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GRAVITATIONALLY COLLAPSED OBJECTS OF VERY LOW MASS

Stephen Hawking

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SUMMARY

It is suggested that there may be a large number of gravitationally collapsed objects of mass 10^{-5} g upwards which were formed as a result of fluctuations in the early Universe. They could carry an electric charge of up to ± 30 electron units. Such objects would produce distinctive tracks in bubble chambers and could form atoms with orbiting electrons or protons. A mass of 10^{17} g of such objects could have accumulated at the centre of a star like the Sun. If such a star later became a neutron star there would be a steady accretion of matter by a central collapsed object which could eventually swallow up the whole star in about ten million years.

It has been known for some time that a star of mass M greater than about one and a half times the mass of the Sun cannot support itself against gravity when it has exhausted its nuclear fuel. If therefore it has not ejected sufficient matter to reduce its mass below this figure by the end of its lifetime, it seems that it must undergo gravitational collapse to produce a 'black hole' of radius about the Schwarzschild radius $2GM/c^2$. This collapsed object would produce a gravitational field of the same order as that of the original star and therefore could still be detected by its gravitational effect. The theory of a mass ejection in the later stages of stellar evolution is still uncertain but it seems that there could easily be as many collapsed stars as visible ones in our galaxy. Indeed the recent observations by Weber (**1**)–(**3**) of gravitational wave pulses which appear to come from the galactic centre suggest that objects of stellar mass may be collapsing at a rate of more than one a day in the nucleus of the galaxy.

The aim of this paper however is to suggest that there may also be a large number of collapsed objects of very much smaller mass which were formed in the very early stages of the Universe. The basis for this suggestion is the 'Chaotic Cosmology' proposed by Misner (**4**). This theory is an attempt to avoid having to postulate very special initial conditions for the Universe in order to produce the presently observed features such as the high degree of isotropy and the existence of galaxies. Instead, it is postulated that these were initially large random fluctuations on all length scales but that most of these fluctuations were later damped out by dissipation processes such as neutrino viscosity and photon viscosity.

A comoving volume V, in the early Universe, would have a gravitational binding energy of the order of $G\rho^2V^{5/3}$ where ρ is the density. The kinetic energy of expansion of the matter in the volume would be of the order of $\rho V^{5/3}(\dot{V}/V)^2$ and the potential energy arising from the relativistic pressure would be of the order of $\rho c^2 V$. This can be neglected in comparison with the gravitational energy if $V > (c^2/G\rho)^{3/2}$.

If the picture of large initial fluctuations is correct there must have been many such volumes for which the gravitational energy considerably exceeded the kinetic energy of expansion. These regions would not have continued to expand with the rest of the Universe but would have collapsed again. If these collapsed regions are not to be completely disconnected universes on their own, the mass in them must not be so large as to close them off from our Universe. This, together with the requirement that gravity should be able to defeat the pressure forces implies that the mass M of the collapsed object will be of the order of $(c^6/G^3\rho_0)^{1/2}$ where ρ_0 is the density in the region at the time of maximum expansion.

Since gravitational collapse is essentially a classical process, it is probable that black holes could not form with radii less than the Plank length $(Ghc^{-3})^{1/2} \sim 10^{-33}$ cm, the length at which quantum fluctuations of the metric are expected to be of order unity. A Schwarzschild radius of this length would correspond to a mass of about 10^{-5} g. For lengths larger than 10^{-33} cm it should be good approximation to ignore quantum gravitational effects and treat the metric classically. One might therefore expect collapsed objects to exist with masses from 10^{-5} g upwards.

It might be thought that a collapsed object could not form unless its Schwarzschild radius were greater than the Compton wavelength h/cm of one of the elementary particles which went to form it. This would imply a minimum mass for a collapsed object of about 10^{14} g. However, this does not seem a valid argument since the Compton wavelengths of the photon and other zero rest-mass particles are infinite, yet a sufficient concentration of electromagnetic radiation can cause gravitational collapse. The relevant wavelength to compare with the Schwarzschild radius is not the wavelength at rest but hc/E where E is the typical energy of a particle. This will be much greater than mc^2 as the particles will be ultra-relativistic. In fact if there are q different species of particle present, the temperature T will be of the order of $(\rho c^5h^3/qk^4)^{1/4}$ and so the typical wavelength will be $hc/kT = (hq/\rho c)^{1/4}$. This will be less than the Schwarzschild radius if $M > (c^2/2G)(qh/\rho_0 c)^{1/4}$. But $M \sim (c^6/G^3\rho_0)^{1/2}$. Thus the condition will be satisfied if $M > \frac{1}{4}(chq/G)^{1/2} \sim q^{1/2} \times 10^{-5}$ gm. This again indicates that collapsed objects cannot have masses of less than about 10^{-5} g. Hagedorn (5) has suggested that q might increase exponentially in the early Universe. However it has been claimed (6) that Hagedorn's theory breaks down when the wavelength of a typical particle is greater than the particle horizon. The Schwarzschild radius of a collapsed object is of the same order as the particle horizon at the time of maximum expansion. We shall therefore consider the possibility that may be collapsed objects of any mass from 10^{-5} g upwards.

An upper bound on the number of these objects can be set from the measurements by Sandage (7) of the deceleration of the expansion of the Universe. These measurements indicate that the average density of the Universe cannot be greater than about 10^{-28} g cm^{-2}. Since the average density of visible matter is only about 10^{-31} g cm^{-2}, it is tempting to suppose that the major part of the mass of the Universe is in the form of collapsed objects. This extra density could stabilize clusters of galaxies which, otherwise, appear mostly not to be gravitationally bound.

One might expect these collapsed objects to have velocities in the range 50–1000 km s^{-1}, similar to those of other bodies such as stars and galaxies, which move primarily under the influence of gravity. A collapsed object moving with velocity v through matter of density d would lose energy by gravitational scattering at a rate of the order of $4\pi G^2M^2\, d/v^2$ per unit distance. This is so low that a 1 g object could travel 10^{21} light years through solid lead at 100 km s^{-1} without being appreciably slowed down. However it is possible that a collapsed object could carry an electric charge and so lose energy at a much higher rate by electromagnetic scattering. This charge would arise from there being an unequal number of positive and negatively charge particles in the region which collapsed. If the difference in number is Z, the electrostatic energy will be of the order of $Z^2e^2V^{-1/3}$. For the region to be able to collapse against electrical repulsion this energy must be less than the gravitational energy $GM^2V^{-1/3}$, i.e. $|Z|$ must be less than $M(G/e^2)^{1/2} = 5 \times 10^5 M$. If this is the case the region can collapse to a 'black hole' which resembles the Reissner–Nordstrom solution with charge Ze. The charge would be later reduced if particles of the opposite charge were to fall into the black hole. This process of neutralization would continue until the temperature fell to the point where the wavelength of the particles was greater than the radius of the hole. If one assumes that the electrostatic energy of the collapsed object at this time was of the order of kT, one has

$$\frac{Z^2e^2c^2}{2GM} \sim \frac{hc^3}{2GM}$$

and so $Z^2 \sim (hc/e^2) \sim 900$. Thus one might expect values of $|Z|$ of up to 30.

A charged collapsed object would behave in many respects like an ordinary atomic nucleus with the same value of Z. If it travelled at high velocity through matter, it would induce ionization and excitation and would lose energy at a rate of the order of $4\pi Z^2e^4\, d/m_e m_p v^2$ per unit distance (**8**) where m_p and m_e are the proton and electron masses respectively. This is a factor $Z^2e^4/G^2M^2m_em_p = (Z^2/M^2) \times 10^{18}$ times greater than the energy loss by purely gravitational scattering. Such an object would produce a track in a bubble chamber similar to that of an atomic nucleus of the same charge. It could however be distinguished from such a particle by the fact that it did not undergo any detectable deflection in a magnetic field. If one suppose that most of the mass of the Universe was in the form of charged collapsed objects of mass 10^{-5} g, travelling at 10 000 km s^{-1}, one would expect one such object a year to strike each 150 square metres of bubble chamber. In any analysis of bubble chamber photographs there are always a few tracks which remain unidentified. It is therefore quite possible that a collapsed object could have been observed but not have been recognized.

A charged collapsed object which was moving through matter at a velocity less than a few thousand km s^{-1} would tend to capture electrons (if $Z > 0$) or protons (if $Z < 0$) to form an electronic or protonic atom respectively. In the former case the radius of the atom would be similar to that of an ordinary atom, i.e. about 10^{-8} cm, but in the latter case it would be 1800 times smaller. From simple geometric considerations one can give an estimate of the probability of an orbiting electron or proton falling down the central black hole and neutralizing its charge. This probability is low for 10^{10} years if $M < 10^3$ g in the case of an electronic 'atom' or if $M < 10^{-2}$ g in the case of a protonic one. One would therefore not expect many such atoms with nuclei heavier than these values.

An 'atom' which was travelling through matter at less than a few thousand km s^{-1} would lose energies by elastic collisions at a rate of the order of $\sigma v^2 d$ per unit length where the cross-section σ is of the order of 10^{-16} sq. cm for an electronic atom of 10^{-23} sq. cm for a protonic one. This latter value is so low that such an atom would probably never be brought to rest. On the other hand an electronic atom with mass in the range 10^{-4}–10^{-5} g would pass right through the Earth but could be stopped by a star of the mass and size of the Sun. If most of the mass of the Universe were in the form of low mass, charged, collapsed objects, the Sun could have acquired a mass of 10^{17} g of such atoms. These would have diffused to the centre and would have probably coalesced to form a single black hole of radius 10^{-11} cm. A mass of 10^{17} g is not much compared to 10^{33} g, the mass of the Sun, but it might possibly have an effect on the small central region in which most of the energy is generated and could, conceivably, be the reason why the flux of neutrinos from the Sun is not what was predicted.

If the Sun or a similar star were later to evolve into a neutron star, the presence of such a black hole at centre would have rather more effect. Since the radius of the hole would be greater than the wavelength of a neutron, there would be a steady accretion of matter by the hole at a rate of the order of 10^{10} g per year. This would produce a slight shrinking of the surface and might possibly be the cause of the recently observed pulsarquakes. As more matter fell in, the black hole would get bigger and would finally swallow the whole neutron star in about ten million years. The last stages would be very rapid and might produce the gravitational waves that Weber observes.

Institute of Theoretical Astronomy, University of Cambridge

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