Changes of period in semiregular variables

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Fourteen well observed variables currently classified as semiregular are tested for change of mean cycle length by the method of contingency tables. Three variables are found to have a significant change of period during the interval of 67 years covered by the data.

INTRODUCTION

FLUCTUATIONS in the length of cycles of variable stars were studied by Eddington and Plakidis (1929). An investigation of cycle lengths of Mira-variables by Plakidis (1932, 1933) and Eddington (1932) found that a normal class of about three-quarters of the long period variables studied had cycle lengths which were distributed as a random gaussian variable. Sterne (1934) showed that a necessary consequence of a random variation in cycle length is the existence of a cumulative increase in the variance of dates of maxima and minima. If the individual cycle lengths are $P+\epsilon_i$, where P is the mean cycle length and the ϵ_i are distributed as a random zero-mean gaussian variable, the observed dates of maxima or minima (neglecting observational errors) are given by Eq. 1.

$$\phi_n=E+\sum_{i=1}^{n}(P+\epsilon_i). \tag{1}$$

These differ from the predicted dates

$$C_n=E+nP \tag{2}$$

by

$$\phi_n-C_n=\sum_{i=1}^{n}\epsilon_i \tag{3}$$

Thus if the cycle lengths vary as a random gaussian variable, the $\phi-C$ diagram (ϕ_n-C_n vs. n) has the structure of a random walk as shown by Balazs-Detre and Detre (1965). The random walk process is nonstationary with the variance increasing linearly with n. Because of this nonstationarity, determining changes in the mean period P from the $O-C$ diagram is complicated.

A general method for detecting changes in the mean period was developed by Sterne and Campbell (1937), and applied to the cycle lengths of 377 Mira-variables. It was concluded that only about five stars (1.3%) of this large sample showed progressive period change.

I. DATA

Fourteen well-observed stars on the observing list of the American Association of Variable Star Observers (AAVSO) are currently classified in the General Catalog of Variable Stars (1969) as semiregular variables (see Table I). Due to an error, the Mira-variable RZ Sco was included in this study. Dates of maxima and minima were obtained from three sources: Campbell (1926), Campbell (1955), and Mayall (1970). The observational material thus covers about 67 years from 1904 to 1970 in some cases (see Table I). Interpolation was necessary in some instances to supply missing dates. In no cases were more than 5% of the dates obtained by interpolation. Due to a long series of missing dates, the data for T Cen was broken into two series labeled T Cen-1, T Cen-2.

II. ANALYSIS

The dates of maxima and minima were used to derive the cycle lengths by subtracting consecutive dates. The cycle lengths were tested for change by the same method of contingency tables used by Sterne and Campbell (1937). Semiregular variables are well suited for this method since they are defined in the General Catalog of Variable Stars (1969) as "possessing an appreciable periodicity . . ." The results are shown in Table II, which refers to a contingency table of the form

Cycle$>P$	a_{11}	a_{12}	R_1
Cycle$\le P$	a_{21}	a_{22}	R_2
	C_1	C_2	S

$$\chi_c^2=\sum_{i,j}\frac{(a_{ij}-R_iC_j/S)^2}{R_iC_j/S},$$

where a_{11} is the number of cycle lengths greater than the "average" period P, and falling in the first half of the data series (of length n); a_{12} is the number of cycle lengths greater than P, and falling in the second half of the data series; and similarly for a_{21}, and a_{22} with cycle lengths less than P. The period used for P is the least-squares period. The marginal quantities R_i and C_i are defined as follows:

$$R_1=a_{11}+a_{12},\quad R_2=a_{21}+a_{22},$$
$$C_1=a_{11}+a_{21},\quad C_2=a_{12}+a_{22},$$
$$S=a_{11}+a_{12}+a_{21}+a_{22}.$$

The null hypothesis is that the cycle lengths are distributed randomly in time about P. If this is true, the values of χ_c^2 should be distributed as a χ^2 variable with one degree of freedom. The probability that any single value of χ_c^2 will be greater than 6.635 (called the

TABLE I. Miscellaneous data on the stars and data series.

Desig	Star	Period from GCVS (days)	Mag at maxima	Mag at minima	Spectral type GCVS	Class from GCVS	Dates in series	Time interval covered
042215	W Tau	260.65	9.9	11.4	M5	SRb	63	1904–48
053068	S Cam	326.4	8.4	10.2	R8e	SRa	74	1904–69
055353	Z Aur	134.8	9.9	10.7	G0e–G6e	SRd	131	1916–59
065208	X Mon	155.7	7.6	9.0	gM3e–M4	SRb	121	1913–64
082405	RT Hya	253	7.6	9.0	M6e–M7	SRb	37	1927–52
094836	U LMi	272.2	10.9	12.7	M6e	SRa	25	1935–53
133633	T Cen-1	90.60	6.1	8.0	K0e–M4IIe	SRa	41	1921–30
—	T Cen-2						93	1932–55
142539	V Boo	258.23	7.9	10.2	M6e	SRa	93	1904–69
155823	RZ Sco	159.59	8.9	11.8	M3e–M4e	M	116	1919–69
194659	S Pav	386.12	7.3	9.4	M7IIe	SRa	35	1928–64
200715a	S Aql	146.42	9.5	11.4	M3e	SRa	126	1919–69
204846	RZ Cyg	275.69	10.4	13.1	M7	SRa	36	1921–48
213753	RU Cyg	234.45	8.0	9.4	M6e	SRa	75	1922–70
220843a	RS Lac	237.5	10.7	11.8	K0	SRd	53	1935–69
233335	ST And	328.0	8.9	10.9	R3e(C3$_1$)	SRa	68	1908–69

a priori probability p) is 0.01, if the above is true. The probability that one or more values of χ_c^2 out of a set of N values of χ_c^2 will be greater than 6.635 (called the *a posteriori* probability α) is much greater than 0.01. It was shown by Eddy (1968) that

$$p=\alpha/N \tag{4}$$

for small values of α. Thus, the *a posteriori* confidence limits at the level of significance α are equivalent to the *a prior* confidence limits at the level of significance p given by Eq. 4. If a set of more than one value of χ_c^2 is being compared with the expected χ^2 distribution, the *a posteriori* confidence limits should be used. If one or more values from the set of values of χ_c^2 exceeds these limits, the probability is $1-\alpha$ that the cause in every case is a change in period and not random statistical fluctuations.

TABLE II. Results of contingency tables.

Star	P(days)	a_{11}	a_{12}	a_{21}	a_{22}	S	χ_c^2
W Tau	max 259.7	24	14	7	17	62	6.798
	min 259.2	23	9	8	22	62	12.658
S Cam	326.9	16	22	20	15	73	1.648
	326.8	17	21	19	16	73	0.665
Z Aur	120.4	6	50	59	15	130	60.734
	120.5	7	52	58	13	130	62.843
X Mon	155.7	27	31	33	29	120	0.534
	155.7	29	29	31	31	120	0.000
RT Hya	253.6	10	9	8	9	36	0.111
	253.2	7	8	11	10	36	0.114
U LMi	273.0	6	4	6	8	24	0.686
	273.0	6	5	6	7	24	0.168
T Cen-1	90.7	11	10	9	10	40	0.100
	90.7	12	9	8	11	40	0.902
T Cen-2	90.3	21	19	25	27	92	0.177
	90.3	20	24	26	22	92	0.697
V Boo	258.2	29	16	17	30	92	7.351
	258.2	27	15	19	31	92	6.309
RZ Sco	159.1	18	30	39	28	115	4.798
	159.2	21	27	36	31	115	1.115
S Pav	385.7	13	12	4	5	34	0.151
	385.8	11	11	6	6	34	0.000
S Aql	146.6	32	37	30	26	125	0.640
	146.7	30	33	32	30	125	0.199
RZ Cyg	274.5	9	9	8	9	35	0.030
	274.5	8	10	9	8	35	0.253
RU Cyg	233.9	17	17	20	20	74	0.000
	233.7	18	20	19	17	74	0.216
RS Lac	237.6	13	17	13	9	52	1.261
	237.9	12	18	14	8	52	2.836
ST And	332.0	23	18	10	16	67	1.980
	332.0	21	16	12	18	67	1.861

III. DISCUSSION

The χ_c^2 for maxima and minima are so highly correlated that there are essentially only 16 independent values in the set of 32 computed values of χ_c^2. Thus the upper *a posteriori* confidence limit is 7.51 at $\alpha=0.10$, and 8.64 at $\alpha=0.05$. In three instances, the $\alpha=0.05$ confidence limit is exceeded. There can be no doubt about a change in the period of Z Aur since both maxima and minima give very large values of χ_c^2. The χ_c^2 for the maxima of W Tau is significant at $\alpha=0.14$, and for the minima at $\alpha=0.005$, so that a change in period for W Tau is fairly certain. The χ_c^2 for V Boo are significant at $\alpha=0.11$ and $\alpha=0.19$, so

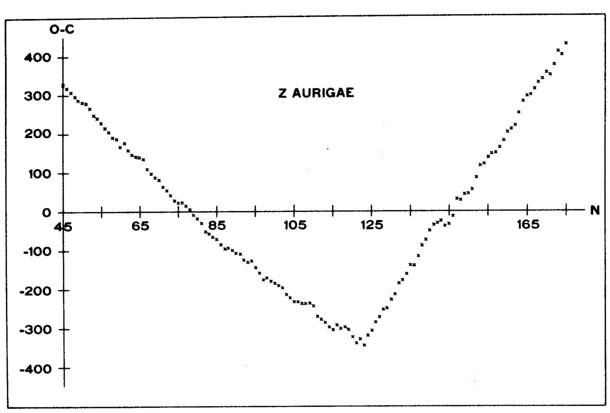

FIG. 1. $O-C$ (days) diagram for dates of maxima of Z Aur computed from $C_n=2428658+(n\text{-}110)120.40$

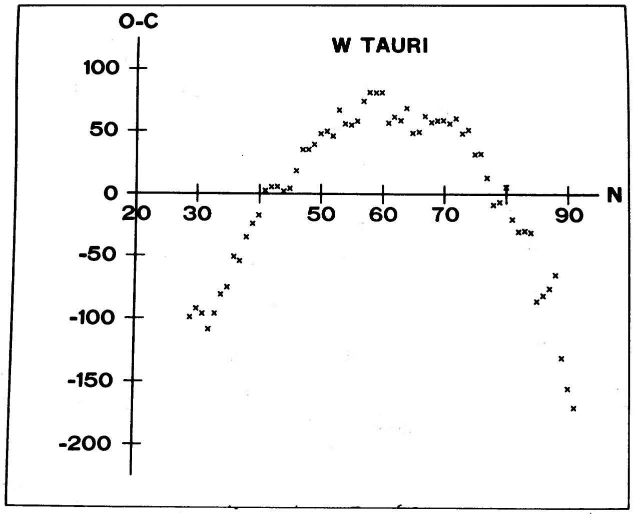

FIG. 2. $O-C$ (days) diagram for dates of maxima of W Tau computed from $C_n=2424812+(n\text{-}55)259.73$

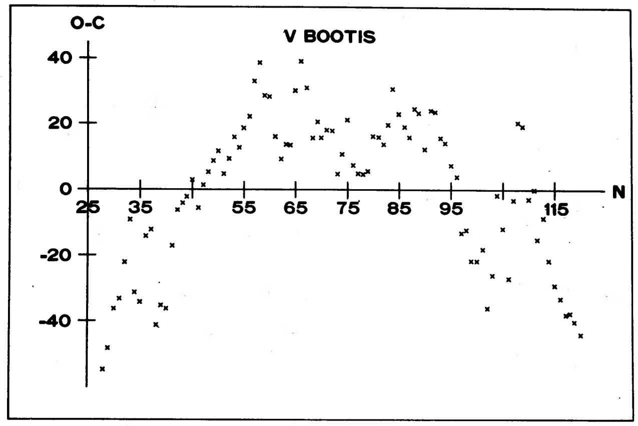

FIG. 3. $O-C$ (days) diagram for dates of maxima of V Boo computed from $C_n=2428556+(n\text{-}74)258.21$

that a change in period for V Boo is less certain but strongly suspected.

The $O-C$ diagrams for the maxima of Z Aur, W Tau, and V Boo are shown in Figs. 1, 2, and 3. The period of W Tau appears to have decreased by about 20 days, either continuously or through one or more abrupt changes. Without more data it would be presumptuous to fit sine or quadratic terms to the period. The period of V Boo also appears to have decreased, but with less regularity than in the case of W Tau.

The appearance of the $O-C$ diagram of Z Aur is striking. At first glance there appears to be one major change in period of about 23 days (20% of the mean period) which occurred abruptly about epoch No. 122 (in the year 1939–1940). Close scrutiny shows two other possible minor changes in period at epoch Nos. 81 and 143. In order to determine whether these suspected changes were significant, the data was divided into two parts at epoch No. 122, and each part tested for period change using contingency tables as above. The resultant χ_c^2 for maxima and minima were 6.858 and 3.749 for the first part, and 0.702 and 0.310 for the second part. The χ_c^2 for the first part is significant at the *a priori* levels $p=0.009$ and 0.052; for the second part the χ_c^2 is not significant enough to accept a change in period. The three significant periods as derived from the least-squares line and from the cycle lengths are given in Table III. The mechanism for producing these abrupt changes in period is not known, but a change in pulsational mode must be considered as a possibility.

The Mira-variable RZ Sco, which was accidentally included in this study, was suspected of having a changing period by Sterne and Campbell (1937). The χ_c^2 calculated (see Table II) are not found to be significant using the criterion of the $\alpha=0.20$ *a posteriori* confidence limits.

In conclusion, it appears that of the fifteen variables studied, three stars (20%) had a significant change of mean cycle length during the period covered by the data. This study differs from that of Sterne and Campbell (1937) in that only stars currently classified as semiregular variables were included (except RZ Sco); also, the data included new published and unpublished observations taken by the AAVSO from 1937 to 1970.

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TABLE III. Significant periods of Z Aur.

Time interval covered (J.D. 2400000+)	21159-25235	25235-29890	29890-36910
Least-squares line			
Max	$110.4(n\text{-}63)+23148$	$113.3(n\text{-}102)+27498$	$134.7(n\text{-}149)+33402$
Min	$110.5(n\text{-}63)+23207$	$113.1(n\text{-}102)+27558$	$134.8(n\text{-}149)+33472$
Cycle lengths			
Max Mean (days)	110.50	113.52	135.00
Max Standard deviation (days)	6.37	7.86	11.47
Min Mean (days)	110.50	113.30	134.83
Min Standard deviation (days)	7.58	9.57	13.11

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Parallaxes of 23 stars determined from plates taken with the McCormick 26-inch refractor

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This 41st list of trigonometric parallaxes by the Leander McCormick Observatory is a continuation of previous similar publications in this Journal. Changes in the reduction procedures have been made and are briefly summarized here. Attention is called to the list star number 2134 as an interesting star. This star has been reported on in detail (Appelbaum 1972).

THIS list (Table I) is a continuation of the previous lists of parallaxes from the McCormick Observatory (Alden 1957; Osvalds 1966; Lü and Fredrick 1968). The entire procedure has been changed to that of using four or more reference stars (excepting star no. 2122), carefully selected to have little or no proper motion, and a linear plate constant reduction routine. All exposures are reduced individually to a mean trail plate, oriented to the equator of 2000, and assigned unit weights in the first parallax solution. If there is no evidence of orbital motion, a second iteration is made with a relative weight assigned to each exposure in proportion to the size of its residual compared with the mean residual in the first solution. The weights then fall in the range from about 0.7 for a large residual to about 1.8 for a plate having a small residual. Note that all errors in the Table are mean errors and not probable errors. Stars 2122, 2128, 2129, 2130, 2132, and 2133 were reduced with an earlier version of this program. A more complete description of current procedures will appear in the Publications of the McCormick Observatory.

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