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BLACK HOLES IN BINARY SYSTEMS: INSTABILITY OF DISK ACCRETION*

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ABSTRACT

We have tested the stability of a thin, orbiting accretion disk near a black hole. Under conditions appropriate for a binary X-ray source, with the usual (ad hoc) assumptions about viscosity, the disk is always secularly unstable on time scales of a few seconds or less. Therefore current thin-disk models for such X-ray sources are self-inconsistent. We mention possibilities for alternative models; perhaps the secular instability explains chaotic time variations in Cygnus X-1.

Subject headings: binaries — black holes — instabilities

Current models (Pringle and Rees 1972; Shakura and Sunyaev 1973; Novikov and Thorne 1973) for binary X-ray sources powered by accretion onto a black-hole companion envisage the gas flow near the hole as either a thin, orbiting disk or a thick, perhaps chaotic cloud. If the X-ray luminosity L exceeds the Eddington limit, $L^{\rm ED} \sim (10^{38}$ ergs s$^{-1}) (M_{\rm BH}/M_\odot)$, where $M_{\rm BH} \equiv$ mass of black hole, then the cloud picture is more likely. Moreover, even at luminosities somewhat lower than the Eddington limit, say $L \gtrsim 10^{-2}\, L^{\rm ED}$ (all figures quoted will be for typical parameters of accretion models), thermal instabilities caused by optical thinness (Pringle, Rees, and Pacholczyk 1973) may disrupt the inner region of a thin disk, transforming it into a thick cloud. We wish to point out in this *Letter* that, *with the usual (ad hoc) assumption about the viscosity*, detailed thin-disk models are *always secularly unstable* over the whole "inner region" (that region where radiation pressure dominates gas pressure, $P_R > P_G$, and the dominant opacity is electron scattering). Such an inner region exists near the hole when $L \gtrsim 10^{-4}\, L^{\rm ED}$. Therefore these models are inconsistent. The observational consequences are great since most of the X-ray luminosity originates in the inner region.

The current thin-disk models (Pringle and Rees 1972; Shakura and Sunyaev 1973; Novikov and Thorne 1973) are *stationary* and include two key assumptions:

(*a*) Accreting matter forms a thin, orbiting, non-self-gravitating disk drifting inward on a slow time scale $t_{\rm drift}$ (slow compared with thermal and Kepler time scales). The drift is caused by viscous stress removing angular momentum.

(*b*) Although the viscous stress $t_{\hat\varphi\hat r}$ arises from intricate processes (e.g., turbulent motions on fast time scales, or magnetic fields), it may be approximated on slow time scales $\sim t_{\rm drift}$ and longer by

$$t_{\hat\varphi\hat r} = \alpha P_{\rm tot}\,, \tag{1}$$

where $P_{\rm tot} = P_R + P_G$ and α is a number believed to lie between 10^{-3} and 1.

To investigate stability of the above models we generalize them to allow time-dependence in the radial disk structure on the slow time scale $t_{\rm drift}$ (a few seconds at the outer edge of the inner region; a few milliseconds at the inner edge). We shall sketch the development here. For a complete discussion of the stationary models, see Novikov and Thorne (1973) and Shakura and Sunyaev (1973). For a complete discussion of the time-dependent generalization, see Lightman (1974).

Variables describing the local, instantaneous state of the disk are surface density $\Sigma(r, t)$ (g cm^{-2}), total inward mass flux $\dot M(r, t)$ (g s^{-1}), mean half-thickness $h(r, t)$ (cm), mean pressure $P(r, t)$, mean temperature $T(r, t)$, radiative flux $F(r, t)$ (ergs cm^{-2} s^{-1}) from top of disk (= same from bottom), and vertically-integrated viscous stress $W(r, t) \approx 2ht_{\hat\varphi\hat r}$ (dyne cm^{-1}) (means are vertical averages). The structural equations relating these variables (ignoring relativistic corrections) are:

Equations of radial structure:

$$2\pi r \frac{\partial \Sigma}{\partial t} = \frac{\partial \dot M}{\sigma r} \quad \text{(conservation of mass)} \tag{2a}$$

$$\frac{d(\Omega r^2)}{dr} \dot M = \frac{\partial}{\partial r}(2\pi r^2 W) \quad \text{(conservation of angular momentum);} \tag{2b}$$

here $\Omega \equiv (GM_{\rm BH}/r^3)^{1/2}$. Equations (2) are exact.

Equations of vertical structure (specialized to inner region):

$$F = \tfrac{3}{4}\Omega W \quad \text{(conservation of dissipated energy),} \tag{3a}$$

$$F = \tfrac{2}{3} acT^4/(\kappa_{\rm Compt}\,\Sigma) \quad \text{(vertical radiative diffusion, Compton opacity),} \tag{3b}$$

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$P = \frac{1}{2}h\Omega^2\Sigma$ (vertical pressure balance against out-of-plane gravitational forces of black hole), (3c)

$P = P_R \equiv \frac{1}{3}aT^4$ (equation of state, $P_R \gg P_G$), (3d)

$W = 2\alpha hP$ (source of viscosity, eq. [1]). (3e)

Equations (3) are only approximate, because of uncertainties in averaging over vertical structure.

The stationary models are obtained by setting $\partial\Sigma/\partial t \equiv 0$ in equations (2), (3).

For time-dependent models, it is best to choose $\Sigma(r, t)$ as the sole independent variable characterizing the local, instantaneous state of the disk. Then, at each (r, t), one solves equations (3) algebraically for h, P, T, F, and W as functions of (Σ, r). It is essential to determine $W(r, t)$ self-consistently in this way, rather than to fix W through equation (2b) from a given $\dot{M}$, as one does in the stationary case. Equations (2) yield, as the evolution equation of $\Sigma(r, t)$,

$$\frac{\partial\Sigma}{\partial t} = \frac{\partial}{\partial r}\left[\frac{d(\Omega r^2)}{dr}\right]^{-1}\frac{\partial}{\partial r}[r^2 W(\Sigma, r)] . \tag{4}$$

The instability arises in the inner region for the following reason: Equations (3) give

$$W(\Sigma, r) = \text{const.}/\Sigma . \tag{5}$$

[To justify this paradoxical result: Since $P_R \gg P_G$, P is not determined directly by ρ ($\rho \equiv \Sigma/2h$), but only by T; and in fact T and P turn out to be independent of Σ. Equation (3c) implies $h \propto \Sigma^{-1}$; then equation (3e) shows $W \propto \Sigma^{-1}$.] The integrated stress W is here a *decreasing* function of Σ; hence the nonlinear diffusion equation for Σ, equation (4), has a *negative* effective diffusion coefficient. As a result an initially stationary disk tends to break up into rings $\Delta r \gtrsim h$, on time scales $\sim (\Delta r/r)^2\, t_{\text{drift}}$; alternate rings have high-Σ/low-W and low-Σ/high-W. The density contrast grows because matter is pushed into regions of minimum viscous stress W. Eventually the low-Σ regions become optically thin and hence thermally unstable (Pringle *et al.* 1973). As Σ grows in the high-Σ regions, eventually a regime is reached in which the disk cannot radiate as much energy as it is generating and the vertical structure equations fail to admit a solution. Therefore the growing instability causes a complete breakdown in the thin-disk picture, assumption (*a*). These conclusions are supported by detailed analytic and numerical calculations which one of us (A.P.L.) will report elsewhere (Lightman 1974).

Definitive models must therefore await a better understanding of viscosity: we mention two quite distinct possible alternatives to current models:

1. Assumption (*a*) fails because assumption (b) is roughly correct. Around the hole forms a cloud, which is 10 to 100 times larger than the hole. If dissipation is efficient (expected, since accreting matter must still lose its angular momentum), the cloud may emit X-rays as a hot, thin plasma with Comptonization probably important (Felten and Rees 1972; Illarionov and Sunyaev 1972). Alternatively, synchrotron cooling may be important. Gross time variations, both in intensity and in spectrum, are expected on the hydrodynamical time scale of the cloud $\sim$ tens to hundreds of milliseconds and longer. If the cloud is optically thick to Compton scattering, time variations on time scales shorter than the random walk time of a photon through the cloud $\sim \tau r/c$ (τ = optical depth) may be lost (F. K. Lamb, private communication). In particular, submillisecond time variations in signal, originating very near the hole (Sunyaev 1972), might be hopelessly smeared out by scattering in the translucent cloud.

2. Assumption (*b*) is seriously wrong. With α a function of Σ rather than a constant in the time-dependent case (eq. [1]), a stable, stationary, thin disk is possible if α falls at least as fast as Σ^{-1} in the inner region (*less* efficient viscosity). Such an α leads in turn to a $\Sigma(r)$ that increases steeply toward the hole. For example, (Cunningham 1973), equation (1) might be replaced by

$$t_{\hat{\varphi}\hat{r}} = \beta P_G\,, \quad \beta = \text{const.}\,, \tag{6}$$

even when $P_R \gg P_G$. (Perhaps this relation is preferable for a self-limiting magnetic viscosity, since gas is frozen to the B-field while radiation is not.) The stationary, thin-disk model resulting from equation (6) is stable and is much like current models except that Σ is much greater in the inner region (typically 25 times greater at $r = 10GM_{\text{BH}}/c^2$). The thickness, $2h$, is still $\lesssim 2 \times 10^5$ cm. This dense disk is quite optically thick and is probably immune to thermal or magnetic disturbances on length scales $\sim h$; hence, chaotic variations in the X-ray signal are likely to be negligible.

Observations (Schreier *et al.* 1971) of Cygnus X-1 (and similar sources which have been advanced as black-hole candidates) favor alternative (1), since the observed signal is chaotic on all time scales from tens of seconds to $\sim$50 milliseconds (instrumental limit). For either alternative, we believe that the prospects of seeing characteristic ($\lesssim$ms) time variations originating very near the hole are poorer than has been generally supposed on the basis of current models (Sunyaev 1972).

The same instability arises in a disk around an unmagnetized neutron star. For a magnetized neutron star, a disk does not extend inside the magnetosphere (Pringle and Rees 1972); there is no inner region, hence there is no instability.

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