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Astrometric determination of the gravitational redshift of van Maanen 2 (EG 5)

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The astrometric parameters of van Maanen 2 and twenty-three other stars in its vicinity are determined. The true radial velocity of the white dwarf, +6±15 km sec^{-1}, is determined from its acceleration in position. A comparison with the value measured by Greenstein (1972) indicates a gravitational reshift of +33±16 km sec^{-1}. Adopting the photometrically determined radius of 0.013 $R_\odot$, we find a mass of 0.7±0.3 $M_\odot$. These values are consistent with the mass-radius models of Chandrasekhar and of Hamada and Salpeter. We note a rather large scatter in the ξ coordinate residuals of van Maanen 2; these may indicate the presence of a second body.

INTRODUCTION

OORT (1932) noted that, because of the large proper motion and parallax of van Maanen 2 ($\mu = 2''.975$ yr^{-1} and $\pi = 0''.230$), a determination of the acceleration in position, $\Delta\mu$, would eventually provide a direct astrometric determination of the radial velocity. The position of a star whose image has been measured on a series of plates may be reduced to yield its standard coordinates:

$$(\xi_i, \eta_i);\ i = 1, \text{ number of exposures.}$$

From these positions we may determine the star's standard coordinates (ξ_o, η_o) at some central epoch t_o, the ξ and η components of its proper motion μ_ξ and μ_η, the rate of change of these components $\Delta\mu_\xi$ and $\Delta\mu_\eta$, and the parallax π. The equations relating these quantities are

$$\xi_i = \xi_o + \mu_\xi t + P_\xi \pi + \frac{\Delta\mu_\xi}{2} t^2$$

and (1)

$$\eta_i = \eta_o + \mu_\eta t + P_\eta \pi + \frac{\Delta\mu_\eta}{2} t^2,$$

where t is the plate epoch minus t_o and P_ξ and P_η are the parallax factors in the standard coordinates. The parameters of each of these equations yield estimates of the stars true radial velocity:

$$V_\xi = -4.88 \times 10^5 \Delta\mu_\xi / \mu_\xi \pi$$

and (2)

$$V_\eta = -4.88 \times 10^5 \Delta\mu_\eta / \mu_\eta \pi.$$

The weighted mean of these two quantities is then taken as definitive. Note that, if observations are continued in a given system at a constant rate, the weight of the quadratic coefficient increases with the fifth power of time (van de Kamp 1971). Thus, accurate results depend upon a long history of observations with a constant instrument. Standard coordinates derived via plate constant methods using a knowledge of the proper motions of the reference stars, for example, the central overlap technique developed by Gatewood and Eichhorn (1973), may be substituted directly into the above equations and do not require the corrections (van de Kamp 1935) due to dependence changes and the relative proper motions of the reference stars.

A direct measure of the radial velocity would allow a determination of the contribution of gravitational redshift to the radial velocity found spectroscopically. The redshift (K in km sec^{-1}) is related to the mass (M in solar units) and the radius (R in solar units) by the equation

$$K = 0.635 M R^{-1}. \quad (3)$$

Thus, if either the mass or the radius may be determined independently, the equation may be solved for the remaining unknown. Or, if we further adopt the theoretical mass-radius relationship (Chandrasekhar 1939; Hamada and Salpeter 1961), and assume a composition for the white dwarf, we may show that the redshift uniquely determines the mass, radius, and surface gravity of the star.

An obvious test of the mass-radius relation is to compare the quantities derived from independent data with those implied by the theory. Unfortunately, there are very few opportunities for such a comparison and the validity of the theory is not yet certain. The determination of an accurate parallax and astrometric radial velocity for van Maanen 2 would, in combination with other data, allow such a comparison.

Van Maanen 2 is of spectral type $WD\ G$ with $B-V$ and $U-B$ color indices of +0.56 and +0.02, respectively, and an apparent visual magnitude of 12.37 (Blanco *et al.* 1970). The spectrum is unique in showing only the strongly broadened ultraviolet lines of iron and the H and K lines of Ca II. Greenstein (1972), using 16 plates secured at the prime focus of the 200-in. reflector, finds a radial velocity, based on the positions of the H and K lines, of +39 ±4 km sec^{-1}. Model-atmosphere calculations by Weidemann (1961) indicate that the effective temperature is quite similar to that of the Sun.

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TABLE I. Relative positions, proper motions, and absolute parallaxes of 24 stars in the region of van Maanen 2 on the system of the FK4.

#	m_{pg}	πabs	σ_π	R.A.$_{1950}$	$\sigma_{R.A.}$	μ	σ_μ	Dec.	$\sigma_{Dec.}$	μ'	σ_μ'
121	10.4	...	...	$0^h43^m46.9492$	15	$-0^s.00230$	7	5°03′23″.063	31	−0″.0454	14
122	8.8	...	...	0 44 09.8935	25	−0.00081	11	5 04 21.854	54	+0.0051	25
123	10.4	...	...	0 44 15.6035	24	+0.00199	10	4 40 45.869	36	−0.0010	17
124	11.4	+0″.005	4	0 44 37.4760	2	+0.00111	1	5 09 13.668	2	−0.0042	1
125	13.0	−0.001	6	0 44 50.0653	2	+0.00047	1	4 55 17.748	3	−0.0032	2
126	11.7	−0.002	4	0 45 05.1178	2	+0.00224	1	5 07 45.482	2	+0.0174	1
127	14.0	...	...	0 45 19.5320	19	+0.00092	9	5 20 45.333	42	−0.0173	20
128	9.7	...	...	0 45 27.5176	22	+0.00044	11	4 49 51.077	39	+0.0060	18
129	11.8	+0.011	4	0 45 28.5185	2	+0.00079	1	4 45 28.552	2	+0.0045	1
130	13.0	−0.002	6	0 45 38.8006	3	+0.00033	2	4 59 09.971	3	+0.0067	2
131[a]	6.6	+0.141	5	0 45 45.4553	2	+0.05055	1	5 01 26.054	4	−1.1403	2
132	11.9	+0.006	4	0 46 01.4976	2	+0.00178	1	5 14 57.528	3	−0.0194	1
133	13.6	+0.010	6	0 46 05.2923	3	−0.00021	2	5 14 55.293	4	−0.0235	2
134	12.3	+0.001	4	0 46 27.2593	2	+0.00336	2	4 47 43.617	2	+0.0083	1
135[a]	12.9	+0.230	7	0 46 30.7352	3	+0.08188	2	5 09 14.101	3	−2.7118	2
136	13.4	+0.010	8	0 46 31.4955	3	−0.00077	2	5 01 12.569	4	−0.0234	2
137	11.1	...	...	0 46 31.8642	17	+0.00100	8	4 51 43.444	31	−0.0295	14
138	9.9	...	...	0 46 35.4099	26	+0.00109	14	5 14 14.061	42	+0.0163	20
139	13.6	−0.004	7	0 46 55.7401	4	+0.00087	4	4 51 30.630	5	−0.0256	5
140	14.0	−0.002	9	0 46 56.7694	3	−0.00034	2	5 07 43.002	4	−0.0047	3
141	10.0	...	...	0 46 58.4936	21	+0.00102	10	4 39 19.989	29	+0.0299	14
142	13.1	+0.005	6	0 47 06.2174	3	−0.00231	3	5 23 57.651	6	−0.0240	5
143	13.5	−0.009	10	0 47 24.0089	6	−0.00004	5	5 16 42.499	7	−0.0013	6
144	11.5	...	...	0 47 36.2090	19	−0.00153	11	5 15 04.673	42	−0.0151	20

[a] 131 is Ambron 194; 135 is van Maanen 2. The light of 131 was reduced via rotating sector to that of a star with a photographic magnitude of 12.2.

Van de Kamp (1971), analyzing 511 exposures on 281 plates obtained with the 24-in. Sproul refractor during the years 1937–1970, reported an astrometric radial velocity of +160 km sec^{-1} with a standard error of 67 km sec^{-1}. However, Greenstein (1972) pointed out that this could not be reconciled with the spectroscopically measured value. We find upon inspection of Table 2 of van de Kamp's report that V_η has been computed with the wrong sign. V_ξ and V_η thus differ by 370 km sec^{-1} and no realistic value of the radial velocity is evident from that study.

In Sec. I of this paper we detail the data available for our determination of the astrometric parameters of van Maanen 2 and 23 field stars. In Sec. II we derive the mass, radius, and surface gravity of the white dwarf by several different techniques, and in Sec. III we give our conclusions.

I. DATA AND REDUCTIONS

Two plate series obtained with the Allegheny Observatory 30-in. photographic refractor were available for the present study. The first (centered on van Maanen 2) was comprised of 39 measurable exposures on 19 plates obtained during the years 1951–1973. The second (centered on Ambron 194, 60-mm or 15 arcmin from the white dwarf) consisted of 103 measurable exposures on 35 plates obtained during the years 1917–1922 and 1949–1973.

TABLE II. Acceleration and radial velocity of van Maanen 2.

$\Delta\mu_\xi$ ″/yr^2	$\Delta\mu_\eta$ ″/yr^2	V_ξ km/sec	V_η km/sec
+0.0000259	+0.0000146	−44	+11
±0.0000300	±0.0000212	±52	±16

The plates were measured on the Strand Automatic Measuring Machine (SAMM) at the United States Naval Observatory and reduced as described by Gatewood and Eichhorn (1973). The positions of 11 of the 24 stars in Table I are also listed in the AGK3, and of these 11, the positions of six are listed in the SAOC and one is a FK4 star. Both the AGK3 and the SAOC are ostensibly on the FK4 system. Measurements of the positions of all these stars plus those of other AGK3 and SAOC stars found on Plate 6 (epoch 1896.9) of the Toulouse zone are listed in the *Astrographic Catalogue.*

To form a reference system for the iterative solution we combined the positions and proper motions listed in the two reference catalogs and formed 1896.9 positions (with a 1950.0 orientation) for those stars measured on the Toulouse plate. A least squares solution provided plate constants with which we were able to determine the spherical coordinates of the stars measured on this plate. These were then combined with the data from the AGK3 and SAOC to form an improved catalog. Next, the images of all of the stars in Table I on one early, one midepoch, and one recent plate were measured with the Mann two-screw comparator of the Allegheny Observatory. Finally, these measures were reduced to the improved catalog. The positions and proper motions derived from these reference plates then became the reference system in the first iteration of the central overlap method.

Table I lists the astrometric properties of the stars involved in this investigation referred to the orien-

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TABLE III. Physical characteristics of van Maanen 2.

Method	Mass $M_\odot$	Radius $R_\odot$	Surface gravity 10^8 cm sec^{-2}
Photometric	0.69±0.33	0.0132±0.0040[a]	1.08
Chandrasekhar	0.74±0.26	0.0142±0.0017	1.01±0.61
Hamada–Salpeter Mg^{24}	0.62±0.20	0.0118±0.0021	1.20±0.80
Hamada–Salpeter Fe^{56}	0.53±0.17	0.0102±0.0018	1.39±0.93

[a] Radius based on an adopted temperaure of 5800±900°K.

tation and epoch of 1950.0. The star numbers are a continuing sequence begun in the paper cited above. The apparent photographic magnitudes were estimated with the new Cuffey iris photometer. Their internal standard error is 0.1 mag. Stars listed without parallaxes are measured on the reference plates only. The values of the standard errors are in units of the last significant figure of the quantity to which they refer and are strictly internal.

As a preliminary to a rigorous solution, the two series were reduced independently. The astrometric parameters determined for the stars common to the two series (130, 132, 135, 136, and 140) were found to be free of systematic differences and the individual values determined from one series were in good agreement with those determined from the other series. Thus, the two series were treated as a unit in all further computations. In doing this we were able to enforce what is referred to as "overlap condition" (Eichhorn, Googe, and Gatewood 1967) and thus bring about a single overlap solution of these two parallax series.

Star 131 is Ambron 194. Other parallax determinations for this star are: McCormick Observatory +0".155±0".015 s.e. and Yale Observatory +0".136 ±0".018 (Jenkins 1952, 1963). The weighted mean of these is +0".146±0".012, in excellent agreement with the value listed in Table I. The weighted mean parallax of all three determinations is +0".142±0".005.

Parallaxes of van Maanen 2 observed at other observatories are: McCormick, +0".247±0".018; Mt. Wilson, +0".244±0".018; Yale +0".236±0".021 (Jenkins 1952, 1963); and Sproul, +0".226±0".006 (van de Kamp 1971). The weighted mean of these is identical to the value given in Table I, +0".230; the standard error of the mean of all five determinations is 0".004. From Eqs. (1) and (2) we find the astrometric radial velocity of van Maanen 2, +6±15 km sec^{-1} (see Table II).

II. MASS AND RADIUS

A comparison of the astrometric and spectroscopic radial velocities yields the redshift $K=+33$ km sec^{-1} with a standard error of 16 km sec^{-1}.

Since Weidemann (1961) suggests that the effective temperature of van Maanen 2 is similar to that of the Sun, we may, assuming rather similar bolometric corrections, compare the luminosities of these stars to determine their relative radii. We find the absolute visual magnitude of the white dwarf to be 14.18±0.08. If we assume that the absolute visual magnitude of the Sun is 4.8, we find $R=0.0132R_\odot$. Unfortunately, we can not regard this value with confidence since Wiedemann (1971) indicates that his original estimate of the effective temperature may have been 900° too low, while Oke and Shipman (1971) indicate that it was at least 700° too high. However, if for the moment we adopt the above radius, which is at least near the mean of that indicated by the various photometric studies, and solve Eq. (3) for mass, we find $M=0.69\pm0.33M_\odot$, where the standard error reflects the uncertainty in K.

Another and perhaps more direct approach to the problem is to adopt the mass-radius relation proposed by Chandrasekhar (1939). His approximate solution for completely degenerate configurations is:

$$R=0.0401(\mu_e^{5/3}M^{1/3})^{-1}. \quad (4)$$

If we assume $\mu_e=2.0$, we may, considering Eq. (3) and the equation of surface gravity in cm sec^{-2}, $g=4.31\times10^4\ K/R$, show that $R=0.0336\ K^{-1/4}$, $M=0.0530\ K^{3/4}$, and $g=1.27\times10^6\ K^{5/4}$. Thus, we find $R=0.0142\pm0.0017R_\odot$, $M=0.74\pm0.26M_\odot$, and $g=(1.01\pm0.61)\times10^8$ cm sec^{-2}.

The Chandrasekhar models have been refined by Hamada and Salpeter (1961). Their models are com-

TABLE IV. Yearly mean residuals of van Maanen's Star.

Mean epoch	# obs	ΔX	ΔY
1917.975	7	−0".002	+0".023
1919.977	2	−0.026	−0.080
1920.971	4	−0.022	−0.032
1921.997	20	+0.013	+0.010
1922.619	6	+0.004	−0.005
1949.797	2	+0.045	−0.022
1951.730	18	−0.020	+0.015
1952.697	3	−0.063	−0.017
1953.838	6	+0.015	+0.001
1954.671	4	−0.030	−0.015
1955.780	14	+0.013	−0.009
1957.874	3	−0.012	−0.023
1958.843	3	+0.029	−0.033
1960.965	4	−0.014	+0.002
1961.951	2	−0.044	+0.003
1963.777	8	−0.002	+0.013
1965.748	4	+0.029	−0.053
1969.611	4	+0.001	+0.029
1972.776	27	+0.005	+0.005

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position-dependent and may therefore provide, in the presence of accurate redshift and photometric data, a vehicle for the study of the chemical composition of white dwarfs. Their Mg^{24} and Fe^{56} models can be represented adequately over the range of masses found for white dwarf members of double star systems, $0.4M_\odot \leq M \leq 1.0M_\odot$, by equations of the form

$$R = H - 0.0113M,$$

where H is 0.0188 for a Mg^{24} composition and 0.0162 for a Fe^{56} composition. From this approximation we find: $R = H(1.0+0.0178K)^{-1}$, $M = 1.575HK(1.0+0.0178K)^{-1}$, and $g = (4.31\times10^4K + 7.61\times10^2K^2)H^{-1}$. Hence, a Mg^{24} model yields $R = 0.0118\pm0.0021R_\odot$, $M = 0.62\pm0.20M_\odot$, and $g = (1.20\pm0.80)\times10^8$ cm sec^{-2}, while Fe^{56} model yields $R = 0.0102\pm0.0018R_\odot$, $M = 0.53\pm0.17M_\odot$, and $g = (1.39\pm0.93)\times10^8$ cm sec^{-2}.

III. CONCLUSIONS

The Chandrasekhar and the Hamada–Salpeter models give estimates of the radius, mass, and surface gravity of van Maanen 2 that are in excellent agreement with those values derived with the help of photometric data. This independence of technique lends confidence to these results. We surmise that the values $M = 0.7\pm0.3M_\odot$, $R = 0.013\pm0.002R_\odot$, and $g = (1.1\pm0.6)\times10^8$ cm sec^{-2} are in good agreement with available data (see Table III).

Van de Kamp (1971) reported some evidence of a perturbation in ξ. We too note that the standard error of a single observation of van Maanen 2 was 1.4 times larger in the ξ coordinate that in the η coordinate (the coordinate most involved in the determination of the astrometric radial velocity). In this series of papers, as well as for other stars in this study, the ratio of the standard error of a single observation in ξ to that in η has generally been 1.0 (Gatewood and Eichhorn 1973; Breakiron and Gatewood 1974). The yearly mean residuals listed in Table IV were plotted against a number of different periods without much success; if orbital motion exists we have failed to find the period.

The form of Eq. (2) and the relative accuracy of the astrometrically and spectroscopically measured radial velocities indicates that continued astrometric observation of van Maanen 2 will yield rapid improvement in our understanding of the characteristics of this star.

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REFERENCES

Blanco, V. M., Demers, S., Douglass, G. G. and Fitzgerald, M. P. (1970). Publ. U. S. Naval Obs. **21**.
Breakiron, L. A., and Gatewood, G. (1974). Publ. Astron. Soc. Pac. **86**.
Chandrasekhar, S. (1939). *An Introduction to the Study of Stellar Structure*, edited by (U. Chicago Press, Chicago), p. 431.
Eichhorn, H., Googe, W. D., and Gatewood, G. (1967). Astron. J. **72**, 626.
Gatewood, G., and Eichhorn, H. (1973). Astron. J. **78**, 769.
Greenstein, J. L. (1972). Astrophys. J. **173**, 377.
Greenstein, J. L., and Trimble, V. L. (1967). Astrophys. J. **149**, 293.
Hamada, T., and Salpeter, E. (1961). Astrophys. J. **134**, 683.
Jenkins, L. F. (1952). *The General Catalogue of Trigonometric Stellar Parallaxes* (Yale U. Obs., New Haven).
Jenkins, L. F. (1963). *Supplement to the General Catalogue of Trigonometric Stellar Parallaxes* (Yale U. Obs., New Haven).
Oke, J. B., and Shipman, H. L. (1971). *I. A. U. Symposium*, No. 42, *White Dwarfs*, edited by W. J. Luyten (D. Reidel, Dordrecht), p. 67.
Oort, J. H. (1932). Bull. Astron. Inst. Neth. **238**, 287.
van de Kamp, P. (1935), Astron. J. **44**, 73.
van de Kamp, P. (1971). I. A. U. Symposium, No. 42, *White Dwarfs*, edited by W. J. Luyten (D. Reidel, Dordrecht), p. 32.
Weidemann, V. (1960). Astrophys. J. **131**, 638.
Weidemann, V. (1971). *I. A. U. Symposium*, No. 42, *White Dwarfs*, edited by W. J. Luyten (D. Reidel, Dordrecht), p. 81.

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