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CURVE-OF-GROWTH ANALYSIS OF THE SPECTRUM OF PROCYON

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SUMMARY

The spectrum of Procyon (F5 IV–V) is investigated in the region $\lambda\lambda$ 4000–7500 Å from spectrograms having reciprocal dispersions of 1 to 1·5 Å mm^{-1}. A differential curve-of-growth analysis with respect to the Sun confirms that most elements in the atmosphere of Procyon have abundances, relative to hydrogen, similar to those in the Sun, although heavy elements ($Z \gtrsim 50$) appear to be slightly deficient. Particular problems which arise in the curve-of-growth comparison of these two stars are discussed.

1. INTRODUCTION

A few years ago high-resolution intensity tracings for the wavelength region 3600–8800 Å of the K2 giant α Boo were published in *A Photometric Atlas of the Spectrum of Arcturus* (**1**). In view of the need (**2**) for photometric atlases of stars representing other spectral types, a similar high-resolution investigation of the spectrum of Procyon (F5) has been started.

Procyon (m_V = 0·34) (**3**) was assigned a luminosity class IV–V on the revised MK system (**4**); certainly, its absolute magnitude of +2·64, based on a trigonometric parallax of 0″·288 (**5**), is about a magnitude brighter than that of class V stars with the same $(B–V)$ of 0·40 (**3**). Procyon also belongs to a well-observed binary system with a period of 40·65 yr (**6**); but since the companion, a white dwarf, is 11 magnitudes fainter (**3**) its presence affects only the radial velocity of Procyon A and does not contaminate the observations of line intensities. The primary has a well-determined mass of 1·85 $M_\odot$ (**7**) which, together with a radius of 2·24 $R_\odot$ estimated by Gray (**7**), yields a surface gravity of 10^4 cm s^{-1}.

The earliest published measurements and identifications of the Procyon spectrum were made in the blue by Albrecht (**8**), in the infra-red by Roach (**9**), and throughout the visible by Swensson (**10**); and the first equivalent-width measurements found in the literature were given for a few selected lines by Elvey (**11**) and by Thackeray (**12**). Other lists of equivalent widths covering varying extents of the spectrum include papers by Boyarchuk (**13**), Pannekoek (**14**), Wellmann (**15**), Wright *et al.* (**16**), Edmonds (**17**) and Bogudlow (**18**). The tracings of Procyon published by Hiltner & Williams in their *Atlas of Stellar Spectra* (**19**) were used by Greenstein (**20**) in curve-of-growth analyses of the atmospheres of selected F stars; similar analyses were published in the same year by Wright (**21**). Other curve-of-growth investigations which concern Procyon include those of Boyarchuk (**13**), Wellmann (**15**) and Danziger (**22**).

2. THE SPECTRUM

(i) *Observations*

Seventeen spectrograms, covering the wavelength range 3375–7500 Å at reciprocal dispersions varying from 0·75 Å mm^{-1} in the ultra-violet to about 1·5 Å mm^{-1} in the red, have so far been obtained at the coudé focus of the 100-in. Hooker telescope on Mount Wilson in exactly the same way as in the Arcturus project. All of the spectrograms of Procyon were more than 2 mm wide; most were widened to 3 mm. A description of the calibration and photometric procedures and of the production of direct intensity tracings has been given in the Introduction to the Arcturus Atlas (**1**). More spectrograms of Procyon are required to complete the observational material necessary for a photometric atlas. However, in the investigation of Arcturus (**23**) internal errors of equivalent width measurements were shown to be satisfactorily small and devoid of systematic trends; so that, although there are only single spectrograms of some regions of the Procyon spectrum at present, it seems reasonable to present a curve-of-growth analysis based on the material now available, in the hope that further measurements will reveal accidental errors only and will not substantially affect the results given below.

(ii) *Measurements*

The spectrum of Procyon is particularly suited to high-dispersion spectrophotometry; the lines are sharp and few have central residual intensities smaller than 25 per cent. Furthermore, the frequency of lines is such that the continuum is visible, and blending is not severe enough to cause serious difficulties of interpretation, except in the neighbourhood of the Balmer limit. This study has been limited to $\lambda \geqslant 4000$ Å.

Wavelengths of many lines were calculated from comparator measurements of the original spectrograms; for all other features the wavelengths were interpolated on the tracings. Since the stellar wavelengths were in effect interpolated from a set of standard wavelengths at rest relative to the star, changes in stellar radial velocity did not affect the calculated wavelengths. Equivalent widths of all visible features were measured by counting squares on the tracings. Wavelengths, equivalent widths, central residual intensities and (where possible) identifications of all features have been recorded in a catalogue, which contains data on some 5200 stellar lines. The Revised Multiplet Table (**24**) provided the majority of laboratory data for element and multiplet identifications. Absorption lines due to telluric O_2 and H_2O were measured for wavelength and included in the catalogue.

3. THE ANALYSIS

(i) *Curves of growth*

Differential curves of growth were constructed by plotting, for each atom or ion, graphs of log W/λ against log $X_\odot$, where $X_\odot$ is the 'solar *f*-value' of a line: log $X_\odot$ is the value of the abscissa on the normalized standard solar curve of growth corresponding to the equivalent width of that line in the solar spectrum. 'Partial' curves of growth, drawn for groups of lines with similar lower excitation potentials χ, were fitted together into one 'total' curve for zero-volt lines by applying suitable horizontal shifts; such shifts in abscissae are equivalent to the Boltzmann correction

$\chi\Delta\theta_{ex}$, where θ_{ex} is the reciprocal excitation temperature $(5040/T_{ex})$.* Photoelectric measurements of solar equivalent widths were used where available (**25**), and were used exclusively for an atom or ion if they were sufficiently numerous; photographic measurements (**26**) from the Utrecht Atlas (**27**) were otherwise included, though given lower weight. The Cowley–Cowley solar curve of growth (**28**), modified slightly as described in Section 4(ii) below, was adopted as the standard solar curve. Lines which occurred on the damping region of the solar curve of growth were omitted from the analysis.

Differential excitation temperatures $\Delta\theta_{ex}$ were estimated from the curves for neutral Si, Cr, Ti, Fe and Ni; the most acceptable value appeared to be $-0{\cdot}10$, though this was only at all definite in the case of Fe I. Ionized elements indicated a smaller numerical value for $\Delta\theta_{ex}$ but the partial curves showed rather bad scatter and had insufficient points; a figure of $-0{\cdot}05$ was adopted. Total differential curves of growth for 33 atoms or ions were constructed by plotting log W/λ against log $X_{\odot}-\chi\Delta\theta_{ex}$. A stellar curve drawn freehand for neutral elements resembled very closely Unsöld's theoretical curve for the Schuster–Schwarzschild model (based on Minnaert's empirical formula) (**29**) but with a displacement in the ordinate equivalent to a microturbulent velocity parameter of about 2·4 to 2·7 km s^{-1}; a slightly greater microturbulent velocity (3 km s^{-1}) was found to be more appropriate for ionized lines. Unsöld's curve was fitted to all the differential graphs, and the 'curve-of-growth shifts', or horizontal shifts between the stellar curve and the normalized solar curve, were recorded. The curves of growth are reproduced in Fig. 1. The curve for Fe I represents lines arising from odd parity spectroscopic levels only; the special case of this element is discussed in Section 4(ii).

(ii) *Chemical abundances*

The observed curve-of-growth shift $[X]$ for an element in two stars is related to the differential abundance ratio $[N/H]$ by

$$[X] = [D]+[N/H],$$

where $[D]$ is the horizontal shift, equivalent to strengthening or weakening of weak lines, which is to be expected because of differences in temperature and electron pressure between the two stars, and can be calculated if the sources of opacity in both atmospheres are known. In solar-type stars it is assumed that H^- is the dominant source of opacity. In Pagel's formulation for solar-type stars (**30**), if an element j has a first ionization potential I_j and a mean degree of ionization x_j in a star whose 'representative' (reciprocal) temperature is θ and mean electron pressure is P_e, then for neutral lines of easily ionized elements

$$[D] = [\theta]+[x_j]+\Delta\theta(I_j-0{\cdot}74), \tag{1a}$$

and for ionized lines, and neutral lines of elements which are predominantly neutral in the atmosphere

$$[D] = -1{\cdot}5[\theta]+[x_j]+\Delta\theta(-0{\cdot}74)-[P_e]. \tag{1b}$$

$[P_e]$ is calculated from Saha's equation, applied differentially to both neutral and ionized lines of an element:

$$[X]_{\text{neutral}}-[X]_{\text{ionized}} = \Delta\theta.I_j+2{\cdot}5[\theta]+[P_e]. \tag{2}$$

* Note: ΔQ and $[Q]$ are defined here as Q(Procyon) $-$ Q(Sun), and log Q(Procyon) $-$ log Q(Sun) or log $Q_P-\log Q_{\odot}$, respectively.

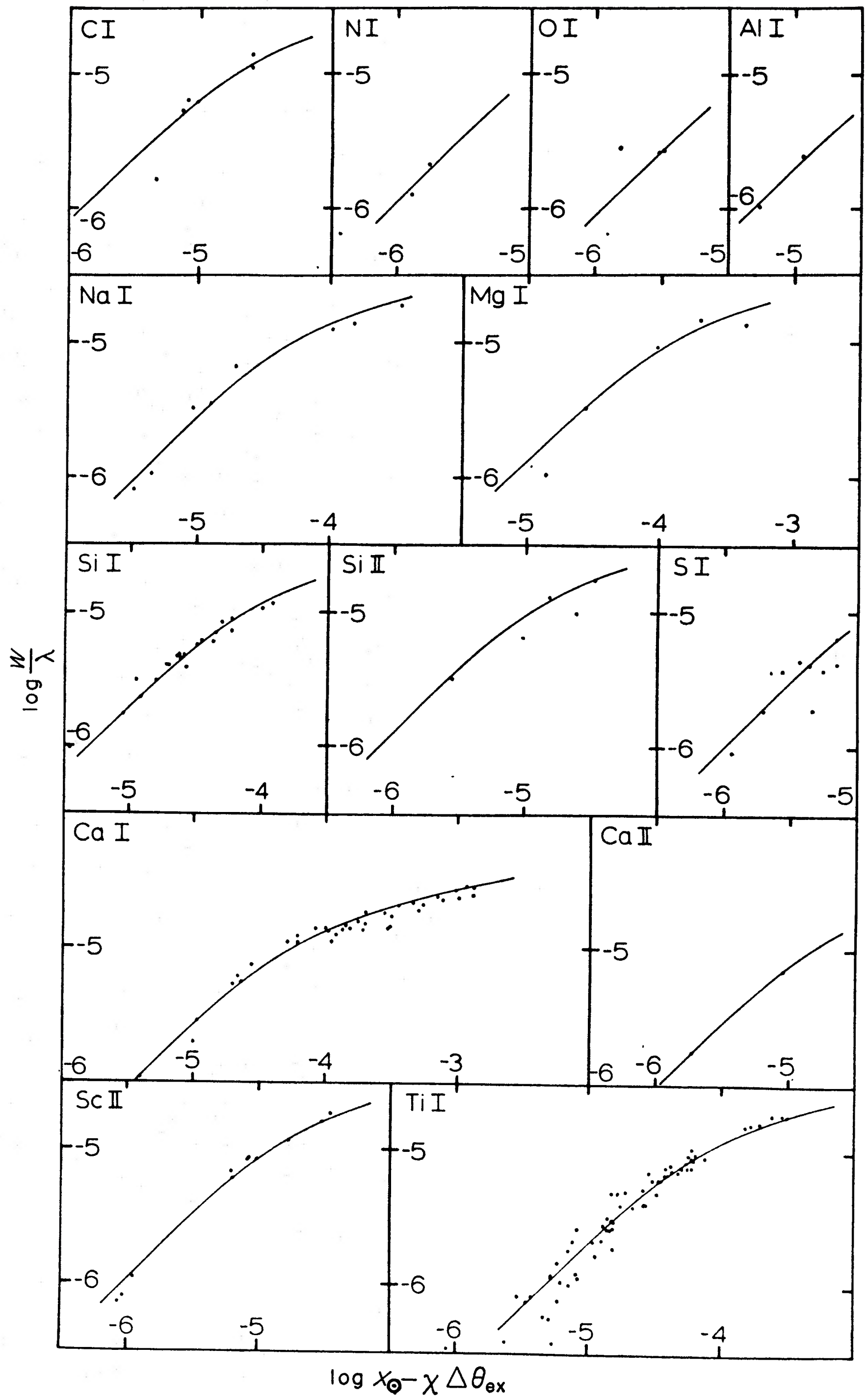

FIG. 1. *Differential curves of growth for Procyon with respect to the Sun.*

As the solutions to these equations depend heavily upon the terms $\Delta\theta.I_j$, the choice of $\Delta\theta$ is critical. In principle the 'representative' temperature refers to the effective temperature θ_{eff}, although it has been argued (**31**) that, where there is sufficient evidence, $\Delta\theta$ in equations (1a) and (1b) should be replaced by the respective value of $\Delta\theta_{\text{ex}}$ for each atom or ion. The distribution of continuous flux in the spectrum of Procyon has been studied in detail by Talbert & Edmonds (**32**). Their values for θ_{eff} of 0·76 or 0·79 are supported by earlier results of 0·78 by Hanbury Brown *et al.* (**33**) and by Gray (**7**) and 0·77 by Oke & Conti (**34**). If θ_{eff} for

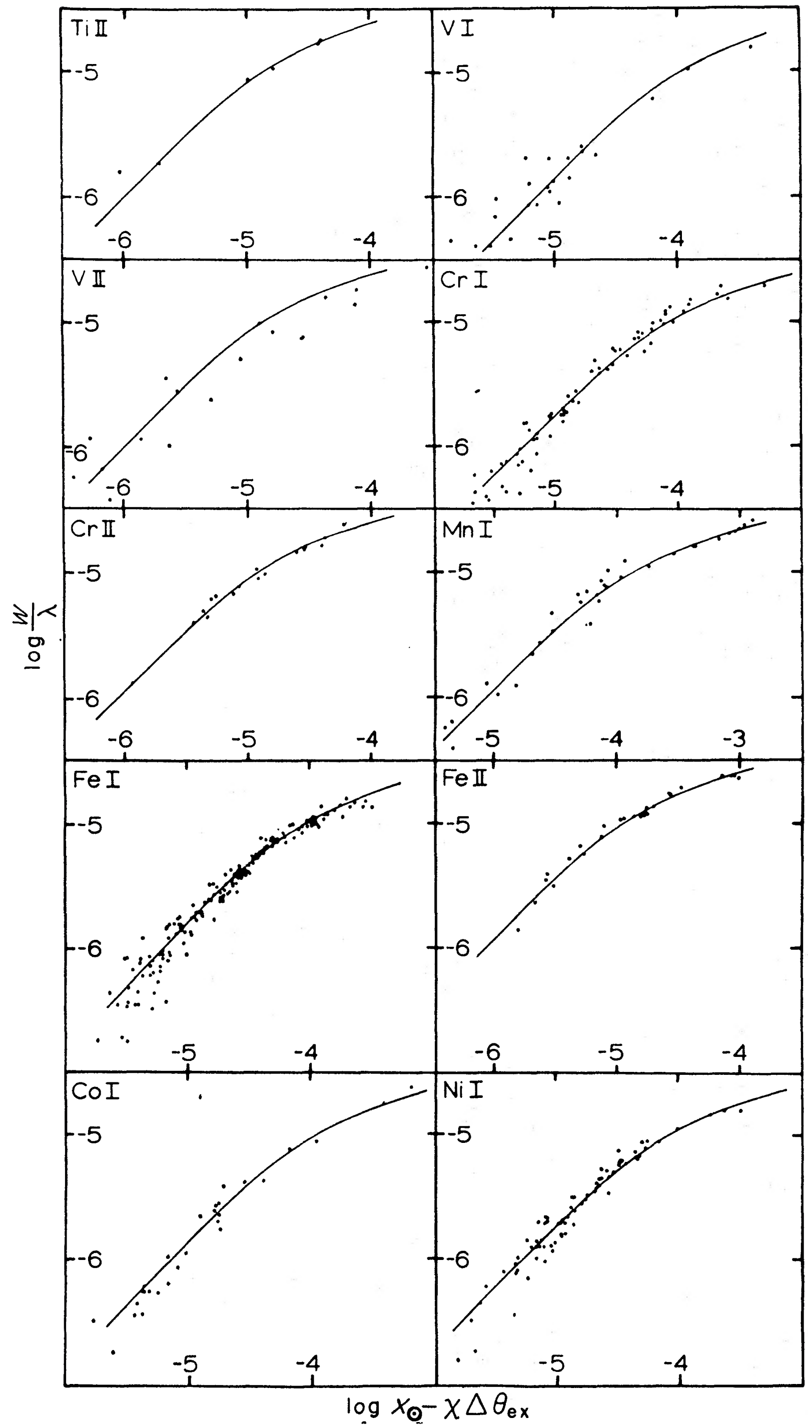

FIG. 1. (*Continued*).

the Sun is assumed to be 0·87 (35), then $\Delta\theta_{eff} = -0{\cdot}10$, the same as $\Delta\theta_{ex}$ for neutral lines. The substitutions $\Delta\theta = \Delta\theta_{eff}$ and $[\theta] = [\theta_{eff}]$ were therefore made; any real difference between $\Delta\theta_{eff}$ and $\Delta\theta_{ex}$ for ionized lines is in any case very small.

The observed ($[X]$) and calculated ($[D]$) curve-of-growth shifts are given in Table I. Equation (2) was solved for Ti, Cr and Fe and yielded $[P_e] = +0{\cdot}07$. For elements with $I_j < 8$ eV ionization is practically complete in both stars. The

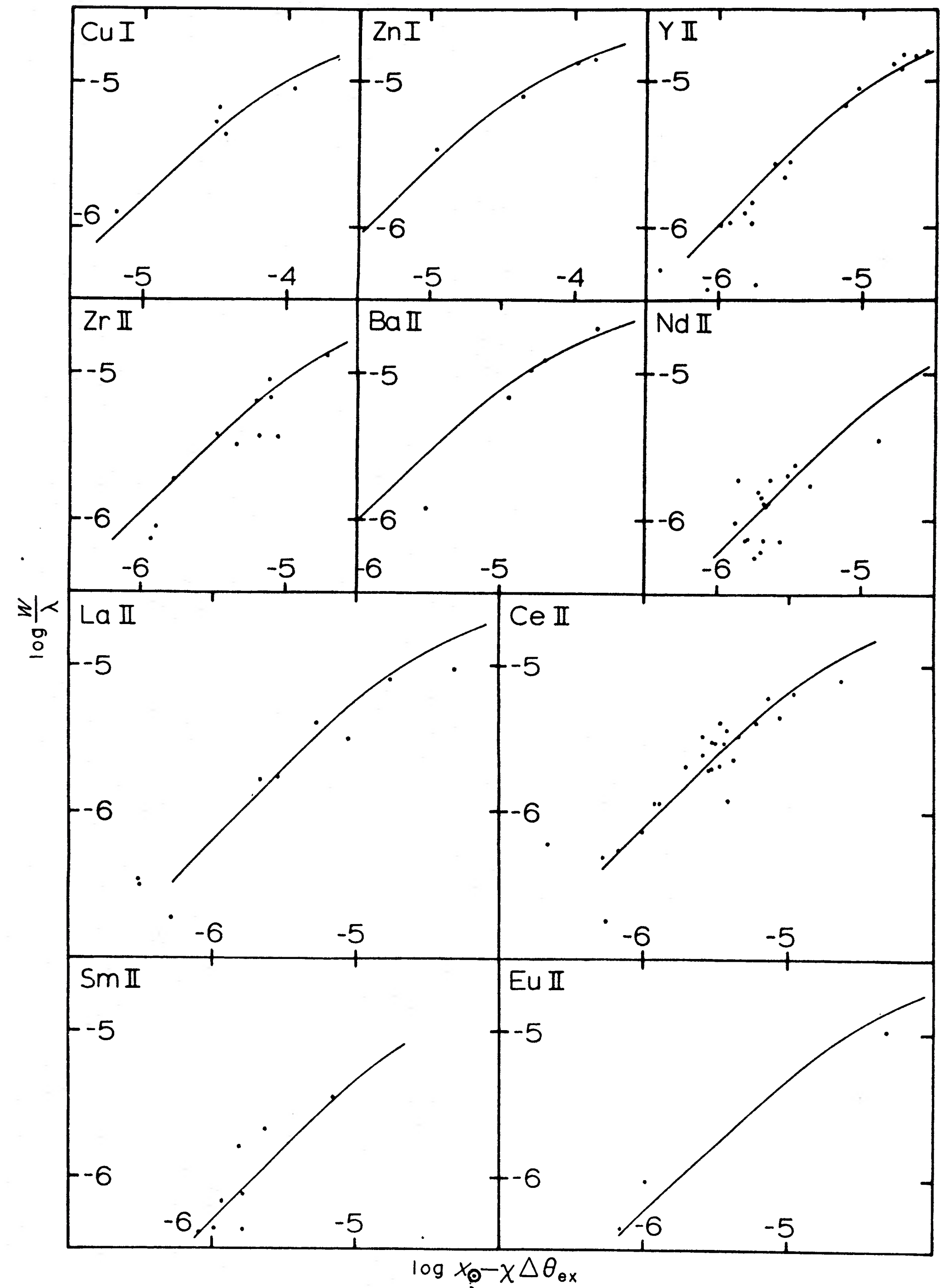

FIG. 1. (*Continued*). *In drawing the curves of growth higher weight has been given to points which refer to photoelectric, as opposed to photographic, measurements of solar equivalent widths but the two types of points have not been distinguished in the diagrams. This explains why some of the curves, in particular that for Zr* II, *may appear to fit rather badly.*

degrees of ionization for elements with $I_j > 8$ eV were determined from the Bilderberg solar model (**34**) and from Edmonds' model VI for Procyon (**36**). Si and Zn were found to be only partially ionized and the values of $[x_j]$ at optical depth $\tau = 0{\cdot}1$ ($+0{\cdot}1$ for Si and $+0{\cdot}3$ for Zn) were adopted as the mean values for the two atmospheres. C, N, O and S were found to be predominantly in the neutral

TABLE I

Abundances of elements in Procyon relative to the Sun

Spectrum	I_j	[*D*]	[*X*]	[*N*/*H*]
C I	11·20	+0·08	−0·1:	−0·2:
N I	14·49	+0·08	+0·1:	0·0:
O I	13·56	+0·08	0·0:	−0·1:
Na I	5·14	−0·49	−0·53:	0·0:
Mg I	7·64	−0·74	−0·86:	−0·1:
Al I	5·98	−0·57	−0·7:	−0·1:
Si I	8·15	−0·7	−0·69	0·0
Si II	—	+0·2	+0·1:	−0·1:
S I	10·31	+0·08	0·0:	−0·1:
Ca I	6·11	−0·59	−0·56	0·0
Ca II	—	+0·08	0·0:	−0·1:
Sc II	6·7	+0·08	+0·03	0·0
Ti I	6·82	−0·66	−0·68	0·0
Ti II	—	+0·08	0·00	−0·1
V I	6·74	−0·65	−0·9:	−0·2:
V II	—	+0·08	0·0:	−0·1:
Cr I	6·76	−0·65	−0·76	−0·1
Cr II	—	+0·08	+0·04	0·0
Mn I	7·43	−0·72	−0·94	−0·2
Fe I	7·87	−0·76	−0·80	0·0
Fe II	—	+0·08	+0·06	0·0
Co I	7·86	−0·76	−0·87	−0·1
Ni I	7·63	−0·74	−0·74	0·0
Cu I	7·68	−0·74	−0·78:	0·0:
Zn I	9·37	−0·6	−0·56:	0·0:
Y II	6·5	+0·08	+0·02	−0·1
Zr II	6·92	+0·08	+0·05	0·0
Ba II	5·19	+0·08	−0·01	−0·1
La II	5·6	+0·08	−0·2:	−0·3:
Ce II	6·54	+0·08	−0·1:	−0·2:
Nd II	6·3	+0·08	−0·2:	−0·3:
Sm II	6·6	+0·08	−0·3:	−0·4:
Eu II	5·64	+0·08	−0·2:	−0·3:

: denotes uncertain entry.

state in both stars. The derived abundance ratios are given in the last column of Table I.

The accuracy of the observed curve-of-growth shifts may be as good as ±0·05 for the neatest curves but can be ±0·1 or worse for curves which show bad scatter or which have very few points; the less accurate results are denoted by a colon in the Table. The error of calculation of [*N*/*H*], as distinct from the errors in the underlying assumptions, is therefore of the order of ±0·1 (except for the uncertain entries). The choice of $\Delta\theta$ affects [*D*] and hence [*N*/*H*], and a small error in $\Delta\theta$ can have quite alarming results (**31**); however, the lack of correlation between I_j and [*N*/*H*] for neutral elements suggests that the choice of $\Delta\theta_{ex}$ was not seriously in error. Although the curve-of-growth shifts for ionized lines are altered by up to +0·1 in the logarithm when allowance is made for a higher microturbulent velocity parameter, [*N*/*H*] is effectively unchanged since [*X*], [P_e] and [*D*] are affected alike. The whole analysis depends upon the validity of such approximations to single-layer models as are implied in the derivation of the quantities [P_e] and $\Delta\theta$; and the results for Si and Zn are further worsened by the introduction of the mean degrees of ionization $[x_j]$. External systematic errors may be present in both the photometry and the method of constructing curves of growth; they are discussed in the following section.

Within the experimental uncertainties the abundance ratio $[N/H]$ in Procyon is generally the same as in the Sun, in agreement with the analyses by Wright (**21**) and Greenstein (**20**). There is a tendency for the abundances of V I and Mn I to be lower, relative to hydrogen, in Procyon. However, lines of these elements are known to be affected by hyperfine broadening (**37**). Hyperfine broadening has the same effect upon a curve of growth as microturbulent broadening in that it increases the height of the Doppler shoulder; moreover, the amount of hyperfine broadening varies from line to line, so that each line will lie on its own curve of growth. Since in this analysis the same solar curve was used throughout, the presence of hyperfine structure would give rise to apparent overabundances of V I, Mn I and Co I in the solar spectrum. There is also a tendency, noted by Greenstein (**20**), for the ratio of heavy elements to hydrogen to be slightly deficient in Procyon compared with the Sun. Ba has a second ionization potential of only 10 eV and the number of Ba II ions may therefore have been depleted by double ionization. Ba is thought to be the only element affected in this way at the temperature of Procyon.

An apparent logarithmic deficiency of 0·1 would arise if the continuous opacity in Procyon were 25 per cent greater than was assumed. A possible contribution to the opacity may come from neutral hydrogen, but according to figures kindly supplied by Dr P. R. Warren the additional opacity caused by neutral hydrogen absorption is negligibly small in both stars as far down as optical depth $\tau = 0{\cdot}5$, and only 7 per cent in Procyon and 1 per cent in the Sun at $\tau = 1$.

It should be pointed out that Procyon is known to be a member of a binary system in which the companion is at an advanced evolutionary stage. The current minimum separation is 10 A.U. (**6**); if a similar separation existed during any period of mass loss which the present white dwarf may have undergone in the past, the wholesale mass transfer which is supposed (**38**) to occur in the case of much closer binaries would not have occurred. Nevertheless, a star only 10 A.U. away from an object shedding substantial mass could be expected to receive some of it; and it is as well to keep in mind that the composition of the surface layers of Procyon A (which are all that we examine in this paper), and even the composition of the whole star, may have been influenced at some past epoch by processes external to the star we now observe.

It is also relevant to mention that Conti & Danziger (**39**), investigating the abundances of Li and Be at high dispersion in F dwarfs and K giants, reported that the Li I and the Be II resonance doublets were both absent in Procyon, and that it was the only star besides Arcturus known to show neither Li nor Be.

4. COMMENTS AND CRITICISMS

(i) *Comparison of equivalent widths*

Many authors who measure equivalent widths like to compare their results with other published lists. The value of such comparisons is not so much to illuminate accidental errors or ragged measurements as to discover whether different operators, working with completely different equipment, can obtain similar values for measurements of the same spectral features. The survey by Wright *et al.* (**16**)

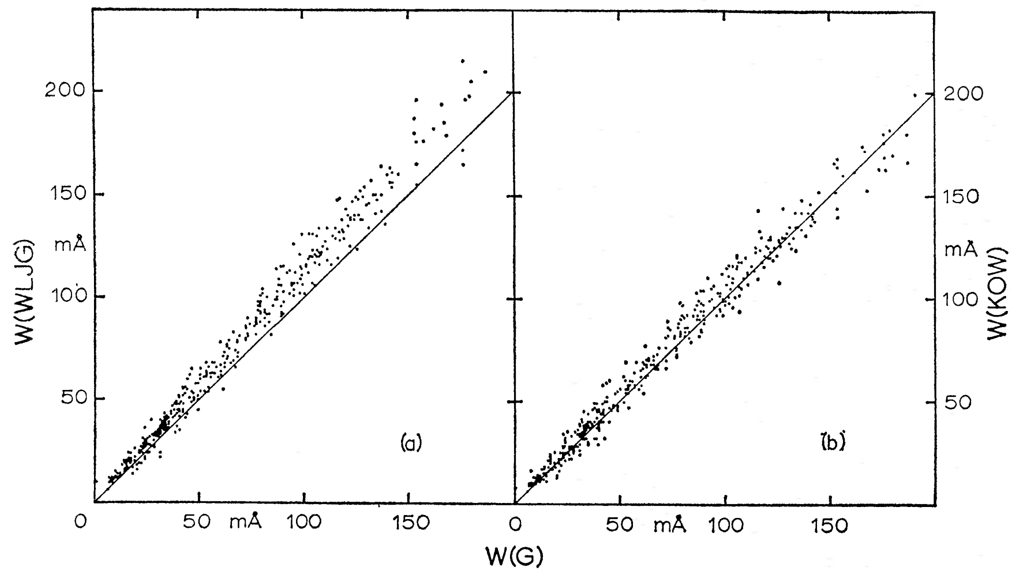

FIG. 2. *Comparison of the author's measurements (abscissae) of equivalent widths in Procyon with those of (a) Wright* et al. (**16**), *and (b) Wright (see text).*

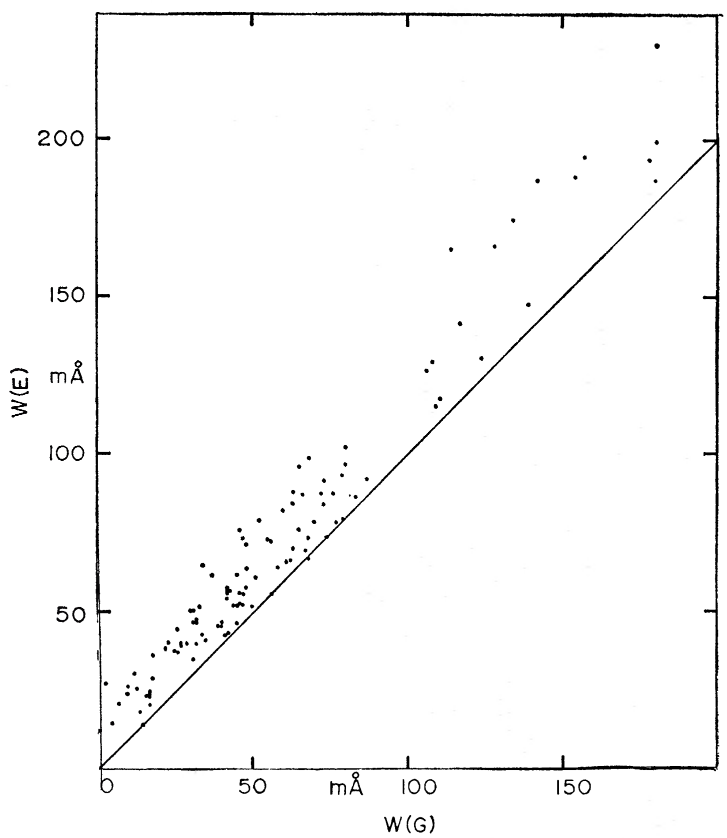

FIG. 3. *As Fig. 2, with Edmonds' measurements* (**40**) *as ordinates.*

presents equivalent widths for stars of types B to G in the wavelength region 3900–4520 Å, obtained from a variety of sources and spectrographs and averaged. In so far as the independent sets of intensities differ systematically, often quite alarmingly, from one another, useful information has undoubtedly been lost through the process of averaging. The measurements used in the present study,

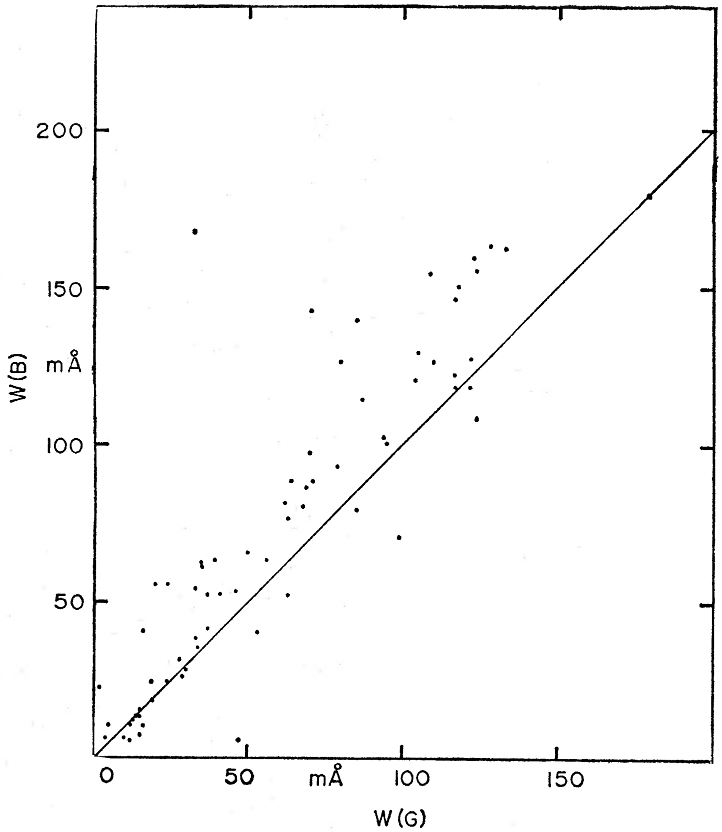

FIG. 4. *As Fig. 2, with Bogudlow's measurements* (**18**) *as ordinates.* 16 *lines in Bogudlow's list appeared too blended on the writer's tracings for reliable measurement of equivalent widths, and have not been included in the diagram.*

W(G), have been compared in Fig. 2(a) with those of Wright *et al.*, W(WLJG); the latter represent the unweighted mean of up to six different investigations. There is clearly a systematic disagreement, amounting to 12 per cent +2 mÅ.

One of the sets, W(KOW), included in the above data, was obtained at Mount Wilson at a dispersion of 2·9 Å mm^{-1} by Wright; the agreement between the two sets of Mount Wilson data is shown in Fig. 2(b) to be good.

After the survey by Wright *et al.*, Edmonds (**40**) published equivalent widths W(E) for selected identified lines in the wavelength region 3260–6610 Å measured from spectrograms having reciprocal dispersions of 3 to 5 Å mm^{-1}; W(E) are compared with W(G) in Fig. 3, where it is seen that Edmonds' data are systematically greater than W(G) by about 12 per cent + 10 mÅ. Part of the difference could be explained if Edmonds were supposed to have positioned the continuum level too high. This would also give rise to some spurious identifications; indeed, at least four lines, of 10, 11, 14 and 29 mÅ equivalent width, which are included in his line list for $\lambda > 4000$ Å, are not visible on the higher dispersion tracings.

A third comparison concerns equivalent widths W(B) of Fe II lines measured by Bogudlow (**18**) from spectrograms obtained with the 102-in. telescope of the Crimean Observatory and having reciprocal dispersions of 1·1 to 3·3 Å mm^{-1}. Although the dispersions of the respective spectrograms are very similar, Fig. 4 shows that the equivalent widths derived from them do not agree at all well. Some of the scatter must be attributed to misidentifications and blends in the line list of the Russian author.

It is clear that systematic errors exist in the published high-dispersion measurements of the spectrum of Procyon; but their magnitude and even their sign cannot be determined without recourse to some superior method of measurement such as is offered in principle by double-pass photoelectric photometry. In a differential analysis such errors will be reduced provided that the photometry for both or all the stars in question is carried out by the same person using the same equipment. In the differential analysis of Procyon presented here this proviso is not met; and any systematic errors in the photometry which affect the results must remain at present unknown.

(ii) *The curve-of-growth technique*

Much has been said about the serious weaknesses of the curve-of-growth method, its gross oversimplifications and its untenable approximations. For two stars such as Procyon and the Sun, whose spectra are not in general overcrowded with lines, it could be hoped that its application to measurements of unblended lines would at least provide some unambiguous answers. This was not in fact the case. Lines of neutral elements, in particular Fe, Ti, Cr and Ni, are very numerous; in the solar spectrum they are well distributed both in strength and in excitation potential and so constitute a reasonable basis for determining, for the Sun, both the excitation temperature and the shape of the curve of growth. In the Procyon spectrum these lines are weakened owing to the star's higher temperature, so much so that $\Delta\theta_{ex}$ can only be determined with any certainty for Fe I. Lines of ionized elements provide neither the same wealth of lines nor adequate ranges of excitation potential. Few of the differential curves of growth for Procyon are therefore well defined and so the same theoretical curve was made to fit all curves alike.

'Absolute' curves of growth for Procyon have also been investigated; these are drawn with laboratory or theoretical values of $\log gf\lambda$ as abscissae, and the relative horizontal shifts between partial curves for various multiplet groups yield information about the stellar excitation temperature, θ_{ex}. Past experience suggested limiting these attempts to Fe I, Ti I and possibly Cr I. To say that published f-values contain gross unidentified errors and to elaborate upon the anomalous effects that these create is to belabour a well-beaten dog; nevertheless, if the physical data were reliable on even a relative scale a value for θ_{ex} could be determined with considerably more certainty than is actually possible. The f-values for iron-group elements given by Corliss & Bozman (**41**) have been corrected by Takens (**42**), whose table for Ti I yielded a temperature θ_{ex} of 0·91 in Procyon. Unfortunately, it is now known that Takens' tables still contain a considerable but unquantified wavelength-dependent error. Fe I f-values have had a chequered history. The extensive list published by Corliss & Warner (**43**) has undergone major corrections and revisions. In the most recent of these (**44**) Allen concludes that 'we can now be sure that the Fe I scale of oscillator strengths is known to within about 0·1 dex'; at the same time, however, his diagrams reveal a large internal raggedness, amounting to about 1·0 'dex', for which he cannot readily account. Accurate experimental f-values for Fe I are now being reported from several laboratories (see references cited in (**44**)); but the lines so far investigated are about ten times too strong in Procyon for a temperature determination, which requires truly weak lines. Cr I f-values that are accredited greatly improved accuracy have recently been measured by Wolnik *et al.* (**45**) but again there are not enough of them for a determination of θ_{ex}; large systematic differences exist between these values and those given by Takens.

The superficial attraction of a differential curve-of-growth analysis is its dispensation with f-values. However, inasmuch as the fundamental assumption beneath the use of stellar f-values is that the intrinsic shape of the standard stellar curve of growth is known, oscillator strengths do still affect the work. The Cowley–Cowley solar curve of growth (**28**), which was derived from the Corliss & Bozman f-values for iron-group elements and photoelectric measurements of solar equivalent widths, represents a thorough appraisal of available data; it is almost identical with van der Held's theoretical curve (**46**), and a microturbulent velocity parameter of 1·4 km s^{-1} is quoted. The present investigation was commenced using, as the standard, the Cowley–Cowley solar curve as tabulated in their paper. However, many of the stellar differential curves revealed a discontinuity between the ' shoulder ' and the weak-line regions, in the sense that weak lines were too weak compared with stronger lines and gave rise to an ambiguity of about 0·1 in the logarithm in fitting the adopted Procyon curve to the points. The solar curve of growth was therefore re-examined, using Takens' f-values, and a downward shift to the Cowleys' curve of 0·08 in the logarithm was judged to be an improvement. The solar curve thus modified was then used for redrawing the differential curves for Procyon, and though the modification had been slight it had a considerable effect upon the stellar curves in that lines falling on the Doppler shoulder now had greater values of $X_\odot$ and so fitted onto the same curve of growth as the weaker lines. It is felt that more attention should be drawn to the nature of the systematic error resulting from the use of an unsuitable standard curve of growth in a differential analysis, particularly when very few weak lines are represented on the differential curve.

The case of Fe I in the solar spectrum is both interesting and unfortunate. It has been demonstrated (**47**) that the curve of growth for Fe I is bifurcated in the region corresponding to strong lines, so that lines arising from spectroscopic levels with odd parity lie on a curve representing greater damping than lines arising from even levels. This ' Carter effect ', which Warner has explained quantitatively (**48**), complicates the concept of ' a ' standard solar curve of growth since the bifurcation begins in the horizontal region where many of the solar Fe I lines occur; but inasmuch as the most recent (and accurate) experimental Fe I f-values all refer to strong solar lines (and demonstrate clearly this marked bifurcation) it is not possible, without weak lines measured on the same scale, to construct the whole curves. It was noticed that the same strong lines in Procyon exhibited practically no bifurcation at all, probably because the effect is obscured by smaller damping in Procyon owing to lower surface gravity and gas pressure. The analysis of weak and medium-strength Fe I lines in Procyon, using the modified solar curve of growth, yielded well-defined curves for lines of odd parity (which nearly all arise from 3 eV and higher) and an accurate determination of $\Delta\theta_{ex}$. On the other hand, lines of even parity, chiefly arising from levels with smaller excitation potentials and most of them strong enough to be affected by damping in the solar spectrum, were evidently not represented satisfactorily by the modified solar curve of growth; those near the Doppler shoulder of the Procyon curve (the great majority) appeared spuriously too strong (log $X_\odot$ too small) and $\Delta\theta_{ex}$ could not be determined accurately from these lines.

The microturbulent velocity parameter ξ_t derived from fitting Unsöld's theoretical curve of growth to differential curves for Procyon was about 2·5 km s^{-1}. Previous determinations range from 1·03 km s^{-1} (**20**) to 3·6 km s^{-1} (**13**). Cowley & Cowley (**28**), using a van der Held theoretical curve, derived 1·4 km s^{-1} for the solar atmosphere. However, these two theoretical curves differ in shape only in the region of the Doppler shoulder, where they are separated by a vertical distance of 0·14 in the logarithm. There is therefore an ambiguity of 40 per cent in any determination of ξ_t owing to the available choice of theoretical curves of growth. It cannot be inferred from this analysis that there is any significant difference in ξ_t between the atmospheres of Procyon and the Sun.

It has been noted that differential curves for ionized lines appear to show a greater microturbulent velocity parameter than that for neutral lines, but effects of this nature probably arise through the use of the same curve of growth for all lines irrespectively in the solar spectrum and may have no astrophysical significance. The differential curve-of-growth technique cannot take into account the real variations of broadening and damping within a stellar atmosphere; the fact that they appear indicates that the observational material is worthy of a more elaborate analysis.

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