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RADIO RECOMBINATION LINES AND NON-LTE THEORY: A REANALYSIS

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ABSTRACT

A graphical method of obtaining non-LTE solutions of data on radio recombination lines for the Orion Nebula and M17 shows the dependence upon different calculations of the non-LTE populations (b_n-factors). Temperature solutions are found to be independent of differences in b_n-factors. Although these differences do affect the solutions for the electron concentration and the emission measure, the fact that high values of the emission measure are always obtained indicates that most of the radiation must come from very dense emitting regions embedded in the nebulae.

I. INTRODUCTION

Recent non-LTE analysis of radio recombination line data for H II regions (Hjellming and Churchwell 1969; Hjellming and Davies 1970) has depended upon a single set of b_n-factors calculated by Sejnowski and Hjellming (1969). Using different collision cross-sections, Brocklehurst (1970*a*, *b*) has recently calculated b_n-factors which are significantly different from those obtained by other authors. Such differences may cause changes in the electron temperature $\langle T_e \rangle$, the electron concentration $\langle N_e \rangle$, and the emission measure $\langle E \rangle$ obtained in the solutions. The interpretations and assumptions underlying these averages are discussed by Hjellming and Davies (1970); they represent averages weighted mainly by N_e^2.

This paper presents a reanalysis of data on radio recombination lines for the Orion Nebula and M17 by using a method designed to show the dependence of the results on the b_n-factors and on uncertainties in the data. We consider three sets of b_n-factors which represent the complete range of cross-sections which have been used. The new form of analysis is more convenient to use than the three-parameter "best fit" method used previously.

The reader should note that we will assume the high-order lines are well understood; that is, their intensities can be described by the same processes used to account for the α-line emission. The alternative, that the high-order lines are altered by some unknown process, has been suggested by Mezger and Ellis (1968) and discussed recently by Shaver (1970). If only the α-lines are considered in the solution, it is found that $T_e \sim 7000°$–$8000°$ K with the lines being formed either in LTE or nearly LTE conditions (cf. Sorochenko and Berulis 1969). However, as shown by Hjellming and Churchwell (1969) and Hjellming and Davies (1970), it is possible to obtain a non-LTE solution consistent with nearly *all* line observations, and therefore we shall include high-order lines in this analysis.

We find that (1) determinations of $\langle T_e \rangle$ are largely independent of the b_n-factors; (2) the observational data for $n \gtrsim 100$ can be used to determine $(\Delta b/b)_n = (b_n - b_{n-1})/b_n$ for any value of E, and, since theoretical calculations of the b_n-factors determine $(\Delta b/b)_n$ as a function of N_e (since $\langle T_e \rangle$ is already determined), a graphical comparison can be used to determine the values of $\langle N_e \rangle$ and $\langle E \rangle$ for which the two determinations of $(\Delta b/b)_n$ agree for all *n;* and (3) all three sets of b_n-factors give large values of $\langle E \rangle$ (ranging from 1.4×10^7 to 2.3×10^7 pc cm^{-6} for the Orion Nebula and from

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6.6×10^6 to 1.1×10^7 pc cm^{-6} for M17) compared with the values obtained from the continuum data (3×10^6 pc cm^{-6} for the Orion Nebula [Gordon 1969] and 2.4×10^6 for M17 [Schraml and Mezger 1969]). Hence, either we must say that all sets of b_n-factors are seriously in error or we must accept the extremely clumped model implied by the high values of $\langle E \rangle$.

II. DETERMINATION OF TEMPERATURE

We now consider the relation between the electron temperature and the parameters of the data. Let ν_L = frequency at the center of a radio recombination line of hydrogen resulting from a transition from an upper level with principal quantum number m to a lower level n, T_C = continuum antenna temperature measured at this frequency, T_L = excess antenna temperature due to the line at the line center, and $\Delta\nu_L$ = line width at the half-intensity level (see Hjellming and Davies 1970 for discussion of assumptions hidden in these definitions). Kardashev (1959) showed that if the line is formed under LTE conditions ($b_n = b_m = 1$) in an optically thin plasma, then the temperature is related to the observed line strength, $\Delta\nu_L T_L/T_C$, by a formula which we will rewrite as

$$(T_e)_{\mathrm{LTE}} = 26630\left(\frac{\nu_L}{\mathrm{GHz}}\right)^{1.826}\left[\!\!\left[\left\{\left(\frac{\Delta\nu_L}{\mathrm{kHz}}\frac{T_L}{T_C}\right)\left[1+\frac{N(\mathrm{He\,II})}{N(\mathrm{H\,II})}\right]\right\}^{-1}[(m-n)f_{nm}/n]\right]\!\!\right]^{0.87} \quad (1)$$

(cf. Mezger and Ellis 1968). In this equation f_{nm} is the absorption oscillator strength and $f_{nm}/n = 0.194$, 0.0271, 0.00841, 0.00365, and 0.00191 if $m - n = 1$, 2, 3, 4, and 5, respectively (Goldwire 1968). We will take $N(\mathrm{He\,II})/N(\mathrm{H\,II}) = 0.1$ for both the Orion Nebula and M17.

Data on recombination-line strengths for the Orion Nebula and M17 are presented in Table 1; in addition, the values of $(T_e)_{\mathrm{LTE}}$ calculated from equation (1) are presented. The errors quoted in Table 1 are uniformly at the 3σ level wherever possible.

With due allowance for the observational uncertainties, it is clear that a wide range of values of $(T_e)_{\mathrm{LTE}}$ is obtained; in particular, we note the well-known result (Zuckerman *et al.* 1967, Mezger and Ellis 1968) that the α-lines tend to give much lower temperatures than the high-order lines. The different temperatures obtained in Table 1 can be interpreted as showing that the emitting plasma is neither in LTE nor optically thin at all frequencies.

A more complicated relation between T_e and $\Delta\nu_L T_L/T_C$ must be used if non-LTE effects are present, as suggested by Goldberg (1966), or if the optically thin assumption is not valid. Under these conditions we can write the following equation (Goldberg 1968; Hjellming, Andrews, and Sejnowski 1969) relating T_L/T_C and $\Delta\nu_L$ to $\langle T_e \rangle$, $\langle E \rangle$, b_n, and $(\Delta b/b)_n$:

$$\frac{T_L + T_C}{T_C} = \frac{1}{1 - \exp(-\tau_C)} \times \left[\!\!\left[\frac{1 + b_m \tau_L{}^{\mathrm{LTE}}/\tau_C}{1 + b_n\beta\tau_L{}^{\mathrm{LTE}}/\tau_C}\left\{1 - \exp\left[-\left(1 + \frac{b_n\beta\tau_L{}^{\mathrm{LTE}}}{\tau_C}\right)\tau_C\right]\right\}\right]\!\!\right], \quad (2)$$

where

$$\tau_L{}^{\mathrm{LTE}} = \frac{1.53 \times 10^{-9}(f_{nm}/n)n^3\nu_L\langle E \rangle}{\Delta\nu_L\langle T_e \rangle^{2.5}[1 + N(\mathrm{He\,II})/N(\mathrm{H\,II})]}, \quad (3)$$

$$\beta = 1 - 20.84\frac{\langle T_e \rangle}{\nu_L}\left(\frac{\Delta b}{b}\right)_n (m - n), \quad (4)$$

and

$$\tau_C = 0.0314\langle E \rangle\langle T_e \rangle^{-1.5}\nu_L{}^{-2}[\ln(0.04955/\nu_L) + 1.5\ln\langle T_e \rangle], \quad (5a)$$

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TABLE 1

RECOMBINATION-LINE DATA

Line	ν_L (GHz)	$\Delta\nu_L T_L/T_C$ (kHz)	$(T_e)_{\mathrm{LTE}}$ (° K)	Reference
			Orion Nebula	
H56α......	36.47	1260 ±300	8400(+2200, −1400)	Sorochenko *et al.* 1969
H65α......	23.40	623 ±230	6900(+3400, −1700)	Churchwell *et al.* 1970
H85α......	10.52	89 ± 5	8710(+450, −400)	Churchwell and Mezger 1970
		99.8 ± 2.9	7890±200	Gordon 1970
H94α......	7.79	61.5 ± 4	6940(+420, −370)	Gordon and Meeks 1968
H104α.....	5.76	28.5 ± 9.4	7820(+3300, −1720)	Gudnov and Sorochenko 1968
		29.4 ± 4.8	7610(+1280, −940)	Dravskikh and Dravskikh 1967
H109α.....	5.01	22.3 ± 1.8	7480(+570, −490)	Palmer 1968*b*
		23.2 ± 1.7	7240(+500, −430)	Churchwell and Mezger 1970
		24.2 ± 0.5	6980±120	Reifenstein *et al.* 1970
		25.2 ± 0.8	6550±180	Mezger and Höglund 1967
H110α.....	4.87	22.8 ± 0.5	6990±130	Davies 1970
H126α.....	3.25	9.74	6980	McGee and Gardner 1968
H134α.....	2.70	5.95	7660	Zuckerman and Palmer 1969
H156α.....	1.72	2.1 ± 0.26	8270(+1010, −800)	Williams 1967
H158α.....	1.65	2.42 ± 0.1	6820(+260, −240)	Dieter 1967
H166α.....	1.42	1.4 ± 0.2	8380(+1200, −920)	DeBoer *et al.* 1968
H149α.....	1.97	3.71 ± 0.19	6480(+300, −280)	Menon and Payne 1969
H150α.....	1.93	3.57 ± 0.19	6460(+320, −290)	Menon and Payne 1969
H151α.....	1.89	3.30 ± 0.19	6670(+350, −320)	Menon and Payne 1969
H152α.....	1.85	3.53 ± 0.19	6060(+300, −270)	Menon and Payne 1969
H153α.....	1.82	3.33 ± 0.19	6160(+320, −290)	Menon and Payne 1969
H154α.....	1.78	3.04 ± 0.19	6430(+370, −330)	Menon and Payne 1969
H155α.....	1.75	2.53 ± 0.19	7280(+510, −450)	Menon and Payne 1969
H156α.....	1.72	2.25 ± 0.19	7790(+620, −530)	Menon and Payne 1969
H157α.....	1.68	2.84 ± 0.19	6140(+380, −340)	Menon and Payne 1969
H158α.....	1.65	2.39 ± 0.19	6890(+520, −440)	Menon and Payne 1969
H161α.....	1.56	2.36 ± 0.19	6290(+480, −410)	Menon and Payne 1969
H162α.....	1.53	1.98 ± 0.19	7080(+650, −540)	Menon and Payne 1969
H163α.....	1.50	2.11 ± 0.19	6480(+550, −470)	Menon and Payne 1969
H164α.....	1.48	1.6 ± 0.19	7970(+930, −740)	Menon and Payne 1969
H165α.....	1.45	1.91 ± 0.19	6610(+630, −520)	Menon and Payne 1969
H166α.....	1.42	1.35 ± 0.19	8650(+1200, −940)	Menon and Payne 1969
H167α.....	1.40	1.39 ± 0.19	8160(+1100, −860)	Menon and Payne 1969
H168α.....	1.37	1.39 ± 0.19	7900(+1080, −830)	Menon and Payne 1969
H170α.....	1.33	1.38 ± 0.19	7450(+1030, −790)	Menon and Payne 1969
H171α.....	1.30	1.28 ± 0.19	7710(+1160, −870)	Menon and Payne 1969
H172α.....	1.28	1.32 ± 0.19	7270(+1050, −800)	Menon and Payne 1969
H176α.....	1.20	1.19 ± 0.19	7010(+1150, −850)	Menon and Payne 1969
H177α.....	1.18	1.26 ± 0.19	6470(+990, −740)	Menon and Payne 1969
H178α.....	1.16	0.56 ± 0.19	12700(+5500, −2900)	Menon and Payne 1969
H106β.....	10.74	23.7 ± 1.1	9430(+400, −360)	Gordon 1970
H137β.....	5.01	4.4 ± 0.4	10120(+880, −740)	Palmer 1968*b*
		4.9 ± 0.65	9220(+1220, −950)	Churchwell and Mezger 1970
H138β.....	4.90	4.72 ± 0.23	9150(+410, −370)	Davies 1970
H158β.....	3.27	1.26 ± 0.2	13830(+2240, −1660)	Gardner and McGee 1967
H197β.....	1.69	0.18 ± 0.04	21700(+3000, −1940)	Williams 1967
H121γ.....	10.74	8.15 ± 1.9	11670(+3200, −2000)	Gordon 1970
H157γ.....	4.96	2.1 ± 0.4	9730(+1960, −1370)	Churchwell and Mezger 1970
H158γ.....	4.86	2.17 ± 0.22	9130(+890, −740)	Davies 1970
H225γ.....	1.70	0.17 ± 0.03	12300(+2300, −1610)	Williams 1967
H133δ.....	10.69	6.1 ± 0.6	9740(+920, −760)	Gordon 1970
H148δ.....	7.80	3.85 ± 0.96	8170(+2310, −1440)	Gordon and Meeks 1968
H172δ.....	4.99	1.15 ± 0.3	10350(+3100, −1890)	Churchwell and Mezger 1970
H173δ.....	4.91	1.12 ± 0.11	10260(+970, −800)	Davies 1970
H186ε......	4.91	0.72 ± 0.11	10430(+1620, −1200)	Davies 1970

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TABLE 1—*Continued*

Line	ν_L (GHz)	$\Delta\nu_L T_L/T_C$ (kHz)	$(T_e)_{\rm LTE}$ (° K)	Reference
		M17		
H56α......	36.47	1178 ±441	8910(+4490, −2150)	Sorochenko *et al.* 1969
H85α......	10.52	98 ± 5	8010(+370, −340)	Churchwell and Mezger 1970
H104α.....	5.76	32.4 ± 7.9	6990(+1920, −1210)	Gudnov and Sorochenko 1968
		35.1 ± 4.9	6520(+910, −700	Dravskikh and Dravskikh 1967
H109α.....	5.01	24.3 ± 1.8	6950(+480, −420)	Churchwell and Mezger 1970
		26.9 ± 0.5	6350±100	Reifenstein *et al.* 1970
		27.6 ± 4.2	6220(+960, −720)	Zuckerman *et al.* 1967
		34.1 ± 1.2	5180±160	Mezger and Höglund 1967
H110α.....	4.87	26.3 ± 0.5	6180±100	Davies 1970
H126α.....	3.25	10.4	6600	McGee and Gardner 1968
H134α.....	2.70	8.03± 0.5	5900(+340, −300)	Zuckerman and Palmer 1969
H156α.....	1.72	2.21± 0.71	7910(+3170, −1700)	Lilley *et al.* 1966
H158α.....	1.65	2.4 ± 0.81	6870(+2960, −1540)	Lilley *et al.* 1966
		5.18± 0.5	3520(+320, −270)	Dieter 1967
		3.83± 1.0	4570(+1380, −840)	Williams 1967
H166α.....	1.42	1.52± 0.56	7800(+3840, −1870)	Palmer and Zuckerman 1966
H137β.....	5.01	5.9 ± 0.8	7840(+1060, −820)	Churchwell and Mezger 1970
H138β.....	4.90	6.07± 0.3	7350(+330, −300)	Davies 1970
H158β.....	3.27	2.29± 0.8	8220(+3730, −1890)	Gardner and McGee 1967
H197β.....	1.69	0.43± 0.14	10600(+4300, −2300)	Williams 1967
H157γ.....	4.96	2.3 ± 0.6	8980(+2700, −1640)	Churchwell and Mezger 1970
H158γ.....	4.86	2.75± 0.25	7430(+640, −540)	Davies 1970
H225γ.....	1.70	0.34± 0.16	6710(+4960, −1910)	Williams 1967
H173δ.....	4.91	1.48± 0.13	8050(+670, −570)	Davies 1970
H186ε......	4.91	0.77± 0.13	9840(+1720, −1250)	Davies 1970

where all units are cgs except for ν_L which is in GHz and $\langle E\rangle$ which is in pc cm^{-6}. Equation (5a), which is taken from Oster (1961), is frequently expressed in the following approximate form (Mezger and Henderson 1967):

$$\tau_C \approx 0.08235\langle T_e\rangle^{-1.35}\nu_L{}^{-2.1}\langle E\rangle\,. \tag{5b}$$

This approximation was used to derive equation (1).

A second-order expansion of equation (2) has been derived by Goldberg (1968); this expansion can be rewritten in the following convenient form:

$$\langle T_e\rangle \cong (T_e)_{\rm LTE}\{b_n[1 + 0.858\langle T_e\rangle^{-0.35}\nu_L{}^{-3.1}\langle E\rangle(\Delta b/b)_n(m-n)]\} \tag{6}$$

(Gordon 1970), where we have used equation (5b) and where $(T_e)_{\rm LTE}$ is obtained from equation (1). The derivation of equation (6) involves assuming that $|\tau_C + b_n\beta\tau_L{}^{\rm LTE}| \ll 1$, $|\beta| \gg 1$, and $(T_L/T_C)_{\rm LTE} \ll 1$.

If we restrict ourselves to large enough n so that $b_n \approx 1$, then equation (6) involves only $\langle T_e\rangle$ and $\langle E\rangle$ $(\Delta b/b)_n$ as unknowns for each measured line; therefore, measurements of two lines for which $(\Delta b/b)_n$ is the same may be used to eliminate $\langle E\rangle(\Delta b/b)_n$ to obtain

$$\langle T_e\rangle \approx (T_e)_{\rm LTE}{}^{n+1,n}\left[1 - \left(\frac{1}{m-n}\right)\left(\frac{\nu_L{}^{n+1,n}}{\nu_L{}^{m,n}}\right)^{-3.1}\right]^{0.87}$$
$$\times \left\{1 - \left(\frac{1}{m-n}\right)\left(\frac{\nu_L{}^{n+1,n}}{\nu_L{}^{m,n}}\right)^{-3.1}\left[\frac{(T_e)_{\rm LTE}{}^{n+1,n}}{(T_e)_{\rm LTE}{}^{m,n}}\right]^{1.15}\right\}^{-0.87}. \tag{7}$$

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We have assumed that the two lines involved are an α-line ($n + 1 \rightarrow n$) and a high-order line ($m \rightarrow n$, $m - n > 1$). A form of equation (7) was first derived by Palmer (1968*a*).

With the large number of α-lines for the Orion Nebula and M17 it is now possible to obtain values of $\Delta\nu_L T_L/T_C$ for α-lines involving the same principal quantum number as many of the observed high-order lines. In both cases the data are sufficient to permit a single smooth curve to be drawn through the data points, with due allowance for measurement uncertainties, to estimate accurately the characteristics of any α-line involving $100 < n < 180$. This may be seen by an inspection of the α-line data points plotted in Figure 6.

Using the data in Table 1, with the most reasonable curve drawn through the α-line data, we have obtained the solutions for $\langle T_e \rangle$ for each high-order line from equation (7) and have plotted them as a function of n in Figure 1. We see that $\langle T_e \rangle = 10000° \pm 500°$ K for the Orion Nebula and $\langle T_e \rangle = 7500° \pm 800°$ K for M17. The error limits indicated in Figure 1 combine the error limits of both the high-order lines and the estimates of the associated α-line strengths.

As discussed by Hjellming and Davies (1970) and Gordon (1970), essentially the same results can be obtained by another method which is also independent of knowledge of the b_n-factors. This technique makes use of the fact that a given frequency in which the nebula is not optically thick, $(T_e)_{\rm LTE}$ must approach $\langle T_e \rangle$ as $m - n$ increases; this "convergence" is directly implied by equation (7).

It is important to note that both the convergence method and equation (7) can be used to determine $\langle T_e \rangle$ without the use of b_n calculations.

Let us now discuss how the neglect of higher-order terms in equation (7) can affect the solution for $\langle T_e \rangle$ when this equation is used. If we write $\langle T_e \rangle = (T_e)_{\rm LTE}\,(1 + C_1 + C_2)^{0.87}$, where C_1 represents the second term used in equation (7) and C_2 represents the higher-order terms neglected in the expansion, then equation (7) would become

$$\langle T_e \rangle \approx (T_e)_{\rm LTE}{}^{n+1,n}\left[1 - \frac{1}{(m-n)}\left(\frac{\nu_L{}^{n+1,n}}{\nu_L{}^{m,n}}\right)^{-3.1}\frac{(1 + C_2/C_1)^{n+1,n}}{(1 + C_2/C_1)^{m,n}}\right]^{0.87}$$
$$\times \left\{1 - \frac{1}{(m-n)}\left(\frac{\nu_L{}^{n+1,m}}{\nu_L{}^{m,n}}\right)^{-3.1}\left[\frac{(T_e)_{\rm LTE}{}^{n+1,m}}{(T_e)_{\rm LTE}{}^{m,n}}\right]^{1.15}\frac{(1 + C_2/C_1)^{n+1,m}}{(1 + C_2/C_1)^{m,n}}\right\}^{-0.87} . \quad (8)$$

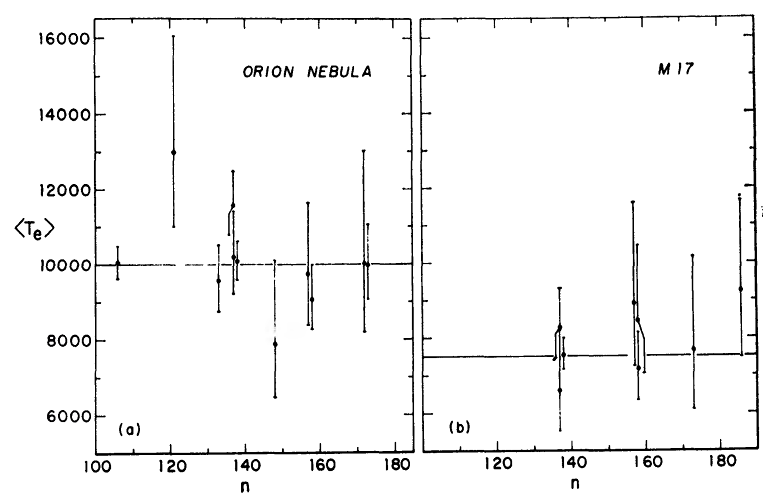

FIG. 1.—Solutions for $\langle T_e \rangle$ obtained by combining data on high-order lines and α-line data for the Orion Nebula and M17. *Horizontal lines*, adopted temperature solutions.

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It can be shown that if $(1 + C_2/C_1)^{n+1,n}/(1 + C_2/C_1)^{m,n}$ differs from unity by 20 percent, then the value of $\langle T_e \rangle$ differs from that obtained by equation (7) by only 4 percent for the H106β and H138β data for the Orion Nebula. Thus equation (7) is relatively insensitive to the neglect of higher-order terms. In any case, the convergence method gives essentially the same $\langle T_e \rangle$ as does equation (7) without making any approximations. Furthermore, the temperatures agree with those obtained by the general "best fit" method used by Hjellming and Davies (1970).

III. SIMPLE DETERMINATION OF $\langle E \rangle(\Delta b/b)_n$

Assuming that $\langle T_e \rangle$ has been determined by methods discussed in § II, we can use equation (6) to solve for $\langle E \rangle(\Delta b/b)_n$, obtaining

$$\langle E \rangle(\Delta b/b)_n = 1.17\{[\langle T_e \rangle/(T_e)_{\rm LTE}]^{1.15} - 1\} \frac{\langle T_e \rangle^{0.35} \nu_L{}^{3.1}}{(m - n)} . \tag{9}$$

Clearly $\langle E \rangle(\Delta b/b)_n$ can then be obtained for each observed line. Figure 2 shows the results obtained for the Orion Nebula and M17 with the data in Table 1. We see that a straight line may be drawn through the resulting points (although the justification of any curve for M17 is very poor), which indicates solutions corresponding to the following:

$$[\langle E \rangle(\Delta b/b)_n]_{\rm Orion} \cong 3.32 \times 10^6 \exp[-0.0665n] \tag{10a}$$

and

$$[\langle E \rangle(\Delta b/b)_n]_{\rm M17} \cong 1.94 \times 10^6 \exp[-0.0683n] . \tag{10b}$$

In Figure 2, and most subsequent figures, the number of observations available is large enough that the scatter of data points is a better indicator of the uncertainties than error bars based upon internal uncertainties. Since $\langle E \rangle$ is assumed to be a constant, the results of Figure 2 and equation (10) indicate the dependence of $(\Delta b/b)_n$ upon n implied by the observations and equation (9). These can be compared with any available theoretical calculations of $(\Delta b/b)_n$ as a function of n. Such a comparison is shown in

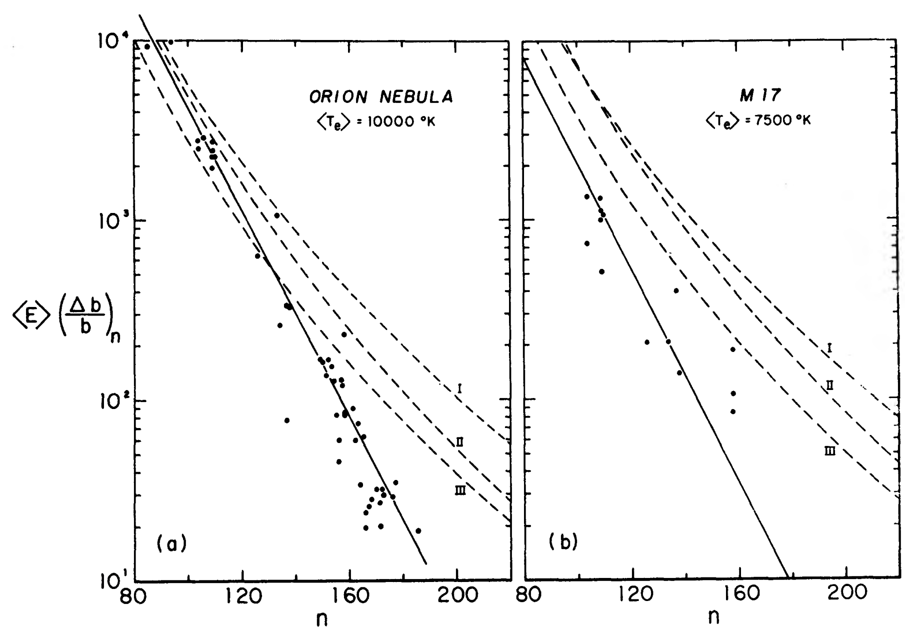

FIG. 2.—Solutions for $\langle E \rangle(\Delta b/b)_n$ obtained by analysis of data by optically thin formula (*dots*). *Solid line*, line drawn through the data points; *dashed lines*, results for theoretical calculations that used three different sets of cross-sections.

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Figure 2, where the dashed lines indicate plots of $C(\Delta b/b)_n$ for $N_e = 10^4$ cm^{-3}, where C is an arbitrary constant chosen to place the curves conveniently in Figure 2. The class I results are obtained from the b_n-factors calculated by Sejnowski and Hjellming (1969), who used cross-sections based upon the dipole approximation; class II b_n-factors were calculated by Sejnowski and Hjellming (1969) using a semiempirical formula based upon the dipole approximation to represent the cross-sections for collisional ionization and impact cross-sections for impact parameters calculated by Seaton (1962); and class III b_n-factors were calculated by Brocklehurst (1970*a*, *b*), who used a combination of cross-sections of different types.

The main conclusion to be drawn from Figure 2 is that for $n > 100$ the values of $(\Delta b/b)_n$ obtained from the data and equation (9) differ substantially in their dependence upon n from that obtained by any of the theoretical calculations of $(\Delta b/b)_n$ as a function of n. We conclude from this either that all the b_n-factors are seriously in error or that equation (9) does not give correct solutions for $\langle E\rangle(\Delta b/b)_n$ for $n > 100$. Because, as we shall show in § IV, the general formula using equation (2) gives $(\Delta b/b)_n$'s which agree well with the theoretical calculations, we conclude that equation (9) is not valid for $n > 100$. The failure of this approximate equation is a qualitative indication that $\tau_C \sim 0.1$ at $n = 100$, as we show later when solutions for $\langle E\rangle$ are obtained. It is also worth noting that the solutions for $\langle E\rangle(\Delta b/b)_n$ by means of equation (9) become parallel to the theoretical $(\Delta b/b)_n$ curves for $n < 100$. In this region the frequencies are large enough that $\tau_C \ll 1$ and the approximations leading to equation (9) become valid.

We would like to emphasize that whereas $\langle T_e\rangle$ can be relatively well determined by a first-order approximation formula, $\langle E\rangle(\Delta b/b)_n$ cannot because it depends sensitively upon the difference between $\langle T_e\rangle$ and $(T_e)_{\mathrm{LTE}}$.

IV. DETERMINATION OF $\langle E\rangle$ AND $\langle N_e\rangle$

Since most of the data for radio recombination lines in H II regions are in the range $n > 100$, the general formula (eq. [2]) should be used to determine $(\Delta b/b)_n$ from the data. Inspection of equations (2)–(5) reveals that, if $\langle T_e\rangle$ is assumed to be known from considerations discussed in § II, the value of T_L/T_C is determined by specification of $\langle E\rangle$, $(\Delta b/b)_n$, b_n, and $\Delta\nu_L$. By confining ourselves to $n > 100$, we can set $b_n \approx 1$, while retaining all terms involving $(\Delta b/b)_n$. Furthermore, $\Delta\nu_L$ is known from the observational data; in general (Hjellming *et al.* 1969), $\Delta\nu_L = 1.665\ \nu_L\ V/c$, where V is a dispersion velocity and c is the speed of light. We will take $V/c = 6.0 \times 10^{-5}$ for both the Orion Nebula and M17; the results are rather insensitive to the value of this quantity. We can then treat equation (2) as a transcendental equation to be used to solve for $(\Delta b/b)_n$ as a function of $\langle E\rangle$ for each measured recombination line. If we then use two well-determined lines with a significant difference in principal quantum numbers (m and n), we can plot $(\Delta b/b)_n$ as a function of $(\Delta b/b)_m$ where $\langle E\rangle$ is a parameter which varies along the line in this plot. For the sake of convenience, we will call such a line, which is determined by the observational data, the "data line." Now the theoretical calculations of b_n-factors give values of $(\Delta b/b)_n$ and $(\Delta b/b)_m$ which are functions of $\langle N_e\rangle$, which is to be determined, and $\langle T_e\rangle$, which has been determined by methods discussed in § II. Thus for each class of b_n calculations we may plot $(\Delta b/b)_m$ against $(\Delta b/b)_n$ for a range of $\langle N_e\rangle$; such a plot we will call a "theoretical line." *The intersection of a data line with a theoretical line will uniquely determine the values of* $\langle E\rangle$ *and* $\langle N_e\rangle$ *for each nebula for each class of* b_n*-factors.*

We have applied this technique to the H110α and H150α data for the Orion Nebula and to the H110α and H134α data for M17. Figure 3 shows the resulting data lines drawn as solid lines; the values of $\langle E\rangle$ corresponding to different points on the data line are indicated. In addition, the theoretical lines obtained for each of the three classes of of b_n-factors discussed in § III are plotted as dashed lines; the values of $\langle N_e\rangle$ corresponding to various points on these curves are indicated. The intersection of the data lines

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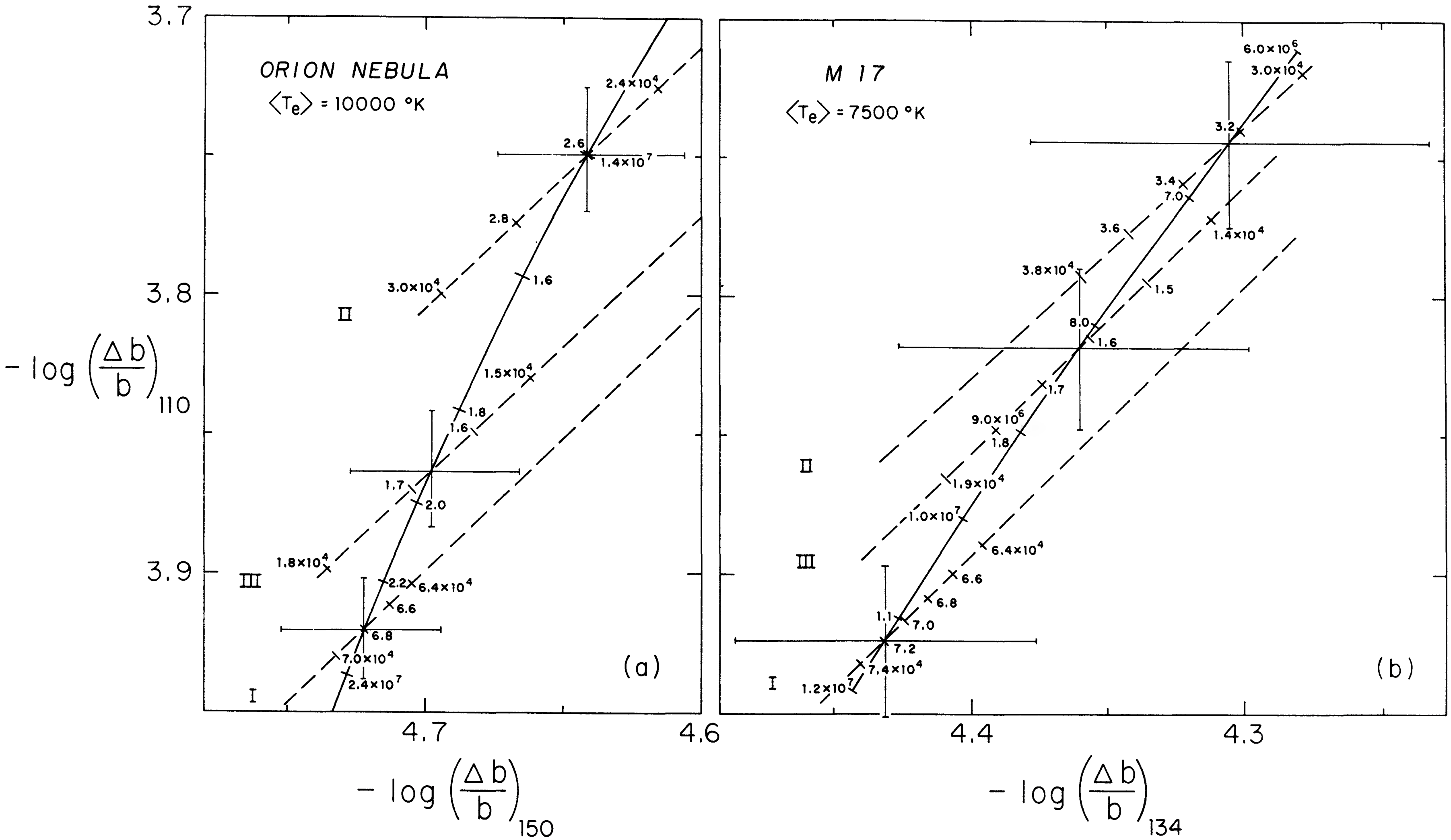

FIG. 3.—Data lines for the Orion Nebula and M17 and theoretical lines for three sets of b_n-factors. Numbers by each curve indicate the emission-measure scales for the data lines and the electron-concentration scales for the theoretical lines.

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with the theoretical lines gives the solutions for $\langle N_e \rangle$ and $\langle E \rangle$ indicated in Table 2. Also indicated in Table 2 is the size scale for the line-of-sight depths of the emitting regions, as obtained from $\langle s \rangle = \langle E \rangle / \langle N_e \rangle^2$. To evaluate the effects of uncertainties in the value of $\langle T_e \rangle$ upon the class II solutions, variations corresponding to $\pm 500°$ K for the Orion Nebula were found to give $\pm 0.3 \times 10^7$ variation in $\langle E \rangle$ and $\pm 0.1 \times 10^4$ variation in $\langle N_e \rangle$; however, a variation of $\pm 800°$ K in $\langle T_e \rangle$ in M17 gives $\langle E \rangle = (7 \times 10^5)$–$(6.2 \times 10^6)$ pc cm^{-6} and $\langle N_e \rangle = (7 \times 10^3)$–$(3.8 \times 10^4)$ cm^{-3}. We conclude that solutions for the Orion Nebula are relatively insensitive to uncertainty in $\langle T_e \rangle$, but solutions for M17 are strongly affected by such uncertainties.

At each intersection point in Figure 3, error bars are drawn which indicate the error limits implied by the uncertainties in the measurements (in addition to that due to uncertainty in $\langle T_e \rangle$). Because measurement uncertainties transfer directly into the position of each point on the data line, a range of $\langle E \rangle$ and $\langle N_e \rangle$ is permitted at each intersection; and this range gives a direct indication of the uncertainties of the solutions. However, we see from Figure 3 that the differences in the solutions for $\langle E \rangle$ and $\langle N_e \rangle$ for different classes of b_n-factors greatly exceed the uncertainties due to the data. For this reason there will always be much more uncertainty in the solutions for $\langle N_e \rangle$ and $\langle E \rangle$ than there is in solutions for $\langle T_e \rangle$ until definitive b_n-factors are available. However, in the case of the Orion Nebula, the value of $\langle E \rangle$ probably cannot be less than 10^7 pc cm^{-6}.

Although at the present time the cross-sections used in calculating the class III b_n-factors are most likely to be correct (Seaton 1970), further improvements will probably be made in the atomic calculations. Therefore the definitive b_n-factors may eventually differ from both the class II and class III results. However, it is clear that class III solutions give higher emission measures and lower electron concentrations than the class II solutions.

To fulfill the requirement that solutions must correctly account for most of the observational data, we present in Figures 4 and 5 the solutions for $(\Delta b/b)_n$ as a function of n for the data in Table 1. The solid lines indicate theoretical solutions for $(\Delta b/b)_n$ for the correct values of $\langle T_e \rangle$ and a range of $\langle N_e \rangle$. The dashed curves drawn through the data points indicate the best solutions for $(\Delta b/b)_n$. We also plot, as dashed lines, the straight-line solutions obtained from the approximate equation (9). We see that with the general formula, the solutions for $(\Delta b/b)_n$ from the data easily have the same dependence on n as the theoretical calculations.

All of the solutions for $(\Delta b/b)_n$ for each data point in Figures 4 and 5 contain the assumption that $b_n \approx 1$; this will cause serious errors for low values of n. Therefore, we have plotted in Figure 6 both the raw data taken from Table 1 and the theoretical

TABLE 2

SOLUTIONS FOR PROPERTIES OF H II REGIONS

$\langle T_e \rangle$ (° K)	Class of b_n-Factors	$\langle E \rangle$ (pc cm^{-6})	$\langle N_e \rangle$ (cm^{-3})	$\langle s \rangle$ (pc)
		Orion Nebula		
10000±500......	I	2.3×10^7	6.8×10^4	0.005
	II	1.4×10^7	2.6×10^4	0.021
	III	1.9×10^7	1.7×10^4	0.066
		M17		
7500±800......	I	1.1×10^7	7.2×10^4	0.002
	II	6.6×10^6	3.2×10^4	0.031
	III	8.2×10^6	1.6×10^4	0.063

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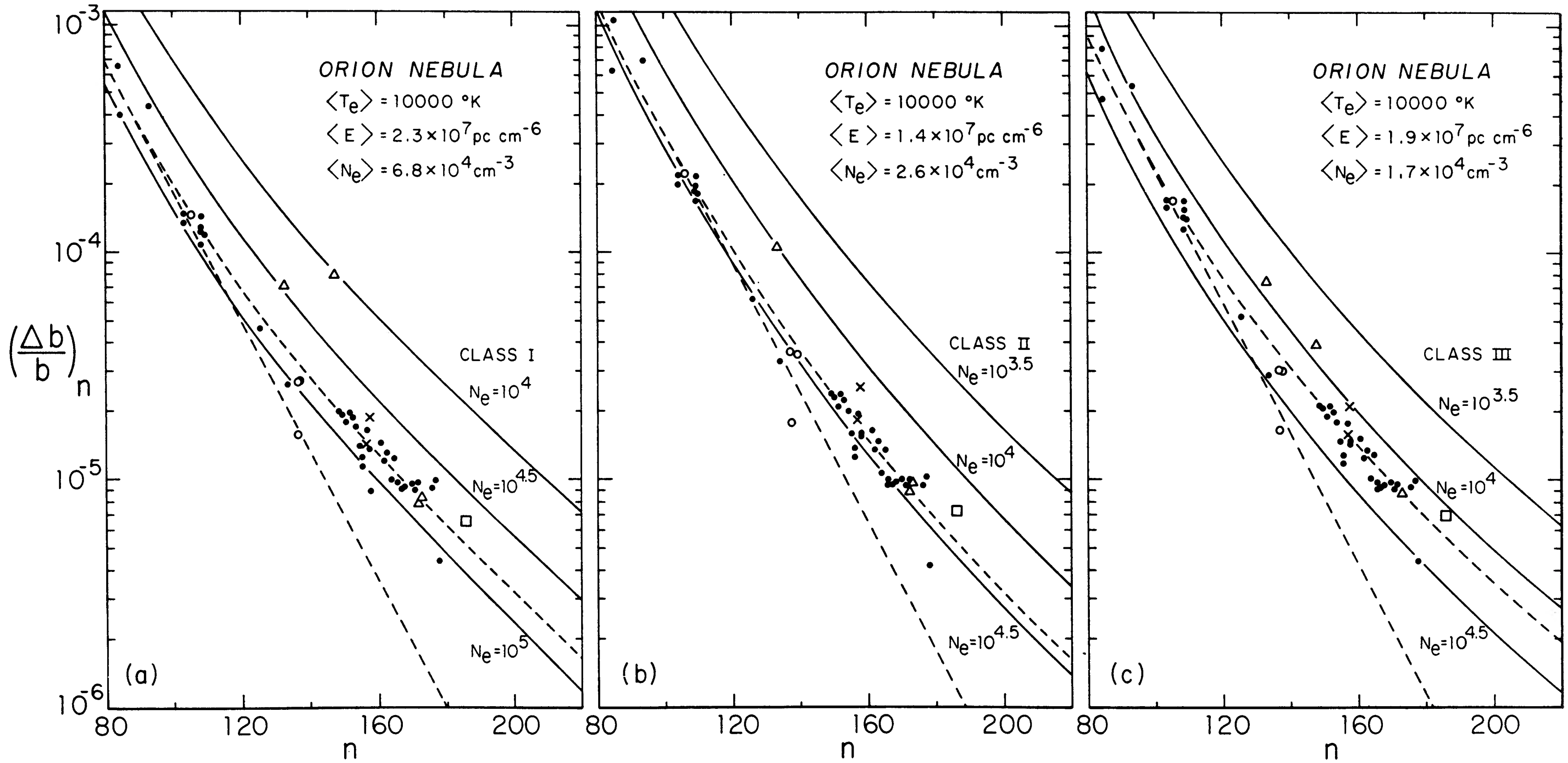

FIG. 4.—Solution for $(\Delta b/b)_n$ as a function of n obtained from data using the general formulae and the indicated solution parameters for the Orion Nebula. *Filled circles*, α-lines; *open circles*, β-lines; *crosses*, γ-lines; *triangles*, δ-lines; *squares*, ϵ-lines.

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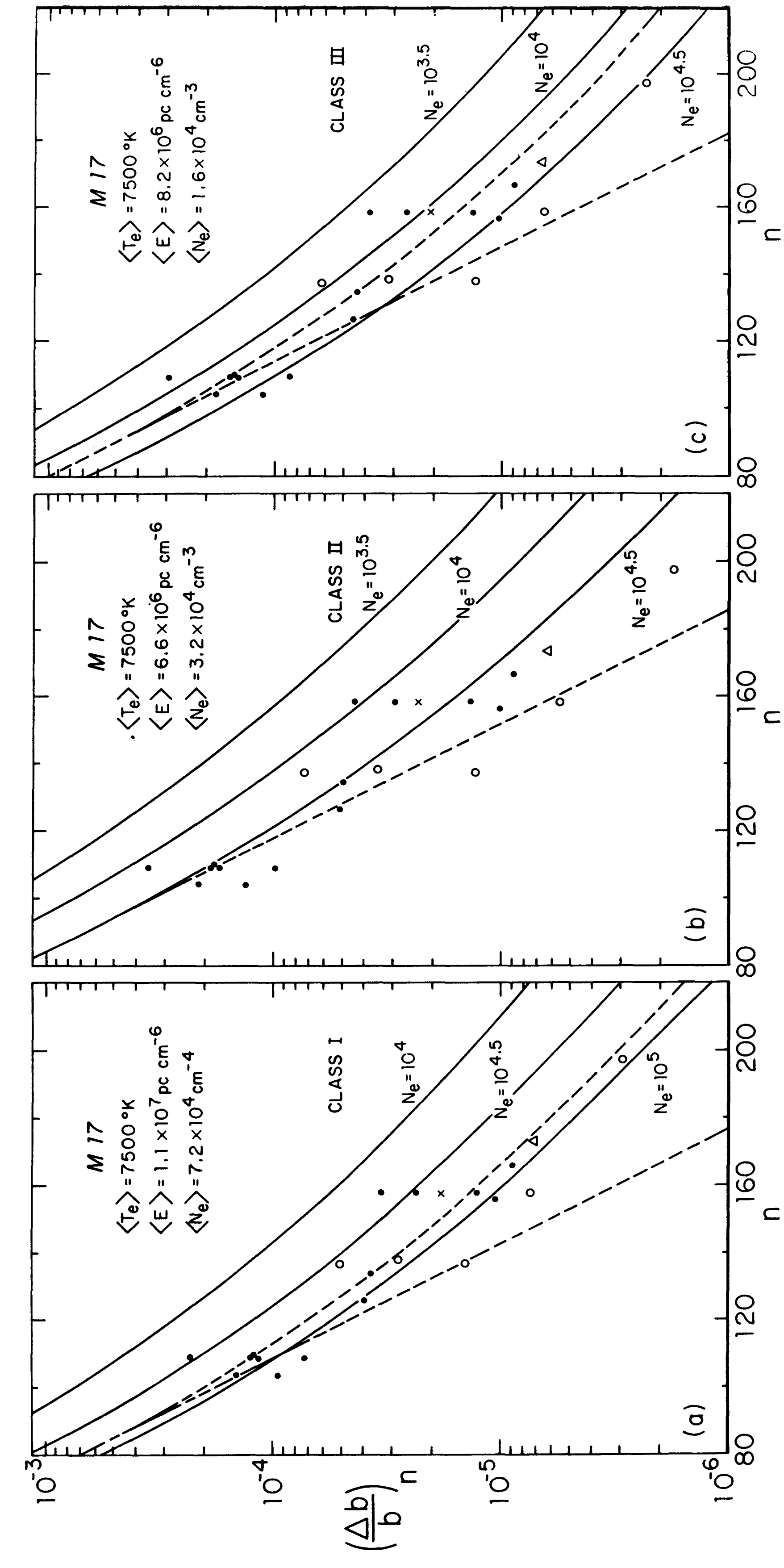

FIG. 5.—Solutions for $(\Delta b/b)$ as a function of n for M17. Symbols are same as in Fig. 4.

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solutions for $\Delta\nu_L T_L/T_C$ as a function of ν_L for the class II solutions. The theoretical curves for class I, II, and III solutions are virtually indistinguishable from the curves shown in Figure 6, since all are chosen to fit the same data. The agreement between the data and the solutions is generally good, with no more than a few discrepant points when the error bars on the data points are considered. While we must be cautious in claiming that specific discrepancies (e.g., H94α, H158β, H197β, etc.) are due to uncertainties in the observations, this is a reasonable possibility which can be established or disproved only by further observations of similar lines. As an example of this, Churchwell *et al.* (1970) have pointed out that, for the three high-frequency α-line measurements for the Orion Nebula (H56α, H65α, H94α), the points lie above the curves, and they have argued that this indicates that a serious discrepancy exists; however, when the strengths of the H85α line recently derived by Gordon (1970) and Churchwell and Mezger (1970) are considered, the measurements bracket the theoretical solution. We conclude that measurement uncertainties are sufficiently large that only when different observations of similar lines agree can firm conclusions be made concerning discrepancies between theoretical solutions and the observational data.

As yet there is no clear indication of a discrepancy that might be expected at low frequencies in Figure 6. These nebulae certainly contain a wide range of densities and emission measures. Recombination lines observed at high frequencies are mainly produced in the densest regions. As frequency decreases and the densest regions became optically thick, the more tenuous gas will make an increasingly large contribution to the

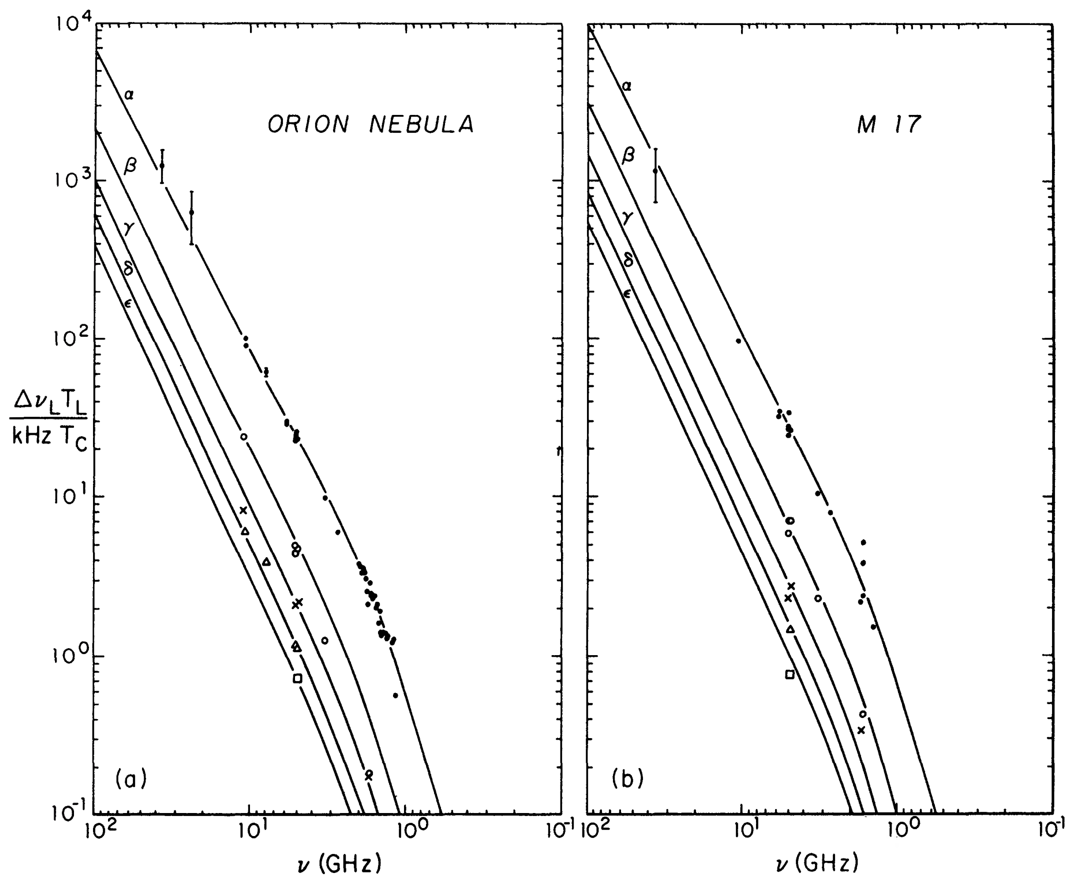

FIG. 6.—Data for $\Delta\nu_L T_L/T_C$ are as a function of ν for α-, β-, γ-, δ-, and ϵ-lines for the Orion Nebula and M17. *Solid curves*, theoretical solutions for all three classes of b_n-factors with the parameters indicated in Table 2.

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III. RESULTS

The results of combining the infrared and polarization data are tabulated in Table 2. The quantities tabulated include the star designation, the average measured polarization, the average deviation of measured polarization, the 3.5-μ observed magnitude designated [3.5 μ], and the magnitude differences, [4.9 μ] − [3.5 μ], [8.4 μ] − [3.5 μ], and [11.0 μ] − [3.5 μ].

The calibration of the infrared magnitude system in units of flux was shown in Table 1; in general, large signals were observed from most stars so that statistical errors were small. Calibrations on stars of known brightness and corrections for differential extinction were made several times during each night's observation. As a result of these procedures, it is estimated that errors of approximately ± 0.1 mag can be assigned to each measured infrared magnitude.

The average polarization is simply the time-averaged amount of polarization in B and V computed from

$$\langle P \rangle = \frac{1}{N} \sum_{i=1}^{N} P(t_i) ,$$

where $P(t)$ is the amount of polarization at time t and N is the number of nights on which observations were made. The average deviation of P is defined by

$$\langle |\Delta P| \rangle = \frac{1}{N} \sum_{i=1}^{N} |P(t_i) - \langle P \rangle|$$

and therefore is a measure of the amplitude of the polarization variation. Both of these quantities will be affected by the completeness of the set of observations and the uniformity in time with which the data were collected and will therefore have unavoidable scatter in a plot of intrinsic polarization against infrared color. The average polarization may, in addition, be affected by a large interstellar component for those objects in the galactic plane, and the amount of the contribution will be difficult to estimate. No attempt has been made to estimate the maximum amount of polarization because of the generally nonsystematic way in which the data have been taken.

As a result of the combination of infrared data and polarization data, a correlation was found to exist between infrared excess in terms of magnitude difference [11.0 μ] − [3.5 μ] and averaged measured polarization $\langle P \rangle$. These data are plotted in Figure 1 for all of the cool stars measured in common. It is clear from Figure 1 that at the very least an upper bound in the plot exists above which no stars are found. That is, no stars with measurable intrinsic polarization are found which do not have an infrared excess at 11 μ. The only possible exception to this is the star 119 Tau that may lie slightly in the area in which no other stars are found. A plot of the same data was made for only those stars with galactic latitude $|b^{\mathrm{II}}| > 20°$, in order to eliminate any possible contamination by stars with interstellar polarization. The diagram under these conditions remained almost completely unchanged except that the possible one exception to the upper bound in Figure 1 (119 Tau) was removed. 119 Tau has a galactic latitude $b^{\mathrm{II}} = -8°$.

The qualitative interpretation of the results shown in Figure 1 is clear. If intrinsic polarization of optical-wavelength light is observed from a star, then that star exhibits an infrared excess at $\lambda = 11\ \mu$. The reverse statement is not so strong. That is, if infrared excess is observed, then most, but not all, stars show intrinsic polarization.

IV. DISCUSSION

The data presented here are open to three possible interpretations. The polarization may be explained by (1) atmospheric scattering, (2) circumstellar scattering by gas, or (3) circumstellar scattering by dust.

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