

W URSAE MAJORIS: THE PARAMETERS OF A CONTACT BINARY

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SUMMARY

Spectroscopic radial velocity observations of W Ursae Majoris, the prototype contact binary, are reported. The parameters (masses, radii, luminosities) of the system have been determined and are given in Table V. There is a real discrepancy between the spectroscopically determined mass-ratio of W Ursae Majoris and the photometrically determined one. BD + 55° 1351 and W Ursae Majoris share the same space motion which provides an estimate of the distance to the system. A theoretical main-sequence contact binary model is constructed which reproduces the observations.

1. INTRODUCTION

W Ursae Majoris (W UMa) is the prototype of a class of short period, late spectral type, eclipsing binary systems whose properties have been reviewed by Kraft (1967) and Moss & Whelan (1970). The peculiar mass-luminosity law ($L \propto M$), the nearly equal surface temperatures of the components and the continuously varying light curves seem to be explained by the contact binary model of Lucy (1968a, b). This model entails energy transfer through a common convective envelope of the binary components. In order to compare predictions based on the Lucy theory with observations, we have undertaken an accurate determination of observable parameters of the W UMa system.

Previous spectroscopic studies of W UMa have produced substantially different results. Values quoted for the mass-ratio range from 0.47 to 0.69; and values of the system velocity range from 0 to -50 km s^{-1} . Recent independent solutions for the mass-ratio determined from light curves using the light curve synthesis technique, have given mass-ratios of 0.41 (Mochnacki 1972) and 0.40 (Hutchings & Hill 1973). Both values are in noteworthy disagreement with the spectroscopically determined mass-ratios.

The wide range of previously observed W UMa system velocities, if real, would indicate the possibility of a third body in the W UMa system. This hypothesis is reasonable in the light of the quasi-sinusoidal period variations (O–C times of light curve minima) noted by Brown & Pinnington (1969), and Rigterink (1972).

To investigate the third body possibility and the mass-ratio discrepancy, we have made a spectroscopic study of W UMa, and have derived the system parameters from the radial velocity and photometric data. From these parameters we have constructed a theoretical contact binary model for W UMa.

2. THE OBSERVATIONS

The Steward Observatory 90-in. (229-cm) reflector Cassegrain spectrograph with an air-spaced Schmidt camera was used to obtain 70 spectra of W UMa

during the period 1972 February 25–28 (U.T.). These spectra had a dispersion of $62 \text{ \AA mm}^{-1}$ on Kodak Ila-O plates, with an average exposure time of less than 5 min (0.01 of a period), covering the entire radial velocity cycle. Additionally, three spectra were taken of BD + 55°1351, a nearby star which has been suggested as a proper motion companion by Eggen (1967a). The spectra were measured using the Kitt Peak National Observatory Grant measuring engine and reductions were obtained using a computer program developed by N. B. Sanwal. Table I

TABLE I
Radial velocity observations of W Ursae Majoris

Plate No.	Hel J.D. 2441370.0+	Phase	Exposure time (min)	Component 1 Velocity $\pm$ P.E. (km s ⁻¹)		Component 2 Velocity $\pm$ P.E. (km s ⁻¹)	
CS 337A	2.670	0.79	3	-144	6	224	6
B	2.674	0.80	3	-157	4	169	5
C	2.679	0.81	3	-149	3	156	4
D	2.681	0.82	3	-161	5	177	6
E	2.687	0.84	4	-160	7	155	7
F	2.692	0.85	5	-150	5	144	5
CS 338A	2.702	0.88	3	-117	4	95	7
B	2.707	0.90	4	-120	6	127	9
C	2.712	0.91	6	-130	11	72	8
D	2.720	0.94	7	-91	6	101	9
E	2.729	0.96	9	-96	5	10	6
CS 339B	2.749	0.02	6	17	13	-98	9
C	2.755	0.04	5	12	16	-116	11
D	2.761	0.06	4	28	8	-164	10
E	2.767	0.08	6	24	7	-181	8
F	2.773	0.10	4	31	12	-206	14
CS 340A	2.788	0.14	5	45	5	-246	7
B	2.794	0.16	4	68	6	-252	7
C	2.800	0.18	4	68	7	-250	10
D	2.805	0.19	3	83	1	-260	4
E	2.809	0.20	4	78	14	-258	9
F	2.814	0.22	3	72	8	-303	6
CS 341A	2.824	0.24	3	83	5	-266	6
B	2.828	0.26	3	85	6	-258	6
C	2.833	0.28	3	58	5	-260	4
D	2.837	0.29	3	90	13	-251	8
E	2.842	0.30	3	86	3	-253	7
F	2.845	0.31	3	46	8	-249	6
CS 342A	2.856	0.34	3	70	5	-218	6
B	2.860	0.36	3	58	12	-211	8
C	2.866	0.37	4	66	7	-183	10
D	2.870	0.39	3	50	8	-171	6
E	2.875	0.40	4	30	7	-195	9
F	2.880	0.42	4	24	9	-150	8
CS 343A	2.894	0.46	5	3	6	-108	6
B	2.901	0.48	5	-7	2	-109	2
C	2.908	0.50	5	-11	2	-104	8
D	2.915	0.52	5	-98	7	33	10
E	2.920	0.54	5	-116	6	42	5
F	2.924	0.55	4	-129	7	46	5
CS 344A	2.935	0.58	5	-161	5	128	7
B	2.942	0.60	4	-133	7	146	7

TABLE I—*continued*

Plate No.	Hel J.D. 2441370.0 +	Phase	Exposure time (min)	Component 1		Component 2	
				Velocity (km s ⁻¹)	± P.E.	Velocity (km s ⁻¹)	± P.E.
C	2.948	0.62	3	-157	5	154	6
D	2.953	0.63	3	-161	7	169	3
E	2.957	0.64	3	-158	5	167	6
F	2.961	0.66	4	-151	5	197	3
CS 345A	2.972	0.69	4	-161	8	235	7
B	2.978	0.70	3	-159	4	209	4
C	2.984	0.73	5	-156	6	178	7
CS 346C	3.619	0.63	5	-144	6	144	7
D	3.625	0.65	5	-168	4	154	6
E	3.630	0.66	4	-173	5	179	6
F	3.636	0.68	5	-186	4	200	7
G	3.642	0.70	4	-172	6	189	4
CS 347A	3.655	0.74	4	-152	8	205	5
B	3.660	0.75	4	-166	4	201	5
C	3.665	0.77	4	-176	5	187	7
D	3.670	0.78	4	-178	6	178	8
E	3.675	0.79	4	-164	6	185	5
F	3.680	0.81	4	-157	6	161	8
G	3.685	0.83	4	-132	3	157	7
CS 348A	3.697	0.86	5	-131	9	126	9
B	3.709	0.90	6	-153	6	82	8
C	3.715	0.92	7	-107	1	83	9
D	3.720	0.93	5	-115	6	50	8
CS 349D	3.804	0.19	3	64	4	-294	5
E	3.810	0.20	4	91	7	-283	6
CS 355C	6.019	0.82	5	-156	7	152	6
D	6.026	0.86	6	-134	7	172	7

contains the data, listing successively the plate number, Heliocentric Julian date, phase, exposure time in minutes, and radial velocities with probable errors for the primary (component 1) and the secondary (component 2) in kilometres per second. Sixty-nine spectra showed measurable double lines. Lines used for measurement were chosen from the Dominion Astrophysical Observatory list (Batten *et al.* 1971) and are the same as those used in our recent study of TX Cnc (Whelan, Worden & Mochnacki 1973, Table 2). Between seven and nine lines were measurable for most spectra. Measurement of IAU radial velocity standard stars taken during the course of observation of W UMa showed agreement with the published values to within $2 \text{ km s}^{-1} \pm 1 \text{ km s}^{-1}$.

The short-period nature of contact binaries causes rotationally broadened line profiles. The extensive blending of absorption lines which results makes it difficult to measure precise Doppler shifts. For this reason individual measurements of W UMa spectra are only accurate to within $\pm 10 \text{ km s}^{-1}$. Remeasurement of a substantial number of spectra gave results which were consistent with this accuracy. We feel that the radial velocity curve shown in Fig. 1 is of good accuracy due to the large number of spectra measured.

The solution to the radial velocity curve was obtained independently for each component using the Kitt Peak National Observatory CDC 6400 computer and a program developed by John C. Varga. As contact binaries would be expected to have nearly circular orbits due to tidal effects, the Stern (1941) method for low

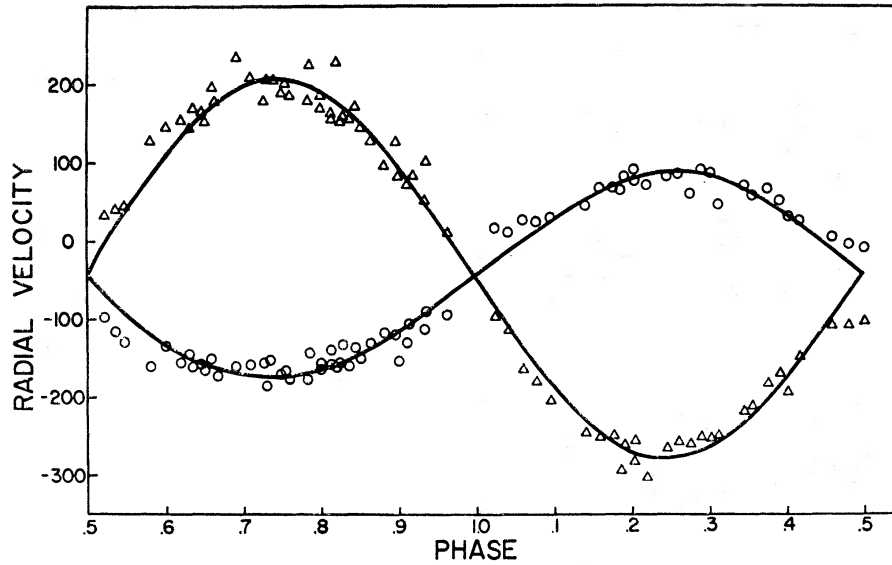


FIG. 1. Radial velocity observations and solution for *W UMa*. Zero phase is at the deeper photometric minimum. Circles represent the primary component and triangles the secondary. The solid lines are the adopted solution.

eccentricity systems was used in the solution. The results are given in Table II, along with the spectroscopic data derived by previous investigators. The notation there and in Table III is standard. The eccentricity of the orbit was indeed found to be insignificantly different from zero.

The resulting system velocity $V_0 = -42.3 \pm 3 \text{ km s}^{-1}$ is in good agreement with our independent solution for each component. This value is also in good agreement with the radial velocity of BD +55° 1351, shown along with *W UMa* in Table III. As BD +55° 1351 is a proper motion companion of *W UMa*, it is reasonable to assume both are at the same distance, enabling us to determine independently the luminosity of *W UMa*.

Whelan *et al.* (1973) noted that, as discussed by Hutchings (1973), the apparent light centres of contact binaries are drawn together as a result of variable surface brightness and non-spherical Roche geometry and rotation of the stars. Such an effect may lead to errors in the radial velocity semi-amplitudes unless

TABLE II

Spectroscopic elements of W Ursae Majoris

Element	Unit	Adams &	Popper (1950)	Struve &		Binnendijk (1967)	Present
		Joy (1919)		Horak (1950)	work (1973)		
V_0	km s ⁻¹	-5 ± 13	-43 ± 2	-25	-50*	0	-42.3 ± 3
K_1	km s ⁻¹	134 ± 5	111 ± 3	155	125	145 ± 3	131.3 ± 2
K_2	km s ⁻¹	188 ± 15	234 ± 3	235	250	210 ± 3	243.0 ± 3
$a_1 \sin i$	10 ⁶ km	0.610	0.509	0.711	0.573	0.665	0.60 ± 0.01
$a_2 \sin i$	10 ⁶ km	0.860	1.074	1.078	1.147	0.964	1.11 ± 0.02
$a \sin i$	10 ⁶ km	1.470	1.583	1.789	1.720	1.629	1.72 ± 0.03
$M_1 \sin^3 i$	Solar	0.67	0.97	1.24	1.22	0.92	1.18 ± 0.05
$M_2 \sin^3 i$	Solar	0.48	0.46	0.82	0.61	0.63	0.64 ± 0.03
M_2/M_1	—	0.71	0.47	0.66	0.50	0.69	0.54 ± 0.03

* Data in this column from measurements of $\lambda 4045$ only.

TABLE III

Proper motion and radial velocity of W UMa and BD + 55° 1351

Star	$\alpha(1950)$	$\delta(1950)$	μ_α	μ_δ	V_0 (km s ⁻¹)
W UMa	9 ^h 40 ^m .3	+56° 11'	+0 ^s .0032	-0 ^{''} .030	-42 ± 3
BD + 55° 1351	9 ^h 46 ^m .6	+55° 19'	+0 ^s .0033	-0 ^{''} .032	-43 ± 2

intrinsically narrow spectral lines are used in the measurements. This effect was calculated and found to be negligible in this system for the lines we used.

Careful analysis of the previous radial velocity solutions listed in Table II showed that the large variations of these values are consistent with inaccuracies and difficulties in measurement. Effects such as poor time resolution, difficulty in measurement, and small numbers of data points seem to appear in most of these solutions. The large number of data points obtained in our investigation, and the consistency of our independent solutions lead us to feel that we have established an improved solution. Consequently, the range in previous values can be attributed to difficulties of observation and analysis rather than real variations in the parameters.

Some of our spectra show evidence of Ca II K line (3933 Å) emission as previously noticed by Struve & Horak (1950); together with a flare observed by Kuhl (1964), this is indicative of activity in the W UMa system, and may indicate free mass near the binary. We estimate the spectral type of W UMa to be about G0 in agreement with earlier estimates of F8–G0.

3. THE PARAMETERS OF W UMa

To derive all observable parameters of the W UMa system we have combined our spectroscopic data with the photometric solutions obtained by other investigators. Recent photometric solutions for W UMa were obtained by Lucy (1971) and Mauder (1972) who adopted the previous spectroscopic mass-ratios shown in Table II for their solutions. Mochnacki (1972) and Hutchings & Hill (1973) determined the mass-ratio directly from the photometry through light curve synthesis techniques. This data is given in Table IV and was taken from the above mentioned analyses, which apart from the mass-ratio, are consistent with each other. In Table IV, the mass-ratio, q , is defined to be less than unity and subscripts P and S refer to the primary (more massive) and secondary components, respectively.

In our analysis, we obtained a significantly different mass-ratio from those of the photometric solutions. We have thus used caution in adopting the photometric solution values for determination of system parameters. The value for the inclination, i , enters the determination as $\sin i$, and depends only weakly on the value of the photometrically determined mass-ratio q , for $80^\circ < i < 90^\circ$, which is the case for W UMa. The temperature values of Mochnacki (1973, private communication) were used in preference to those of Hutchings & Hill (1973), which were about 400° hotter, as the former values appear to give better agreement with the observed colours. The temperature *difference* between the two components is similar for both light curve synthesis solutions, with the secondary being hotter. F is the fill out factor defined by Mochnacki & Doughty (1972), and C is the value of the dimensionless Roche potential of the equipotential surface which the stars fill. The

TABLE IV
Adopted photometric data for W Ursae Majoris

Element	Value
<i>P</i>	0.333638 days
<i>q</i>	0.41 ± 0.01
<i>i</i>	84° ± 1°
<i>T_P</i>	5600 K ± 200 K
<i>T_S-T_P</i>	320 K ± 40 K
<i>F</i>	1.13 ± 0.03
<i>C</i>	3.87

values given imply that W UMa is in shallow contact, sharing a common envelope only in the outermost parts of the stellar convection zone.

Combining the phase information of our radial velocity data with the photometric light curve it is seen that the deeper photometric minimum occurs when the primary component (the more massive, larger star) occults the secondary. W UMa is thus a W-type W UMa system following Binnendijk's classification (1970). This is consistent with the secondary having a higher temperature and energy transfer taking place through a thin outer convection zone, where superadiabatic conditions prevail (Mochnecki & Whelan 1973; Lucy 1971).

The resulting solution, combining photometric and spectroscopic data is shown in Table V, in which our spectroscopic mass-ratio has been used. The

TABLE V
Parameters of W Ursae Majoris

Quantity	Unit	Primary	Secondary
Mass	Solar	1.20 ± 0.06	0.65 ± 0.03
Radius	Solar	1.07 ± 0.03	0.81 ± 0.02
Surface temperature	K	5600 ± 200	5920 ± 200
<i>T_S-T_P</i>	K		320 ± 40
Radiated luminosity	10 ³³ erg s ⁻¹	3.92 ± 0.40	2.78 ± 0.30
Total luminosity	10 ³³ erg s ⁻¹		6.70 ± 0.70
Bolometric magnitude	magnitudes		4.17 ± 0.12
Total mass	<i>M</i> _⊙		1.85 ± 0.09
Period	days		0.333638
Separation of centres	Solar radii		2.48 ± 0.06
Ω ² (synchronous)	10 ⁻⁸ s ⁻²		4.76
Core luminosity	10 ³³ erg s ⁻¹	6.37 ± 0.68	0.32 ± 0.04
Δ <i>L</i>	10 ³³ erg s ⁻¹		2.45 ± 0.3

radii given are the radii of spheres of volume equivalent to the Roche surface volume which the stars fill. Ω² is the square of the angular velocity, assuming the rotation to have been synchronized by tidal effects. The *radiated* luminosities were obtained from the radii and effective temperatures using

$$L_{\text{rad}} = 4\pi\sigma R^2 T_e^4.$$

For W UMa this yields a mass-luminosity law of

$$L \propto M^\beta, \quad \beta \sim 0.8$$

which is in good agreement with the contact binary theory of Lucy (1968a, b,

1971) which predicts $\beta \approx 0.9$. The large radius and luminosity of the secondary component are also explained by this theory as results of energy transfer from the primary to the secondary. By assuming that the component stars of W UMa obey a main sequence mass-luminosity law with respect to the luminosity produced in their cores L_{core} :

$$L_{\text{core}} \propto M^\beta, \quad \beta \sim 4.7$$

we can obtain a value for the core (intrinsic) luminosity of each component and for the luminosity, $|\Delta L|$, transferred between them. The quantities L_{core} , L_{rad} and $|\Delta L|$ are related by the equation

$$L_{\text{rad}} = L_{\text{core}} \pm |\Delta L|.$$

The total system luminosity for W UMa has been estimated above by using the derived parameters of radius and temperature. As a check, however, it may also be obtained in two additional ways. The first method involves using the proper motion companion BD + 55° 1351. By assuming this star to be at the same distance as W UMa, an assumption justified by our observations; and assuming it to be a normal main sequence star, we can derive its distance and from this a distance and luminosity for W UMa. Eggen's (1967a) V , $B-V$, and $U-B$ values for BD + 55° 1351 suggest a spectral type of K3V (Johnson 1966) which classification is in agreement with the appearance of our radial velocity spectra for this star. A main sequence star of this type has an absolute visual magnitude of $M_V = 6^{\text{m}}.5$ (Johnson 1963; Van de Kamp 1971). This gives a distance modulus of $3^{\text{m}} \pm 0^{\text{m}}.3$, which corresponds to $M_{\text{bol}} = 4^{\text{m}}.4 \pm 0^{\text{m}}.3$ for W UMa at maximum.

Parallax data can also be used. A measurement quoted by Binnendijk (1967) is $\pi = 0''.015 \pm 0''.009$. While the accuracy of such an observation is limited, a parallax of $0''.024$ corresponds to a distance of 40.5 pc and a distance modulus of $3^{\text{m}}.0$. Similarly, a parallax of $0''.015$ corresponds to 66 pc and $4^{\text{m}}.1$ respectively.

The maximum visual magnitude for W UMa when it is out of eclipse is $V = 7^{\text{m}}.5$ (Breinhorst 1971). Allowing $0^{\text{m}}.1$ for reddening and the small bolometric correction of a G-type star and using $M_{\text{bol}} = 4^{\text{m}}.2$ from Table V we deduce a distance modulus of $3^{\text{m}}.2$. The agreement between this value, and those derived from the proper motion companion BD + 55° 1351 and the parallax data leads us to believe the parameters listed in Table V are of good quality.

We note that the parameters of W UMa, quoted in Table V, are different from those given by the authors mentioned in Table II. The differences are due to the use of spectroscopic and photometric quantities taken from a variety of investigations. Our results are in reasonable agreement with those quoted by Cester & Gridelli (1967).

4. THEORETICAL MODEL

Using the values of observed parameters from Table V, a main-sequence contact binary model has been constructed using the basic principles of the Lucy (1968a) model, as modified by Moss & Whelan (1970, 1973) and Whelan (1972a, b). Parameters of a model which most closely fit the observations are given in Table VI. The composition X , Y , Z (hydrogen, helium and 'others') has been assumed to be roughly solar. α is the ratio of mixing length to pressure scale height. It is necessary to transfer 70 per cent of the exchanged energy through the super-adiabatic part of the common convective envelope to produce a hotter secondary

as observed. This condition is consistent with shallow contact as mentioned previously. The differences in the convective constants $\log_{10} K$ of the two stars which this entails are of the right sense and of the same size needed to drive the energy flow through the convective zone as suggested by Biermann & Thomas (1972) and Moss & Whelan (1973). The model of Table VI is a main-sequence

TABLE VI
Parameters of a theoretical age-zero model for W Ursae Majoris

Quantity	Unit	Primary	Secondary
X	—	0.75	
Y	—	0.223	
Z	—	0.027	
α	—	1.3	
Energy transfer	—	~70 per cent in superadiabatic part of convection zone	
Mass	Solar	1.20	0.65
Radius	Solar	1.07	0.82
Ω^2	10^{-8} s^{-2}		4.76
L_{rad}	$10^{33} \text{ erg s}^{-1}$	3.94	2.80
L_{core}	$10^{33} \text{ erg s}^{-1}$	6.44	0.33
$ \Delta L $	$10^{33} \text{ erg s}^{-1}$	-2.5	+2.5
T_{eff}	K	~5600	~5900
$\log_{10} K$	cgs	-2.48	-2.06

model, but it is possible that W UMa is slightly evolved as suggested by Hazlehurst (1970). The core-luminosities of the components, however, place them close to the Sun-Sirius mass-luminosity relation (Eggen 1967b).

5. CONCLUSIONS

As a result of this investigation we conclude that W UMa is a W-type W Ursae Majoris system in shallow contact, with the secondary component about 300 K hotter than the primary, in agreement with contact binary theory. This conclusion is substantiated by the construction of an age-zero contact binary model with a reasonable chemical composition obeying the mass luminosity law predicted by Lucy (1968a, b). The theoretical model has an inequality between the adiabatic constants of the two components, indicating the energy exchange occurs in the outermost regions of the convective zone, where superadiabatic conditions hold. This model is based on recent photometric solutions and our new spectroscopic solution for the system parameters, given in Tables II, IV and V.

The key elements of our spectroscopic solution are the mass ratio $q = 0.54 \pm 0.03$ and a system velocity of $V_0 = -42 \pm 3 \text{ km s}^{-1}$. The spectroscopic mass-ratio thus derived differs substantially from the apparent photometric mass-ratio of $q = 0.41 \pm 0.02$. This difference is real and significantly outside the range of errors. Further, the spectroscopic mass-ratio is consistent with previous determinations of it. Differences between this determination and previous determinations of mass-ratio and system velocity appear to be attributable to difficulties in obtaining observational accuracy, and problems in measurement of data.

The fact that BD + 55° 1351 shares a common space velocity with W UMa enabled us to make a second determination of the distance to W UMa. As the two

determinations of the distance are consistent with each other, and within the observed error of the parallax, we feel confident that we have obtained a reasonable solution for the W UMa system.

The above conclusions indicate that a possible third-body does not manifest itself through significant observable variations in the system velocity of W UMa. We will discuss, in a future paper, the possibility of a third-body in the W UMa system especially from the point of view of reconciling the discrepancy between the spectroscopically and photometrically determined mass-ratios (Worden & Whelan 1973).

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