z has been nearly constant because g has been so for the last 10^{10} years. In this model, the gas mass is constant only if $\psi = 0$, but the age distribution of stars shows instead that ψ has been decreasing on a time scale $\sim 5 \times 10^9$ years during the second era [Tinsley 1974] which means in this model that the gas mass, and hence Z, have evolved on similar time scales.) Moreover, the distribution of Z in stars disagrees with point A, since the total mass of stars born with $Z < Z'$ is simply

$$M_*(<Z') = \int_0^{t'} \psi(t)dt , \qquad \text{where } Z(t') = Z' ,$$

i.e.,

$$M_*(<Z') = \frac{m_i}{(1 - R)}\left\{1 - \exp\left[-\frac{Z'(1 - R)}{P_Z}\right]\right\} .$$

This expression varies with Z' when $(1 - R)Z'/P_Z \ll 1$, clearly contradicting point A. In particular, stars born with $Z \leqslant 0.3Z_\odot$ are predicted to comprise about a third of the total mass m_i; a third of G–K dwarfs should therefore have $Z \leqslant 0.3Z_\odot$, a prediction which disagrees with Bond's (1970) data cited in § I.

b) The First Era with VIMF

VIMF models for the first era can be approximated by use of a constant IMF, very deficient in stars below about 2 $M_\odot$ relative to the function needed to explain the local luminosity function. The equations of § III can be used with $f = 0$ and with initial abundances and astration parameters all zero except for D and ^{3}He which may have significant initial abundances (X_{2i} and X_{3i}) from the big bang. The fractions R, P_Z, Q_3, etc., will differ from estimates based on the local IMF; in particular, R and P_Z will be greater since a much smaller fraction of the mass is locked in remnants or stars that have lifetimes longer than that of the Galaxy, and a much greater fraction is in stars that eject newly synthesized metals.

The properties of this model can be seen most clearly by dividing equations (9)–(11) and (13)–(16) by equation (6), and using the variable g defined in (17), to obtain the following:

$$(1 - R)d\alpha/dg = R(1 - \alpha)\,, \tag{20}$$

$$(1 - R)d\beta/dg = R(1 - \beta) - \beta P_Z/Z\,, \tag{21}$$

$$(1 - R)dX_2/dg = -RX_2 + P_2\,, \tag{22}$$

$$(1 - R)dX_3/dg = -(1 - Q_3)RX_3 + P_3 + \tfrac{3}{2}Q_3RX_2\,, \tag{23}$$

$$(1 - R)dX_L/dg = -(1 - Q_L)RX_L + \lambda_1 + \lambda_2 Z + (\gamma_1 + \gamma_2 Z)m_i e^{-g}\,, \tag{24}$$

$$(1 - R)d\sigma/dg = S(1 - \sigma) - \sigma P_Z/Z\,, \tag{25}$$

$$(1 - R)dX_S/dg = SX_S{}^*Z/Z_\odot\,. \tag{26}$$

Z is given by equation (19). Equations (20)–(26) are each of the form $dx/dg + ax = F(g)$, where a is a constant and F a function, so they can be readily solved with an integrating factor e^{ag} which gives $xe^{ag} = \int e^{ag'}F(g')dg'$, with the constant of integration chosen to satisfy the initial conditions. To simplify the notation, let

$$r = R/(1 - R)\,, \quad q_3 = (1 - Q_3)R/(1 - R)\,, \quad q_L = (1 - Q_L)R/(1 - R)\,. \tag{27}$$

In addition to the exact solutions, it is instructive to note the limits in each case as $g \to 0$, i.e., near the beginning as $m_g \to m_i$. These show clearly how each abundance is affected by astration and the various contributions.

The results from (20) and (21) are

$$\alpha = 1 - e^{-rg} \to Rg/(1 - R)\,, \tag{28}$$

$$\beta = 1 - (1/rg)(1 - e^{-rg}) \to \tfrac{1}{2}\alpha\,. \tag{29}$$

From equation (22), with (28) and (29),

$$X_2 = X_{2i}e^{-rg} + (P_2/R)(1 - e^{-rg}) = X_{2i}(1 - \alpha) + (P_2 Z/P_Z)(1 - \beta)\,. \tag{30}$$

The second form of this solution, which also holds in the limit $g \to 0$, shows that a fraction α of primordial D has been destroyed, and a fraction β of the mass of D expected from supernovae if it were unaffected (as metals are) by astration. From equation (23),

$$\begin{aligned} X_3 = {} & X_{3i}\exp(-q_3 g) + \frac{P_3}{(1 - Q_3)R}[1 - \exp(-q_3 g)] + \tfrac{3}{2}X_{2i}[\exp(-q_3 g) - \exp(-rg)] \\ & + \frac{3}{2}\frac{P_2}{R}\left[\frac{Q_3}{1 - Q_3} - \frac{1}{1 - Q_3}\exp(-q_3 g) + \exp(-rg)\right] \\ & \to X_{3i}[1 - (1 - Q_3)\alpha] + (P_3 Z/P_Z) + \tfrac{3}{2}Q_3\alpha X_{2i} + \tfrac{3}{2}Q_3(P_2 Z/P_Z)\,, \end{aligned} \tag{31}$$

where the successive terms give the contributions from the fraction of primordial ^{3}He that has survived astration, stellar production, burning of primordial D and burning of D from supernovae. For the other light elements, equation (22) gives

$$\begin{aligned} X_L = {} & \left[\frac{\lambda_1}{(1 - Q_L)R} + \frac{\lambda_2 P_Z}{(1 - Q_L)^2 R^2}\right][1 - \exp(-q_L g)] + \frac{\lambda_2 P_Z g}{R(1 - R)(1 - Q_L)} \\ & + \left[\frac{\gamma_1 m_i}{(1 + Q_L R - 2R)} + \frac{\gamma_2 m_i P_Z}{(1 + Q_L R - 2R)^2}\right][\exp(-q_L g) - \exp(-g)] - \frac{\gamma_2 m_i P_Z g e^{-g}}{(1 - R)(1 + Q_L R - 2R)} \\ & \to \lambda_1 P_Z/Z + \lambda_2 Z\beta/R + \gamma_1 m_g\alpha/R + \gamma_2 m_g Z\beta/R\,, \end{aligned} \tag{32}$$

where the contributions given by each term are clear from the definitions of λ_1, λ_2, γ_1, γ_2 (§ III*f*). From equation (25),

$$\sigma = 1 - \frac{1-R}{Sg}\left[1 - \exp\left(\frac{-Sg}{1-R}\right)\right] \rightarrow \frac{Sg}{2(1-R)} . \tag{33}$$

Finally, from equations (26) and (33),

$$X_S = \frac{SX_S{}^* P_Z g^2}{2(1-R)^2 Z_\odot} = \frac{SX_S{}^* Zg}{2(1-R)Z_\odot} \rightarrow \sigma X_S{}^* Z/Z_\odot . \tag{34}$$

Because of the empirical constraint $\sigma \ll 1$, the last approximation is close to the value of X_S even when g is not very small. As expected, X_S is simply the abundance produced in exposed material ($X_S{}^*Z/Z_\odot$) multiplied by the fraction of interstellar metals that has been exposed (σ).

The first era ends, by definition, at time t_1 when $Z = Z_1 \sim Z_\odot$. The IMF in the first era in the model of Biermann and Tinsley (1974) leads to $R \sim 0.8$, $P_Z \sim 0.08$, so by equation (19) the gas content at t_1 is given by $g_1 \sim 0.05$, i.e., $m_{g1} \sim 0.95 m_i$. In this case, the limiting forms of equations (28)–(34) apply throughout the era. The time t_1 depends on the efficiency of star formation, and it will be short if the efficiency is high—a point beyond the scope of the present discussion.

c) *Infall with a Constant Mass of Gas*

Both eras can be described if infall forms the disk from an initially small mass while star formation keeps m_g approximately constant; the results of this section also apply to the inhomogeneous-collapse model (§ II*a*[ii]). In both cases, the infalling gas is assumed primordial, so the only possibly significant abundances in the infall (and initially) are those of D and ^{3}He. The initial mass is taken to be m_g.

The condition for constant m_g is (eq. [6])

$$f = (1 - R)\psi , \tag{35}$$

which is consistent with estimates of the present birthrate and infall rate (if observed in the high-velocity clouds). The total mass at time t is

$$M(t) = m_g + \int_0^t f dt = m_g + (1 - R)\int_0^t \psi dt . \tag{36}$$

A convenient independent variable for this model is

$$\mu = \frac{M(t) - m_g}{m_g} = \frac{1-R}{m_g}\int_0^t \psi dt , \tag{37}$$

which has initial value zero and present value ~ 20. Equation (7) becomes

$$(1 - R)dZ/d\mu = -Z(1 - R) + P_Z ,$$

which has the solution

$$Z = \frac{P_Z}{1-R}(1 - e^{-\mu}) \rightarrow \frac{P_Z}{1-R} \sim 0.03 \quad \text{as } \mu \rightarrow \infty . \tag{38}$$

As emphasized by Larson (1972), Z attains an equilibrium value in the very short time needed for $M(t)$ to increase to a few times the constant m_g, so point B is readily explained. The value of the limit is also in excellent agreement with observation. This model also explains point A with a constant IMF, as shown by Searle (1972): equations (37) and (38) lead to the distribution

$$M_*(<Z') = \int_0^{t'} \psi(t)dt = \frac{m_g}{1-R}\ln\left[1 - \frac{(1-R)Z'}{P_Z}\right]^{-1} .$$

In this case, stars born with $Z \leqslant 0.3Z_\odot$ comprise a third of the *gas* mass, which is only about 2 percent of the total present mass, so the distribution in G–K dwarfs does not conflict with that observed.

It can be shown similarly, from equations (9)–(11) and (13)–(16) with (35), (37), and (38), that the other abundance parameters tend exponentially to equilibrium values before $M(t)$ is a significant fraction of its present value.

The values predicted by this model, for the solar system and for the present interstellar medium, are the following limits:

$$\alpha \to R\,, \quad \beta \to R\,, \quad X_2 \to P_2 + X_{2f}(1 - R)\,, \quad X_3 \to \frac{X_{3f}(1 - R) + P_3 + \frac{3}{2}Q_3RX_2}{1 - Q_3R}\,,$$

$$X_L \to \frac{\lambda_1 + \lambda_2 Z + \gamma_1 m_g + \gamma_2 m_g Z}{1 - Q_L R}\,, \quad \sigma \to \frac{S}{1 + S - R} \sim \frac{S}{1 - R}\,, \quad X_S \to \frac{SX_S{}^*P_Z}{(1 - R)^2 Z_\odot} \sim \sigma X_S{}^* Z/Z_\odot\,. \quad (39)$$

In models with infall of gas lost by halo stars (§ II*a*[ii]), the equilibrium Z is achieved even sooner because the early gas is enriched. The significant difference from primordial infall is that all the gas is astrated just once, so $\alpha_f = 1$ in equation (9), and in (38) the limiting value of α is 1, while $X_{2f} = 0$, and X_{3f} is a fraction of the primordial abundance depending on the halo IMF.

d) *The Second Era with Infall*

Infall may be postulated during the second era to explain point B (§ II*a*[ii]), with any model for the first era providing the initial abundance $Z_1 \sim Z_\odot$ at time t_1. Let the other initial values be $m_{g1} = m_i e^{-g_1}$, α_1, β_1, etc. The only infall abundances with possibly significant values are X_{2f} and X_{3f}, with $\alpha_f = 1$ if the infall is from halo stars (see § IV*c*); Z_f is negligible from halo stars during this era, and is zero in primordial gas. Equation (6) may now be used to reduce equations (7), (9)–(11), and (13)–(16) to the following:

$$\begin{aligned}
m_g dZ/dt &= P_Z\psi - Zf\,,\\
m_g d\alpha/dt &= R(1 - \alpha)\psi + (\alpha_f - \alpha)f\,,\\
m_g d\beta/dt &= R(1 - \beta)\psi - \beta(P_Z/Z)\psi\,,\\
m_g dX_2/dt &= -RX_2\psi + P_2\psi + (X_{2f} - X_2)f\,,\\
m_g dX_3/dt &= -(1 - Q_3)RX_3\psi + P_3\psi + \tfrac{3}{2}Q_3RX_2\psi + (X_{3f} - X_3)f\,,\\
m_g dX_L/dt &= -(1 - Q_L)RX_L\psi + (\lambda_1 + \lambda_2 Z)\psi + (\gamma_1 + \gamma_2 Z)m_g\psi - X_L f\,,\\
m_g d\sigma/dt &= S(1 - \sigma)\psi - \sigma(P_Z/Z)\psi\,,\\
m_g dX_S/dt &= SX_S{}^*(Z/Z_\odot)\psi - X_S f\,.
\end{aligned} \quad (40)$$

Since $P_Z \sim Z_\odot$ and, at present $f \sim \psi$ (assuming that the high-velocity clouds represent infall), the first member of equation (40) shows that Z is changing very slowly. In order to satisfy point B, Z must tend early to an approximate equilibrium value, so that f and ψ are roughly proportional with $f/\psi \sim P_Z/Z$. The other quantities in equation (40) then also tend to equilibrium values. Since the data do not necessarily demand that Z be strictly constant, it might be more realistic to consider the limits obtained if $m_g \to$ constant. Physical arguments for this have been discussed by Larson (1972) and Quirk (1972). Since m_{g1} may be much greater than the present m_g (as it is in the example at the end of § IV*b*), m_g will at first decrease during the second era, but eventually the birthrate will balance the rates of infall and stellar mass loss. Equation (35) holds in this limit, and the other quantities tend to the limiting values given in equations (38) and (39), with the same modifications if infall is from halo stars. The time scale for approaching these values depends on the efficiency of star formation in the early second era. The physical reasons for expecting a high efficiency will not be discussed here, but plausible estimates lead to time scales on the order of 10^9 years (Quick and Tinsley 1973).

e) *The Second Era with Comets*

In this type of model (§ II*a*[iv]) there is no infall, so equations (20)–(25) can be used. The equations for condensable elements, which will be taken to be those included in Z and X_S, must be modified to allow for the postulated sink. In unit time, star formation consumes a mass $Z\psi$ of metals and a mass $X_S\psi$ of *s*-process elements. Let the additional masses lost from the gas by condensation be $cZ\psi$ and $cX_S\psi$. In the comet model (Tinsley and Cameron 1974), the range $0 \leq c \lessapprox 1$ seems plausible. Equations (7) and (16) become

$$\begin{aligned}
d(Zm_g)/dt &= -(1 + c - R)Z\psi + P_Z\psi\,,\\
d(X_S m_g)/dt &= -(1 + c - R)X_S\psi + SX_S{}^*(Z/Z_\odot)\psi\,,
\end{aligned}$$

so that equations (18) and (26) are modified to

$$(1 - R)dZ/dg = -cZ + P_Z\,, \quad (41)$$

$$(1 - R)dX_S/dg = -cX_S + SX_S{}^*(Z/Z_\odot)\,. \quad (42)$$

The comet model cannot explain point A, so it must start at the beginning of the second era with initial conditions as in § IV*d*. The solution to equation (41) is

$$Z = Z_1 \exp\left[\frac{-c(g - g_1)}{1 - R}\right] + \frac{P_Z}{c}\left\{1 - \exp\left[\frac{-c(g - g_1)}{1 - R}\right]\right\}$$
$$\sim P_Z/c \quad \text{as } g \to \infty \quad (m_g \to 0)\,. \tag{43}$$

Since $Z_1 \sim P_Z$ (within about 50%), the limiting value of Z is reached very quickly if $c \sim 1$. Thus if there is a significant sink, in the sense $c \sim 1$, point B can be explained.

Setting $Z =$ constant in equations (20)–(25) and (42), limits are obtained for the other quantities; the detailed solutions show that relative present departures from the limiting values are $\sim m_g/m_{g1} \sim 0.05$:

$$\alpha \to 1\,, \quad \beta \to R/(R + c)\,, \quad X_2 \to P_2/R\,, \quad X_3 \to \frac{P_3 + \frac{3}{2}Q_3 R X_2}{(1 - Q_3)R}\,,$$
$$X_L \to \frac{\lambda_1 + \lambda_2 Z}{(1 - Q_L)R} + \frac{(\gamma_1 + \gamma_2 Z)m_g}{(1 - Q_L)R}\,, \quad \sigma \to \frac{S}{S + c} \sim \frac{S}{c}\,, \quad X_S \to \frac{S X_S{}^* Z}{c Z_\odot} \sim \sigma X_S{}^* Z/Z_\odot\,. \tag{44}$$

These limits will be compared with the predictions of other models in § V.

f) *The Second Era with Black Holes*

This type of model can be approximated by setting P_Z to zero in equations (18) and (20)–(26), and using initial conditions at time t_1 as in § IV*d* (see § II*a*[v]). The stellar production factors for D, ^{3}He, Li, Be, B, and *s*-process elements are not set to zero since they are thought to be produced in the envelopes of red giants or supernovae, rather than in stellar cores. The quantities R, Q_3, Q_L, and the finite stellar production factors will be different from usual, however, because there is less total stellar mass loss. The solution of the above equations with finite initial values is straightforward. For example, $Z = Z_1$ (of course), and

$$\alpha = \alpha_1 \exp[-r(g - g_1)] + 1 - \exp[-r(g - g_1)]\,,$$

which tends to the finite limit 1 as $m_g \to 0$. The quantities that tend to limiting values are

$$\alpha \to 1\,, \quad \beta \to 1\,, \quad X_2 \to P_2/R\,, \quad X_3 \to \frac{P_3 + \frac{3}{2}Q_3 P_2}{(1 - Q_3)R}\,, \quad X_L \to \frac{\lambda_1 + \lambda_2 Z_1}{(1 - Q_L)R}\,, \quad \sigma \to 1\,. \tag{45}$$

However, the solution to equation (26) is

$$X_S = \frac{S X_S{}^* Z_1}{(1 - R)Z_\odot}(g - g_1) = \frac{S X_S{}^*}{(1 - R)}\frac{Z}{Z_\odot}\ln\frac{m_{g1}}{m_g}\,. \tag{46}$$

An important difference between this and other models for the second era is that here the ratios of secondary to primary abundances (excemplified by X_L/Z) increase steadily with time. This will be discussed in § V*a*.

g) *MESF*

Although most of the equation in § III cannot be used in this inhomogeneous model, α is given by equation (28), and its present value (using $R \sim 0.3$, $g \sim 3$) is predicted to be about 0.5. Approximately half of the primordial D will remain. Talbot and Arnett (1973*b*, and private communication) have determined numerically that, after an initial rapid enrichment consistent with point A, both primary and secondary abundances increase very slowly.

V. COMPARISON WITH OBSERVED ABUNDANCES

In all of the models for the second era that are consistent with point B, the predicted solar-system and present abundances are independent of those at the end of the first era (except for Z, which must be $\sim Z_\odot$ at that time, by definition of the eras), or of the process postulated to describe the first era. This section will consider only models for the second era, comparing their predictions with theoretical and empirical constraints on abundances other than Z.

a) *Ratios of Secondary to Primary Abundances*

Truran (1973) has stressed that an important constraint on models for chemical evolution is provided by the growing evidence that certain CNO isotope ratios have not changed substantially during the last 4.5 billion years. For example, the ratio $(^{14}N/^{15}N)/(^{12}C/^{13}C)$ is approximately 3 in the solar system *and* in the interstellar medium, whereas the value characteristic of core CNO hydrogen burning is $\sim 3 \times 10^4$. This result is a sensitive demonstration that the ratios of isotopes produced in primary, secondary, and possibly even tertiary processes of nucleosynthesis have evolved negligibly, between the time the solar system formed and the present. The conclusion of Harmer and Pagel (1973) that most Population I stars have the solar abundance ratio of nitrogen to iron also suggest that secondary-to-primary abundance ratios are almost constant for disk stars. The implied constraint on the models discussed here is that the representative secondary-to-primary abundance ratio X_S/Z must tend to equilibrium in the second era. This is the case with infall (eq. [39] and § IV*d*), comets (eq. [44]), and approximately so with MESF (§ IV*g*), but not with black holes (eq. [46]). This shortcoming of the model with black holes was pointed out by Truran (1973). The discrepancy will be found as long as one of the isotopes considered is produced in stellar envelopes, and not locked up in black holes, while the other isotope is locked up. Several of the ratios discussed by Truran (1973) are of this type, so the black-hole model appears to be invalidated. It will not be considered further here.

b) *Neutron Exposure and s-Process Abundances*

The arguments of Peters *et al.* (1972) suggest that $\sigma \leqslant 0.01$, which implies that $S \leqslant 0.01$, and the limiting values of σ obtained in the consistent models are $S/(1 - R)$ with infall (eq. [39]) and S/c with comets (eq. [44]). In both types of model, $X_S \to \sigma X_S{}^* Z/Z_\odot \sim \sigma X_S{}^*$. Therefore the exposed material must be enriched by the *s*-process to an overabundance $X_S{}^*/X_S \geqslant 100$. Similar predictions are made by the MESF model (Talbot, private communication). Schramm and Tinsley (1974) find similar results using numerical models. This indicates that in spite of the nonnegligible lifetimes of the stars involved, the instantaneous recycling approximation is not too misleading in its predictions as to the scaling of solar-system abundances with production parameters. However, the numerical models show that the time-dependence of X_S is given incorrectly by this approximation if low-mass stars produce most of the *s*-process elements. In this case, X_S increases steadily with time, whereas if massive stars are the main site of the *s*-process, X_S behaves like Z (and as predicted by the instantaneous recycling approximation), rising quickly to an almost constant value. Schramm and Tinsley (1974) also find that the requirements $\sigma \leqslant 0.01$, $X_S{}^*/X_S \geqslant 100$ are consistent with the very uncertain theoretical stellar models that have been proposed for the *s*-process. The present results show that refined theoretical estimates can be compared in the future with observed abundances, with uncertainties no more than about a factor 2 due to alternative possibilities for the effects of Galactic evolution. The model independence of the abundance-to-production ratios is due directly to the constraint that both primary and secondary abundances have evolved very little during the second era.

c) *Lithium, Beryllium, and Boron*

The cosmic-ray contribution to abundances of these elements tends to $(\gamma_1 + \gamma_2 Z)m_g/(1 - Q_L R)$ in the infall models (eq. [39]), but eventually to zero in the comet model because the gas mass is asymptotically zero. Production rates by cosmic rays, essentially equivalent to the quantity $(\gamma_1 + \gamma_2 Z)\psi(t_0)$, have been calculated by Meneguzzi, Audouze, and Reeves (1971), who found that the theoretical rates, if maintained for about 10^{10} years, could explain all of the then accepted abundances except ^{7}Li. Audouze and Tinsley (1974) found similar agreement in numerical models, provided that the abundance of B is not much greater than previously believed (as postulated by Cameron, Colgate, and Grossman [1973] but shown to be unlikely by Andouze, Lequeux, and Reeves [1973]). For comparison with the present approximations, denote by X_{L0} the cosmic-ray contributions obtained with constant m_g, Z, and ψ, and no astration (i.e., the abundances found by Meneguzzi *et al.* [1971]); equation (14) gives $X_{L0} = (\gamma_1 + \gamma_2 Z)\psi(t_0)t_0$. Thus the infall models predict cosmic-ray contributions differing from X_{L0} by the ratios $X_L/X_{L0} = (1 - Q_L R)^{-1} m_g/t_0\psi(t_0) \sim 0.25(1 - Q_L R)^{-1}$. Using the estimates for R and Q_L given in § III, this ratio is ~ 0.25 for Li, ~ 0.3 for B, and intermediate for Be. Within the theoretical and observational uncertainties, spallation of interstellar gas by cosmic rays could explain all of the solar system ^{6}Li, Be, and B in this type of model. Lithium-7 is not explained by this process (cf. references in § III*f*).

The uncertainties, however, do not rule out a substantial contribution by stars (including supernovae) for ^{6}Li, Be, and B; and stellar production must account for all of the Li, Be, and B in the comet model. The limiting abundances from this process (eqs. [39] and [44]) are respectively $(\lambda_1 + \lambda_2 Z)/(1 - Q_L R) \sim \lambda_1 + \lambda_2 Z$, and $(\lambda_1 + \lambda_2 Z)/(R - Q_L R) \sim 5(\lambda_1 + \lambda_2 Z)$. The production factors, λ_1 and λ_2, are as yet too poorly determined to say whether these final abundances agree with observation (cf. references in § III*f*).

Abundances of the light elements have not been calculated in the MESF model, but they are espected to be similar to those predicted by the infall model since the extents of astration are similar in the two models.

d) *Deuterium*

Because D is so vulnerable to astration, its predicted abundances are significantly different in the various models for the second era. The surviving fractions of big-bang D_e(cf. eqs. [39] and [44], and § IV*g*) are $1 - R$ (~ 0.7) with infall of primordial gas, ~ 0.5 with MESF, but zero with infall from halo stars or with comets. Abundances on the order of twice the interstellar value can be made in a universe whose density is consistent with the lower limit set by galaxies (Gott *et al.* 1974), so the primordial infall and MESF models may not require any D from supernovae. The contribution from supernovae tends to P_2 in all infall models, to $P_2/R \sim 3P_2$ in the comet model, and (although not explicitly calculated) would be $\sim P_2$ in MESF. The value of P_2 depends chiefly on the very uncertain energy per nucleon available in supernova explosions (Colgate 1973; Reeves 1973), so that it cannot be said whether this source is sufficient. Note that because of the equilibrium nature of the second stage demanded by point **B**, the predicted present abundance from supernovae is $\sim P_2$, and should *not* be multiplied by a factor ~ 10 to allow for the faster supernova rate in the past, as has often been done; the faster production rate is exactly canceled by the faster astration rate that accompanied the higher stellar birthrate! (An exception to this would be if the early rapid production were all in the very first generation of stars.)

An argument against supernovae as a significant source of the observed D, which is almost independent of the supernova energies, has been given by Epstein *et al.* (1974). They find that supernova shocks produce Li, Be, and B in amounts relative to D that far exceed the observed abundance ratios; i.e., that $(\lambda_1 + \lambda_2 Z_\odot)/P_2 \gg X_L/X_2$, in the present notation. Equations (39) and (44) show that the models predict present values of X_L/X_2 from supernovae alone greater than the production ratios, simply because D is at least as rapidly destroyed by astration as the other light elements; the limiting ratios in the infall and comet models, respectively, are, in obvious notation,

$$\frac{X_L(\mathrm{SN})}{X_2(\mathrm{SN})} \to \frac{(\lambda_1 + \lambda_2 Z)}{(1 - Q_L R)P_2} \sim \frac{(\lambda_1 + \lambda_2 Z_\odot)}{P_2}, \qquad \frac{X_L(\mathrm{SN})}{X_2(\mathrm{SN})} \to \frac{\lambda_1 + \lambda_2 Z}{(1 - Q_L)P_2} \sim \frac{2(\lambda_1 + \lambda_2 Z_\odot)}{P_2}.$$

The MESF model would give similar ratios. This result is as yet uncertain because it depends on the exact nature of supernova shocks, but if substantiated it would argue strongly against the comet type of model or infall from halo stars, because it says that even if supernovae produce all of the Li, Be, and B, they contribute negligibly to the observed D abundance.

e) *Helium-3*

The various models predict significantly different contributions from primordial ^{3}He, D-burning, and stellar ^{3}He production, again because of their different amounts of astration. For comparison with observation, the solar-system abundance may be considered, using $X_3 = (2.0 \pm 1.0) \times 10^{-5}$ (Geiss and Reeves 1971), and X_2 essentially the same as its present interstellar value, $(2.0 \pm 0.3) \times 10^{-5}$ (Rogerson and York 1973). The tentative estimate $Q_3 = 0.45$ will also be used, and $R = 0.3$ (§ III).

First consider the infall model (eq. [39]). The above estimates give $X_3 = 0.8X_{3f} + 1.2P_3 + (0.47 \pm 0.07) \times 10^{-5}$. If all the D comes from the big bang, then infall must be primordial and (from § V*d*) the big-bang abundance is $X_2/(1 - R) = (2.9 \pm 0.4) \times 10^{-5}$. In the standard big bang (Wagoner 1973), the density consistent with these D abundances gives $X_{3f} = (1.6 \pm 0.1) \times 10^{-5}$. As a result, the model predicts $X_3 = (1.8 \pm 0.2) \times 10^{-5} + 1.2P_3$, so that, within the uncertainties, it cannot be said whether a significant stellar contribution is needed or even precluded. Similar conclusions were reached numerically by Audouze and Tinsley (1974). On the other hand, if there is negligible primordial D or ^{3}He in the infalling gas, the destruction of D (from an alternative source which must be postulated) does not provide enough ^{3}He, and a stellar contribution is required.

In the comet model (eq. [44]), the above parameters lead to $X_3 = 6.1P_3 + (2.5 \pm 0.4) \times 10^{-5}$. A stellar contribution appears to be unnecessary in this case, since enough ^{3}He is made by astration of D, the source of which must be Galactic (§ V*d*).

The uncertainties in the abundance of ^{3}He, in the destruction parameter Q_3 (whose uncertainty is not included in the above error estimates), and in models for Galactic evolution mean that the possible importance of ^{3}He production by red giants (Iben 1968) will be difficult to determine.

VI. CONCLUSION

The paucity of metal-poor stars in the local population can be explained by several models for a brief initial period of evolution. These include variation of the IMF so that a greater yield of metals per unit mass of star formation was obtained when the gas had a low metal abundance; efficient star formation in a disk which grew by infall from an initially small mass; pregalactic enrichment of the gas; and enhancement of star formation in regions of above-average metal abundance, either in small pockets during collapse, or in the disk afterwards. (Original references for these hypotheses have been given in § II.) It has been shown that the choice of model for the first brief era has very little effect on the predictions made about abundances in disk stars, the solar system, or the present interstellar gas. The relative merits of the alternative models depend on physical arguments, which lie beyond the scope of this paper. More than one of the processes could of course have occurred.

Consistent models for chemical evolution after this initial enrichment are strongly limited by the slow increase of mean stellar metal abundance with epoch of birth. Among the alternative hypotheses which can explain this result are infall of metal-poor gas; a sink of heavy elements, such as formation of a significant mass of comets around the average star; locking of newly-synthesized metals into massive stellar remnants (black holes); and metal-enhanced star formation. (References to the authors of these theories have been given in § II.) Models of the black-hole type are inconsistent with the further empirical constraint that certain ratios of abundances of secondary to primary products of nucleosynthesis do not differ significantly between the solar system and the present interstellar medium.

A further important constraint is imposed if the observed deuterium abundance must be all due to synthesis in the big bang. The consistent models are then limited to those in which a significant fraction of the interstellar gas has not been astrated by the present time. Models in which the infalling gas is mass lost by halo stars, or in which there is a sink of metals, are invalidated by this constraint, and the only consistent types of models have infall of primordial gas or metal-enhanced star formation. If the other processes have occurred, they cannot have contributed importantly to the slow evolution of metal abundance.

Radial gas flows in the Galaxy can be included by reinterpretation of the inflow terms in the general equations of § III, but not in any of the specific models. It can be shown (Biermann, private communication) that infall of gas with low angular momentum must give rise to inward radial flows. If our Galaxy is like other spirals in which a decreasing interstellar metal abundance with increasing radius has been observed (Searle 1971), such a flow will enhance the net rate of metal-poor inflow. (In addition, the radial flow driven by infall will enhance the abundance gradient that is expected to be established during the initial collapse [Larson 1974]). If the abundance gradient in our Galaxy can be observed with sufficient accuracy, it will be a challenging problem in the future to incorporate radial flows in a self-consistent way.

It is interesting to note now the solutions to the equations of chemical evolution, in the instantaneous recycling approximation, decouple the time-dependence of the birthrate from the abundances as a function of gas content (except that time is implicit in the production rates of light elements by cosmic rays). Independent evidence on time scales can be derived from nucleocosmochronology (Schramm 1974) and the distribution of stellar ages (Tinsley 1974).

Finally, it should be repeated that the physical processes underlying the alternative models are not mutually exclusive, and several may have operated together to give rise to the distribution of chemical elements observed today in the solar neighborhood.

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