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ARISTOTELIAN PLANETARY THEORY IN THE RENAISSANCE: GIOVANNI BATTISTA AMICO'S HOMOCENTRIC SPHERES

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In 1536 a young man of 23, Giovanni Battista Amico, published a treatise on planetary theory called "On the Motions of the Heavenly Bodies according to Peripatetic Principles without Eccentrics or Epicycles".[1] The work contains descriptions of models for planetary motions employing only homocentric spheres, and since Amico was a student at Padua, as was Girolamo Fracastoro, who also wrote a description of homocentric sphere models, one may see in this some sort of Aristotelian school of astronomy. The treatise has received little attention although it is not entirely unknown. Dreyer praised Amico highly, and gave an almost correct but hardly intelligible description of the models.[2] However, he seemed to miss the most interesting point (some might say the only interesting point), namely, that Amico employs the same geometrical construction used by Copernicus for producing a rectilinear oscillation from the combined movements of two circles. But while Copernicus used this device only for the irregular precession of the equinoxes, the change of the obliquity of the ecliptic, and the approach and withdrawal of Mercury's epicycle, Amico used it to account for every single irregularity in the planetary motions.

The same device was used in the thirteenth and fourteenth centuries by the astronomers of the Marāgha school, Naṣīr ad-Dīn aṭ-Ṭūsī, Quṭb ad-Dīn ash-Shīrazī, and Ibn ash-Shāṭir, who also employed another mechanism that produced the same result of rectilinear oscillation from two circular motions.[3] Since the two mechanisms are frequently confused, it seems appropriate to explain their differences.

The device used by Copernicus and Amico, which will be treated in more detail shortly, consists of a deferent-epicycle construction with the radii of both circles equal. It is, in fact, a special case of a method of generating an ellipse, as Copernicus points out in a famous (and often ridiculously interpreted) cancelled passage in his manuscript.[4] The Marāgha astronomers used not only this device, but also an equivalent principle of a circle rolling on the internal circumference of another circle of twice its radius. This is a special case of the generation of a hypocycloid, and it was never used by Copernicus or Amico. These two limiting cases of both models produce identical rectilinear oscillations.

Such devices would be of interest to astronomers attempting to adhere to Aristotelian principles since they facilitate the development of models composed only of uniform circular motions. The motivation for the models developed by the Marāgha astronomers and Copernicus was that, believing as they did that the motions of the planets were controlled by solid spheres, it was apparent by simple mechanical sense that the rotation of a sphere can be uniform only with respect to its diameter, but not with respect to any other straight line. Hence they found alternatives to the bisection of the eccentricity in Ptolemy's models. The reciprocation principle allows the introduction of a rectilinear motion at will wherever it is required, and it is perfectly consonant with the mechanics of building models by the combination of uniform circular motions.

The similarities of the planetary models of the Marāgha astronomers to those of Copernicus are well known, and they are indeed striking. The present discovery of the extensive use of one of the devices by Amico would indicate, I believe, that the transmission of the planetary theory, if it occurred, came through Italy, perhaps by way of Byzantine sources.[5] There is, however, no reason to assume any connection between the work of Amico and Copernicus. Certainly by the time Amico's book was published in 1536 Copernicus had more-or-less finished his own work, and the reciprocation device was already described in the *Commentariolus* which, whatever its date, was surely much before 1536. However, in the dedicatory preface of *De revolutionibus* to Pope Paul III, Copernicus mentions "those who have sworn by homocentrics, although they have shown that some irregular motions can be made by joining them together, yet they have been able to establish nothing definite that corresponds accurately with the appearances".[6] This would be an adequate appraisal of Amico's work, but it could apply equally well to Fracastoro's *Homocentrica*, published in 1538, or, I suppose, to any other homocentric planetary theory.

Amico's treatise is for the most part clearly written, and those aspects of the application of his models that he handles cursorily are easily reconstructed. His descriptions of the geometrical mechanisms, however, are inelegant, plodding, and sometimes careless. Further, his models do not, in fact, perform as he believes they do since, as we shall see, the reciprocation mechanism works properly only in the plane, but not on the surface of a sphere. He states in his preface that he took up planetary theory because he was concerned about the discrepancy between Aristotelian natural philosophy, the principles of the "Philosophers", and Ptolemaic planetary theory, the planetary theory of those he calls the "Astrologers". The resulting homocentric sphere models show the usual harmful effect of taking philosophy before accuracy since they are distinctly inferior to Ptolemy's and really do not work at all. Aristotle and truth were both Amico's friends, but Aristotle was surely his better friend. The models themselves are probably entirely his own invention, but there is no reason to assume that he independently discovered the reciprocation mechanism that underlies them.

The treatise begins with a review of the homocentric theory of Eudoxus, Callippus, and Aristotle, drawn from the twelfth book of the *Metaphysics* and Simplicius's commentary on *De caelo*. Needless to say, Amico can make little of it. He then explains his device for producing a rectilinear oscillation from circular motions, and develops the models for the motions of the Sun, Moon, and planets in longitude. Here he also takes up the question of why the planets vary in brightness and the Sun and Moon vary in size if their distances do not change. He also criticizes traditional planetary theory, arguing at one point that the Moon cannot move on an epicycle since it always shows the same face to the Earth and this would not be so if it were revolving around the centre of an epicycle. This argument is extended by analogy to show the impossibility of any planets' moving on epicycles. He then explains models for the planetary motions in latitude, using the same reciprocation device, and last of all the motion of the eighth sphere which is treated without reciprocal motion. In fact, he takes his description of the precession directly from George Peurbach's *Theoricae novae planetarum*, and thus he reproduces Peurbach's not very successful attempt to make some kind of geometrical sense out of the precession and trepidation tables in the *Alphonsine Tables*. Had he chosen to apply the reciprocation device to the precession, he would doubtless have developed a model identical to that of Copernicus although the parameters would have been Alphonsine. Amico gives hardly any numerical parameters, and these few suggest that he was relying upon the *Alphonsine Tables*. In this paper I shall consider only the models for longitude.

The Reciprocation Mechanism

Amico begins with three principles (Chapter 7):

(1) A single sphere has only one motion, and higher spheres set lower spheres in motion unless the lower spheres are moved in the opposite direction. Thus, motion is transmitted only from higher spheres to lower, and will always be transmitted unless a counteracting sphere with an opposite motion is placed between the spheres.

(2) If the axes of two contiguous spheres intersect at right angles in the centre of the Earth, and the poles of the higher sphere oscillate back and forth in small arcs, a point on the equator of the lower sphere, which is revolving uniformly, will be made faster and slower by the back and forth oscillations of the higher sphere. The small arcs in which the higher sphere oscillates are called 'arcs of reciprocation' (*arcus reciprocationis*).

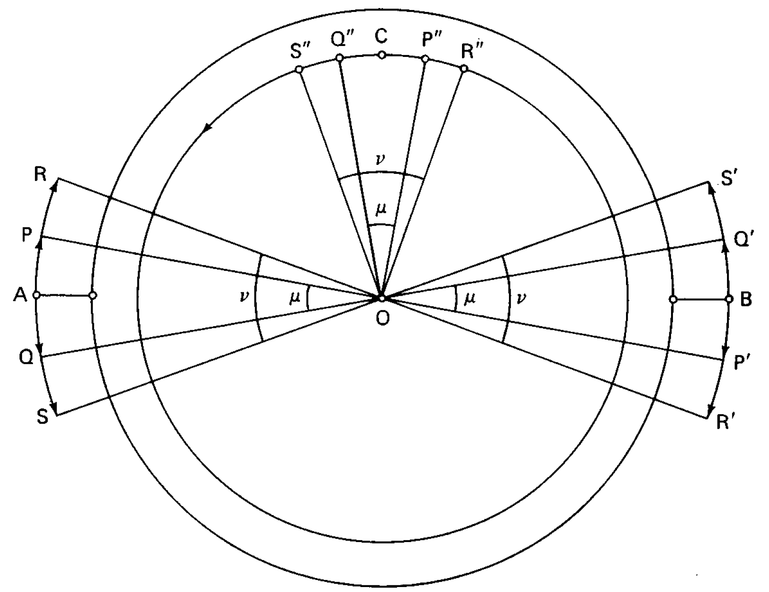

FIG. 1.

(3) Assuming that the back and forth oscillations occur in equal times, the larger the arcs of reciprocation, the greater will be the acceleration or retardation of the motion of the lower sphere.

These principles are illustrated in Figure 1 in which O is one pole of the lower sphere, on the equator of which a point C is moving uniformly. The poles A and B of the higher sphere oscillate back and forth in the plane of the equator of the lower sphere through the smaller arcs of reciprocation

$$\mu = PQ = P'Q'$$

or through the larger arcs of reciprocation

$$\nu = RS = R'S'$$

in equal times. The oscillation of the higher sphere is transmitted to the lower sphere, apparently through the pole O which is fixed in the higher sphere, and thus causes the uniform motion of C to be accelerated or retarded by receiving in addition to its uniform motion an oscillation through the smaller arc of reciprocation

$$\mu = P''Q''$$

or the larger arc of reciprocation

$$\nu = R''S''.$$

Since the periods of oscillation are equal, the amount of acceleration or retardation depends only upon the amplitude of the arcs of reciprocation. During the forward part of the oscillation C will move faster than the mean motion. When

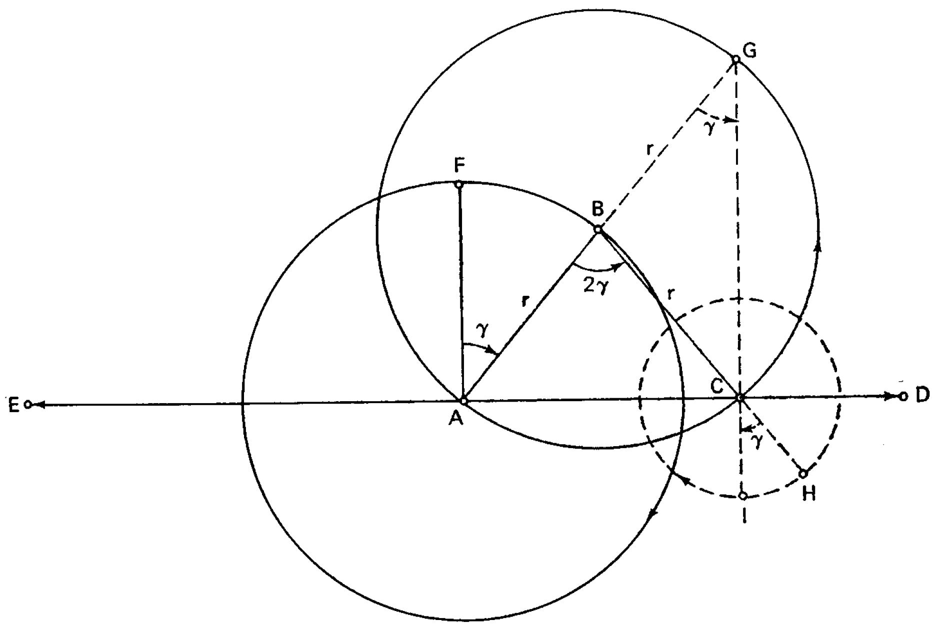

Fig. 2.

the backward part of the oscillation is faster than the mean motion, C will move retrograde, when it is slower, C will move more slowly, and when both motions are equal, C will be stationary.

So far there are two spheres, a lower sphere revolving uniformly and a higher sphere oscillating back and forth. The oscillation of the poles of the higher sphere is controlled by two still higher spheres. It is the movement of these next two spheres that is identical to Copernicus's libration mechanism (Chapters 9_1 and 9_2). In Figure 2, we let A be one pole of the outermost sphere, which is located in the plane of the arc of reciprocation. This sphere revolves uniformly through an angle γ. We then let B be the pole of a contiguous lower sphere at a distance $AB=r$, and we let the lower sphere revolve in the opposite direction through angle 2γ. Now, were this in a plane, as illustrated in the figure, a point C on the lower sphere at the distance $BC=AB=r$ would oscillate back and forth on the straight line DAE through the distance $DE=4r$, and the distance AC would be given by

$$AC=2r \sin \gamma. \tag{1}$$

Amico apparently believes that this also occurs on a sphere such that C oscillates back and forth on great circle DAE. For small values of r, Equation (1) is approximately true on a sphere, but where r is large, and it is about 20° in the case of Mars and Venus, C will not move on a great circle. Thus, if Figure 2 be considered spherically, and if angle $FAB=\gamma$ and angle $ABC=2\gamma$, then in isosceles triangle ABC

$$\text{angle } BAC+\text{angle } ABC+\text{angle } BCA>180°, \tag{2}$$

and therefore angle $FAC>90°$, angle $AGC>\gamma$, and angle $GCA>90°$. Equation (1) is consequently invalid, and this, of course, ruins Amico's models. Nevertheless, we shall describe them as though they actually performed as circles in a plane, and this is evidently the way Amico thinks of them. Our figures and equations will be in the plane, but it is to be understood that they are only an approximation to the strict geometric behaviour of the models.

The two spheres performing the oscillation are called the 'spheres of progression and regression' (*orbes accessus et recessus*), their name being taken from the theory of the trepidation of the eighth sphere (*motus accessus et recessus octavae sphaerae*) which also involves a forward and backward motion although without the simplicity of a linear oscillation.

The relation of the reciprocation to an epicyclic model is quite simple. $AE=AD$ is the maximum equation of the anomaly, δ_{max}, subtended by the radius of the epicycle when it is at the point of tangency with the line drawn from the centre of the Earth tangent to the epicycle. In Amico's models the maximum equation always occurs at 90° of anomaly, whereas in an epicyclic model the maximum equation occurs at $90°+\delta_{max}$. This in itself will produce an error of nearly 8° for Mars and more than 10° for Venus at 90° of anomaly. Amico does not mention this discrepancy. The entire arc of reciprocation $DE=2\delta_{max}$, and since $DE=4r$, thus $r=\frac{1}{2}\delta_{max}$. This will be somewhat modified in the planetary models to take account of the zodiacal anomaly, corresponding to the movement of the centre of the epicycle on the eccentric.

Now, the pole of the higher of the two spheres in Figure 1 is fixed at point C, and thus it will oscillate back and forth through the great circle arc *DE*. Amico then points out that, due to the motion of the two higher spheres, the pole at *C* will not only be moved back and forth on *DE*, but will also receive a rotational motion, as a point *H* on the sphere with pole *C* will lie on the line *FA* when *C* and *A* coincide, but will be turned 90° and lie on the line *AD* when *C* coincides with *D*. Therefore it is necessary that this sphere move in the opposite direction through angle γ from *H* to *I*. This third sphere is called a 'withstanding' (*obsistens*) sphere.[7]

The reciprocation mechanism thus requires three spheres, and directly within them is the fourth sphere, revolving uniformly and carrying the planet fixed on its equator. This is Amico's one and only geometrical model, and it is highly flawed. By combining reciprocating mechanisms in various ways he attempts to account for all planetary motions in longitude and latitude. We may now proceed to the individual models.

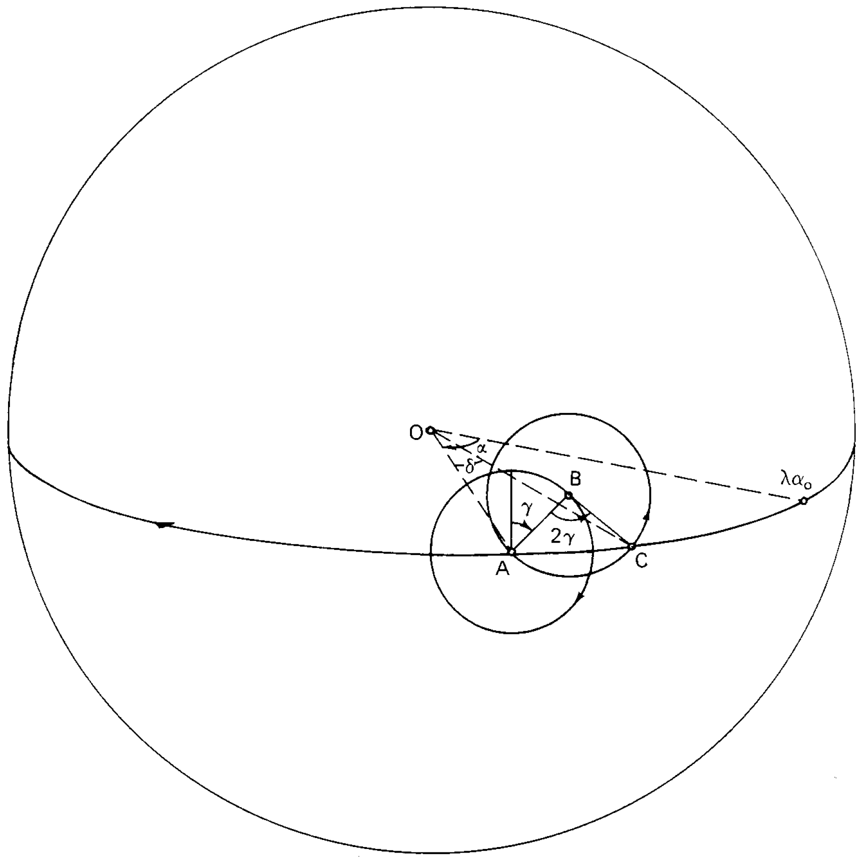

FIG. 3.

The Sun and the Simple Lunar Model (Chapters 8 and 9_2)

The model is shown in Figure 3. In this and all further drawings and descriptions, the third 'withstanding' sphere is omitted, but it must always be understood to be present directly below the two 'spheres of progression and regression'. Further, the reciprocation mechanism is shown as if it were moving along with the planetary body and we shall speak of it in this way, although in fact it is located at opposite sides of the sphere carrying the planet and simply gives the reciprocal motion to the entire lower sphere. Finally, Amico treats the Moon very cursorily along with the planets, but for the sake of greater clarity I shall treat it in two parts.

We imagine the observer at O, and the centre of the reciprocation mechanism A moving from west to east through angle α of mean motion measured from the point $\lambda\alpha_0$ which corresponds to the longitude of the apogee in an eccentric or epicyclic model. In the same time, AB revolves through angle γ of mean anomaly, and BC through angle 2γ, both in the directions indicated in the figure. Thus, C will oscillate back and forth about A in the angle δ where

$$\delta = 2r \sin \gamma, \tag{3}$$

so that the true longitude λ will be given by

$$\lambda = \lambda\alpha_0 + \alpha - \delta. \tag{4}$$

In the case of the Sun, $\alpha = \gamma$, so the return to $\lambda\alpha_0$ will be equal to the period of the anomaly. In the case of the Moon, $\alpha > \gamma$, so the return in mean motion will occur before the return in anomaly, and the result will be, in effect, an eastward movement of $\lambda\alpha_0$ just as in Ptolemy's lunar model. Some of this is not explained very clearly by Amico, but this is surely what he has in mind. There is a further complication of the lunar model corresponding to Ptolemy's variation of the eccentricity which will be taken up along with the complex model for the planets.

The Planets and the Complex Lunar Model (Chapters 11 and 12)

We first consider the simple case of taking only the solar anomaly as though, in an epicyclic model, the epicycle moved on a circle concentric to the Earth with a radius equal to the distance of the apogee, that is, as though the epicycle were always at greatest distance. The model is shown in Figure 4. Again, imagine the centre of the reciprocation mechanism moving from west to east through angle α of mean motion in longitude measured from $\lambda\alpha_0$ in a plane inclined to the plane of the ecliptic by the same amount as the inclination of the plane of the deferent in an epicyclic model. In the case of Venus and Mercury, α is the mean motion of the Sun so the longitude of A is always that of the mean Sun, and the mean anomaly γ is independent of α. For the superior planets, γ is always equal to the distance from A to the mean Sun, that is, $\gamma = AOS$, so that at mean conjunction A and S are at the same longitude and $\gamma = 0°$. As the Sun moves ahead of A after mean conjunction through angle γ, AB revolves through γ, and BC through 2γ, both in the directions shown in Figure 4, which are opposite to the motions of γ for the Sun and Moon in

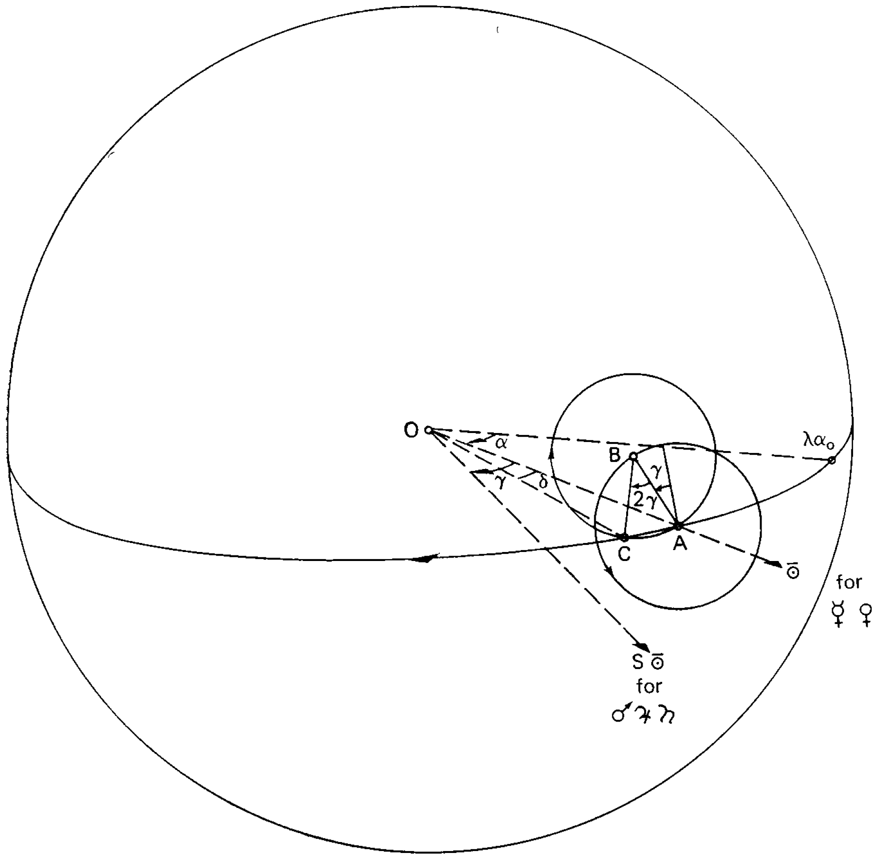

FIG. 4.

Figure 3, just as in Ptolemy's models. As before, δ follows from Equation (3), and the true longitude λ is given by

$$\lambda = \lambda\alpha_0 + \alpha + \delta. \tag{5}$$

Now, this model is as though the epicycle were always at the distance of the apogee, and thus the maximum equation is only the least value of the maximum equation, which is equal to the maximum equation of the anomaly minus the maximum difference for the epicycle at greater distance. Denoting the least maximum equation by $\delta_{\min}$, and column numbers in the equation tables of the *Alphonsine Tables* by c,

$$\delta_{\min} = c_{6\max} - c_{5\max}. \tag{6}$$

It follows that the arc of reciprocation, which Amico calls the 'lesser arc' (*arcus minor*), is

$$\mu = 2\delta_{\min} = 2(c_{6\max} - c_{5\max}), \tag{7}$$

and the radii of the circles

$$r = \tfrac{1}{4}\mu = \tfrac{1}{2}\delta_{\min} = \tfrac{1}{2}(c_{6\max} - c_{5\max}). \tag{8}$$

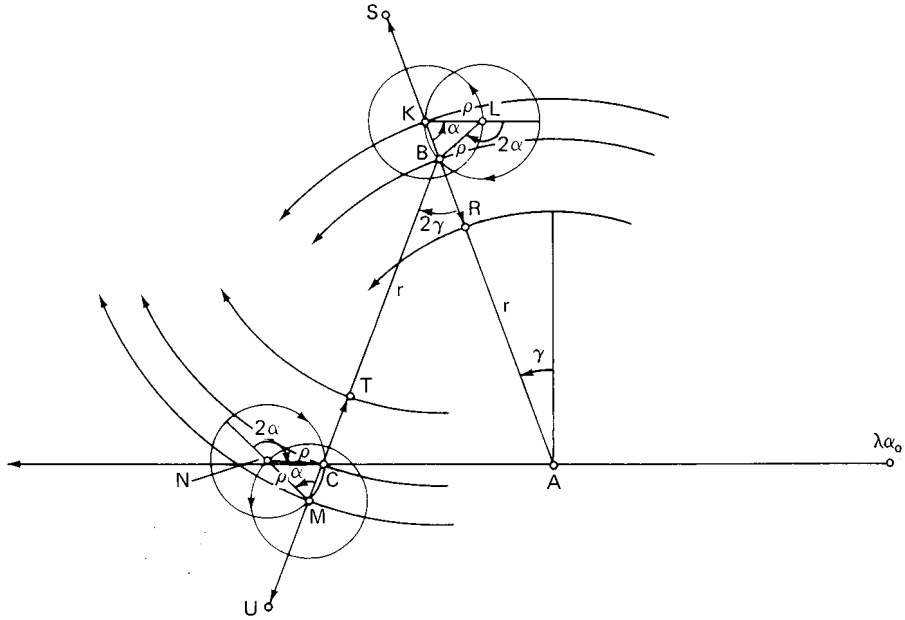

FIG. 5.

In order to represent the zodiacal anomaly Amico employs two additional reciprocation mechanisms in such a way that the equation of the anomaly is increased as a function of the mean distance of the planet from $\lambda\alpha_0$, that is, as a function of α. This is exactly equivalent to varying the radius of the epicycle as in certain Indian planetary models, and has the same effect as the change in the distance of the epicycle as it moves on an eccentric circle. It works in much the same way as the device used by Ibn ash-Shāṭir and Copernicus for moving Mercury's epicycle in and out.

The complex model is shown in Figure 5. The highest sphere moves through the mean anomaly γ about pole A, and fixed in this sphere is pole K of the higher sphere of another reciprocation mechanism. This sphere revolves about K through angle α of the mean motion in longitude which is equal to zero when A is at $\lambda\alpha_0$, and in the same time the sphere under pole L, which is carried about K through α, revolves through 2α, all motions taking place in the directions indicated in the figure. The pole B of the second sphere of the original reciprocation mechanism is carried about L through 2α at the distance

$$\rho = KL = LB.$$

The result is that B will move in and out between R and S on line ARS so that the distance AB will vary from

$$AB_{\min} = AR = r$$

to

$$AB_{\max} = AS = r + 4\rho.$$

The sphere under B then revolves through 2γ as before, but in order to keep C oscillating on a straight line it is necessary to place yet another reciprocation device between B and C, namely, the spheres with poles M and N, which also move through α and 2α just as K and L. In this way BC will always be equal to AB. It must be understood that there is also a 'withstanding' sphere below each pair of reciprocating spheres, and thus this construction requires ten spheres with poles and motions in the descending order $A_{(\gamma)}$, $K_{(\alpha)}$, $L_{(2\alpha)}$, withstanding $_{(\alpha)}$, $B_{(2\gamma)}$, $M_{(\alpha)}$, $N_{(2\alpha)}$, withstanding $_{(\alpha)}$, $C_{(\gamma)}$, which is itself a withstanding sphere, and finally the sphere bearing the planet through its mean longitudinal motion.

The distances vary such that when A is at $\lambda\alpha_0$,

$$AB=AR=BC=BT=r,$$

so the conditions of the simple model hold and the equation of the anomaly will reach

$$\delta_{\min}=2r.$$

But when A has moved 180° from $\lambda\alpha_0$,

$$AB=AS=BC=BU=r+4\rho.$$

At this time the equation of anomaly will be

$$\delta=2(r+4\rho)\sin\gamma, \tag{9}$$

the maximum value will reach

$$\delta_{\max}=2(r+4\rho),$$

and the arc of reciprocation, which Amico calls the 'greater arc' (*arcus maior*),

$$\nu=4(r+4\rho).$$

From the *Alphonsine Tables*

$$\delta_{\max}=c_{6\max}+c_{7\max}, \tag{10}$$

and

$$\nu=2(c_{6\max}+c_{7\max}). \tag{11}$$

It follows as before that

$$r=\tfrac{1}{2}\delta_{\min}=\tfrac{1}{2}(c_{6\max}-c_{5\max}), \tag{8}$$

and further that

$$\rho=\tfrac{1}{8}(\delta_{\max}-\delta_{\min})=\tfrac{1}{8}(c_{5\max}+c_{7\max}). \tag{12}$$

The radii of the small circles are thus very small. From the equations in the *Alphonsine Tables*, the largest, Mars, will be 1;42,37,30°, and the smallest, Saturn, only 0;5,45°.

Amico gives no explanation of how to tabulate the equations; he does not even give the equations. If one wanted to solve directly by trigonometry, one would first observe that the increase in the length of AB and BC is a function of the versed sine of α. In this case, since ρ is small, there is no error worth noting if this be treated in the plane. Thus,

$$RB = TC = 2\rho \text{ vers } \alpha = 2\rho(1-\cos\alpha), \quad (13)$$

so that

$$AB = BC = r + 2\rho \text{ vers } \alpha = r + 2\rho(1-\cos\alpha), \quad (14)$$

and the equation for any values of α and γ will be given by

$$\delta = 2(r + 2\rho \text{ vers } \alpha) \sin\gamma$$
$$= 2[r + 2\rho(1-\cos\alpha)] \sin\gamma. \quad (15)$$

This could be tabulated in three columns by placing in the first the simple equation for $\alpha=0°$, that is, for each value of γ

$$c_1 = \delta_{\min} = 2r \sin\gamma, \quad (3)$$

and in the second the difference for $\alpha=180°$, that is, for each value of

$$c_2 = \delta_{\max} - \delta_{\min} = 2(r+4\rho) \sin\gamma - 2r \sin\gamma$$
$$= 4\rho \sin\gamma. \quad (16)$$

The third column would contain the proportional minutes for the increase in the length of AB and BC as a function of α; that is, if the entire variation is put equal to 60 minutes, the correction would be simply half the versed sine

$$c_3 = 30 \text{ vers } \alpha = 30(1-\cos\alpha), \quad (17)$$

and c_3 would vary from 0 to 60 as α varied from 0° to 180°. The equation of the anomaly would then be computed from

$$\delta = c_{1(\gamma)} + (c_{3(\alpha)} \cdot c_{2(\gamma)}), \quad (18)$$

and the sign of c_1, positive or negative, would immediately indicate whether δ is to be added to or subtracted from the mean motion.

Since Amico does not discuss this at all, it is possible that he believes that one should continue using ordinary tables for the computation of planetary positions (which, after all, is the business of the 'Astrologers') although envisioning the motions of the planets as he has described them. At no point, however, does he indicate that the models are, at best, only rough, qualitative representations of the planetary motions. He is surely unaware that the reciprocation device only works in the plane, and even if it is assumed that the plane representation is an adequate model of planetary motions there are still gross deficiencies that render the models greatly inferior to Ptolemy's. The principal discrepancies with epicyclic models are as follows:

(1) The equations of anomaly are symmetrical about 90° of anomaly which is not true of an epicyclic model and leads to severe errors for Mars and Venus.

(2) The correction of the equation of anomaly as a function of α is symmetrical about 90° which also is not true for tables computed for the movement of the centre of an epicycle on an eccentric.

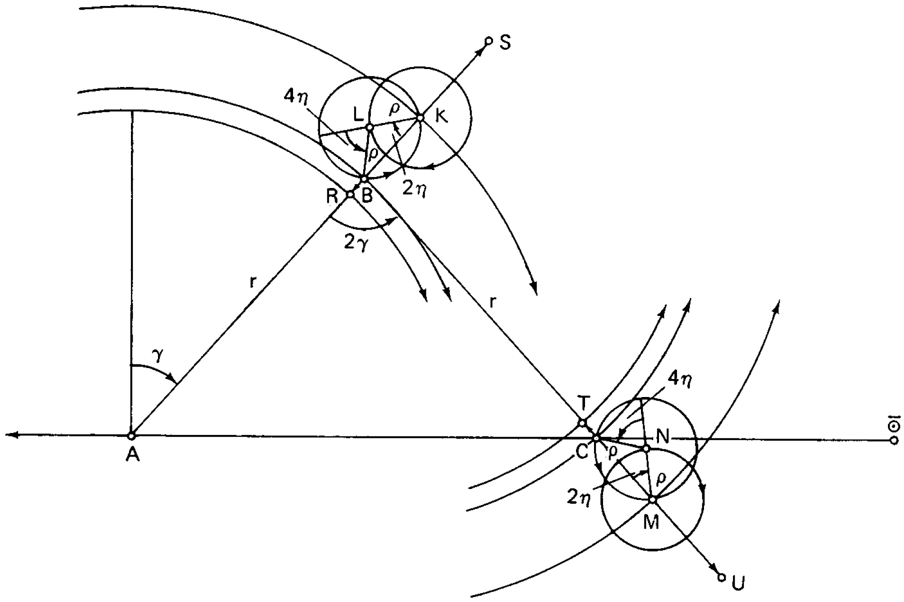

FIG. 6.

(3) The refinements of Ptolemy's models, that is, the bisection of the eccentricity and the direction of the mean apogee of the epicycle towards the equant, are completely eliminated. There is really no equation of centre in Amico's models, as the correction of the equation of anomaly as a function of α is equivalent only to the variation of the distance of the centre of the epicycle on the eccentric. In a sense, Amico has developed models that are probably equivalent in effect to a pre-Ptolemaic state of planetary theory.

(4) Mercury is given no special consideration. This means that Amico is disregarding or rejecting Ptolemy's construction for reducing the distance of the epicycle at $\pm 120°$ from the apogee. It would be possible to approximate this by two additional sets of reciprocating circles with motions 2α and 4α, which would be equivalent to the solution used by Copernicus and Ibn ash-Shāṭir, but Amico does not so much as mention that there is anything unusual about Mercury.

Amico treats the Moon along with the rest of the planets without pointing out that the model must differ in some respects. Figure 6 shows what I take to be the lunar model. It is identical to the planetary models except that:

(1) We imagine the centre of the reciprocation mechanism A moving with the velocity of mean elongation from the Sun through distance $\bar{\odot}A=\eta$.

(2) The directions of the movements of AB and BC through γ and 2γ are opposite to the corresponding movements for the planets.

(3) The two additional sets of reciprocating circles move with velocities 2η and 4η, that is, LK and MN revolve through 2η, and KB and NC through 4η. In this way the equation of anomaly will be increased at quadratures and unchanged at syzygies.

There is nothing in the model equivalent to Ptolemy's direction of the mean apogee of the epicycle towards the point opposite the centre of the deferent. On the brighter side, the principal defect of Ptolemy's model, the enormous variation in distance, is certainly solved since the distance does not vary at all.

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2. J. L. E. Dreyer, *A history of astronomy from Thales to Kepler* (2nd ed., New York, 1953), 301–3.
3. The principal literature is: Carra de Vaux, *Les sphères célestes selon Nasîr Eddîn Attûsî*, Appendice VI in P. Tannery, *Recherches sur l'histoire astronomie ancienne* (Paris, 1893), 337–59; V. Roberts, "The Solar and Lunar Theory of Ibn ash-Shāṭir: A Pre-Copernican Copernican Model", *Isis*, xlviii (1957), 428–32; E. S. Kennedy and V. Roberts, "The Planetary Theory of Ibn al-Shāṭir", *Isis*, l (1959), 227–35; F. Abbud, "The Planetary Theory of Ibn al-Shāṭir: Reduction of the Geometric Models to Numerical Tables", *Isis*, liii (1962), 492–9; V. Roberts, "The Planetary Theory of Ibn al-Shāṭir: Latitudes of the Planets", *Isis*, lvii (1966), 208–19; E. S. Kennedy, "Late Medieval Planetary Theory", *Isis*, lvii (1966), 365–78; W. Hartner, "Naṣir al-Dīn al-Ṭūsī's Lunar Theory", *Physis*, xi (1969), 287–304.
4. *De revolutionibus* III, 4; *Nikolaus Kopernikus Gesamtausgabe*, i (München and Berlin, 1944), f. 75r.
5. O. Neugebauer has found in MS *Vat. Gr.* 211, f. 116r, figures showing a model employing Ṭūsī's rolling circle device. One may wonder if more of this material is to be found in Greek manuscripts in Italy, and further, if any Italian astronomers and natural philosophers of the late-fifteenth and early-sixteenth centuries knew of the Marāgha astronomers' models and wrote treatises on planetary theory incorporating them. I would guess that if there are any surviving intermediaries between Marāgha and Copernicus they are to be found in Italy. The discovery of such works would place the *Commentariolus* clearly in perspective as an adaptation of Marāgha models to a heliocentric arrangement and confirm its suspected early date, as it would then appear the result of Copernicus's years in Italy.
6. *De revolutionibus* (Nuremberg, 1543), f. iiib; *Nikolaus Kopernikus Gesamtausgabe*, ii (München, 1949), 5, lines 1–3.
7. Such an additional sphere to counteract a rotational motion is also used by Ṭūsī in his lunar model; see Carra de Vaux and Hartner, *op. cit.*