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SURFACE CHARACTERISTICS OF THE MAGNETIC STARS*

GEORGE W. PRESTON
Hale Observatories, Carnegie Institution of Washington, California Institute of Technology

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I. Introduction

A. *General Remarks.* The magnetic stars, first identified as spectroscopically peculiar objects (Ap stars) characterized by enhanced lines of Mn, Si, Sr, Cr, Eu, and occasionally other elements, lie on or near the main sequence with effective temperatures in the range $8000 \lesssim T_e \lesssim 15{,}000°$ K. Analyses during the past two decades generally have confirmed the overabundances inferred from "enhanced" lines, have added much more detail about elements whose lines cannot be seen on low-dispersion spectrograms, and have shown that the overabundances of the heavier elements are accompanied by deficiencies of some elements, notably He and O (Sargent and Searle 1967). The explanation of these anomalies in terms of surface nuclear reactions (Burbidge and Burbidge 1955), interior nuclear reactions (Fowler et al. 1965), diffusion processes (Michaud 1970), and, most recently, selective magnetic accretion (Havnes and Conti 1971) is a subject of apparently unending debate. It will not be pursued here. The status of this and other topics concerning magnetic stars as of 1965 is discussed in detail in *The Magnetic and Related Stars* (Cameron 1967) and the references contained therein. The present review is confined primarily to more recent developments.

Some Ap stars were early discovered to be spectrum variables: that is, lines of certain elements were found to vary in strength in a periodic manner that cannot be understood in terms of changes in temperature or electron pressure. The variations in line strength are often accompanied by periodic variations in radial velocity and light. When such is the case, the velocity curve is in quadrature with the line strength variation, while the extrema of the light and spectrum variations coincide in phase. All elements do not participate in these variations to the same degree, and in some stars the elements can be divided into groups that vary in antiphase with one another. In a given star large variations in line strength tend to be associated with elements that are greatly overabundant, though the converse is not also true.

During the first half of this century the eccentric behavior of these strange Ap stars was studied from a strictly phenomenological point of view. There were many measurements, classification of stars, classification of lines in their spectra, but little understanding — so little that Otto Struve (1942), in the concluding remarks of a 1942 review paper on the subject, could do little more than take solace in the fact that the phenomena were not supernatural! This somewhat downcast view was replaced by a more hopeful one a few years later following Babcock's (1947) discovery of a magnetic field in 78 Virginis. In the decade that followed, Babcock (1958) was able to show that virtually all Ap stars (with qualifications as noted below) possess surface magnetic fields, that the fields are always variable with or without polarity reversal, and that many of them vary periodically. The magnetic periods were identical to the periods of the light and spectrum variations.

B. *The Oblique Rotator Model.* The discovery of a reversing magnetic field in HD 125248 (Babcock 1951) found a natural interpretation in terms of the oblique rotator — a model in which the axis of a static magnetic field is inclined to the rotation axis of the star. Babcock (1949) first considered it as a possibility, Stibbs (1950) worked out its consequences for a dipole field, and Deutsch (1954) became its most ardent advocate. Stibbs showed how the model could account for

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periodic magnetic variations with or without polarity reversals. By hypothesizing "spectroscopic patches," i.e., concentrations of certain elements near the magnetic poles, Deutsch was able to explain the spectrum variations and the phase relation between equivalent width and radial velocity. Further, it could be shown that the model predicts the order of magnitude and sign convention of the cross-over effect discovered by Babcock (1951). In early qualitative discussions, Babcock (1956) questioned whether the "matching velocity" required by the cross-over effect could be reconciled with the equatorial rotational velocity inferred from the magnetic period, but more recent detailed studies (Pyper 1969; Hockey 1971) indicate that this probably is not a serious problem.

Finally, the oblique rotator received strong support from Deutsch's (1956) discovery that line widths of periodic Ap stars tend to vary inversely as their periods. A contemporary version of this relation based on as yet unpublished $v \sin i$ values for all well-studied periodic Ap stars (Preston, in preparation) is shown in Figure 1. For an oblique rotator, the equatorial rotational velocity v, the period P, and the stellar radius R are related by $2\pi R = Pv$, or

$$V = \frac{50.6R}{P} \quad (1)$$

if R, v, and P are expressed in solar radii, km $\sec^{-1}$, and days, respectively. Because the stars are viewed at unknown inclinations i, equation (1) should form the upper envelope of the $v \sin i$ vs. P diagram if R is chosen properly. A recent discussion (Preston 1970*b*) led to $R \sim 3.2 R_\odot$ for *all* Ap stars and gave weak evidence that R may increase with T_e among the Ap stars as might be expected. The curve in Figure 1 is based on this mean value of R.

In retrospect most of the observational ingredients of the oblique rotator model were available long ago — periods, line widths, the spectrum variations, and the suggestive phase lags of the velocity curves. But prior to the discovery of magnetic fields in the Ap stars and the recognition of departures from spherical symmetry that such fields necessarily imply, no one was prepared to seriously entertain the notion that a star could possess permanent nonuniformities on its

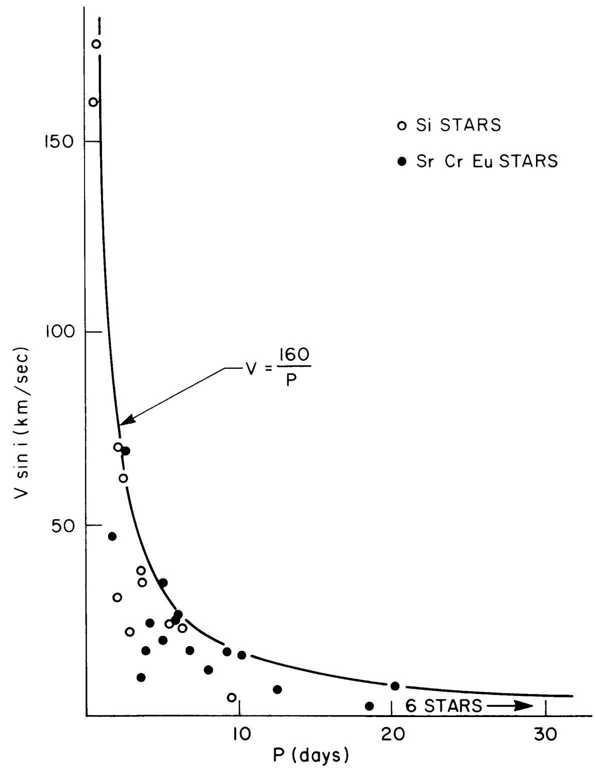

FIG. 1 — A plot of projected rotational $v \sin i$ versus period P for periodic Ap stars. The curve represents the relation between equatorial rotational velocity and period for stars with radius $R = 3.2R_\odot$, as discussed in the text. The curve is a reasonable upper envelope for the plotted points.

surface, other than those axisymmetric ones induced by axial rotation. The model still does not meet with universal acceptance — in part a consequence of the fact that the evidence for it is largely circumstantial; no crucial test has ever been devised. It remains, however, an attractive model and the *only* model developed in sufficient detail to order and predict the variety of observational phenomena encountered among the magnetic stars.

C. *Frequency of Variability Among Ap Stars.* The oblique rotator model has been devised to account for the periodic variations of the Ap stars. To what fraction of all Ap stars does the model apply? An indication is provided by the apparent-magnitude distribution of known Ap stars shown in Figure 2. Only those stars are included for which Ap characteristics have been verified by classification of slit spectrograms, largely due to Osawa (1965) and Cowley et al.

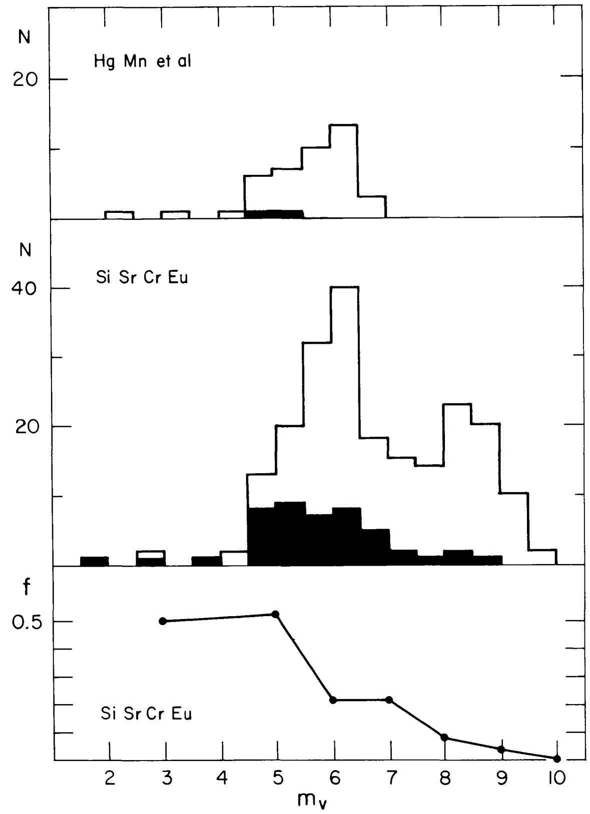

FIG. 2 — (Upper) The distribution of known Hg-Mn stars with apparent magnitude. (Middle) The distribution of known SiSrCrEu stars with apparent magnitude. The shaded portions of the latter histogram denote known periodic Ap stars; the two shaded boxes in the upper panel are questionable, as discussed in the text. The fraction of Ap stars known to be periodic is plotted versus m_v in the lower panel.

(1969). The shaded portions of the histograms represent those stars known to be periodic variables on the basis of light, spectrum, or magnetic variations. The division of the sample into two groups, Hg-Mn stars and SiSrCrEu stars, is justified by a number of considerations, one of which is apparent in Figure 2 itself.

(1) There are no confirmed periodic variations in light, spectrum, or magnetic field among the Hg-Mn stars. Deutsch (private communication) regretted his announcement for π Bootis A (Deutsch 1947). Preston, Stepień, and Wolff (1969) reported a periodic magnetic variation for κ Cancri, but noted that the variation (± 200 gauss) was so small as to be questionable. Inasmuch as few of the field determinations differ from zero by more than two probable errors, their report of a periodic variation was probably premature. The small shaded area in the Hg-Mn star histogram represents these two questionable periodicities.

(2) Conti (1970) was unable to find evidence for fields greater than a few hundred gauss in any of the components of four sharp-lined Mn spectroscopic binaries. This result, coupled with that for κ Cnc above and the small fields reported for Mn stars previously by Babcock (1958), raises serious question as to whether the Hg-Mn stars are magnetic stars at all. This doubt arises partly from the recognition (Preston 1969*b*) that errors in effective field determinations may be substantially larger than those calculated from the internal scatter in measured Zeeman displacements.

(3) The abundance peculiarities of the Hg-Mn stars differ systematically from those of all the other Ap stars. For example, no rare earths have been found in the Hg-Mn group (this is not true, for example, for the Si stars) and they do not share the He and O deficiencies of the SiSrCrEu stars. For these reasons alone a division of the Ap stars into a Mn group and a "main" (SiSrCrEu) group has been suggested by Sargent and Searle (1967) and Guthrie (1969).

(4) Finally, the incidence of spectroscopic binaries is significantly higher (by a factor of as much as three to five times) among the Hg-Mn stars than is the case for the remainder of the Ap stars.

On the basis of these considerations, the Hg-Mn stars will be excluded from all of the discussions that follow.

The increase in the fraction of periodic Ap stars from 0.0 at $m_v \sim 10$ to about 0.5 for $m_v \lesssim 5.5$ reflects the understandable tendency for observers to work on the brightest stars. Is the observed fraction for the latter a good approximation for the true value? Probably not. For reasons discussed in section III below, it is highly likely that photometric, spectrum, and magnetic variations are intimately related, and of these the first are the easiest to detect because of the sensitivity of photoelectric devices — particularly for stars with rotationally broadened lines that render estimates of spectrum variability difficult and magnetic field determination impossible by present techniques. Thus, it is not surprising that a major fraction of the known periodicities are photometric. But even photoelectric photometry has its limits, as indicated in Figure 3, which shows the frequency of visual ranges ΔV for periodic light variables. The apparent maximum in the distribution between $0^{m}01$ and $0^{m}02$ is not likely to be real. A range $\sim 0^{m}02$ is about where the photometrists give up: or, if they don't give up, it is where they begin to fail. It seems more likely that the true distribution continues to rise below $\Delta V = 0^{m}02$ in such a way that the true number of photometric variables is not greatly different from the total number of SiSrCrEu stars. When one takes account of the fact that stars with long periods ($\gtrsim 50$ days) would have been missed by many observing programs designed to detect shorter periods, it is tempting to conclude that perhaps all of the SiSrCrEu stars will prove to be variable. The accidental discovery of the very slow variation of HD 9996 (HR 465) is a case in point (Preston and Wolff 1970).

II. Stellar Magnetic Geometries

The title of this section refers to those modest conclusions that can be drawn about the distributions of magnetic fields on stellar surfaces.

A. *Model Independent Conclusions.* Babcock (1960*b*) and others have pointed out certain general features of stellar magnetic-field distributions that are independent of the properties of models chosen to represent the surfaces of magnetic stars. Toroidal magnetic fields cannot easily produce a longitudinal Zeeman effect in the integrated light of a stellar disk, so the fields probably have a general poloidal character. Higher order multipoles also produce small integrated effects, so the dominant component is probably dipolar. Thus, in the most favorable (pole-on) aspect the ratio of effective field to polar field H_e/H_p is ~ 0.3 for a dipole and ~ 0.05 for a quadrupole (Schwarzschild 1950). For the latter, effective fields $\sim$ 2kG would be accompanied by Zeeman broadening and component resolution appropriate for mean surface fields $H_s \sim 20$ to 40 kG, a circumstance not generally observed. Further, Babcock has remarked that the sharpness of the line profiles and the purity of the observed Zeeman effect in many cases rules out the possibility that the observed effect arises in small areas of the disk that carry intense fields (i.e., fields orders of magnitude larger than the net field). The depths of the Zeeman-analyzed lines alone in many cases require that they be produced by most of the stellar surface, and in one star, HD 126515 (Preston 1970*a*), resolved Zeeman patterns of Fe II and Cr II lines can be seen throughout a magnetic cycle in which the effective field reverses polarity. So much for magnetic "spots."

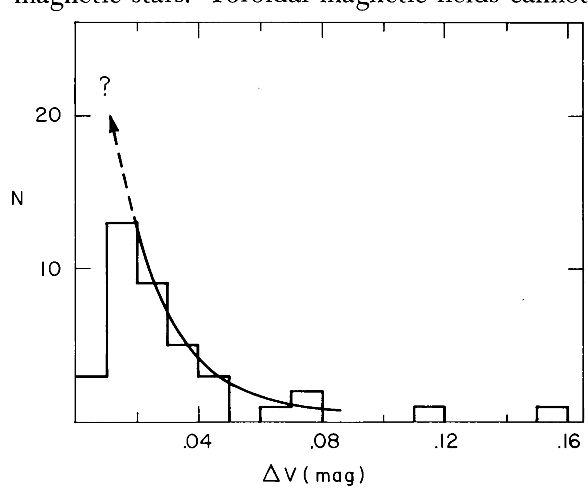

FIG. 3 — The frequency distribution of visual light ranges for Ap stars. The likelihood that the true distribution continues to increase below $\Delta V = 0.02$ as indicated by the dashed curve is discussed in the text.

B. *The Orientation of Magnetic Axes.* Almost all other conclusions about stellar magnetic geometries are model-dependent: In particular, they are based on the assumption that magnetic stars are oblique rotators. The writer's interest in magnetic geometries arose during the course of a study (Preston 1967*b*) of the null line Fe I $\lambda 4065.40$ in the spectrum of β Coronae Borealis (Plate I). Comparison of the width of this line (which is unaffected by the presence of a magnetic field because the Landé *g* factors of both the upper and lower levels of the transition are zero) with all others showed immediately that the Zeeman effect is the predominant line-broadening agent in the atmosphere of that star and that previous estimates of the projected rotational velocity were far too large. An upper limit of $v \sin i \lesssim 3$ km sec^{-1} derived from $\lambda 4065.40$, coupled with a rotational velocity of ~ 6 km sec^{-1} inferred from the magnetic period of $18^{d}5$, led to the conclusion that $i \lesssim 30°$. But the polarity reverses almost symmetrically about zero as described by $r \equiv H_e(\min)/H_e(\max) \approx -0.8$. (If $r = +1.0$, the field is constant; if

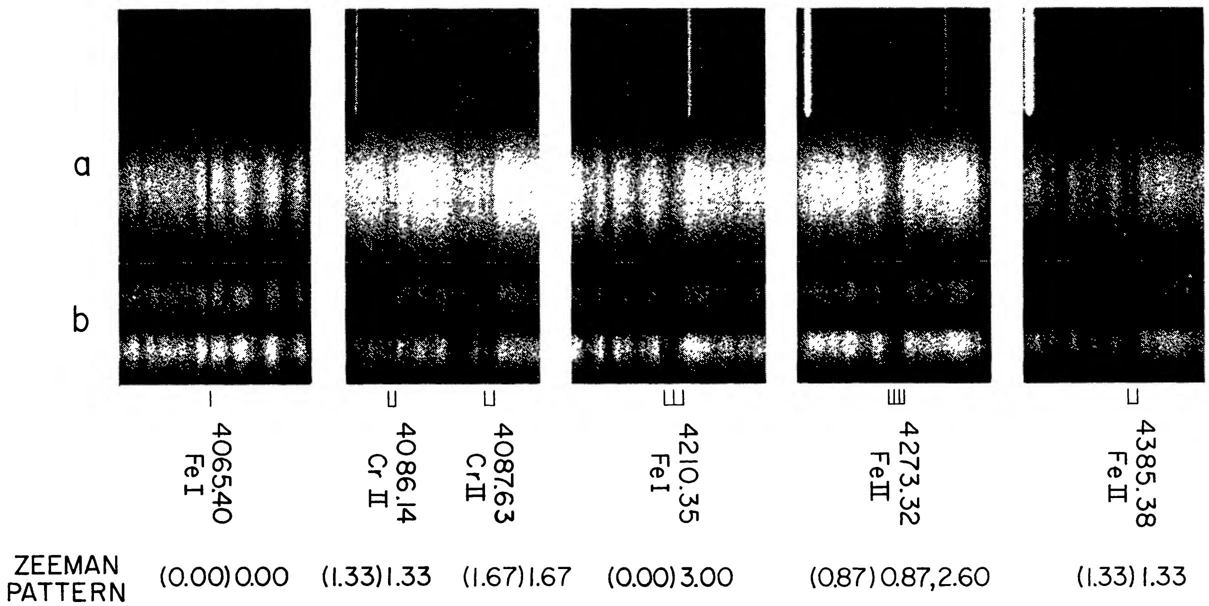

PLATE I

Lines with Zeeman patterns of unusual interest on a 1.3 Å mm^{-1} spectrogram of β CrB. The null line Fe I λ4065.40 is much narrower than adjacent lines, all of which are broadened by the Zeeman effect. The width of the profile of λ4065 provides an upper limit of 3 km sec^{-1} for the projected rotational velocity of β CrB. The resolved Zeeman doublets Cr II λ4086.14, λ4087.63, and Fe II λ4385.38 provide direct evidence for a mean surface field H_s = 6.0 kG. The effective field H_e of β CrB never exceeds 1 kG. (G. W. Preston 1969, *Ap. J.* **158**, 1081; Copyright 1969, The University of Chicago Press.)

$r = -1$, the field oscillates symmetrically about zero.) Stibbs' (1950) result for dipole rotators gives

$$r = \frac{\cos(\beta + i)}{\cos(\beta - i)} \tag{2}$$

or

$$\tan\beta = \left(\frac{1 - r}{1 + r}\right)\cot i \quad , \tag{3}$$

in which β is the inclination of the magnetic axis to the rotation axis. With $i \lesssim 30°$ and $r = -0.8$, we obtain $\beta > 86°$. At first sight, the writer concluded that such a special condition (orthogonality of the rotation and magnetic axes) constituted an argument against the rotator model; but as it turned out, he came to jeer and stayed to "whitewash the fence." One can just as well invert the problem and inquire as to the distribution of β required to produce the observed distribution of r for *all* magnetic stars, not just β CrB. For dipole geometry the answer is simple. For a family of rotators with magnetic inclination β and random inclinations of their rotation axes to the line of sight, the probability of observing r and dr is

$$p_\beta(r) = \frac{2(1 - r)\tan\beta}{[(1 - r)^2 + (1 + r)^2 \tan^2\beta]^{3/2}} \quad . \tag{4}$$

A plot of this distribution for several values of β is shown in Figure 4, together with a histogram of r values for all known periodic magnetic stars. In an earlier discussion (Preston 1967*c*), the predominant feature of the observed distribution, the pronounced peak near $r = -1$, led to the conclusion that the magnetic axes tend to lie near the rotational equators. However, in view of the addition of several more stars with $r > 0$ in the past three years (HD 215441, 78 Vir, HD 24712, HD 111133, and 52 Herculis), this conclusion should be modified. The observed distribution appears to contain a component with small β, as suggested by the 50-50 admixture of distributions for $\beta = 20°$ and $\beta = 80°$ shown in Figure 4. Landstreet (1970) has constructed $p_\beta(r)$ distributions for the cylindrically symmetric fields of Böhm-Vitense (1965) and decentered dipole fields. The shapes of these

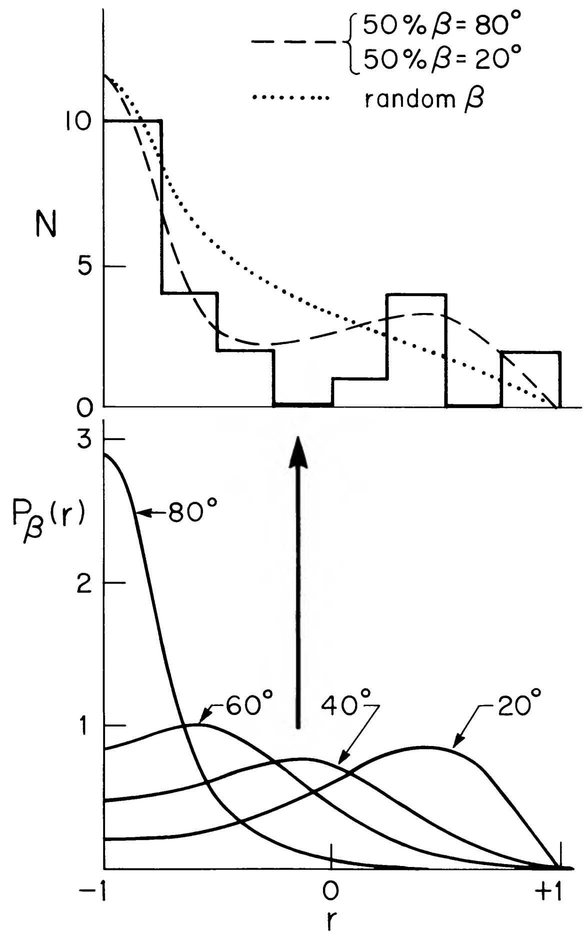

FIG. 4 — (Upper) The observed frequency distribution of magnetic ranges as defined by the quantity $r = H_e(\text{min})/H_e(\text{max})$ discussed in the text. (Lower) The probability distributions $p_\beta(r)$ given by equation (4) in the test for four values of the magnetic inclination β. The curves in the upper panel indicate two possible interpretations of the observed distribution. The vertical arrow calls attention to the apparent deficit of stars with intermediate values of β.

distributions differ from the centered dipole case in detail, but the general conclusions based on comparison with observations is similar for all of them.

C. *Mean Surface Fields.* Until recently, HD 215441 (Babcock 1960*a*) was the only star known to possess a magnetic field large enough to produce resolved Zeeman patterns. A necessary condition for such resolution is that the Zeeman splitting $\Delta\lambda_Z$ must exceed the Doppler (or instrumental) width $\Delta\lambda_D$. With H in kilogauss, λ in angstroms, and v, in km sec^{-1}, this condition is

$$1.4 \times 10^{-7} \frac{\lambda H z}{v} > 1 \quad , \tag{5}$$

where z is the σ-splitting of the pattern in units of the splitting of a normal Zeeman triplet. At λ4300, this expression becomes

$$H > 1.7 \frac{v}{z} \quad , \tag{6}$$

and for $z = 1$ and $v = 3$ (a limiting v set by rotation and/or the limiting resolution of typical coudé spectrographs) we have $H \gtrsim 5$ kG.

The mean surface fields derived from the separation of resolved components in such a case is the surface brightness (I) weighted average of the scalar magnitude of the surface field averaged over all area elements dA of the disk, i.e.,

$$H_s = \frac{\int HIdA}{\int IdA} \quad . \tag{7}$$

The effective field is a similar average of the longitudinal component of the field $H \cos \gamma$,

$$H_e = \frac{\int H \cos\gamma \; IdA}{\int IdA} \quad , \tag{8}$$

where γ is the angle between the line of sight and the field vector at dA. While H_e is a signed quantity that vanishes for some value of α, the angle between the line of sight and the magnetic axis of an axisymmetric geometry, H_s is much less sensitive to aspect. For a dipole, for example, H_s/H_p varies from 0.80 to 0.64, or by about 25 percent, as α varies from 0° (pole-on aspect) to 90° (equator-on aspect), while H_e/H_p varies from 0.3 to 0.0. With these considerations in mind it is of interest to reconsider the geometrical arguments concerning β CrB in the previous section. If, as inferred from $v \sin i$, the period and the ratio r, β is so large (>86°), and i is so small (<30°), then the polar field calculated from Stibbs' formula,

$$H_p = \frac{3.3\, H_e(\text{max})}{\cos(\beta - i)} \quad , \tag{9}$$

should be >6 kG even though the effective field does not exceed 1 kG. In view of the insensitivity of H_s to aspect, we might expect to detect resolved Zeeman patterns in the spectrum of

β CrB. This is indeed the case, as examination of the resolved patterns in Plate I reveals. The value of $H_s \sim 6.0$ kG deduced from the splitting of favorable Zeeman doublets (Preston 1969*d*) is in excellent agreement with expectations and constitutes one of the most compelling arguments in favor of the conceptual framework of the oblique rotator model.

A search among other sharp-lined magnetic stars has resulted in the discovery of resolved patterns in ten stars up to the present time. For other stars the differential broadening of lines with large and small z values leads to crude estimates of H_s (Preston 1971) and a first attempt to construct the distribution of H_s among the most slowly rotating magnetic stars ($v \sin i \leqq 10$ km sec^{-1}), as shown in Figure 5. The distribution, characterized by a peak at about 2–3 kG and a tail extending well beyond 10 kG, comprises one of the elementary properties of magnetic stars to be explained by theories of stellar magnetism.

D. *Representations of Stellar Magnetic Geometries.* Much of the discussion up to this point has been couched in terms of dipolar magnetic configurations. Now it is necessary to admit that stellar magnetic fields in general are *not* dipolar—within the framework of the oblique rotator model. This was an early deduction based on Stibbs' analysis, which showed that magnetic variations due to oblique dipole rotators must be sinusoidal as contrasted with actual H_e variations that frequently exhibit marked anharmonicity. In view of the peculiarities of the line profiles near the phases of polarity reversal (Babcock's cross-over effect), I would issue caution against overinterpretation of the "observed" slopes of the ascending and descending branches of the H_e variations. However, the tendency of one extremum of the variation to be wider than the other is a real effect that indicates departures from dipole geometry. These departures have been studied from two points of view. On the one hand, Deutsch (1958) has presented a method by which the magnetic-field distribution, regarded as a sum of centered dipole and quadrupole components with adjustable orientations and moments, can be determined from a spherical harmonic analysis of the equivalent widths, radial velocities, and effective field variations observed for lines of selected ions in the stellar atmosphere. The field distribution is *determined*, in principle, from observation, but, as persual of the equations (Deutsch 1970) reveals, it is difficult to estimate the sensitivity of the result to errors in the various observed quantities. Analyses of two stars by this method, HD 125248 (Deutsch 1958) and α^2 Canum Venaticorum (Pyper 1969), require significant quadrupole contributions to the fields.

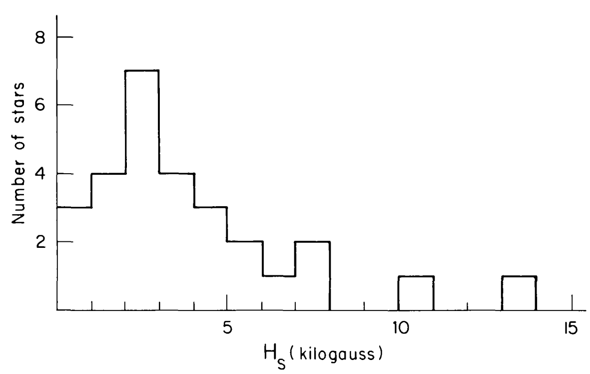

FIG. 5—The frequency distribution of mean surface fields H_s among slowly rotating magnetic stars ($v \sin i \leqslant 10$ km sec^{-1}) as inferred from resolved or broadened Zeeman patterns.

Alternatively, one may try to reproduce the shapes of observed effective magnetic-field variations by adjustment of the parameters of plausible but arbitrary magnetic configurations. This has been done with some success by Böhm-Vitense (1965), who used modified dipole geometries. More recently Landstreet (1970) has shown that even better fits can be achieved for some stars, e.g., 53 Camelopardalis, with decentered dipole geometry. Recent studies of the cyclic variations of both H_e and H_s in two stars, β CrB (Wolff and Wolff 1970) and HD 126515 (Preston 1970), provide strong support for the latter geometry. In both cases the mean surface field varies with a single wave in the period of the effective field variation. The extrema of both variations are almost (but not quite) in phase. Centered dipole geometry would produce a double wave in H_s during a single cycle of H_e, but decentered dipoles can readily reproduce the observed ranges in H_s and H_e. Further, such magnetic geometry explains the tendency for lines to be broader (due to a larger Zeeman effect) at one H_e extremum than at the other, it accounts for the asymmetrical cross-over effect observed in 53 Cam (see, for example the discussion by Preston 1967*a*), and it holds promise of providing reasons for other behavior of the magnetic stars to be discussed below.

III. The Photometric Variability of the Ap Stars

A. *Possible Causes of the Light Variations.* The photometric variations of the Ap stars defy summary description, as the reader can varify by inspection of the beautiful but confusing results of Rakos (1962, 1963), Stępień (1968*b*), Wolff and Wolff (1971), van Genderen (1971), and others referred to by Hardie (1967). In spite of the photometric diversity, one generalization mentioned earlier can be made: The periods of the photometric, spectrum, and magnetic variations are always equal and the extrema of the variations always (or *almost* always) coincide in phase. These conclusions have been used as arguments against the oblique rotator in the past for lack of a physical reason why surface brightness should depend on the polarity of the field. The decentered dipole model offers a possible way out of this dilemma. For such models the local surface field varies with z as

$$\frac{H(z)}{H(1)} = \frac{(1-a)^3}{2} \frac{[3(z-a)^2 + D^2]^{1/2}}{D^4} , \quad (10)$$

where a is the displacement of the dipole from the center of the star in units of the stellar radius and $D^2 = 1 + a^2 - 2az$, as indicated in Figure 6. If $a \lesssim 0.2$ field strength has a primary maximum at $z = +1$ and a secondary maximum at $z = -1$, while for $a \gtrsim 0.2$ field strength declines monotonically from the strong "pole" at $z = +1$, steeply near $z = +1$, and slowly for $z < 0$. If field strength and surface brightness were correlated in such configurations, light variations with single waves ($a \gtrsim 0.2$) or double waves ($a \lesssim 0.2$) could be produced during a rotation cycle, and such meager data as are available (see table below)

Star	a	Source for a
β CrB	0.10	Wolff and Wolff (1970)
HD 126515	0.36	Preston (1970)
53 Cam	0.67	Landstreet (1970)

indicate that a can have an appreciable range of values. Before pursuing such speculation further, we should inquire whether it is reasonable on any grounds to expect that field strength and surface brightness should be correlated. We can list the possible effects:

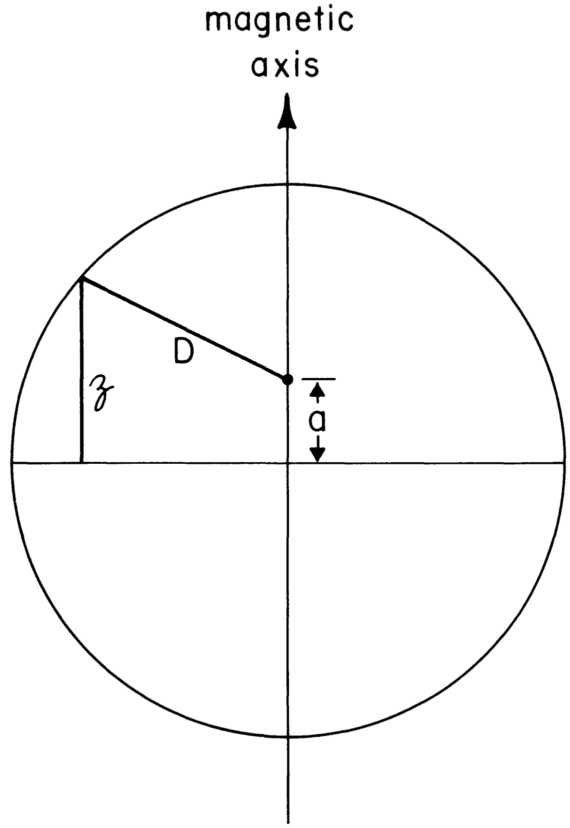

FIG. 6 — Schematic diagram of a decentered dipole model described by equation (10) in the text.

(1) *Temperature effect*: This has been proposed on empirical grounds by Stępień (1968*b*). It is necessary to suppose that a magnetic pressure term alters the atmospheric structure in such a way that a temperature variation over the surface of the star is produced.

(2) *Gravity effect:* It is conceivable that magnetic stresses provide enough hydrostatic support at some places on the stellar surface to produce a variation in the effective gravity.

(3) *Geometrical distortion:* Mestel (1967) has noted that light variations will result if a magnetic star suffers distortions symmetric about a magnetic axis inclined to the rotation axis. He finds that, depending on aspect, spheroidal configurations can produce light variations with both single and double waves in a rotation cycle. Such geometric distortions may or may not be accompanied by effects (1) and (2) above.

(4) *Blanketing effect:* Line blanketing is severe in the violet regions of many of the cooler Ap stars. For example, Wolff (1967) found that the blanketing fraction for HD 188041 increased from 10 to 25 percent between 5000 Å and 4000 Å. In those stars that are spectrum variables, blanketing variations may produce photometric effects shortward of 5000 Å. The concentration of elements that produce such blanketing near magnetic poles is an empirical deduction for which there is no physical explanation at present.

(5) *Backwarming effect:* A redistribution of flux with wavelength will occur as a consequence of item (4) above. Peterson (1970) has pointed out that the principal effects in observable spectral regions are likely to be produced by abundant atomic species with bound-free continua in the spectral region below 3000 Å that carries a significant fraction of the flux. Peterson was concerned primarily with the effects of Si in the Si stars. More recently Wolff and Wolff (1971) have noted that discrete absorption by hosts of doubly ionized rare-earth lines in $\lambda\lambda$2000–3000 might play a similar role in the cooler Ap stars.

B. *The Basic Data.* The U, B, and V light ranges of approximately 30 Ap stars are plotted in the form of (ΔV, ΔU) and (ΔB, ΔU) diagrams in Figure 7. The data were taken from all available sources, most of which are referred to at the beginning of section III(A). Important information that may be contained in the shapes of the light variations is ignored, and four stars, for which marked double waves make the definition of ranges hopelessly ambiguous, were omitted from the diagram. We will consider double waves later in this section. Filled circles denote *UBV* observations while open circles and crosses denote observations (*uvy*) made in the Strömgren four-color system by Wolff and Wolff (1971) and in the Walraven five-color system by van Genderen (1971), respectively. Lines connect ranges for the same star obtained in two different photometric systems. The ranges are signed quantities reckoned from the phase of maximum visual light, i.e., all ΔVs are positive by definition, but ΔU and ΔB can be either positive or negative.

It is clear from inspection of Figure 7 that most of the stars lie along bands in the two diagrams that extend up and to the right, and that a few stars depart markedly from the correlation defined by the majority. This is particularly noticeable for α^2 CVn at $\Delta U = +0.04$ and HD 9996 and HD 221568 in the upper left corner of the (ΔV, ΔU) plane. We now inquire as to which of the possible effects listed above might produce the correlations in Figure 7.

C. *Peterson's Theory for Si Stars.* Peterson (1970) has calculated the emergent fluxes at λ3647, λ4000, and λ6000 for model atmospheres with $T_e = 9000°$, 11,000°, and 13,000° K ($\theta_e=$ 0.56, 0.46, and 0.39, respectively) as a function of [Si/H], the Si/H abundance ratio is in solar units. I have taken the liberty of treating his monochromatic magnitudes as if they were U, B, and V magnitudes and have plotted the light ranges that would result if [Si/H] varied from 10 to 300. I ignored the first power of 10 for [Si/H] in Peterson's calculations because the fluxes change little in that range. The results for $\theta_e = 0.39$ and 0.46 are compared with the data for Si stars in Figure 8. In view of the crude correspondence between Peterson's wavelengths and the *UBV* system a detailed comparison is impossible, but it is clear, as Peterson concluded, that reasonable variations in [Si/H] can produce the approximate slopes of the observed correlations and the photometric ranges encountered for individual stars. However, as noted by Wolff and Wolff (1971), Si spectrum variables do not have ranges that are particularly larger than those that are *not* Si spectrum variables, and the Si star with the largest light variation (HD 215441) is not a recognized spectrum variable, though it has been examined for such variability (Babcock 1960*a*; Preston 1969*a*). It is difficult to believe that the required order-of-magnitude variation in [Si/H] could be undetectable in the Si II λ4128-30 doublet. Therefore, we are led to consider that, in some stars at least, the light variations may be due to other causes.

D. *Temperature and Gravity Effects.* Mihalas (1966) has published the U, B, and V magnitudes of hydrogen-blanketed model atmospheres with *normal* composition for a set of log g and θ_e values. From these it is possible to estimate the differential coefficients $\partial M_\lambda/\partial\theta$ and $\partial M_\lambda/\partial \log g$

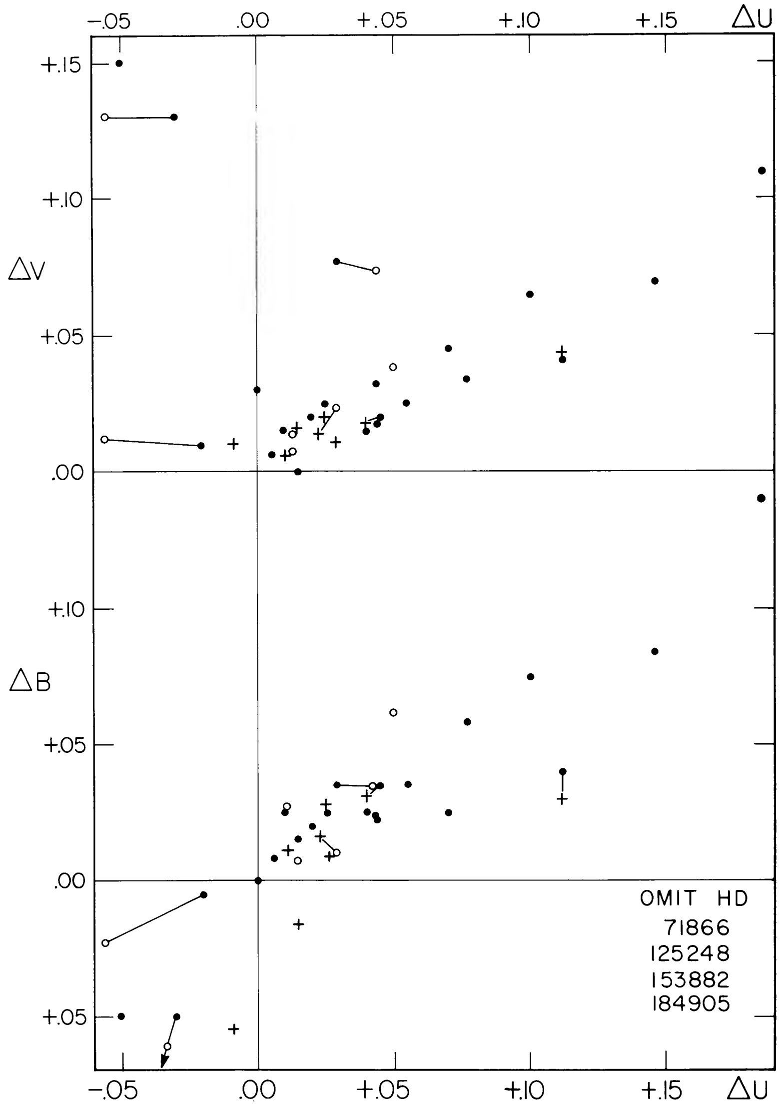

FIG. 7 — Photometric range diagrams for the Ap. stars. The ranges are signed quantities reckoned from maximum visual light. Thus, ΔV is always positive but ΔU and ΔB may be positive or negative. Filled circles denote *UBV* observations; open circles and crosses denote ranges in the nearest equivalent bandpasses of Strömgren four-color and Walraven five-color photometry, respectively. Lines connect ranges for the same star in two photometric systems.

(as functions of θ_e as shown in Figure 9) required to compute the photometric variations that would result from temperature and gravity variations, viz.:

$$\Delta M_\lambda = \left(\frac{\partial M_\lambda}{\partial \theta_e}\right) \Delta \theta_e + \left(\frac{\partial M_\lambda}{\partial \log g}\right) \Delta \log g \,. \quad (11)$$

Equation (11) and the curves in Figure 9 have been used to compute the grids of *U*, *B*, and *V* ranges for $\theta_e = 0.45$ that are plotted with the Si star data in Figure 10. The shapes of the grids change with temperature but, as inspection of Figure 9 shows, the changes are not great so long as $\theta_e \lesssim 0.5$, and this is the case for the Si stars. Temperature variations $\lesssim 400°$ K at (approximately) constant gravity are capable of explaining the range correlations in both the (ΔV, ΔU) and (ΔB, ΔU) planes as Stępień suggested. Thus we have at least two mechanisms, Si-induced backwarming and temperature variations, to explain the light variations of the Si stars. Whether the gravity variations inferred from literal interpretation of Figure 10 are to be taken seriously is best deferred until other possible effects have been investigated.

E. *Blanketing and Backwarming Due to Ionized Rare Earths.* The need for another effect is well-exemplified by α^2 CVn, which sticks out like a sore thumb in Figures 7, 8, and 9, apparently because of its abnormally large ΔV. Put

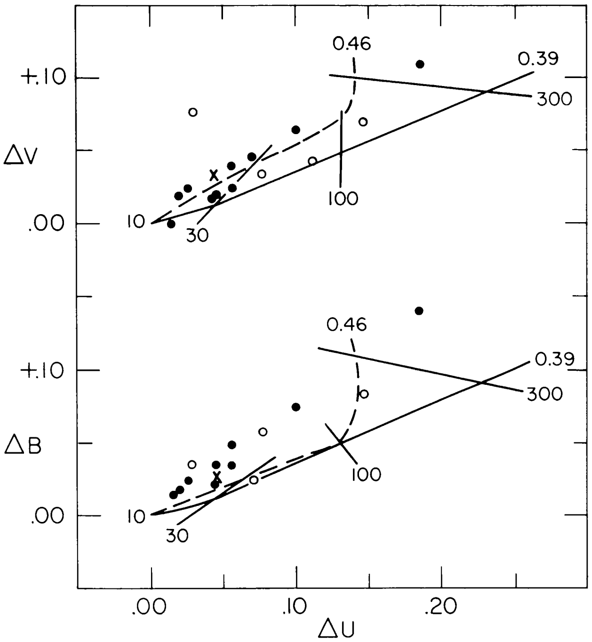

FIG. 8 — Photometric range diagrams for the Si stars only. The curves represent the loci of ranges produced by variations in [Si/H] from 10 to 300 as calculated by Peterson (1970) for θ_e = 0.39 and 0.46. The comparison of theory and observation is discussed in the text. Open circles denote known Si spectrum variables.

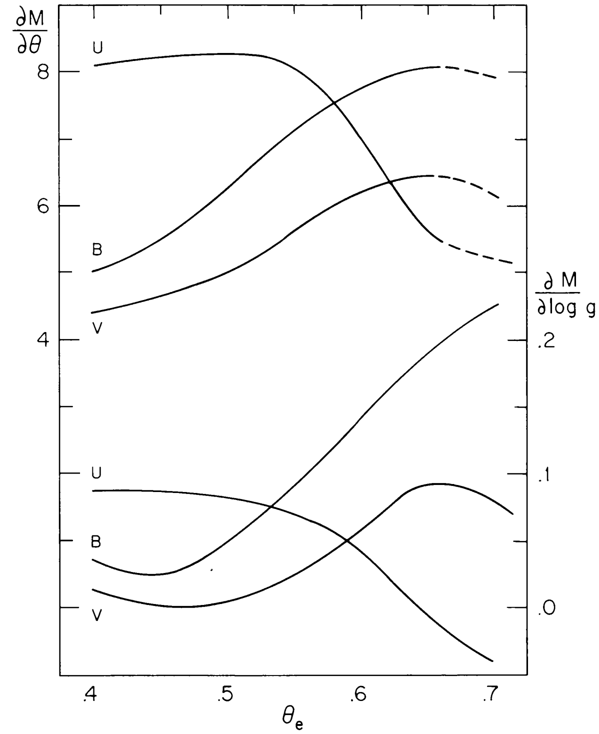

FIG. 9 — The partial derivatives of U, B, and V with respect to θ_e. The derivatives are based on Mihalas' (1966) calculations for hydrogen-blanketed model atmospheres of normal chemical composition.

another way, α^2 CVn is an exaggerated example of those Ap stars — usually the cooler ones *unlike* α^2 CVn — that get "red" when they get bright (see the *UBV* photometry by Pyper 1969). It's an old problem. Now to be sure α^2 CVn is a Si star, but it is also an Eu star and a pronounced rare-earth spectrum variable. This is in fact what attracted Belopolsky's (1913) attention to it 60 years ago. Such rare-earth variability is also a characteristic of the badly behaved stars in Figure 7, principally the cooler ones that possess negative ΔVs and ΔBs, i.e., that become fainter in the violet and ultraviolet as they brighten at visual wavelengths. Though the photometric ranges are much smaller, in general, for the SrCrEu stars than for the Si stars, it is clear from the distribution of observed points for SrCrEu stars in the $\Delta\,\theta_e$, Δ log g grids of Figure 11 that thermal variations are not likely to be the *principal* cause of their light variations.

The Wolffs (1971) recently have suggested that ultraviolet line absorption by doubly ionized rare earths may produce a backwarming effect in some stars analogous to that calculated for Si by Peterson. Dieke, Crosswhite, and Dunn

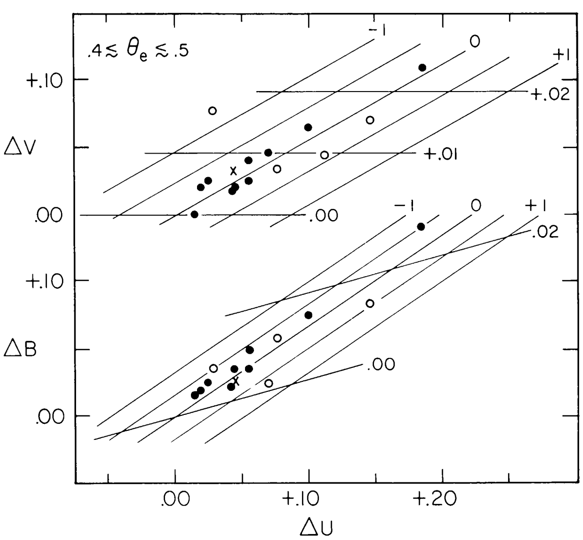

FIG. 10 — Photometric range diagrams for the Si stars, compared with a grid of ranges produced by changes in θ_e of $\Delta\theta_e = +0.01$ and $+0.02$ and $\Delta \log g$ values from -1 to $+1$. The grid was constructed from the curves in Figure 9 at $\theta_e = 0.45$.

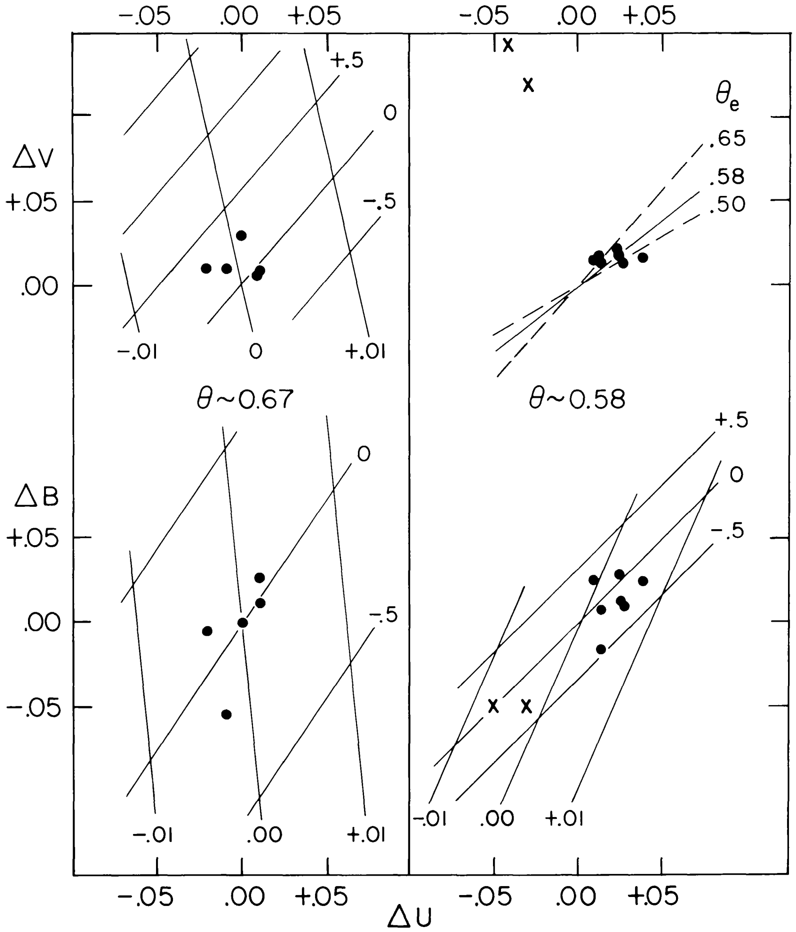

FIG. 11 — Photometric range diagrams for the SrCrEu stars grouped approximately by effective temperature. Ranges generated by temperature and gravity variations are plotted as in Figure 10. At $\theta_e \sim 0.58$, lines of constant θ and log g are nearly parallel in the (ΔV, ΔU) plane. Therefore, the gravity variations are omitted in the upper right panel and are replaced by slopes of the temperature variations for three values of θ_e. The crosses denote the two long-period large-amplitude stars HD 9996 and HD 221568.

(1961) have called attention to the great concentration of lines of doubly ionized rare earths in the spectral regions λλ2000–3000, a circumstance of some interest since the emergent flux of A-type stars is large in this spectral region and double ionization of the rare earths should predominate in A-type and late B-type stars. Variable concentration of such ions on the stellar surface may produce a brightening in the visual region via the backwarming effect, while *line blanketing* due to the singly ionized species of these elements in the violet and near ultraviolet overpowers the backwarming effect to produce a diminution in brightness in the U and B passbands.

The relative concentrations of singly ionized and doubly ionized rare earths are strongly temperature dependent. If the former dominate as one would expect for the coolest Ap stars, then negative ΔUs and ΔBs should occur, while if the latter dominate one might expect small (negative or positive) ΔU and ΔB but large ΔV. This may be the explanation of why the photometric ranges of α^2 CVn, one of the hottest rare-earth spectrum variables, are unlike either the Si stars with which it is grouped by temperature or the cooler SrCrEu stars to which it is related spectroscopically.

The Wolffs' suggestion can be subjected to a direct observational test: Maximum visual light should always occur when the rare-earth lines are strongest, and, if there is a double wave in the rare-earth line strengths, it should be accompanied by a double wave in visual light that may or may not be superposed on variations due to another (e.g., thermal) effect. Such a test is provided by the list of photometric-spectrum variables in Table I for which it is possible to compare phases of maximum visual light and maximum line strength for spectrum-variable elements. The results are encouraging. For ten of the twelve rare-earth (RE) spectrum variables, maximum visual light and maximum line strength for the rare earths coincide in phase. For one of the remaining two, HD 188041, a small phase shift of ~ 0.1 cycle may or may not be significant, while for the other, 17 Comae Berenices A, the phase shift of 0.25 cycle must be regarded as a serious discrepancy. The six stars (21 Persei, HD 71866, HD 125248, 73 Draconis, HD 221568, and HD 224801) that possess double waves in their visual light variations all possess double waves in their rare-earth variations. HD 125248 has not usually been counted among these objects, but the beautiful demonstration of a double wave in visual light by Maitzen and Rakosch (1970) prompted the writer to reexamine the observed equivalent-width variation for the rare earths published by Deutsch (1958). A weak secondary maximum is clearly visible: It resembles the weak, secondary maximum seen in the other five stars with double waves. For all of the remaining stars in Table I except one, ϵ Ursae Majoris, maximum visual light coincides with maximum line strength of the most strongly variable element. In a few cases only the K line of Ca II (78 Vir) or the Sr II lines (χ Serpentis) are known to be variable. For these stars the light variations are weak, and one may ask whether the bound-free continua of Ca II and Sr II alone could be responsible, or whether there are other subtle variations as yet undetected in their spectra.

Apart from the comments about double waves

TABLE I

PROPERTIES OF Ap PHOTOMETRIC-SPECTRUM VARIABLES AT MAXIMUM VISUAL LIGHT

		Line Strength*			
HD	Name	Max.	Min.	Remarks†	Phase Relation Taken From
9996	HR 465	RE	(Cr)	----------	Preston and Wolff (1970)
18296	21 Per	RE,Ti,Mn	(Cr)	DW in Sp(RE) and V	Preston (1969*c*)
19832	56 Ari	Si	He	DW in Sp(Si,He) and V	Peterson, B. (1966); Hardie and Schroeder (1963)
65339	53 Cam	(RE),Ti,Mg	----	----------	Babcock (1958); Preston and Stepień (1968)
71866	----	RE	----	DW in Sp(RE) and V	Babcock (1956); Stepień (1968*b*)
108662	17 Com A	See Remarks	----	Ca, (Eu) max. at mean light	Preston, Stepień, and Wolff (1969)
112185	ε UMa	----	Ca	DW in V	Provin (1953)
112413	α^2 CVn	RE,(Ti)	(Cr),(Fe)	----------	Pyper (1969)
118022	78 Vir	Ca	----	----------	Preston (1969*b*); Wolff and Wolff (1971)
124224	HR 5313	Si	He	DW in Sp(Si,He) and V	Peterson, B. (1966); Hardie (1958)
125248	HR 5355	RE	Cr	DW in V and probably in Sp(RE)	Wolff and Wolff (1971; Deutsch (1958)
140160	χ Ser	Sr	----	----------	Provin (1953)
173650	HR 7058	RE,Fe-peak	----	----------	Rice (1970)
188041	HR 7575	RE:	----	RE max $0\overset{P}{.}1$ before max. light	Babcock (1954); Wolff (1969); van Genderen (1971)
196502	73 Dra	RE,Ti,Mn,Sr	Mg,(Cr),(Fe)	DW in Sp(RE) and V	Preston (1967*d*); Stepień (1968*a*)
221568	----	RE,Si	(Fe-peak),Sr	DW in Sp(RE,Si) and in V	Osawa (1967); Kodaira (1967)
224801	HR 9080	RE	----	DW in Sp(Eu) and V	Stepień (1968*b*); Preston (unpublished)

NOTES TO TABLE I

*Parentheses indicate that the spectrum variation is weak or uncertain.

†DW = double wave.

Sp = Spectrum variation of element in parentheses.

V = visual light variation.

RE = rare earths.

above, no attempt has been made in this review to interpret the *shapes* of light curves. However, mention must be made of Rice's (1970) conclusion that the shape of the V light variation of HD 173650 closely resembles that of the equivalent-width variations of lines in that star. Similar conclusions can be drawn for several other spectrum variables in Table I. These are strongly indicative of a causal relation between light and spectrum variations along the lines suggested by Peterson and by the Wolffs. Hopefully, the time will come when light variations will be well-enough understood so that they can be used to map the temperature and/or abundance features that are associated with magnetic geometries of the Ap stars. At present, unfortunately, research in this complex subject proceeds at a phenomenological level not greatly different from the one that frustrated Struve 30 years ago.

REFERENCES

Babcock, H. W. 1947, *Ap. J.* **105**, 105.
—— 1949, *The Observatory* **69**, 191.
—— 1951, *Ap. J.* **114**, 1.
—— 1954, *Ap. J.* **120**, 66.
—— 1956, *Ap. J.* **124**, 489.
—— 1958, *Ap. J. Suppl.* **3**, 141 (No. 30).
—— 1960*a*, *Ap. J.* **132**, 521.
—— 1960*b*, in *Stellar Atmospheres,* J. L. Greenstein, ed. (Chicago: University of Chicago Press), p. 282.
Belopolsky, A. 1913, *A.N.* **196**, 1.
Böhm-Vitense, E. 1965, in *Magnetism and the Cosmos,* W. R. Hindmarsh, F. J. Lowes, P. H. Roberts, and S. K. Runcorn, eds. (New York: American Elsevier Publishing Co., Inc.), p. 179.
Burbidge, G. R., and Burbidge, E. M. 1955, *Ap. J. Suppl.* **1**, 431 (No. 11).
Cameron, R. C. 1967, in *The Magnetic and Related Stars,* R. C. Cameron, ed. (Baltimore: Mono Book Corporation).
Conti, P. S. 1970, *Ap. J.* **160**, 1077.
Cowley, A., Cowley, C., Jaschek, C., and Jaschek, M. 1969, *A.J.* **74**, 375.
Deutsch, A. J. 1947, *Ap. J.* **105**, 283.
—— 1954, *Trans. I.A.U.* **8**, 801.
—— 1956, *Pub. A.S.P.* **68**, 92.
—— 1958, in *Electromagnetic Phenomena in Cosmical Physics, I.A.U. Symposium No. 6,* B. Lehnert, ed. (Cambridge: Cambridge University Press), p. 209.
—— 1970, *Ap. J.* **159**, 985.
Dieke, G. H., Crosswhite, H. M., and Dunn, B. 1961, *J. Opt. Soc. Am.* **51**, 820.
Fowler, W. A., Burbidge, E. M., Burbidge, G. R., and Hoyle, F. 1965, *Ap. J.* **142**, 423.
Guthrie, B. N. G. 1969, *Ap. and Space Sci.* **3**, 542.
Hardie, R. H. 1958, *Ap. J.* **127**, 620.
—— 1967, in *The Magnetic and Related Stars,* R. C. Cameron, ed. (Baltimore: Mono Book Corporation), p. 481.
Hardie, R. H., and Schroeder, N. M. 1963, *Ap. J.* **138**, 350.
Havnes, O., and Conti, P. S. 1971, *Astr. and Ap.* (in press).
Hockey, M. 1971, *M.N.R.A.S.* **152**, 97.
Kodaira, K. 1967, *Ann. Tokyo Astr. Obs.* 2nd Series, **10**, 157.
Landstreet, J. 1970, *Ap. J.* **159**, 1001.
Maitzen, H. M., and Rakosch, K. D. 1970, *Astr. and Ap.* **7**, 10.
Mestel, L. 1967, in *The Magnetic and Related Stars,* R. C. Cameron, ed. (Baltimore: Mono Book Corporation), p. 101.
Michaud, G. 1970, *Ap. J.* **160**, 641.
Mihalas, D. 1966, *Ap. J. Suppl.* **13**, 1.
Osawa, K. 1965, *Annals Tokyo Astr. Obs.*, Ser. 2, **9**, 123.
—— 1967, in *The Magnetic and Related Stars,* R. C. Cameron, ed. (Baltimore: Mono Book Corporation), p. 363.
Peterson, B. A. 1966, *Ap. J.* **145**, 735.
Peterson, D. M. 1970, *Ap. J.* **161**, 685.
Preston, G. W. 1967*a*, in *The Magnetic and Related Stars,* R. C. Cameron, ed. (Baltimore: Mono Book Corporation), p. 1.
—— 1967*b*, *Ap. J.* **147**, 804.
—— 1967*c*, *Ap. J.* **150**, 547.
—— 1967*d*, *Ap. J.* **150**, 871.
—— 1969*a*, *Ap. J.* **156**, 967.
—— 1969*b*, *Ap. J.* **158**, 243.
—— 1969*c*, *Ap. J.* **158**, 251.
—— 1969*d*, *Ap. J.* **158**, 1081.
—— 1970*a*, *Ap. J.* **160**, 1059.
—— 1970*b*, in *Stellar Rotation,* A. Slettebak, ed. (Dordrecht, Holland: D. Reidel Publishing Co.), p. 254.
—— 1971, *Ap. J.* **164**, 309.
Preston, G. W., and Stępień, K. 1968, *Ap. J.* **151**, 583.
Preston, G. W., and Wolff, S. C. 1970, *Ap. J.* **160**, 1071.
Preston, G. W., Stępień, K., and Wolff, S. C. 1969, *Ap. J.* **156**, 653.
Provin, S. 1953, *Ap. J.* **118**, 489.
Pyper, D. 1969, *Ap. J. Suppl.* **18**, 347 (No. 164).
Rakos, K. D. 1962, *Lowell Obs. Bull.* **5**, 227.
—— 1963, *Lowell Obs. Bull.* **6**, 91.
Rice, J. 1970, *Astr. and Ap.* **9**, 189.
Sargent, W. L. W., and Searle, L. 1967, in *The Magnetic and Related Stars,* R. C. Cameron, ed. (Baltimore: Mono Book Corporation), p. 209.
Schwarzschild, M. 1950, *Ap. J.* **112**, 222.
Stępień, K. 1968*a*, *Ap. J.* **153**, 165.
—— 1968*b*, *Ap. J.* **154**, 945.
Stibbs, D. W. N. 1950, *M.N.R.A.S.* **110**, 395.
Struve, O. 1942, *Proc. Am. Phil. Soc.* **85**, 349.
van Genderen, A. M. 1971, *Astr. and Ap.* (in press).
Wolff, S. C. 1967, *Ap. J. Suppl.* **15**, 21 (No. 134).
—— 1969, *Ap. J.* **158**, 1231.
Wolff, S. C., and Wolff, R. J. 1970, *Ap. J.* **160**, 1049.
—— 1971, *A.J.* **76**, 422.