ON THE SYSTEMS OF SELENOGRAPHIC COORDINATES, THEIR DETERMINATION AND TERMINOLOGY

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Abstract. Attention is drawn to the absence in literature of the precise definitions of selenographic and celestial selenocentric coordinate systems. In certain cases inaccuracies in the formulation of the first Cassini law occur. This is due to the fact that the principal directions dealt with in the theory of lunar rotation are being constantly confused. A clear-cut definition of the principal coordinate systems concerned with the lunar rotation is given. It is indicated that there is no necessity in a special astronomical time service on the Moon. Since the future expeditions to the Moon will be able to keep terrestrial time, the problem of the hour angle is simply solved by the Formula (11).

There are frequent errors in selenographic papers. The principal directions considered in the lunar rotation theory are often confused.

The following directions are distinguished in the theory of lunar rotation around the mass centre: (1) the direction of the minor axis of inertia; (2) that of the lunar rotation instantaneous vector; (3) that of the Cassini axis; (4) that of the mean lunar rotation axis.

In accordance with these directions the equator of the figure of the Moon, the true (instantaneous) lunar equator, the Cassini circle (equator) and the mean lunar equator are determined.

The above-mentioned circles are confused even in Astronomical Almanacs. Thus the lunar equator of the figure is called 'the true equator' while the Cassini circle, 'the mean equator'. Since the definitions of the main selenocentric circles (directions) lack accuracy, we deem it necessary to agree on a standard terminology.

We should like to recall that in the lunar rotation theory the ecliptical coordinate system $OXYZ$ with the origin in the lunar centre of mass is taken as a stable reference system. Then the moving coordinate system $Oxyz$ is selected which is rigidly connected with the Moon's body (Figure 1). The principal lunar inertia axes are most conveniently taken for this purpose so that the axis Ox should coincide with the major axis of inertia, the axis Oy with the mean one, and the axis Oz with the minor axis of inertia. The axis Oz is normally referred to as the axis of the figure of the Moon. The line of the intersection of the lunar surface with the plane xy is frequently referred to as the equator of the figure. The axis Ox is referred to as the 'first radius' of the Moon. The position of the inertia axes trihedron $Oxyz$ relative to the (stable) system is normally determined by the Eulerian angles ψ, θ, φ.

It will be noted that practically all the authors dealing with the physical libration of the Moon erroneously refer to the angles ψ and θ as the longitude of the descending

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node and the inclination of the true equator of the Moon, respectively; whereas, in fact, they are the longitude of the node and the inclination of the equator of the figure of the Moon. Thus inaccurate use of terms causes difficulties in the understanding of the literature.

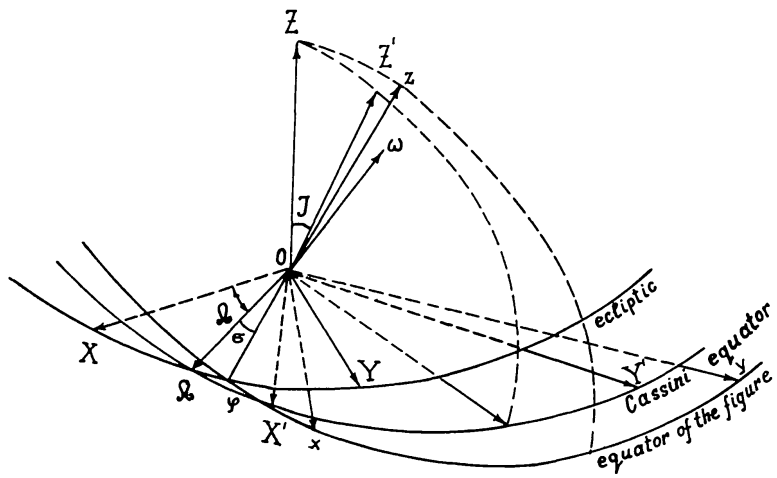

Fig. 1.

The motion of the Moon about the centre of mass is determined by its rotation about the instantaneous vector of rotation $\boldsymbol{\omega}$. The circle whose plane is perpendicular to this instantaneous axis is normally referred to as the true equator of the Moon. The rotation axis $\boldsymbol{\omega}$ and the inertia axis Oz of the Moon do not coincide (the angle between them being about 100″). Therefore, the true equator of the Moon is not to be confused with the equator of its figure.

The projection of the vector $\boldsymbol{\omega}$ along the inertia axes, or, in other words, the position of the rotation axis in the lunar body, is obtained by the well-known Eulerian kinetic equations

$$\begin{aligned}
\omega_x &= -\,\dot{\psi}\sin\varphi\sin\theta - \dot{\theta}\cos\varphi\,,\\
\omega_y &= -\,\dot{\psi}\cos\varphi\sin\theta + \dot{\theta}\sin\varphi\,,\\
\omega_z &= \dot{\psi}\cos\theta + \dot{\varphi}\,.
\end{aligned} \tag{1}$$

To obtain the expressions of these components in terms of time functions we must solve the dynamical Eulerian equations:

$$\begin{aligned}
A\dot{\omega}_x + (C - B)\,\omega_y\omega_z &= N_x\,,\\
B\dot{\omega}_y + (A - C)\,\omega_z\omega_x &= N_y\,,\\
C\dot{\omega}_z + (B - A)\,\omega_x\omega_y &= N_z\,,
\end{aligned} \tag{2}$$

where A, B, C are the principal moments of lunar inertia; N_x, N_y, N_z are the components of the resulting moment of external forces.

Due to the Earth gravitation and, to a far lesser degree, solar gravitation, the lunar rotation axis does not remain parallel to itself but changes its position both relative to the stable reference system $OXYZ$ and within the body of the Moon, relative to the moving coordinate system $Oxyz$. As a result of this motion of the instantaneous rotation axis, the movement of the true poles of the Moon can be observed among the stars on the celestial sphere and over the lunar surface.

The lunar pole displaces slowly among the stars with an approximately constant speed and simultaneously oscillates periodically. Thus the motion of the true pole (the instantaneous axis of rotation) can be expanded into two components: secular and periodic. The lunar pole (axis of rotation) possessing only secular motion is called the mean pole (mean rotation axis). Obviously the true pole is characterized by both the secular and the periodic motions. The motion of the mean lunar pole about the ecliptic pole is precessional, whereas the motion of the true pole about the mean one is nutational.

In this connection one should remember that the rotation of the Moon is studied by a somewhat different method from the rotation of the Earth. In the lunar rotation theory the secular (precessional) motion of its axis is assumed to be given by the Cassini laws and, consequently, the Eulerian angles ψ, θ, φ are from the very start expressed as a sum of two parts, namely,

$$\begin{aligned} \psi &= \psi_0 + \sigma , \\ \varphi &= \varphi_0 + (\tau - \sigma) , \\ \theta &= \theta_0 + \rho , \end{aligned} \tag{3}$$

The first (secular) terms are determined by the equations

$$\begin{aligned} \psi_0 &= \Omega , \\ \varphi_0 &= 180^\circ + (l_☾ - \Omega) , \\ \theta_0 &= I , \end{aligned} \tag{4}$$

expressing the Cassini laws, where $l_☾$ denotes the mean lunar longitude, Ω the mean longitude of the ascending node of its orbit on the ecliptic, and I the inclinantion of the Cassini equator to the ecliptic.

The second terms are small, nearly-periodic functions of time. They are referred to as the physical libration of the Moon, σ being called physical libration in the node, τ the physical libration in longitude and ρ libration in inclination.

The Eulerian angles (4) determine in space a certain trihedron of the axes $OX'Y'Z$ (Figure 1), which as we suggested earlier, should be called the Cassini coordinate system. We named the circle formed by the intersection of the plane $X'Y'$ with the lunar surface the Cassini equator. If the Moon rotated strictly in accordance with the Cassini laws, its axes of inertia would coincide with the axes $OX'Y'Z'$.

The motion of the Cassini trihedron about the point O would take the form of the rotation about the instantaneous axis $\boldsymbol{\omega}_0$. The components of the rotation vector $\boldsymbol{\omega}_0$ along the Cassini axes will be expressed by the formulas

$$\begin{aligned} \omega_{OX'} &= - \dot{\Omega} \sin \varphi_0 \sin I , \\ \omega_{OY'} &= - \dot{\Omega} \cos \varphi_0 \sin I , \\ \omega_{OZ'} &= \quad \dot{\Omega} \cos I + \dot{l}_☾ - \dot{\Omega} . \end{aligned} \tag{5}$$

The vector $\boldsymbol{\omega}_0$ is the mean lunar rotation axis, its intersection with the celestial sphere being the mean selenocentric celestial pole.

One error is typical of practically the entire selenographic literature: the mean rotation axis $\boldsymbol{\omega}_0$ is identified with the Cassini axis OZ', although, in fact, the angle between them constitutes

$$\alpha = -\frac{\dot{\Omega}\sin I}{\omega_{OZ'}} \approx 22''. \tag{6}$$

This error is caused by the fact that up to the present almost all the authors use a not-quite-accurate formulation of the first Cassini law. Hayn (1923), for instance, stresses in his formulation that the Moon rotates about its axis whose position in the body remains unchanged. The period of its full revolution is equal to the mean period of the revolution of the lunar centre of mass about the terrestrial centre of mass.

This formulation would be correct if the rotation axis were not precessing, i.e., if the lunar nodal line did not move. Indeed, if $\dot{\Omega}=0$, then

$$\begin{aligned} \omega_{OX'} &= 0\,, \\ \omega_{OY'} &= 0\,, \\ \omega_{OZ'} &= \dot{l}_{☾}\,. \end{aligned} \tag{7}$$

In this case the position of the rotation axis in the lunar body would be stable and the period of a full revolution would be equal to a sidereal month $\dot{l}_{☾}$. But in accordance with the Cassini laws the body moves in accordance with equations (5) which describe the regular

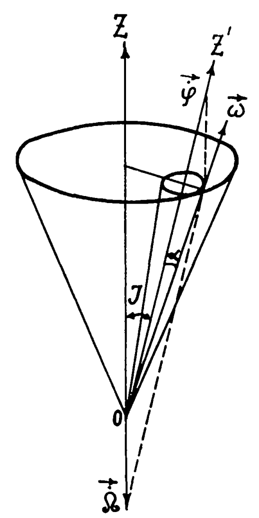

Fig. 2.

procession of the Cassini axis. In accordance with the Equations (5) a body's motion is composed of proper rotation about the axis OZ' with the constant angular velocity $\dot{\varphi}_0 = \dot{l}_{☾} - \dot{\Omega}$ equalling a draconic month and the rotation of this axis about the fixed direction OZ with the constant angular velocity $\dot{\Omega}$. Such body motion about the centre of mass can be described as rolling without sliding of a loose axoid with the axis OZ' along a fixed axoid with the axis OZ (Figure 2).

The precession will have a retrograde character. The angle between the precession velocity vector $\dot{\Omega}$ and that of the velocity of proper rotation $\dot{\varphi}$ will constitute $(180° - I)$ The flare angle of the loose axoid will equal $\alpha = 22''$.

Thus there are two inaccuracies in the formulation of the first Cassini law. First, the mean rotation axis is not fixed in the lunar body but describes the surface of an axoid; second, the period of proper rotation equals a sidereal rather than draconic month.

The Moon's rotation about the mass is virtually studied in relation to the uniformly rotating system of coordinates, i.e., the Cassini trihedron. It is the components of the physical libration of the Moon σ, ρ, τ that determine the position of the lunar axes of inertia $Oxyz$ relative to the rotating coordinate system $OX'Y'Z'$. The rotation of the trihedron of the inertia axes $Oxyz$ about the instantaneous vector $\boldsymbol{\omega}$ is the motion of the true Moon around the centre of masses. The motion of 'the mean Moon', however, is the rotation of the Cassini trihedron $OX'Y'Z'$ about the vector $\boldsymbol{\omega}_0$.

Thus we can see that the kinematic picture of the Moon's rotation can be described by four principal directions: namely,

(1) the instantaneous rotation axis, determined by the vector $\boldsymbol{\omega}$;

(2) the axis of inertia Oz, rigidly connected with the lunar body;

(3) the mean rotation axis, determined by the vector $\boldsymbol{\omega}_0$;

(4) the Cassini axis OZ'. In connection with the given directions four spherical coordinate systems can be established.

Here we must not confuse selenographic coordinate systems, meant for the determination of the position of points (craters) or the Moon's surface, with selenocentric celestial coordinate systems necessary for astrometric observations from the Moon.

Let us first consider the selenographic coordinate systems.

(1) The true selenographic coordinate system. The principal circle is the instantaneous equator of the Moon.

(2) The dynamical coordinate system. The principal circle is the equator of the figure. The coordinates of points on the lunar surface (craters) in this system are firmly connected with the rotating Moon. In almanacs and catalogs the positions of craters in this system are erroneously called 'true selenographic coordinates'.

(3) The mean selenographic coordinate system. This reference system is connected with the mean equator of the Moon.

(4) The Cassini selenographic coordinate system. In this system the Cassini equator is taken as the principal circle. In almanacs these crater coordinates are called 'mean selenographic coordinates'.

The true selenographic coordinate system has not been employed up to the present. But if future astrometric observations, based on the monthly motion of the selenocentric celestial sphere, are to be made from the Moon, the coordinates of the observation point will be defined in terms of this system, similar to the instantaneous latitude observed on the Earth.

On the contrary, the dynamical system (in literature called 'true selenographic coordinates') has found wide application. The catalogs of crater positions and the coordinates of the crater Mösting A are given in terms of this system which is quite expedient, since the dynamical coordinate system is connected with the lunar body and, therefore, the coordinates of the craters do not change with time.

The coordinate system related to the mean equator of the Moon is not employed in selenographic papers. Instead, the Cassini coordinate system is used, which in literature is referred to as 'the mean selenographic coordinate system'. Since the Cassini axis displaces within the body of the Moon, the Cassini coordinates of craters change with time. The formulas of the reduction of the Cassini selenographic coordinates λ, β ('mean coordinates' according to the almanac) to the dynamic coordinates λ_0, β_0 ('true coordinates' according to the almanac) are of the form

$$\begin{aligned} \lambda_0 - \lambda &= -\tau + \operatorname{tg}\beta\left[\rho\cos(\lambda + l_{☾} - \Omega) + I\sigma\sin(\lambda + l_{☾} - \Omega)\right], \\ \beta_0 - \beta &= -\rho\sin(\lambda + l_{☾} - \Omega) + I\sigma\cos(\lambda + l_{☾} - \Omega). \end{aligned} \tag{8}$$

The question of the plotting of selenocentric celestial coordinates attracted attention only recently in connection with the future astronomical observations from the Moon. Here a number of problems are to be solved. Thus no zero point for longitudes has been fixed by appropriate agreements, no final methods of reducing astrometric observations have been worked out, the most effective method of time count is yet to be found, etc.

If we adhere to terrestrial observation methods, the principal spherical coordinate systems will be:

(1) The true selenoequatorial coordinate system. Under this system the principal direction will be the instantaneous lunar rotation axis. The celestial lunar equatorial plane will be the principal plane. The points at which the rotation axis intersects the celestial sphere may be termed 'true selenocentric celestial poles'.

(2) The mean selenoequatorial coordinate system. The mean axis $\boldsymbol{\omega}_0$, whose components are determined by Equations (5) will serve as the principal direction for this system. The mean selenocentric celestial pole will regularly precess about the ecliptic pole in a circle with an angular radius equalling $I+\alpha$ and the revolution period 18.6 yr.

The Cassini celestial coordinate system can be used instead of the mean selenoequatorial coordinate system. The Cassini axis will be the principal direction in this system.

(3) The ecliptic selenocentric coordinate system. This system needs no further explanation.

It should be pointed out that since the selenocentric celestial sphere rotates very slowly, selenocentric astrometry methods may differ considerably from the terrestrial astrometry methods. Owing to the slow rotation of the celestial sphere on the Moon, the system of celestial coordinates, related to the lunar inertia axes (of the equator of the figure) could prove suitable. A collective discussion of all these questions would be highly desirable.

We should like to make a point in connection with the determination of hour angles of stars on the Moon. Some authors suggest that lunar sidereal time should be determined for the solution of parallactic triangles on the lunar celestial sphere. In our opinion there is no need in a special astronomic time service on the Moon. Lunar expedition will always be able to keep terrestrial time by maintaining radio-link with the Earth. Besides, these expeditions will be provided with a special 'Lunar Almanac', containing the necessary astronomical data for observations from the Moon.

Among other things such an almanac should contain selenoequatorial coordinates of a certain number of stars. These coordinates can be taken in terms of either the true or the mean selenoequatorial system or the Cassini coordinate system. Below we shall adhere to the latter. We shall term the angle between the direction to a star and the Cassini celestial equatorial plane the lunar mean declination '*d*' of the star. The other coordinate taken on the equatorial are from the ascending node Ω of the lunar orbit to the declination circle plane shall be termed the mean lunar right ascension '*a*' (see Figure 3).

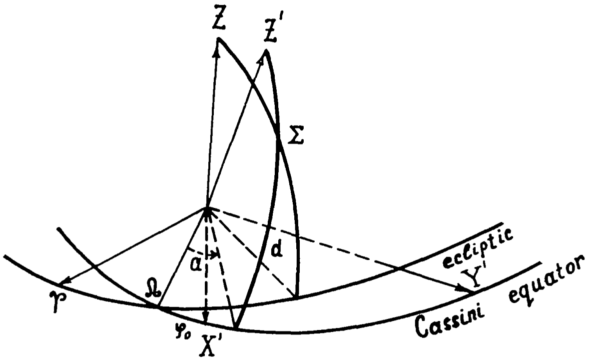

Fig. 3.

Let (β, λ) be the latitude and the longitude of an observation point of the Moon taken in terms of the Cassini system. In this system the selenographic longitude λ is taken from the axis OX' of the Cassini trihedron.

Let the moment of the observation of the star Σ be fixed by the terrestrial time T. Then by the ordinary terrestrial almanac according to (4) it would be easy to find the angular distance φ_T of the axis OX' from the node Ω

$$\varphi_T = 180° + (l_☾ - \Omega)_T . \tag{9}$$

At the moment of the stars culmination the angle φ_c will be

$$\varphi_c = a - \lambda . \tag{10}$$

Hence, the problem of the hour angle can be easily solved; it can be obtained by the formula

$$t = 180° + (l_☾ - \Omega)_T - (a - \lambda) . \tag{11}$$

Should greater accuracy be required, a correction for the physical libration of the Moon can be introduced in the hour angle, i.e., employ the true lunar declination and right ascention. To relate the position of stars to the true (instantaneous) equator, one must give certain values to the parameter (f, I) of the physical libration of the Moon.

The present paper wishes to draw attention to the fact that there are no precise definitions or terms in selenography. To rectify this situation we suggest that a provisional working group should be established within the framework of the 17th IAU Commission. This working group could be entrusted with the task of working out suggestions aimed at standardizing the terms and formulations concerning the principal notions of the selenocentric spherical astronomy.

References

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