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Distortion mechanisms for lunar occultation diffraction patterns

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Relative magnitudes of the factors causing distortion of optical lunar occultation observations are compared. Curves are provided to facilitate the design of equipment for occultation observations.

INTRODUCTION

THE theoretical diffraction pattern resulting from diffraction of monochromatic coherent light at an infinitely thin, perfectly smooth black screen is well known for its characteristic shape. The intensity of the light I passing the screen oscillates about the incident intensity of the light I_0 and asymptotically approaches I_0 as the distance from the screen increases. This ideal theoretical diffraction pattern cannot be observed in lunar occultations because of interference effects resulting from the instrumentation, the light path to the observer, the nature of the lunar limb, and the nature of the light emitted from the star.

A number of papers (Evans 1970; Morbey and Hutchings 1971; Morbey and Fletcher 1974) have been published in recent years which outline attempts to fit data from occultation observations to theoretical models that take into account effects of stellar size, telescope diameter, bandwidth, etc. In general, these attempts have been successful, but a comparison of the relative magnitudes of these effects on the theoretical diffraction pattern has not been made. It is the purpose of this paper to provide this comparison for the majority of factors which affect the light from the star and distort the diffraction pattern.

DISCUSSION

Suppose the abscissa scale of an ideal theoretical diffraction pattern M_1 is expanded and the resulting pattern is called M_2. If the positions of the fringes of M_2, measured from the 1/4-intensity point, are compared with the positions of the fringes of M_1, it will be seen that the maximum intensity at some fringe N of M_1 is coincident with the minimum intensity of the corresponding fringe of M_2. The number N at which this coincidence first occurs will be adopted as a number which represents the degree of distortion casued by the interference of one diffraction pattern with another. Note that a larger N means less distortion.

The Cornu spiral is a geometrical representation of Fresnel's integrals. The positions of the maxima and minima of the diffraction pattern can be found from the Cornu spiral curve and are defined by the following tangent angles to the curve:

$$3/4\pi+2N\pi,\quad \text{maxima},$$
$$7/4\pi+2N\pi,\quad \text{minima},\quad N=0,1,2,\ldots \tag{1}$$

In general, the tangent angle θ to the Cornu spiral is given by

$$\theta=(\pi/2)X^2, \tag{2}$$

where X is the arc distance along the Cornu spiral. X (dimensionless Fresnel number) can be scaled to the corresponding distance on the lunar limb perpendicular to the line of sight by the relation

$$x=\sqrt{\frac{\lambda D}{2}}X, \tag{3}$$

where λ is the wavelength of the light and D is the distance to the lunar limb. In what follows we shall assume $\lambda=5000$ Å and $D=4\times10^{10}$ cm. The maximum and minimum positions are

$$\text{maxima: } X_{\max}(N)=\sqrt{3/2+4N},$$
$$\text{minima: } X_{\min}(N)=\sqrt{7/2+4N},\quad N=0,1,2,\ldots \tag{4}$$

The factors causing distortion of the diffraction pattern will be discussed in the following paragraphs.

A. Observing Equipment

1. Telescope Diameter

As the diffraction pattern sweeps past the telescope aperture the entire telescope primary simultaneously receives different parts of the pattern and a "smoothing" effect results. The approximate magnitude of the distortion can be found by displacing two patterns the diameter of the telescope and then by determining N, the degree of distortion defined above. Figure 1 shows a curve representing the degree of distortion N as a function of the telescope diameter T according to the relation

$$E=\{X_{\min}(N)-X_{\max}(N)\}\sqrt{\frac{D\lambda}{2}}. \tag{5}$$

2. Equipment Bandwidth

The telescope optics and the detection system with its photomultiplier have associated with them an optical bandpass $\Delta\lambda$. Figure 2 shows the distortion caused by the bandpass and is computed from

$$X_{\max}(N)\sqrt{\frac{D(\lambda+\Delta\lambda)}{2}}=X_{\min}(N)\sqrt{\frac{D\lambda}{2}}, \tag{6}$$

$$N=\lambda/2\Delta\lambda-3/8. \tag{7}$$

The amplifiers and other electronics also have a particular signal bandpass associated with them not to mention their added noise. Attention should be paid to ensure that the bandpass is sufficiently wide to allow all the diffraction signal to pass.

B. Light Path to Observer

As the starlight travels through the Earth's atmosphere the wavefront becomes distorted. The observable effects of this distortion are (a) scintillation and (b) image motion or blurring. The scintillation is caused by diffraction effects through the thickness of the atmosphere whereas image motion and blurring is caused by refraction effects.

(1) Scintillation interferes directly with occultation observations since its power spectrum is similar to the power spectrum of the occultation signal. A qualitative degree of distortion can be made by considering it to be the factor by which the excursion of the first fringe of the occultation pattern is greater than the scintillation amplitude.

(2) Image motion or blurring in large telescopes distorts the diffraction pattern significantly if the diaphram size is not sufficiently large to contain the excursions or the size of the image. The deviation of star light which is caused by atmospheric irregularities amounts to a small fraction of a meter and Fig. 1 shows that the diffraction pattern is not affected.

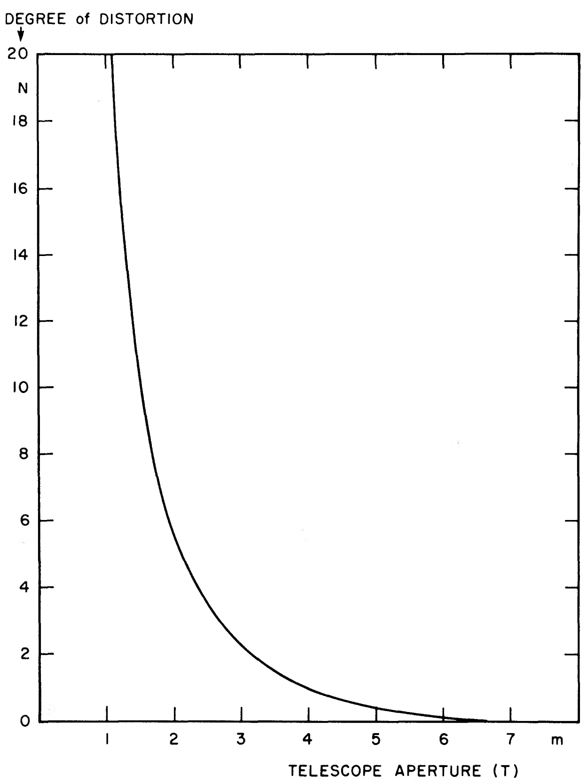

FIG. 1. Degree of distortion N as a function of the telescope diameter T.

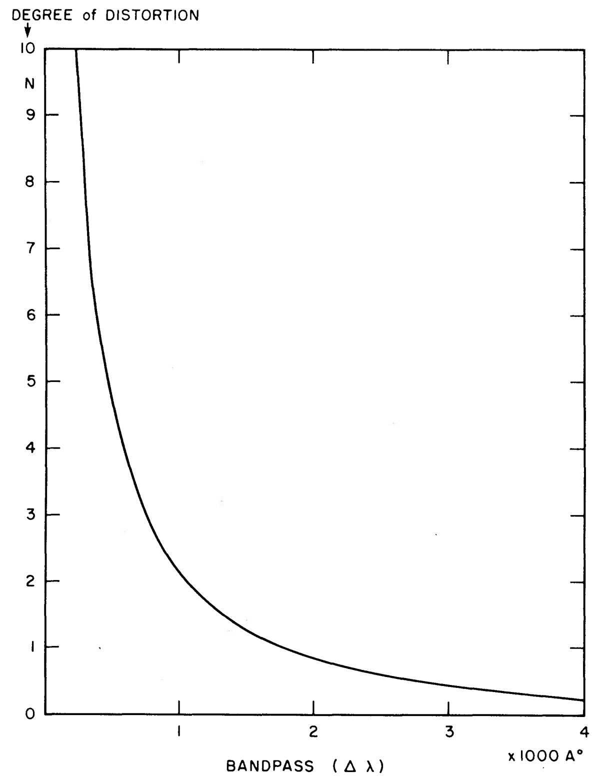

FIG. 2. Degree of distortion N as a function of the light bandwidth $\Delta\lambda$.

C. Nature of the Lunar Limb

1. Perpendicular to the Line of Sight

Diffraction and interference effects can only occur with coherent or partially coherent light. The degree of coherence of the light received from an extended uniformly illuminated source of angular radius ϕ steadily decreases to complete incoherence at a point distant $0.61\ \lambda/\phi$ from the line of sight; λ is the mean wavelength of the light. Evidently, for a very small source the distance on the lunar limb affecting the diffraction pattern is substantial and the number of fringes is large. Because of bandwidth effects only the first few fringes are visible. From Fig. 2 it can be seen that for a bandwidth of 500 Å the degree of distortion is $N\sim5$. The fifth fringe for a point source is at $X\sim4$ and from Eq. 3 the corresponding distance on the lunar limb normal to the line of sight is about 40 m. Irregularities beyond this distance do not distort the occultation pattern significantly assuming that the bandpass is 500 Å.

2. In the Line of Sight

Murdin (1971) has calculated the effect on occultation patterns of lunar surface structure in the line of sight. An approximate and more simple way of estimating the effect can be realized from an equation similar to Eqs. 6 and 7:

$$X_{\max}(N)\sqrt{\frac{(D+\Delta D)\lambda}{2}}=X_{\min}(N)\sqrt{\frac{D\lambda}{2}}, \quad (8)$$

$$N=\frac{D}{2\Delta D}-\frac{3}{8}\sim\frac{D}{2\Delta D}. \quad (9)$$

Here we have related the degree of distortion N and a change in D, the distance to the lunar limb. For $\Delta D=10$ km, $N=2\times10^4$ and no distortion can be detected.

D. Nature of Light Emitted from Star

We have discussed the effects of optical bandwidth above. The resultant bandwidth observable at the detector depends on the spectral energy distribution of the star, the optical bandpass of the interstellar medium and atmosphere, and the telescope optics.

If the source has a finite size of angular radius ϕ, this corresponds to a coherence length on the lunar limb of $1.22\,\bar{\lambda}/\phi$. For a source of angular radius 0.001 arcsec this distance is approximately 50 m which corresponds to $N\sim9$. In order to detect this angular radius it is necessary that the bandpass and the telescope diameter (if greater than 1 m) be taken into consideration in the reduction procedure.

The total contribution to the diffraction pattern from the disk of the star is usually computed from a strip model such as that described by Morbey and Hutchings (1971). A convolution of the diffraction intensity with the brightness distribution of the source is made and the diffracted light intensity is found by integrating over the infinite half plane bordering the lunar limb. This convolution assumes that the light passing perpendicularly through the half plane is coherent.

An alternate method of computing the diffraction pattern from an extended source is to consider the degree of coherence about the line of sight to the source and weight the contribution to the diffraction pattern accordingly in the integration over the half plane. The area of the half plane contributing to the diffraction is defined by

$$F(v)=\frac{2J(v)}{v}e^{i\psi} \quad (10)$$

and the lunar limb. Equation 10 is given by Born and Wolf (1970).

The assumption is that only the coherent light contributes to the diffraction and the magnitude of the contribution at a point on the plane is determined by the degree of coherence at that point. If a form of F in Eq. 10 can be extracted from the occultation data the in-intensity distribution can be determined since it is defined by the inverse Fourier transform of the coherence function. Theoretical work on this latter technique is continuing.

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