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Comets and nongravitational forces. VI. Periodic comet Encke 1786–1971

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The orbit of P/Encke during the whole of the comet's observed history, 1786–1971, is investigated on a uniform basis. Full perturbations by all nine planets are included, and the nongravitational secular acceleration is allowed for by means of the procedure discussed in Paper V of this series. It is confirmed that the secular acceleration (or, equivalently, the transverse nongravitational parameter A_2) has been decreasing since the early part of the nineteenth century, and it is shown that the decrease seems generally to have been remarkably smooth. An important new result is that the secular acceleration evidently attained a maximum around the year 1820, the curve being strikingly symmetrical on each side of this maximum: the physical implication of this is briefly considered. Dates of perihelion passage (on two assumptions concerning the secular acceleration) are calculated back to the year 1570, and predicted elements are provided for the returns 1974–1984.

INTRODUCTION

PAPER III of this series (Marsden 1970) included an investigation of the nongravitational forces acting on periodic comet Encke during the 12 revolutions 1927–1967. Although extrapolations were made back to the time of the comet's first observed perihelion passage in 1786, no actual observations before 1927 were considered, comparison only having been made with the previously published times of perihelion passage.

The nongravitational character of the orbit of P/Encke before 1927 has been investigated by a number of authors [for details see the references in the *Catalogue of Cometary Orbits* (Marsden 1972a)] with the help of a method developed in its basic form by Encke (1831). The method consists of adding a quadratic term to the expression for the mean anomaly and of solving, by least squares, for this secular acceleration coefficient, along with the differential increments in the usual six orbital elements. Similar nongravitational terms in the eccentricity and/or other elements were sometimes included.

Although it was shown that for the period 1927–1967 the older method gave results similar to those obtained in Paper III, and that there is a dynamically derivable relation between the parameters of both methods (Sekanina 1969), it is clearly of value to extend the method of Paper III to the pre-1927 observations. The need for this is emphasized by the following circumstances:

(1) Virtually all the old calculations were done manually (with logarithms or desk calculators); the possibility of error was thus practically unavoidable. Errors have indeed been found in some of these studies.

(2) In the differential-correction procedure, normal places were invariably used instead of individual observations, and these are known to have an adverse effect on the precision of the nongravitational parameters.

(3) Planetary perturbations were not handled properly. Not all the planets were considered, differential perturbations were completely ignored, and incorrect values were often used for the planetary masses. Specifically, a strong correlation is known to exist between the value of the secular acceleration and the value used for the mass of Mercury (see, e.g., Sekanina 1968).

(4) Various assumptions were made in the past for the variation of the nongravitational forces with heliocentric distance (and sometimes with the comet's velocity, too).

(5) Diverse conclusions were drawn concerning the long-term behavior of the forces. Backlund (1910) claimed that there were several sudden decreases in the secular acceleration, thus implying a step-like character for the long-term changes. On the other hand, in the calculations presented in our Paper III, satisfactory results were obtained by assuming that the nongravitational forces varied in a continuous manner, at least since 1927.

(6) Large systematic trends remained in the residuals within individual apparitions (particularly just before and just after a perihelion passage).

(7) The character of the nongravitational acceleration of the comet before 1819 is virtually unknown, because Encke's (1823) investigation of the early apparitions was very rough and never subsequently refined.

I. INITIAL PROCESSING OF THE OBSERVATIONS

The collection of the pre-1927 observations of P/Encke has been an immense task, in spite of the fact that it could be simplified somewhat by examination of the residuals given by previous investigators. Observations already shown to be clearly inconsistent

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with neighboring observations were ignored from the start. Further observations were removed in the course of preliminary differential corrections, apparition by apparition. We purposely did not use all the available observations, for we preferred to have them distributed along the orbit as uniformly as possible. Particular care was taken in handling the observations at unfavorable apparitions (when the comet was observable only after perihelion and usually in the southern hemisphere).

Random checks were made on the positions of the comparison stars that had been utilized. The differences between the original and the modern star positions [the latter being mostly from the SAO star catalog (Staff 1966) corrected of course for proper motion] were found to be very small as far back as the mid-nineteenth century, and only rarely were there differences of a much as five arcsec. Before about 1830, the star positions were worse, and checking was more imperative. Unfortunately, a number of the observations from these times were published without specification of the comparison stars.

The observations of the comet at its apparitions in 1786, 1795, and 1805 were very bad. The positions of the comparison stars were often off by a significant fraction of one arcmin, and displacements of several arcmin were not uncommon. Furthermore, there were large uncertainties in the comet-star offsets. This is understandable, not only because the mechanical quality of the micrometric equipment used during those times was poor, but also because the comet's position was generally referred to naked-eye stars, and the offsets were often as large as 10°.

It is frequently stated that only two observations of P/Encke were made at its 1786 apparition. Inspection of the observational records indicates, however, that there was one observation by Méchain on 17 January and there were four independent observations on 19 January, one by Méchain and three by Messier.

The solar coordinates necessary for calculating the residuals were taken from the U.S. Naval Observatory tape that corresponds to the printed tables by Herget (1953) for the period 1800–2000. In order to discuss the 1786 and 1795 observations, we extended the tape backward by numerical integration of the motion of the Earth–Moon barycenter, with allowance being made analytically for the departure of the center of the Earth from the barycenter.

II. COMPUTATIONS AND RESULTS

The main computations in the present study follow the pattern thoroughly discussed in Paper V (Marsden *et al.* 1973). The differential-correction process is iterated until convergence is attained. Perturbations by all nine major planets are fully included. The variation of the nongravitational acceleration with heliocentric distance r is assumed to take the form

$$g(r)=\alpha\left(\frac{r}{r_0}\right)^{-m}\left[1+\left(\frac{r}{r_0}\right)^{n}\right]^{-k}, \tag{1}$$

where r_0 is a distance beyond which the nongravitational acceleration drops very rapidly, $m=2.15$, $n=5.093$, $nk=23.5$, and α is a normalizing constant. We have generally used $r_0=2.808$ a.u., so that $\alpha=0.1113$. The radial and transverse components of the nongravitational acceleration are then given by

$$R=A_1 g(r),$$
$$\text{and} \tag{2}$$
$$S=A_2 g(r),$$

and the parameters A_1 and A_2 are determined from time spans long enough that the parameters become significant and short enough that they can be regarded as constant. In practice, this generally means considering five consecutive returns of P/Encke, which correspond to about 13 yr. Since the calculations in Paper III were based on a different nongravitational law, we have extended the new calculations to the observations of 1927 and later.

The 13-yr fits were usually made so that the first and last apparitions overlapped with neighboring runs. In the course of this process, it was evident that several more observations should be discarded, although at about a half-dozen apparitions we were still left with 30 or more observations. The maximum number of observations retained at an individual apparition was 45, and this was as long ago as 1852. On the other hand, and as is well known, there were no observations at the comet's returns in 1789, 1792, 1799, 1802, 1809, 1812, 1815, and 1944. The 1944 return came in the middle of the 1937–1951 solution, so the absence of observations had a minimal effect on the nongravitational parameters derived from this span. In the case of the very early apparitions, the long gaps suggested that it was wise to link only three apparitions; so the first three runs were made for 1786–1795–1805, 1795–1805–1819, and 1805–1819–1822. The series of overlapping 13-yr runs then began with 1819–1832.

Particular difficulties were experienced with the observations in 1898 and 1868. In 1898 there were only two observations, both by Tebbutt at Windsor (New South Wales), and they cannot be made to fit with the other observations of the 1885–1898 run. Accordingly, the run was limited to 1885–1895 (which left systematic residuals of almost one arcmin in the 1898 observations). The 1898–1911 run was similarly curtailed to 1901–1911, although the representation of the 1898 observations was better in this case (with residuals no larger than about 20 arcsec). In an attempt to minimize the effect of ignoring the 1898 observations, an additional fit was made by utilizing the 1891, 1895, 1901, and 1905

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TABLE I. Nongravitational parameters for P/Encke.

Time span	Number of observations	Mean residual	A_1	A_2
1786–1805	20	19".5	−2.17 ± 0.90	−0.03608 ± 0.00030
1795–1819	28	16.6	−0.29 ± 0.11	−0.03847 ± 0.00014
1805–1822	30	8.5	+0.69 ± 0.21	−0.03962 ± 0.00011
1819–1832	87	7.3	+1.14 ± 0.06	−0.03996 ± 0.00006
1832–1845	48	6.5	−0.63 ± 0.10	−0.03759 ± 0.00016
1845–1858	113	2.64	+0.38 ± 0.07	−0.03503 ± 0.00006
1858–1871	64	2.44	−0.78 ± 0.03	−0.03170 ± 0.00004
1871–1885	87	3.14	+0.07 ± 0.03	−0.02657 ± 0.00005
1885–1895	86	2.61	−0.14 ± 0.04	−0.02263 ± 0.00005
1891–1905	86	3.91	−0.18 ± 0.04	−0.02109 ± 0.00004
1901–1911	41	2.70	+0.77 ± 0.21	−0.01683 ± 0.00008
1911–1924	55	1.98	−0.09 ± 0.04	−0.01461 ± 0.00003
1924–1937	87	2.62	−0.09 ± 0.02	−0.01144 ± 0.00004
1937–1951	48	2.47	−0.05 ± 0.01	−0.00822 ± 0.00011
1951–1964	70	1.55	+0.03 ± 0.01	−0.00589 ± 0.00007
1957–1971	85	2.41	+0.03 ± 0.01	−0.00474 ± 0.00004

apparitions; however, this was not particularly satisfactory, for there were systematic trends of five to ten arcsec in the residuals—and the corresponding 1898 residuals were greater than one arcmin. It is conceivable that P/Encke experienced around this time a small "jump" of the type discussed in Paper IV (Marsden and Sekanina 1971). The 1868 observations were more numerous but seemed to be systematically off by 10–20 arcsec; they were omitted from the final fit over 1858–1871.

Barnard (1914) described a possible observation of P/Encke obtained in September 1913, only $4\frac{1}{2}$ months after aphelion. Since the position measured from the $3\frac{1}{2}$-hr exposure by Pease with the Mt. Wilson 152-cm reflector seemed to differ by 1.5 arcmin from the expected position of the comet, there was some question as to its validity. We did not include this observation in our differential corrections, although after completing the 1911–1924 run, we found that it gave (O-C) residuals of only +2".0 in right ascension, and +4".6 in declination. It thus seems quite probable that the observation does, in fact, refer to P/Encke. Until recently, the most distant confirmed observations of P/Encke were made some ten months from aphelion (i.e., midway in time between aphelion and perihelion), although in 1972 the comet was photographed at almost exactly an aphelic opposition (Roemer 1972) and in very close agreement with the 1957–1971 orbit; the magnitude was about 20.5 and the comet's appearance stellar.

Our results for the nongravitational parameters A_1 and A_2 are summarized in Table I. The formal mean errors suggest that, in the case of P/Encke, little significance should be attached to the radial component A_1. Indeed, if A_1 is set equal to zero, there is scarcely any deterioration in the representation of the observations, and the values of A_2 are changed by no more than about ± 0.0002. It is perhaps of interest to note that the mean residuals of the recent fits are much the same as those of more than a century ago. As could be expected, the residuals for the fits before 1845, and especially before 1819, are significantly worse, although even then the values of A_2 seem to be established with some reliability.

In Table II, we give osculating orbital elements for epochs near the times of all the perihelion passages of P/Encke (including those at which the comet was not observed) from 1786 to 1971. The figures correspond to the solutions in Table I, and the blank lines indicate transitions from one fit to the next.

III. DISCUSSION

It is permissible to refer the values of A_2 to the middle of the time spans from which they were derived (Sekanina 1969). The plot, shown in Fig. 1, of A_2 as a function of the time, clearly confirms the general decrease (in absolute magnitude) over the last 150 yr and suggests that the decrease has been remarkably smooth. There is perhaps some slight irregularity around the year 1900, and we have already remarked on some difficulty in representing the observations near that time.

The most striking new feature of our results is that before about 1820, $|A_2|$ was *increasing* with time; in

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TABLE II. Orbital elements 1786–1971 (equinox 1950.0).

T (E.T.)	q (A.U.)	e	P (yr)	ω	Ω	i	Epoch
1786 Jan. 31.3600	0.336028	0.848422	3.30	182.2017	336.4689	13.6874	1786 Jan. 18.0
*1789 May 19.2646	0.334958	0.848809	3.30	182.2056	336.4706	13.7055	1789 May 2.0
*1792 Sept. 4.7291	0.335458	0.848601	3.30	182.2623	336.4512	13.6930	1792 Sept. 23.0
1795 Dec. 21.9630	0.335492	0.848591	3.30	182.2579	336.4476	13.6903	1796 Jan. 6.0
*1799 Apr. 11.5297	0.340512	0.846699	3.31	182.3313	336.4273	13.5964	1799 Apr. 20.0
*1802 Aug. 2.3878	0.339544	0.847092	3.31	182.3532	336.4180	13.6113	1802 Aug. 3.0
1805 Nov. 21.9864	0.340645	0.846528	3.31	182.3163	336.4447	13.5874	1805 Nov. 15.0
*1809 Mar. 12.3701	0.334985	0.848686	3.29	182.4266	336.4057	13.6425	1809 Feb. 27.0
*1812 June 26.8396	0.334594	0.848816	3.29	182.4059	336.4130	13.6503	1812 June 11.0
*1815 Oct. 13.1446	0.334417	0.848885	3.29	182.4325	336.3996	13.6508	1815 Sept. 24.0
1819 Jan. 27.7540	0.335087	0.848631	3.29	182.4219	336.3971	13.6368	1819 Feb. 15.0
1822 May 24.4577	0.345886	0.844439	3.32	182.7715	336.2136	13.3549	1822 May 30.0
1825 Sept. 16.7717	0.344765	0.844869	3.31	182.7789	336.2057	13.3732	1825 Sept. 11.0
1829 Jan. 10.2434	0.345447	0.844640	3.32	182.7983	336.1930	13.3590	1828 Dec. 24.0
1832 May 4.4790	0.343404	0.845466	3.31	182.7903	336.2074	13.3849	1832 May 17.0
1835 Aug. 26.8553	0.344362	0.845103	3.31	182.7864	336.2077	13.3696	1835 Aug. 30.0
1838 Dec. 19.5091	0.343962	0.845246	3.31	182.8196	336.1885	13.3728	1838 Dec. 12.0
1842 Apr. 12.5200	0.344933	0.844850	3.31	182.8208	336.1832	13.3551	1842 Mar. 26.0
1845 Aug. 10.1164	0.338078	0.847444	3.30	183.4035	335.8018	13.1411	1845 Aug. 17.0
1848 Nov. 26.5805	0.336956	0.847838	3.30	183.4049	335.7990	13.1580	1848 Nov. 29.0
1852 Mar. 15.2020	0.337565	0.847586	3.30	183.4381	335.7808	13.1430	1852 Mar. 13.0
1855 July 1.5386	0.337202	0.847719	3.30	183.4212	335.7844	13.1467	1855 June 26.0
1858 Oct. 18.8658	0.340769	0.846410	3.30	183.4568	335.7684	13.0832	1858 Oct. 8.0
1862 Feb. 6.7469	0.339950	0.846739	3.30	183.4799	335.7579	13.0944	1862 Jan. 20.0
1865 May 28.4221	0.340970	0.846316	3.30	183.5001	335.7484	13.0748	1865 June 13.0
†1868 Sept. 15.1216	0.333602	0.849163	3.29	183.6556	335.6802	13.1217	1868 Sept. 25.0
1871 Dec. 29.3036	0.332985	0.849412	3.29	183.6425	335.6831	13.1328	1872 Jan. 8.0
1875 Apr. 13.4901	0.332964	0.849373	3.29	183.6580	335.6754	13.1310	1875 Apr. 22.0
1878 July 26.6677	0.333513	0.849144	3.29	183.6532	335.6719	13.1191	1878 Aug. 4.0
1881 Nov. 15.8034	0.343434	0.845347	3.31	183.9111	335.5391	12.8911	1881 Nov. 16.0
1885 Mar. 8.1371	0.342372	0.845756	3.31	183.9143	335.5318	12.9086	1885 Feb. 28.0
1888 June 28.4699	0.343093	0.845460	3.31	183.9386	335.5178	12.8938	1888 June 12.0
1891 Oct. 18.4254	0.340450	0.846476	3.30	183.9537	335.5171	12.9232	1891 Nov. 4.0
1895 Feb. 5.2496	0.341076	0.846242	3.30	183.9516	335.5150	12.9125	1895 Feb. 16.0
†1898 May 27.3690	0.340757	0.846377	3.30	183.9784	335.5022	12.9145	1898 May 31.0
1901 Sept. 15.9651	0.341678	0.846014	3.31	183.9811	335.4935	12.8987	1901 Sept. 13.0
1905 Jan. 12.3806	0.338883	0.847100	3.30	184.6305	335.0542	12.5795	1904 Dec. 26.0
1908 May 1.4098	0.337794	0.847523	3.30	184.6238	335.0550	12.5945	1908 May 19.0
1911 Aug. 19.5341	0.338420	0.847238	3.30	184.6527	335.0343	12.5802	1911 Sept. 1.0
1914 Dec. 5.4301	0.337858	0.847486	3.30	184.6420	335.0435	12.5872	1914 Dec. 14.0
1918 Mar. 24.8441	0.340795	0.846373	3.30	184.6704	335.0346	12.5358	1918 Mar. 28.0
1921 July 13.5895	0.340005	0.846662	3.30	184.6956	335.0202	12.5456	1921 July 10.0
1924 Oct. 31.9355	0.341067	0.846255	3.30	184.7110	335.0051	12.5261	1924 Oct. 22.0
1928 Feb. 19.8372	0.332629	0.849505	3.29	184.9161	334.9030	12.5520	1928 Feb. 4.0
1931 June 3.1131	0.331868	0.849788	3.28	184.8993	334.9104	12.5651	1931 May 19.0
1934 Sept. 15.2835	0.331865	0.849813	3.28	184.9331	334.8939	12.5617	1934 Aug. 31.0
1937 Dec. 27.7540	0.332410	0.849603	3.29	184.9395	334.8781	12.5485	1937 Dec. 13.0
1941 Apr. 17.1460	0.341381	0.846164	3.31	185.1633	334.7593	12.3497	1941 May 6.0
* 1944 Aug. 6.2263	0.340272	0.846592	3.30	185.1681	334.7552	12.3656	1944 Aug. 18.0
1947 Nov. 26.3273	0.341028	0.846292	3.30	185.1923	334.7403	12.3504	1947 Dec. 1.0
1951 Mar. 16.2077	0.338014	0.847457	3.30	185.2137	334.7370	12.3799	1951 Mar. 15.0
1954 July 2.5178	0.338413	0.847295	3.30	185.1999	334.7448	12.3735	1954 June 27.0
1957 Oct. 19.8486	0.338123	0.847396	3.30	185.2273	334.7301	12.3746	1957 Oct. 9.0
1961 Feb. 5.5949	0.339009	0.847061	3.30	185.2301	334.7189	12.3594	1961 Jan. 21.0
1964 June 3.4906	0.339260	0.846992	3.30	185.9096	334.2384	11.9754	1964 June 14.0
1967 Sept. 22.0605	0.338209	0.847400	3.30	185.9141	334.2367	11.9889	1967 Sept. 27.0
1971 Jan. 9.9823	0.338900	0.847155	3.30	185.9443	334.2163	11.9737	1971 Jan. 9.0

*Comet not observed at these returns.

†All observations discarded at these returns.

fact, the variation of $|A_2|$ seems to be remarkably symmetrical about its maximum.

With the help of the approximate conversion formula between A_2 and the classical nongravitational parameter κ,

$$\kappa = -1580 A_2, \qquad (3)$$

Fig. 1 can immediately be compared with a plot based on the old-style computations (see, e.g., Sekanina 1969, Fig. 3). The secular trend between the 1830s and the 1960s is rather similar in the two plots, although the dispersion of the A_2 values is seen to be considerably smaller than that of the κ values from the old computations. Encke's approximate treatment of the pre-1819 apparitions seems to have been inadequate for showing that κ should then have been increasing with time. The values of κ were also particularly unreliable when the adopted mass of Mercury was significantly larger than 1/6 000 000 that of the Sun. When the time spans

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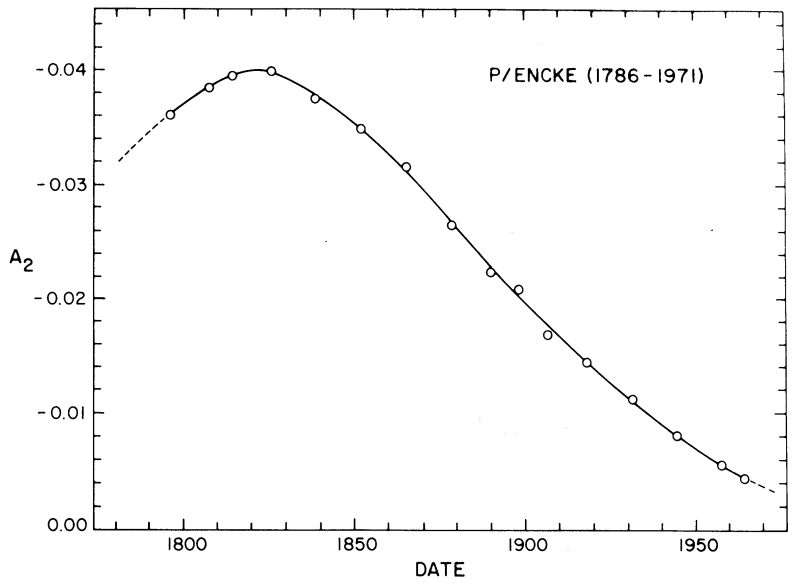

FIG. 1. Plot of the transverse nongravitational parameter A_2 versus the time, 1786–1971.

of the old fits were substantially longer than 13 yr, somewhat inferior values of κ were obtained; there were also unsatisfactorily large trends in the residuals, which some authors tried to remove by using completely unrealistic values for the masses of the Earth and/or Jupiter [e.g., the solution for 1819–1868 by Asten (1878)]. We further note that a recent investigation by Kastel' (1971), covering the returns 1954–1964, gives a value of κ that is about three times larger than follows from Fig. 1.

It has been reported before (Sekanina 1969) that the variations in the secular acceleration of P/Encke, derived from the old-style computations, seem basically to follow a $(1-\beta t)^2$ law, where t is the time, and β^{-1} is the comet's 'death date,' or cessation time of the nongravitational effects. Such a law is indeed expected during the advanced core phase of the "core-mantle model," and if the secular acceleration is entirely due to the effect of mass loss from the comet's nucleus (Sekanina 1969, 1972a). The results of the present study indicate that the $(1-\beta t)^2$ law has been in effect since the 1860s and give a death date around the year 2026, which is in reasonably good agreement with previous determinations.

The general expression for the secular acceleration in the core phase contains in addition a cubic term in

TABLE III. Perihelion times 1786 and earlier.

A_2 = -0.0335	A_2 = -0.0180	A_2 = -0.0335	A_2 = -0.0180	A_2 = -0.0335	A_2 = -0.0180
1786 Jan. 31	1786 Jan. 31	1713 Mar. 7	1713 Mar. 19	1640 Feb. 25	1640 Apr. 23
1782 Oct. 4	1782 Oct. 4	1709 Nov. 14	1709 Nov. 27	1636 Oct. 28	1636 Dec. 25
1779 June 10	1779 June 10	1706 July 23	1706 Aug. 5	1633 June 30	1633 Aug. 28
1776 Feb. 13	1776 Feb. 13	1703 Mar. 31	1703 Apr. 14	1630 Mar. 4	1630 May 2
1772 Oct. 19	1772 Oct. 19	1699 Nov. 24	1699 Dec. 10	1626 Nov. 7	1627 Jan. 5
1769 June 25	1769 June 26	1696 July 27	1696 Aug. 14	1623 July 12	1623 Sept. 9
1766 Mar. 1	1766 Mar. 1	1693 Mar. 29	1693 Apr. 19	1620 Mar. 15	1620 May 14
1762 Nov. 4	1762 Nov. 5	1689 Nov. 30	1689 Dec. 24	1616 Nov. 9	1617 Jan. 10
1759 July 5	1759 July 6	1686 Aug. 3	1686 Aug. 29	1613 July 7	1613 Sept. 15
1756 Mar. 14	1756 Mar. 17	1683 Apr. 5	1683 May 2	1610 Mar. 6	1610 May 22
1752 Nov. 22	1752 Nov. 25	1679 Dec. 5	1680 Jan. 4	1606 Nov. 4	1607 Jan. 27
1749 Aug. 3	1749 Aug. 7	1676 Aug. 3	1676 Sept. 6	1603 July 5	1603 Oct. 3
1746 Apr. 12	1746 Apr. 16	1673 Apr. 13	1673 May 20	1600 Mar. 2	1600 June 7
1742 Dec. 19	1742 Dec. 23	1669 Dec. 20	1670 Jan. 30	1596 Oct. 29	1597 Feb. 9
1739 Aug. 26	1739 Aug. 31	1666 Aug. 30	1666 Oct. 13	1593 June 25	1593 Oct. 16
1736 May 3	1736 May 9	1663 May 7	1663 June 23	1590 Mar. 2	1590 June 29
1733 Jan. 16	1733 Jan. 22	1660 Jan. 11	1660 Feb. 29	1586 Nov. 6	1587 Mar. 11
1729 Sept. 29	1729 Oct. 6	1656 Sept. 15	1656 Nov. 6	1583 July 14	1583 Nov. 22
1726 June 13	1726 June 21	1653 May 23	1653 July 16	1580 Mar. 7	1580 July 20
1723 Feb. 18	1723 Feb. 27	1650 Feb. 1	1650 Mar. 29	1576 Nov. 8	1577 Mar. 26
1719 Oct. 25	1719 Nov. 4	1646 Oct. 13	1646 Dec. 9	1573 July 11	1573 Dec. 1
1716 June 30	1716 July 11	1643 June 24	1643 Aug. 21	1570 Mar. 15	1570 Aug. 9

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TABLE IV. Close approaches to the Earth.

A_2 = -0.0335		A_2 = -0.0180	
Date	Minimum distance (A.U.)	Date	Minimum distance (A.U.)
1742 Nov. 6	0.24	1742 Nov. 11	0.28
1736 June 17	0.26	1736 June 22	0.22
		1699 Oct. 30	0.23
		1683 June 15	0.25
1679 Oct. 26	0.25		
1663 June 20	0.23		
		1646 Oct. 29	0.22
		1630 June 15	0.25
		1620 June 25	0.25
		1573 Oct. 20	0.23

the denominator. If the cubic term is retained, it is possible to fit the A_2 curve in Fig. 1 back to the 1820s, but there is no way to fit the earlier points.

We have no rigorous explanation for the observed profile of the peak of the A_2 curve for P/Encke. The above-mentioned theory does not, of course, take into account two other effects that may contribute to the form of the curve, namely, long-term variations in the effective lag angle of the mass ejection from the nucleus and in the inclination of the nuclear rotation axis to the orbit plane. As far as the former effect is concerned, it was suspected (Sekanina 1971) that the process of exposing the rigid matrix of meteoric materials on the comet's nuclear surface should lead to increasing lag effects as the free vaporization from the surface is being replaced by slow vaporization via diffusion from within the nucleus, which complicates the dynamical effect of the escaping gas during this 'dismantling' phase. As a matter of fact, the nucleus can perhaps be described in this phase by Shul'man's (1972) 'spotted model.'

As for the changes in the direction of the axis of rotation, we have discussed this possibility previously (Paper IV; Sekanina 1972b), most notably in regard to P/Faye. It has also been suggested (Marsden 1972b) that a damped sine wave, such as was considered by Michielsen (1968), could describe the combined effect of the deactivation process and the motion of the axis of rotation. Michielsen's expression, developed originally on a completely empirical basis and intended to fit the various published perihelion times of P/Encke (and other comets), appears to be a remarkably good approximation to the curve in Fig. 1. Extrapolation of this expression into the future suggests that P/Encke would develop a slight secular *deceleration* after around the year 2020, the coefficient A_2 then attaining a maximum positive value of about +0.001 around the year 2075.

Other representations of A_2 can clearly be made, and it seems premature to attempt to explain in detail the variation of the nongravitational parameters of P/Encke. At present, it still appears likely that progressive deactivation is the major mechanism that has been affecting the character of the nongravitational activity of P/Encke since the time of the comet's discovery. Nevertheless, we must be prepared to accept that other effects, such as those mentioned above, are going to complicate any quantitative analysis.

IV. THE PAST AND FUTURE MOTION OF P/ENCKE

Whipple and Hamid (1972) have searched for possible observations of P/Encke in the ancient Chinese dynastic records. As they remark, the existence of the nongravitational effects precludes linking observations of likely candidates with the known observations of P/Encke from 1786 onward. Nevertheless, we have attempted to extrapolate the motion of the comet back for some two centuries before 1786. In view of uncertainties in the extrapolation of A_2 backward in time, we have carried the orbit back on the assumption of constant A_2. We adopted two values, −0.0335 and −0.0180; the resulting times of perihelion passage are listed in Table III. Around the year 1670, the two sets of dates differ by only some six weeks, and the difference is not larger than five months in 1570. (Old Style dates are used before 1582.) Close approaches to the Earth are indicated in Table IV. There seem not to be any very likely candidates for P/Encke during the two centuries; and if the comet of November 1569 (Ho Peng Yoke 1970), the most recent of the candidates considered by Whipple (1970), was in fact P/Encke, it would appear that the curve for $|A_2|$ experienced a pronounced upswing some time before 1786. The remaining five orbital elements for the epoch 1572

TABLE V. Predicted orbital elements 1974–1984 (equinox 1950.0).

T (E.T.)	q (A.U.)	e	P (yr)	ω	Ω	i	Epoch
1974 Apr. 28.9935	0.338125	0.847450	3.30	185°.9288	334°.2224	11°.9820	1974 Apr. 23.0
1977 Aug. 16.9930	0.340663	0.846470	3.31	185.9530	334.2095	11.9389	1977 Aug. 5.0
1980 Dec. 6.5470	0.339941	0.846756	3.30	185.9792	334.1978	11.9460	1980 Nov. 17.0
1984 Mar. 27.6333	0.341003	0.846329	3.31	185.9925	334.1845	11.9274	1984 Apr. 10.0

It is indicated that the perihelion times T will require correction by the following amounts ΔT (in days): 1974, +0.005; 1977, +0.015; 1980, +0.03; 1984, +0.06.

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June 19.0 E.T. were $\omega = 177.^{\circ}6$, $\Omega = 338.^{\circ}7$, $i = 14.^{\circ}8$ (all for equinox 1950.0), $q = 0.342$ a.u., and $e = 0.846$.

There is currently much interest in P/Encke as the possible objective of a space mission in 1980 or 1984. From the point of view of predictability, there can be few better choices; and even though P/Encke may have shown some slight 'erratic' behavior around the year 1900, the fact that observations are evidently possible at *every* opposition eliminates the problem of whether or not the comet will be recovered before the probe is due to be launched. The predicted elements in Table V were derived by running forward the fit to the observations during 1957–1971. The anticipated further decrease in $|A_2|$ means that the prediction will be running off by 1984. The principal effect of the error is in the time of perihelion passage, of course, and we can estimate rather accurately the amount by which this element should be corrected. These corrections, labeled ΔT, are given in the footnote to Table V. The corrected figures should be quite adequate for the planning stage of a mission to P/Encke: any refinements can presumably be made by the time the probe has to be launched.

REFERENCES

Asten, E. v. (1878). Mem. Acad. Imp. Sci. St. Petersb. Ser. 7 **26**, No. 2.
Backlund, O. (1910). Mon. Not. Roy. Astron. Soc. **70**, 429.
Barnard, E. E. (1914). Popular Astron. **22**, 609.
Encke, J. F. (1823). Berliner Astron. Jahrbuch **1826**, 124.
Encke, J. F. (1831). Astron. Nachr. **9**, 317.
Herget, P. (1953). Astron. Papers Amer. Ephemeris **14**.
Ho Peng Yoke (1970). Oriens Extremus **17**, 63.
Kastel', G. R. (1971). Bull. Inst. Teor. Astron. **12**, 724.
Marsden, B. G. (1970). Astron. J. **75**, 75.
Marsden, B. G. (1972a). *Catalogue of Cometary Orbits*, IAU Central Bureau for Astronomical Telegrams, Smithsonian Astrophysical Observatory, Cambridge, Mass.
Marsden, B. G. (1972b). IAU Symp. No. 45, 135.
Marsden, B. G., and Sekanina, Z. (1971). Astron. J. **76**, 1135.
Marsden, B. G., Sekanina, Z., and Yeomans, D. K. (1973). Astron. J. **78**, 211.
Michielsen, H. F. (1968). Private communication.
Roemer, E. (1972). IAU Circ. No. 2435.
Sekanina, Z. (1968). Bull. Astron. Inst. Czech. **19**, 47.
Sekanina, Z. (1969). Astron. J. **74**, 1223.
Sekanina, Z. (1971). IAU Colloq. No. 12, 423.
Sekanina, Z. (1972a). IAU Symp. No. 45, 301.
Sekanina, Z. (1972b). IAU Symp. No. 45, 294.
Shul'man, L. M. (1972). IAU Symp. No. 45, 271.
Staff, Smithsonian Astrophysical Observatory (1966). *Smithsonian Astrophysical Observatory Star Catalog*. U.S. Govt. Pr. Of., Washington, D.C., 4 vols.
Whipple, F. L. (1970). Private communication.
Whipple, F. L., and Hamid, S. E. (1972). IAU Symp. No. 45, 152.

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