JHA iii (1972), 27–35

PRECESSION AND TREPIDATION IN INDIAN ASTRONOMY BEFORE A.D. 1200

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The process by which various non-Ptolemaic elements of the Greek astronomical tradition were transmitted to India and were there transformed into the astronomy of the *siddhāntas* is a subject of complexity and of obscurity. Its elucidation, however, is of great historical importance, both for the understanding it will afford us of the motivation for particular Indian solutions of problems in mathematical astronomy, and for the insight we will obtain from it into those areas of Hellenistic astronomy that, being almost totally eclipsed in Greek by the brilliance of Ptolemy's *Almagest*, can be discerned, though dimly, in the poetry of *jyotiḥśāstra*. The present paper contains an investigation into one aspect of this process, that in which the ideas both of the precession and of the trepidation of the equinoxes were introduced into India and there interpreted in terms of an older Indian tradition of the position of the solstices relative to the *nakṣatras* and in other ways. This example, like that of the planetary model previously discussed in this journal (ii (1971), 80–85), beautifully illustrates the failure of the Greeks to communicate and of the Indians to grasp the full significance of the concepts transmitted.

The *Jyotiṣavedāṅga*[1] of Lagadha (5/4th century B.C.?) states (Ārca 6 = Yājuṣa 7): "The Sun and the Moon begin their northern [course] at the beginning of Śraviṣṭhā [Dhaniṣṭhā]; the southern [course] of the Sun [begins] in the middle of Sarpa [Āśleṣā]. [The beginnings of these two courses occur] always in [the months] Māgha and Śrāvaṇa [respectively]." One also finds this scheme in, for example, the *Parāśaratantra* cited by Utpala (A.D. 966) on the *Bṛhatsaṃhitā*[2] of Varāhamihira (*ca* A.D. 550).

By the time of Varāhamihira a fixed sidereal zodiac was in use in India.[3] In this zodiac the beginning of Aries was identified with the beginning of the nakṣatra Aśvinī; in the fifth and sixth centuries the beginning of Aries was further said to be the point of the vernal equinox. Varāhamihira recognized the discrepancy with the statement of Lagadha (*Bṛhatsaṃhitā* 3, 1–2):

> Once, according to what is said in ancient treatises, the southern ayana of the Sun was from the middle of Āśleṣā, and the northern began with Dhaniṣṭhā. Now [one] ayana of the Sun begins at the beginning of Cancer, the other at the beginning of Capricorn. This is a negation of what was said; the difference is made manifest by direct observations.

In his *Pañcasiddhāntikā*[4] (3, 20–2) Varāhamihira explains this change by a theory of trepidation over an arc of 46;40°—23;20° (identified with the Sun's maximum declination) to either side of the equinox:

> When the sum [of the longitudes] of the Sun and Moon is a revolution, it is called Vaidhṛta [yoga]; but if it is a revolution plus 10 nakṣatras [133;20°],

> Vyatipāta. The time is to be ascertained by means of the degrees attained [by the luminaries]. When the return of the Sun was from the middle of Āśleṣā [at 113;20°], then the ayana [-correction] was positive; now the ayana is from Punarvasu [at 90°]. When the falling away [from the mean position] of the ayana is reversed, then the correction [kṣepa] for the Sun and Moon [equals] the degrees of the maximum declination [kāṣṭhā] of the Sun [23;20°]. There is Vyatipāta if the sum [of the longitudes] of the Sun and the Moon is 180°.

This is the earliest datable reference to a theory of trepidation or precession in India; unfortunately no rate is given. A theory of trepidation was known to Theon of Alexandria (A.D. 361) and to Proclus (A.D. 410–485), and a theory of precession to Hipparchus (*ca* −126); Hipparchus's length for a tropical year was used by Sphujidhvaja (A.D. 269/70) in his *Yavanajātaka* (79, 34) and in the *Romakasiddhānta* summarized by Varāhamihira in his *Pañcasiddhāntikā* (1, 15 and 8, 1). It is not unreasonable to suppose that the *idea* of trepidation or precession was introduced into India by the Greeks, though the *parameters* chosen by the Indians are their own, and that the arguments presented in favour of the hypothesis of a motion of the colures are derived from a particular interpretation of the *Vedāṅgajyotiṣa*. This hypothesis of a Greek influence is strengthened by a passage quoted by Bhāskara (A.D. 629) in his commentary[5] on the *Āryabhaṭīya* of Āryabhaṭa (A.D. 499):

> Here the Romakas [*i.e.*, Romans], who do not know the ultimate purpose, read: 'The sages say that an ayana [begins] from the beginning of Vasudeva [Dhaniṣṭhā] [and another] from the middle of Sarpa [Āśleṣā], whereas it is observed [that they begin] from the beginnings of Capricorn and Cancer. How can this be without a motion [of the solstices]?'

Several of these sources and some new ones, including Maṇindha,[6] are cited by Govindasvāmin (*ca* A.D. 850) in his commentary on the Uttarakhaṇḍa (2, 21b–25a) of Parāśara's *Horāśāstra*[7] (*ca* A.D. 650/750):

> The motion of the ayana has been described by former teachers. In this [matter] Maṇindha [says]: 'Hence the planets move "up" from the prime vertical [= equinoctial colure] 27° in 1800 years'. [It is said] by Sūrya [*Sūryasiddhānta* 3, 9a–b]: 'The circle of the constellations lags to the east 600 [times] in a [Mahā-]yuga'. On the other hand Bhāskarācārya, who completely adheres to the opinion of Āryabhaṭa, says [as shown above, he attributes this to the Romakas]: 'The sages say that an ayana [begins] from the beginning of Vasudeva [Dhaniṣṭhā] [and another] from the middle of Sarpa [Āśleṣā]'. And Varāhamihira [*Bṛhatsaṃhitā* 3, 1a–b]: 'The southern ayana of the Sun was from the middle of Āśleṣā, and the northern began with Dhaniṣṭhā'. In this [matter] Haridatta[8] [A.D. 684] [says]: 'They are 24°; the planets move from that'. The meaning of 'from that' is 'from the ecliptic'.

Govindasvāmin goes on to explain these obscure passages in terms of two schools advocating different rates and arcs of trepidation:

> This is what was said. There is one opinion of Āryabhaṭācārya and so on, another opinion of Maṇindha, Sūrya, and so on. Here Āryabhaṭa and so on [say] that the fact that the [vernal] equinox was at the beginning of Aries

> in Śaka 444 [A.D. 522] is established by such instruments as the gnomon.[9] They determined that after this it moves [at the rate of] 0;1° in one year. So it moves 'down' 24°. Then one should add the minutes of trepidation. Afterwards it moves 'up'. Then, subtracting 0;1° a year from 24°, one should add [the remainder].
>
> The opinion of the *Sūryasiddhānta* and so on [is this]. It moves 'down' 27° in 1800 years. Then it moves 'up'. In this [theory] this is the mathematical operation. In 1800 years 27° are obtained. One should operate with the given [years]. Divide the lapsed years of the Kaliyuga by 1800, multiply the remainder by 3, and divide [the product] by 200. [The result] is degrees, and the remainder is minutes. Or else divide the lapsed years of the Kaliyuga by 1800 and multiply the remainder by 54: these are seconds. It is to be known that in this school 0;0,54° represents the amount of trepidation of one year. It was shown by the previous commentator, Bhagadatta [*ca* A.D. 800], that this school is the better; this also is our tradition. Here it is to be understood that, if the quotient of the division by 1800 is an even number, the motion [of trepidation] is 'down', and if it is odd, 'up'.

This opinion of Maṇindha or Manetho seems the older theory of trepidation. The arc of 27° approximates the 26;40° in the difference of two equal nakṣatras between the Vedic lists (of unequal nakṣatras) beginning with Kṛttikā and the fixed sidereal zodiac beginning with Aśvinī; the 26;40° is probably rounded off to 27° to reflect the number of nakṣatras and to provide nice parameters. For the vernal point is assumed to have coincided with the beginning of Aśvinī and of Aries at the beginning of the Kaliyuga in −3101 and again 3600 years later in A.D. 499; the second part of this assumption is very nearly correct. Then the Vedic nakṣatra-lists beginning with Kṛttikā are explained by a progressive motion of the vernal point to fixed Aries 27° (= fixed Kṛttikā 0;20°) over half of this period, and a subsequent retrograde motion back to fixed Aries 0° over the second half; the retrograde motion continues at the same rate after A.D. 499 so that the trepidation will reach its limit at fixed Pisces 3° in A.D. 2299. The rate of trepidation, as Govindasvāmin correctly remarks, is:

$$\frac{27^\circ}{1800^y} = \frac{3^\circ}{200^y} = 0;0,54^\circ/^y.$$

This, of course, is the rate of precession "established" by Yaḥyā ibn Abī Manṣūr on the basis of an observation of the autumnal equinox that he made on 19 September 830.[10]

Govindasvāmin correctly refers to the *Sūryasiddhānta* as adhering to the opinion of Maṇindha; in the modern versions the relevant verses are (3, 9–10b)[11]:

> The circle of the constellations lags 600 [times] to the east in a [Mahā-]yuga. Multiply this by the ahargaṇa and divide [the product] by the civil days [in a Mahāyuga]. Multiply the resulting arc by three and divide [the product] by ten; the [resulting] degrees are to be known as the [amount of] trepidation.

A total oscillation of the vernal point, including both the progressive and the retrograde motions, occurs in 7200 years over an arc of 108°. In a Mahāyuga

of 4 320 000 years there are 600 such oscillations; and 108° is $\frac{3}{10}$ of 360°.

A number of later texts follow this original version of Maṇindha's trepidation: see, for example, *Vṛddhavasiṣṭhasiddhānta* (2, 36) and the *Śākalyasaṃhitā's Brahmasiddhānta* (1, 192 and 194).[12] Some scholars, however, did advocate a modification of the parameters in the direction of a slowing down of the velocity of the trepidational motion. Thus Āmarāja (*ca* A.D. 1200) cites in his commentary[13] on the Khaṇḍakhādyaka (3, 11) of Brahmagupta (A.D. 665) a verse which he attributes to the *Uttara*,[14] which is an appendix to the *Khaṇḍakhādyaka* expounding non-ārdharātrika views: "Add 3179 to the Śaka-years and divide [the sum] by 7380; having added or subtracted [the quotient] from the beginning of the [fixed] zodiac, the degrees of declination north or south [are to be computed]".

The addition of 3179 to the epoch of the Śaka era results in the epoch of the Kaliyuga, −3101, when the current oscillation is assumed to have begun. But division by 7380 instead of 7200 decreases the rate of trepidation by 180/7200 or 1/40. Another result of such a division is to place the end of half an oscillation not at Kali 3600 (A.D. 499), but at Kali 3690 (A.D. 589). Āmarāja's further discussion shows that he understands this much; he states that there is no trepidation in Śaka 511 (A.D. 589), but he assumes that the limit of trepidation is 24° from fixed Aries 0°, a value which is connected with the second opinion of Govindasvāmin.

Āmarāja goes on to explain wrongly several other passages which in fact utilize the 24°-limit and the rate of 0;1° per year. Thus, on a passage of the *Karaṇakutūhala*[15] in which Bhāskara (A.D. 1183) states that the trepidation in Śaka 1105 was 11°, he remarks: "Because it makes a 'revolution' in 7380 years, there are 585 336 'revolutions' of the vernal point in a Kalpa [of 4 320 000 000 years]". Indeed, $585\,366 \times 7380 = 4\,320\,001\,080$, so that this number is nearly correct. Āmarāja further explains *Bṛhatsaṃhitā* (3, 1–2) on the grounds that Varāhamihira died in Śaka 509 (A.D. 587), when the trepidation was nil.

In the later Āryabhaṭa's (between *ca* A.D. 950 and 1100) *Mahāsiddhānta*[16] (1, 11–12) the 'revolutions' are numbered 578 159; this implies an oscillation in very nearly 7472 years. In his summary of the *Parāśarasiddhānta* (2, 9), this Āryabhaṭa gives the number of 'revolutions' as 581 709; this implies an oscillation in about 7426½ years.

The second opinion Govindasvāmin claims to belong to the school of Āryabhaṭa[17] because of the verse cited from Bhāskara; but we have seen that Bhāskara quotes this with disapproval from the Romakas. Nevertheless, the opinion does belong to the school of Āryabhaṭa, though the connection is through the date Śaka 444 (A.D. 522) at which the vernal point was, according to Govindasvāmin, demonstrated by means of a gnomon to be at fixed Aries 0°. The date seems rather to have been derived from a misunderstanding of a verse in the *Āryabhaṭīya* (Kālakriyā 10): "When three quarters of a [Mahā-]yuga and 3600 years had passed, then 23 years from my birth here had passed". This was quite early interpreted to mean that Āryabhaṭa was born in Kali 3600 and wrote his *Āryabhaṭīya* in Kail 3623 or Śaka 444.[18]

Another parameter, however, is derived from the tradition preserved by Varāhamihira in the verses of the *Pañcasiddhāntikā* quoted above, and apparently belongs to the school of Pauliśa, which was certainly strongly

influenced by Greek methods. Pauliśa, noting the difference of 23;20° between the middle of fixed Āśleṣā and fixed Cancer 0°, concluded that the limit of the arc of trepidation from fixed Aries 0° was 23;20°, which he identified with the maximum declination of the Sun. The usual obliquity of the ecliptic noted in Indian texts is 24°, and it is this amount that the second system takes as the limit of trepidation.

In this system, then, a coincidence of the vernal point and the fixed Aries 0° occurs in A.D. 522, the limit of the trepidation is 24° to either side of the fixed Aries 0°, and the rate of trepidational motion is 0;1° a year or 1° every 60 years. A period of oscillation, then, is 5760 years, and there are 750 such oscillations in a Mahāyuga; but the vernal point and the fixed Aries 0° did not coincide at the beginning of either the Mahāyuga or the Kaliyuga.

The earliest reference to this system seems to be in the Pūrvakhaṇḍa (*ca* A.D. 600/700) of the *Horāśāstra*[19] of Parāśara (3, 31a–b): "Subtract 444 from the Śaka-year and divide [the remainder] by 60; [the result is] the degrees of trepidation". Thereafter it is often repeated. Āmarāja (on *Khaṇḍakhādyaka* 3, 11) says that Pṛthūdakasvāmin (A.D. 864) claimed that the trepidation in Śaka 800 (A.D. 878) would be 6;30°;[20] this should be corrected to 6° as there are 356 (60 × 5;56) years between Śaka 444 and Śaka 800. Muñjāla (A.D. 932) is quoted by several sources[21] as giving a trepidation of 6;50° for Śaka 854, which is 410 (60 × 6;50) years after Śaka 444. Śrīpati (A.D. 1056), in his *Dhruvamānasa*,[22] gives the same rule as appears in Parāśara's *Horāśāstra*. Śatānanda (1099), in his *Bhāsvatī* (8, 1),[23] gives the rate of precession as 1° in 60 years, but makes the year of coincidence Śaka 450 (A.D. 538). And Bhāskara (A.D. 1183) in his *Karaṇakutūhala* (2, 17)[24] gives a trepidation of 11° for Śaka 1105, which is 661 (60 × 11;1) years after Śaka 444.

But there are some modifications of these parameters which occur; motivations for them are not apparent. Thus, Āmarāja cites two verses from a *Karaṇottama* (on *Khaṇḍakhādyaka* 3, 11). The first states that the trepidation for the epoch of this work is 600 minutes; at the usual rate of trepidation this would date the text in Śaka 1044 (A.D. 1122). The next verse is somewhat corrupt, but the most reasonable interpretation that occurs to me is that one should subtract 4217 from the lapsed years (since the beginning of Kaliyuga) and divide (the remainder) by 61. Kali 4217 corresponds to A.D. 1116—just 6 years before the date toward which the first verse pointed. The rate of trepidation—1° every 61 years—means that the traversal of an arc of 24° takes 1464 years, a complete oscillation of 96° 5856 years. There is not an integral number of complete oscillations in a Mahāyuga.[25]

The second variant is quoted by Parameśvara (*ca* A.D. 1380–1460) in his commentary on the *Āryabhaṭīya* (Kālakriyā 10): "Subtract 3600 from the Kali-year and divide [the remainder] by 5808; divide the remainder by 1152. Multiply the remaining years by 2 and divide [the product] by 121. The remainder is the equation in degrees and so on". Here Kali 3600 is substituted for Kali 3623 as the year in which the coincidence of the vernal point and the fixed Aries 0° occurred; in this Parameśvara follows Maṇindha. The rate of trepidation also is changed from 1° in 60 years to 2° in 121 years (1° in $60\frac{1}{2}$ years). As a complete oscillation is 96° (24 × 4), the period is 5808 years (96 × $60\frac{1}{2}$) and the time required for the traversal of an arc of 24° is 1152 years

$(24 \times 60\frac{1}{2})$. There is not an integral number of oscillations in a Mahāyuga.

Finally, Āmarāja (on *Khaṇḍakhādyaka* 3, 11) ascribed to "others" the following line: "Subtract 441 from the Śaka-year, multiply [the remainder] by 2, and divide by 119; the result is in degrees". Śaka 441 or Kali 3620 becomes the year of coincidence, and the rate is changed to 2° in 119 years or 1° in $59\frac{1}{2}$ years. The traversal of an arc of 24°, then, takes 1428 years, a complete oscillation of 96° 5712 years. Again there is not an integral number of oscillations in a Mahāyuga.

So far we have been examining theories of trepidation; but the theory of precession was not unknown in the period we have been investigating. The earliest reference is probably the corrupt number preserved by Bhāskara in his commentary on the *Āryabhaṭīya* and said to be the number of revolutions of the vernal point in a yuga according to the Romakas: viyadrudrakṛtanavadhṛtī or 1 894 110. If dhṛti (18) is emended to a word signifying 1, the resulting 194 110 could be the revolutions in a Kalpa; for the annual motion then would be $\frac{194\,110 \times 360^\circ}{4\,320\,000\,000}$ or approximately 0;0,58°. In fact, for a motion of 0;1° a year or 1° in 60 years there should be 200 000 revolutions of the vernal point in a Kalpa. The deviations from 200 000 that occur in this Romaka-passage and in others shortly to be discussed are presumably due to efforts to make the parameters yield a desirable coincidence of the vernal point and the fixed Aries 0° in about A.D. 500. Without knowing the interval each system would place between the beginning of the Kalpa and A.D. 500 we cannot reconstruct their computations.

The next such parameter of precession was given by Viṣṇucandra (*ca* A.D. 550/600) whose verse is cited by Pṛthūdakasvāmin (on *Brāhmasphuṭasiddhānta* 11, 54):[26] "The yuga of the ayana is said to be 189 411 [revolutions]; this was formerly the opinion of Brahmā, the Sun [Sūrya], and so on". It scarcely seems useful to compute the annual motion resulting from this parameter. The versions of the *Brahmasiddhānta* and the *Sūryasiddhānta* to which Viṣṇucandra refers are no longer available to us.

Finally, Bhāskara (A.D. 1150) has a very difficult passage (*Siddhāntaśiromaṇi*, Golādhyāya 7, 17–18):[27]

> The intersection of the equinoctial and the declinational circles is 'the node of the declination'. Its retrograde revolutions are said by the Saura [*Sūryasiddhānta*] to be 30 000 in a Kalpa. But the precessional motion proclaimed by Muñjāla and so on is correct; in this school its revolutions in a Kalpa are 199 699.

Bhāskara's reference here to the *Sūryasiddhānta* has caused much learned comment as it clearly conflicts with the verse from that work (3, 9) cited as early as Govindasvāmin;[28] it also conflicts with the statement regarding Sūrya's theory of precession made by Viṣṇucandra. The expression of 600 in the *Sūryasiddhānta* is "triṃśatkṛtyo" (thirty twenties); in some versions the word for twenties, "kṛtyo", is corrupt. Perhaps Bhāskara's statement reflects someone's misunderstanding of the *Sūryasiddhānta*, which was interpreted as saying that there are thirty trepidations in a Mahāyuga only, and therefore 30 000 in a Kalpa.

Bhāskara's statement regarding Muñjāla is not without difficulties either, though the number of revolutions in a Kalpa is closer to 200 000 than any other that is attested. Muñjāla, as was shown above, is known to have followed Govindasvāmin's second theory. Unless he wrote a third work besides the *Bṛhanmānasa* and the *Laghumānasa*, Bhāskara's reference must be wrong.

In conclusion, it has been established that before the twelfth century there existed three systems of accounting for the shift in the point at which the ecliptic crosses the equator which became evident when, in the fifth and sixth centuries, it was assumed that the nakṣatras in the Vedic literature and the *Vedāṅgajyotiṣa* referred to the equal nakṣatras of the fixed Indian zodiac. Two solutions involved trepidation over arcs whose limits were determined by an interpretation of the indigenous Indian traditions, the third simple precession; the rates were either 0;0,54° or 0;1° per year, with various modifications chosen to fit into particular contexts which, by and large, cannot now be reconstructed. Both trepidation and precession appeared in Greek before they appear in Sanskrit, and the origins of all three Indian systems are expressly connected with texts dependent on Greek sources: Maṇindha or Maṇittha, Pauliśa, and Romaka. The supposition is quite strong, then, that the theories of trepidation and precession were transmitted to India along with other astronomical theories between the second and fifth centuries A.D., although the specific parameters employed arise out of the adaptation of Greek astronomy to the Indian tradition.

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1. I have used the edition of both the Yājuṣa (with Somākara Śeṣa's *Bhāṣya*) and the Ārca recensions, prepared by Sudhākara Dvivedin, *The Pandit*, NS xxix (1907, repr. Benares, 1907); and the edition by R. Shamasastry (Mysore, 1936). The standard article on precession and trepidation in India remains that written by H. T. Colebrooke a century and a half ago ("On the Notion of the Hindu Astronomers concerning the Precession of the Equinoxes and Motions of the Planets", *Asiatic researches*, xii (1816), 209–50, reprinted in his *Miscellaneous essays*, ii (London, 1837), 374–416). This was used by T.-H. Martin, *Mémoire sur cette question: La précession des équinoxes a-t-elle été connue des égyptiens ou de quelque autre peuple avant Hipparque?* (Paris, 1869), 179–88; Martin is misunderstood by P. Duhem, *Le système du monde*, ii (Paris, 1914), 212–14 and 223–6. Later articles on the subject do not really advance our knowledge of the history of precession and trepidation in India; they represent a continuation of the misunderstanding of the implications of Vedic literature that, as this paper attempts to demonstrate, originated in about the fifth century A.D. Nonetheless, it may not be useless to mention the more significant of these: D. N. Mookerjee, "Notes on Indian Astronomy", *Journal of the Department of Letters, University of Calcutta*, v (1921), 277–302; J. Ghatak, "The Conception of the Indian Astronomers Concerning the Precession of the Equinoxes", *Journal of the Asiatic Society of Bengal*, NS xix (1923), 311–21; S. K. Das, "Precession and Libration of the Equinoxes in Hindu Astronomy", *Journal of the Asiatic Society of Bengal*, NS xxiii (1927), 403–13; R. Krishnamurthy, "Precession or Ayanamsa", *The mathematics student*, xiii (1945), 77–81; and K. V. Abhyankar, "The Precession of the Equinoxes and its Discovery in India", *Acharya Dhruva Smaraka Grantha* (Ahmadabad, 1946), iii, 155–64.
2. I have used the edition of the *Bṛhatsaṃhitā* with the *Vivṛti* of Utpala prepared by Sudhākara Dvivedin, *Vizianagram Sanskrit series*, xii (Benares, 1895–97).
3. It is to this zodiac, whose beginning lies in the vicinity of the star ζ Piscium, that I hereafter refer with the adjective "fixed"; in it the nakṣatras are equal arcs of 13;20° each. In the zodiac of the *Vedāṅgajyotiṣa* the nakṣatras are also equal arcs of 13;20° each, but the initial point of the system is not known to us. In the Vedic texts the nakṣatras are individual stars or groups of stars whose identity is not certain. Much confusion is introduced into the history of Indian astronomy by reading all texts as though they refer to the fixed zodiac; the specific parameters used for the limits of trepidation in Indian astronomy also result from this misunderstanding on the part of Indian astronomers in the fifth and sixth centuries A.D.

4. I have used the edition of O. Neugebauer and D. Pingree, *Det Kongelige Danske Videnskabernes Selskab*, Hist.-Filos. Skr., vi, 1 (Copenhagen, 1970).
5. Unfortunately still unpublished, though an edition is promised by K. S. Śukla of Lucknow. I cite this quotation from T. S. Kuppanna Sastri's edition of Bhāskara's *Mahābhāskarīya*, (with the *Bhāṣya* of Govindasvāmin and the *Siddhāntadīpikā* of Parameśvara), *Madras Government oriental series*, cxxx (Madras, 1957), p. XXVI.
6. Maṇindha is the prevalent South Indian orthography for Maṇittha, which represents the Greek Manethōn. Maṇittha's work was known to Varāhamihira, and so must have been written in the fourth or fifth century. The opinion is also ascribed to Maṇittha by Nīlakaṇṭha (on *Āryabhaṭīya*, Kālakriyā 10; see below, ref. 17).
7. I have used transcripts of MS 3166 in the Mysore Government Oriental Library and of MS D 11498 in the Sarasvati Mahal Library in Tanjore.
8. This verse is not found in his *Grahacāranibandhana* edited by K. V. Sarma, *Journal of oriental research, Madras*, xxiii (1953/54, repr. Madras, 1954).
9. For this method see, for instance, Parameśvara (A.D. 1443) in his second *Goladīpikā* (4, 85–90), ed. K. V. Sarma, *Brahmavidyā*, xx (1956), 119–86 and xxi (1957), 87–144; repr. as *Adyar library series* Paper 32 (Madras, 1957).
10. For the possible connection see *Dumbarton Oaks papers*, xviii (1964), 138.
11. I have used the editions by F.-E. Hall and Bāpū Deva Śāstrin with the *Gūḍhārthaprakāśaka* of Raṅganātha, *Bibliotheca Indica*, xxv (Calcutta, 1854–58); by Sudhākara Dvivedin, *Bibliotheca Indica*, clxxiii (Calcutta, 1910–11, repr. Calcutta, 1925); and by K. S. Śukla (with the *Vivaraṇa* of Parameśvara) (Lucknow, 1957). An apparent misunderstanding of this passage is found in Bhāskara's *Siddhāntaśiromaṇi* (see ref. 27), and Sumatiharṣa (A.D. 1619) in his commentary on Bhāskara's *Karaṇakutūhala* (2, 17; see below, ref. 24), ignoring 3, 9c–d, interprets *Sūryasiddhānta* 3, 9a–b to refer to a trepidation over an arc 60° (30° on either side of Aries 0°) at a rate of 0;1° per year. One complete oscillation, then, takes 7200 years as it should, but the rate of precession is that of Govindasvāmin's second school. Sumatiharṣa ascribes this interpretation to a ṭīkā on the *Sūryasiddhānta*. See also below, ref. 25.
12. Both of these texts are edited by V. P. Dvidevin, *Jyautiṣasiddhāntasaṅgraha, Benares Sanskrit series*, 2 fasc. (Benares, 1912).
13. I have used the edition of the *Khaṇḍakhādyaka* (with the *Vāsanābhāṣya* of Āmarāja) prepared by Babua Miśra (Calcutta, 1925).
14. The *Uttarakhaṇḍakhādyaka* is apparently by Brahmagupta himself. In his *Brāhmasphuṭasiddhānta* (11, 54; I have used the edition of Sudhākara Dvivedin (Benares, 1902)), written in A.D. 628, he denies a motion of the solstices; see also Bhāskara's *Vāsanābhāṣya* (on *Siddhāntaśiromaṇi*, Golādhyāya 7, 17–18; see below, ref. 27). But the *Uttarakhaṇḍakhādyaka* was written almost 40 years later.
15. See below, ref. 24.
16. I have used the edition of Sudhākara Dvivedin, *Benares Sanskrit series*, cxlviii–cl, 3 fasc. (Benares, 1910), and the edition of the Pūrvagaṇita by S. R. Sarma (2 vols., Marburg, 1966). Munīśvara (b. A.D. 1603) in his *Marīcī* on the *Siddhāntaśiromaṇi* (Golādhyāya 7, 17–19) refers to this Āryabhaṭa, not to the author of the *Āryabhaṭīya* as imagined by Colebrooke.
17. There is no mention of a motion of the equinoxes in the *Āryabhaṭīya*; I have used the editions of H. Kern (with the *Bhaṭadīpikā* of Parameśvara) (Leiden, 1874), and of K. Sāmbaśiva Śāstrī (1 and 2) and Suranad Kunjan Pillai (3) (with the *Bhāṣya* of Nīlakaṇṭha), *Trivandrum Sanskrit series*, ci, cx and clxxxv (3 vols., Trivandrum, 1930, 1931, and 1957).
18. See, *e.g.*, the texts collected by K. V. Sarma in his edition of Haridatta's *Grahacāranibandhana* (see above, ref. 8), pp. V–VI.
19. I have used the edition of G. L. Śarman and G. Śarman (Bambaī Saṃ. 2008, Śaka 1873 (A.D. 1951)).
20. I do not find this in Pṛthūdakasvāmin's *Vivaraṇa*, edited with the *Khaṇḍakhādyaka* by P. C. Sengupta (Calcutta, 1941); but only the beginning of his commentary on the *Uttarakhaṇḍakhādyaka* is preserved.
21. Apparently in his lost *Bṛhanmānasa*; it is quoted by the commentators on his extant *Laghumānasa* (1, 2), *e.g.*, by Praśastadhara (A.D. 958) as quoted in Majumdar's edition, p. 5, and by Parameśvara, and was noted by al-Bīrūnī (*India* (ed. Hyderabad, 1958), 308; trans. E. C. Sachau (London, 1910), i, 366–7) as occurring in the *Laghumānasa*. For the *Laghumānasa* I have used the editions of N. K. Majumdar (Calcutta, 1951), and of B. D. Āpaṭe, *Ānandāśrama Sanskrit series*, cxxiii (Poona, 1952). For another opinion attributed to Muñjāla by Bhāskara see below, ref. 27.

22. Quoted by Babuāji Miśra in his edition of Śrīpati's *Siddhāntaśekhara* (2 vols., Calcutta, 1932–47), i, 12.
23. I have used the edition of Ṭīkārāma Dhanañjaya (Benares, n.d.).
24. I have used the edition (with the *Gaṇakakumudakaumudī* of Sumatiharṣa) (Bambaī Saṃ. 1958, Śaka 1823 (A.D. 1901)).
25. Another possible interpretation would be: one should add 4217 to the lapsed years (since the epoch of the work) and divide (the result) by 61. One would then assume that the initial year of the Kaliyuga (−3101) was a year of coincidence, that the annual rate of trepidational motion is 0;1,1°, and that the limit of trepidation is 30° from fixed Aries 0° as in the case of Sumatiharṣa's interpretation of the *Sūryasiddhānta* (see above, ref. 11). Then the motion in 4217 years would be about 70°, so that the actual position of the vernal point relative to fixed Aries 0° would be −10°.
26. I have used MS Sanskrit 2769 of the India Office Library, London; see also the *Pañcasiddhāntikā*, i, 8.
27. I have used the edition of Dattātreya Āpaṭe (with Bhāskara's own *Vāsanābhāṣya* and Munīśvara's *Marīcī*), *Ānandāśrama Sanskrit series*, cxxii (2 vols., Poona, 1943–52).
28. See Munīśvara's *Marīcī*, *ad loc.*, and Colebrooke's summary thereof.

Note added in proof:

Since this article was written, the appearance of the complete text of the *Khaṇḍakhādyaka* in the edition by B. Chatterjee (2 vols, Calcutta, 1970) has made it clear that Āmarāja means by *Uttara* (see ref. 14) not Brahmagupta's *Uttarkhaṇḍakhādyaka*, in which the cited verse does not appear, but the *Khaṇḍakhādyakottara* of his (Āmarāja's) teacher Trivikrama (A.D. 1180).