

Comets and Nongravitational Forces. IV

B. G. MARSDEN AND Z. SEKANINA

Smithsonian Astrophysical Observatory, Cambridge, Massachusetts

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Orbital elements and nongravitational parameters are derived from observations at every apparition of the periodic comets Honda–Mrkos–Pajdušáková, Faye, Tempel 2, Biela, Brorsen, and Tempel–Swift. For all except the first comet, the observations go back a century and more, although the last three comets have failed to reappear for some considerable time. The circumstances of the splitting of P/Biela are studied, and it is shown that the motion of the primary component was scarcely affected; it is also demonstrated that, if the primary still exists, it may pass only 0.05 A.U. from the Earth in November 1971. An up-to-date list of mass-loss rates from comets is presented. It is found that, while most of the reliable determinations indicate that the cometary nongravitational effects decrease with time, there are a few cases where the effects increase slightly. The former situation is discussed in terms of a nuclear core–mantle model, implying that these comets will eventually evolve into inert, asteroidal objects, while the nuclei of the other comets are interpreted as coreless, eventually to disappear completely (or almost completely). A list is given of seven “erratic” comets known or suspected to have experienced large and relatively sudden anomalies in their motions. There is no conflict with observed phenomena when one interprets these anomalies in terms of collisions of coreless (snowball) comets with other objects, specifically the interplanetary boulders postulated by Harwit; it is also proposed that these boulders may themselves be of cometary origin.

THE studies of nongravitational effects on comets presented in Papers I, II, and III of this series (Marsden 1968, 1969a, 1970), as well as the similar studies by Yeomans (1971), were based almost exclusively on observations since 1900—in fact, generally on photographic observations since 1925. The single exception was a discussion of the motion of P/Pons–Winnecke in 1858, 1869, and 1875 (Paper III), which revealed the rather startling result that the sign of the transverse component of the nongravitational force had recently changed, i.e., that the secular variation in mean motion had altered from an acceleration to a deceleration; although it was not conclusively proved, we felt that the change of sign was somehow connected with the repeated large perturbations by Jupiter around the beginning of this century. We have therefore turned our attention to nineteenth-century observations of other comets, in the hope of gaining further insight into the nature of the nongravitational forces and the general evolution of the short-period comets. By modern standards, these old visual observations are decidedly inferior, but they can in fact give information useful to our purpose provided that one takes pains to eliminate all but the few that are reasonably consistent with one another.

In this paper we present calculations on the orbits of six comets. Two of them, P/Faye and P/Tempel 2, are among the best observed of all comets (only P/Encke, P/Halley, and P/Pons–Winnecke having been seen at more perihelion passages) but are relatively little affected by nongravitational forces; we have considered observations right back to their discovery apparitions. The other nineteenth-century comets discussed here are among the so-called long-lost comets investigated earlier by the first author (Marsden 1963) but without consideration then of nongravitational effects. Of the seven original long-lost comets (each of which had previously made at least two

apparitions but had been missed at its six or more most recent returns), P/Holmes and P/de Vico–Swift have been recovered, and there is a possibility that P/Tempel 1 may have been too (Roemer 1968). In spite of recent searches, P/Neujmin 2 is still lost, and since this comet has made only two apparitions, we cannot determine its nongravitational parameters. It seems desirable to investigate further the motions of the three remaining long-lost comets—P/Biela, P/Brorsen, and P/Tempel–Swift—and such investigations are included here. For the equations of motion and the definition of the nongravitational parameters used, the reader is referred particularly to Paper II. As usual, we adopt $C=2$, $\alpha=3$.

Most of the computer runs and final eliminations of erroneous observations have been performed by the first author. The second author has collected from the literature the old visual observations of P/Faye, P/Tempel 2, and P/Brorsen, converted them to mean 1950.0 positions (although not applied corrections to the original star catalogues), and attended to the preliminary elimination of erroneous observations, generally by means of unperturbed differential corrections at the individual apparitions. For P/Biela and P/Tempel–Swift, the positions have mostly been extracted from previous investigations on the orbits. Most of the physical interpretation is due to the second author.

But first, in the following section, we extend our earlier calculations on P/Honda–Mrkos–Pajdušáková, a comet discovered as recently as 1948.

I. PERIODIC COMET HONDA–MRKOS–PAJDUŠÁKOVÁ

The result given for this comet in Paper II was based on observations in 1948–1949, 1954, and 1964. It indicated a large negative value for the transverse component A_2 of the nongravitational force (i.e., a

TABLE I. P/Honda-Mrkos-Pajdušáková.*

1948 Nov. 25.0	1954 Jan. 18.0	1959 Apr. 22.0
1948 Nov. 17.7172	1954 Feb. 5.1055	1959 Apr. 22.7262
184.1235	184.1345	184.1442
233.0953	233.0871	233.0984
13.1599	13.1955	13.1812
0.559267	0.555636	0.556860
0.814204	0.815096	0.814755
1964 July 24.0	1969 Sept. 16.0	1974 Dec. 19.0
1964 July 6.5608	1969 Sept. 22.9781	1974 Dec. 28.1355
184.1308	184.1647	184.5674
233.1272	233.1056	232.9814
13.1963	13.1686	13.1327
0.555746	0.558686	0.578869
0.815017	0.814285	0.809079

* The successive lines give the epoch of osculation and the time of perihelion passage (in E.T.), the argument of perihelion, the longitude of the ascending node and the inclination (in degrees and referred to the ecliptic and mean equinox 1950.0), the perihelion distance (in A.U.), and the eccentricity.

large secular acceleration, as was originally noted in Paper I). Further observations were made in 1969, and in order to fit the four apparitions, it was necessary to make a solution also for the exponential time-variation factor B_2 . The new solution, which satisfies a total of 49 observations with a mean residual of 1.42 arc sec, has nongravitational parameters

$$\left. \begin{aligned} 10^5 A_1 &= +0.202 \pm 0.045 \\ 10^6 A_2 &= -0.6258 \pm 0.0010 \end{aligned} \right\} \text{Epoch 1954 Jan. 18.0 E.T.}$$

$$B_2 = -0.4252 \pm 0.0080.$$

The osculating elements for various epochs are listed in Table I. The radial component A_1 is much better determined than before, and, as for most comets, it is positive and numerically larger than A_2 . The surprise is that B_2 is negative, implying that the nongravitational force (or at least the transverse component) is *increasing* with time. The comets we discussed previously have all given zero or positive values of B_2 , although Yeomans (1971) has found that B_2 was negative in the case of P/Giacobini-Zinner.

II. EXTENSIVELY OBSERVED COMETS

P/Faye

Discovered in 1843, this comet has passed unobserved at only two returns, those of 1903 and 1918. Our initial collection consisted of 1380 observations at the total of 16 apparitions, although our orbit computations are based on 439 observations only. In 1873, only four observations were made and all had to be discarded. In contrast, after our initial eliminations we had retained 193 good (visual!) observations at the 1910 return, and although 100 of them were then randomly discarded, we were still left with many more observations than at any other return.

As mentioned in Paper I, previous investigators (Möller 1872; Khanina and Barteneva 1961) have failed to detect unequivocally nongravitational effects from normal places at as many as four apparitions. Experience with other comets suggests that these effects should show up when orbital solutions are made over sufficiently long time spans and when individual observations are used rather than normal places. Furthermore, although P/Faye passed within 0.51 A.U. of Jupiter in 1899 and within 0.60 A.U. in 1959, the resulting perturbations have been small and should not cause any great difficulties in the orbital solutions.

A purely gravitational orbit fitted to individual observations at the four most recent apparitions confirmed that the nongravitational forces were very small, although there were systematic trends of up to 5 arc sec. The following nongravitational parameters are from a fit to observations at the six returns 1932–1969, the mean residual being 1.56 arc sec:

$$\begin{aligned} 10^5 A_1 &= +2.62 \pm 0.17 \\ 10^6 A_2 &= -0.040 \pm 0.016 \quad \text{Orbit I (1932–1969)}. \end{aligned}$$

An attempt to incorporate also the 1925–1926 observations produced systematic residuals of up to 7 arc sec and thus cannot be regarded as satisfactory. However, it indicates that A_2 was about 3 times larger than in Orbit I *and also of the opposite sign*. We therefore made overlapping solutions at the returns 1910–1947 and 1888–1925:

$$\begin{aligned} 10^5 A_1 &= +3.81 \pm 0.27 \\ 10^6 A_2 &= +0.687 \pm 0.026 \quad \text{Orbit II (1910–1947)} \\ 10^5 A_1 &= +2.90 \pm 0.32 \\ 10^6 A_2 &= +1.044 \pm 0.020 \quad \text{Orbit III (1888–1925)}. \end{aligned}$$

The mean residuals were 1".91 and 2".20, respectively—quite satisfactory since a large proportion of visual observations was involved. Comparison of Orbits I, II, and III shows that, as with P/Pons-Winnecke, A_2 has indeed recently changed sign. While A_1 has remained essentially unchanged, A_2 is by comparison extremely small and has varied in a systematic (and definitely not exponential!) manner.

Orbits IV and V are based on observations at the perihelion passages 1858–1896 and 1843–1881:

$$\begin{aligned} 10^5 A_1 &= +5.60 \pm 0.68 \\ 10^6 A_2 &= +0.608 \pm 0.129 \quad \text{Orbit IV (1858–1896)} \\ 10^5 A_1 &= +6.01 \pm 0.59 \\ 10^6 A_2 &= +1.026 \pm 0.046 \quad \text{Orbit V (1843–1881)}. \end{aligned}$$

These two final solutions confirm that A_2 used to be positive. The ratio A_2/A_1 seems to behave in a quasi-sinusoidal manner, and this can be interpreted in terms of the comet's nuclear precession (Sekania 1971a).

TABLE II. P/Faye.

1843 Oct. 7.0	1851 Mar. 19.0	1858 Aug. 29.0	1866 Feb. 8.0
1843 Oct. 17.6437	1851 Apr. 2.4433	1858 Sept. 13.3850	1866 Feb. 14.4757
200.0304	200.1492	200.1425	200.2062
211.0140	210.9368	210.9735	210.9074
11.3648	11.3506	11.3607	11.3605
1.692255	1.699909	1.694100	1.682119
0.555816	0.554866	0.555725	0.557598
1873 July 21.0	1881 Feb. 9.0	1888 Aug. 31.0	1896 Mar. 22.0
1873 July 18.9847	1881 Jan. 23.1579	1888 Aug. 20.1804	1896 Mar. 19.5194
200.3644	201.1880	201.2122	201.2682
210.7973	210.5951	210.5730	210.5238
11.3557	11.3209	11.3019	11.3213
1.682547	1.738122	1.747990	1.736934
0.557381	0.548965	0.547457	0.548973
1903 June 15.0	1910 Oct. 16.0	1918 Mar. 28.0	1925 July 29.0
1903 June 5.7301	1910 Nov. 2.3531	1918 Mar. 29.6184	1925 Aug. 7.6510
199.4288	199.5310	199.5312	199.9272
206.8401	206.7773	206.8022	206.5719
10.5906	10.5739	10.5881	10.5867
1.645918	1.655198	1.646497	1.618016
0.566129	0.564949	0.566141	0.570769
1932 Nov. 29.0	1940 May 11.0	1947 Sept. 12.0	1955 Feb. 22.0
1932 Dec. 6.2349	1940 Apr. 24.8936	1947 Sept. 28.4107	1955 Mar. 4.6577
200.0676	200.5088	200.5293	200.5792
206.4665	206.3701	206.3500	206.3116
10.5809	10.5523	10.5339	10.5542
1.619998	1.653929	1.663263	1.651837
0.570415	0.565188	0.563723	0.565384
1962 May 16.0	1969 Oct. 26.0	1977 Feb. 26.0	1984 June 29.0
1962 May 14.6933	1969 Oct. 7.5844	1977 Feb. 27.8318	1984 July 9.9144
203.5507	203.6666	203.6587	203.8298
199.1253	199.0496	199.0854	198.9804
9.0951	9.0823	9.0910	9.0921
1.608080	1.616284	1.609767	1.593504
0.575714	0.574670	0.575613	0.578252

In any case, it is reasonable to suppose that P/Faye and P/Pons-Winnecke are comets whose rotational axes are near their orbital planes, and it will be interesting to see whether their A_2 values show any significant increase in magnitude in the future.

In Table II we give a series of osculating elements corresponding to Orbits V, III, II, and I, breaks between orbits indicated by vertical lines.

P/Tempel 2

In Paper I we showed that this comet has a slight secular deceleration. This is confirmed here, the following nongravitational parameters being based on 113 observations at the six apparitions 1930–1967 (with mean residual 1.41 arc sec):

$$10^5 A_1 = -0.179 \pm 0.041$$

$$10^6 A_2 = +0.0508 \pm 0.0073 \quad \text{Orbit I (1930–1967).}$$

Altogether, P/Tempel 2 has made 14 appearances. The best observed return was that of 1899, from which we have retained 83 observations. On the other hand, we found an aggregate of only 21 reliable observations for all the other apparitions 1873–1915, the comet

being missed entirely at its returns in 1883, 1889, and 1910 (and 1935 and 1941).

Although there was no indication that the transverse component of the nongravitational force had changed sign, we used the technique employed in the case of P/Faye and solved for A_1 and A_2 (as well as the orbital elements, of course) over various discrete intervals. The results that follow are from solutions fitted over the intervals 1915–1957, 1904–1946, and 1873–1915, and they give mean residuals of $1''.98$, $2''.31$, and $1''.40$, respectively:

$$10^5 A_1 = -0.215 \pm 0.059$$

$$10^6 A_2 = +0.0677 \pm 0.0040 \quad \text{Orbit II (1915–1957)}$$

$$10^5 A_1 = +0.226 \pm 0.041$$

$$10^6 A_2 = +0.0997 \pm 0.0040 \quad \text{Orbit III (1904–1946)}$$

$$10^5 A_1 = +0.325 \pm 0.107$$

$$10^6 A_2 = +0.1162 \pm 0.0066 \quad \text{Orbit IV (1873–1915).}$$

It seems quite clear that A_2 has been decreasing with time, and the results suggest that we might be able to combine all the observations and make a solution for

TABLE III. P/Tempel 2.

1873 June 11.0	1878 Sept. 13.0	1884 Dec. 10.0	1889 Feb. 7.0
1873 June 25.8350	1878 Sept. 7.7717	1883 Nov. 20.5458	1889 Feb. 2.6289
185.1889	185.1666	185.1149	185.0327
121.9836	121.9806	121.9723	121.9535
12.7517	12.7633	12.7496	12.7500
1.344165	1.339700	1.345035	1.345653
0.552594	0.553688	0.552312	0.552276
1894 May 12.0	1899 Aug. 14.0	1904 Nov. 16.0	1910 Feb. 18.0
1894 Apr. 23.7444	1899 July 29.0368	1904 Nov. 10.9387	1910 Feb. 10.9049
185.1154	185.6322	185.7697	186.6654
121.9175	121.6501	121.6148	121.1655
12.7370	12.6442	12.6454	12.7509
1.350670	1.388620	1.387862	1.322164
0.551106	0.542083	0.542223	0.557914
1915 Apr. 13.0	1920 June 5.0	1925 July 29.0	1930 Sept. 21.0
1915 Apr. 14.5701	1920 June 10.9278	1925 Aug. 7.0388	1930 Oct. 5.7875
186.7133	186.6556	186.5804	186.5974
121.1503	121.1417	121.1387	121.1213
12.7624	12.7618	12.7711	12.7569
1.317561	1.316420	1.313283	1.318688
0.559072	0.559301	0.560096	0.558898
1935 Dec. 24.0	1941 Feb. 15.0	1946 June 29.0	1951 Nov. 10.0
1935 Dec. 7.5618	1941 Feb. 13.9086	1946 July 2.3420	1951 Oct. 25.2002
186.5667	186.6792	190.8561	190.9958
121.0640	121.0261	119.4143	119.3790
12.7337	12.7270	12.4310	12.4326
1.327692	1.330088	1.393328	1.391237
0.556607	0.556022	0.542240	0.542619
1957 Feb. 11.0	1962 May 16.0	1967 Aug. 18.09	1972 Nov. 19.0
1957 Feb. 5.1977	1962 May 12.6901	1967 Aug. 14.250	1972 Nov. 15.0388
191.0334	191.0528	190.9790	190.8739
119.2812	119.2769	119.2719	119.2701
12.4701	12.4818	12.4738	12.4803
1.369265	1.363960	1.366510	1.364388
0.547684	0.548989	0.548397	0.548889

B_2 as well. We have not attempted to do this, although comparison of the four orbits indicates that $B_2 \approx +0.4$.

Orbits I and II involve an approach to Jupiter (to 0.63 A.U.) in 1943. Orbit IV involves a moderate approach (to 1.3 A.U.) in 1907. Orbit III involves both approaches, and this may be the reason for its poorer quality. Elements corresponding to Orbits IV, II, and I are listed in Table III, the vertical lines as before denoting breaks between orbits.

P/Biela

An extensive, but evidently little known—since it is not mentioned in the orbit catalogues—study of the motion of P/Biela was made around the turn of the century by Hepperger (1898, 1900a, b, 1903, 1906). In the first three papers cited, he determined an orbit from the observations in 1826 and 1832 and established that in order to represent those in 1805 and 1846, it was necessary to postulate a nongravitational secular acceleration amounting to more than 0.2 day per (period)². We confirm his results rather closely, having obtained the following nongravitational parameters from fits to the returns 1806–1826–1832 and 1826–1832–1846, respectively:

$$10^5 A_1 = +2.78 \pm 0.10$$

$$10^6 A_2 = -2.5037 \pm 0.0044 \quad \text{Orbit I (1806–1832)}$$

$$10^5 A_1 = +3.86 \pm 0.23$$

$$10^6 A_2 = -2.5370 \pm 0.0067 \quad \text{Orbit II (1826–1846).}$$

Orbit II was fitted to observations of the primary nucleus in 1846, and the mean residual of all the observations used was 1.70 arc sec. Orbit I is not so satisfactory, the residuals at the 1806 return rising to 80 arc sec, although since the comet was less than 0.04 A.U. from the Earth at the time, this is not of so much consequence as might be supposed. The above results suggest that we could have linked all four apparitions; it would probably have been necessary to solve for B_2 , and the value obtained would have been something like -0.02 . A solution similar to Orbit II, but using instead observations of the *secondary* nucleus in 1846, gave for the nongravitational parameters

$$10^5 A_1 = +3.66 \pm 0.42$$

$$10^6 A_2 = -2.2162 \pm 0.0118 \quad \text{Orbit III (1826–1846).}$$

In 1846 the primary component was situated south-east of the secondary and passed perihelion 0.08 day

earlier. There has been some question as to the correct identification of the components in 1852. Since both components varied considerably in brightness, it is not possible to make a decision from physical appearance alone. Following the conclusion by Hubbard (1860), Hepperger (1900a, 1903) assumed that the primary component in 1846 was identical with the northwestern component in 1852, which passed perihelion 0.68 day later than the other component. On carrying our Orbit II forward to 1852 we found that the northwestern component gave residuals of some 1900 arc sec; on the other hand, the southeastern component gave residuals of only 30 arc sec, suggesting very strongly that this is the correct identification, and we do not hesitate to adopt it. Hubbard had previously discussed, but dismissed, this possible identification; Hepperger, in his last paper, repeated his calculations on this assumption, but he evidently made some mistake, for the final elements he gives for both components in 1852 leave residuals of about 500 arc sec.

An orbit fitted to the observations of the primary component in 1846 and 1852 and the total comet in 1826 and 1832 caused the maximum residual to be reduced to 10 arc sec. We could perhaps have eliminated this by solving for B_2 ; however, we obtained somewhat similar residuals even when the 1826 observations were excluded, so we are inclined to attribute it to a slight impulse experienced by the comet at the time of splitting. The nongravitational parameters corresponding to the 1832–1846–1852 solution are

$$10^6 A_1 = +3.56 \pm 0.52$$

$$10^6 A_2 = -2.5965 \pm 0.0061 \quad \text{Orbit IV (1832–1852).}$$

We also integrated Orbit III (fitted to the secondary component in 1846) up until 1852. The northwestern

TABLE IV. Possible orbits for the secondary component of P/Biela.

Transverse nongravitational force A_2^*	Least distance between components (km)	Date	Separation velocity (m sec ⁻¹)
+1.0	103 000	1844 middle	
0.0	90 000	1843 late	
-1.0	31 000	1842 Jan.	0.7
-1.25	34 000	1843 Jan.	1.1
-1.5	31 000	1843 July	1.4

* Measured in terms of A_2 for the primary component; A_1 was taken to be the same as for the primary component.

component (with which the secondary should evidently be identified) gave residuals of some 1400 arc sec, while the residuals of the southeastern component were 500 arc sec (in the opposite direction). We conclude that the splitting of the comet resulted in a strong impulsive change in the motion of the secondary, presumably because this component was much less massive than the primary.

It is of interest to examine the circumstances of the splitting. Hepperger's suggested dates of splitting, September 1844 or November 1843, are unacceptable for reasons mentioned above. It is possible, of course, to fit an orbit to the observations of the secondary in 1846 and 1852. This component would be subject to the systematic nongravitational forces that presumably affect the motions of all comets, but since it made only two apparitions, we cannot derive the nongravitational parameters. However, we can assume particular values for them, fit the observations, and then run the orbit back to establish the minimum distance from the primary component, the motion of which can be defined by Orbit II. The results of several

TABLE V. P/Biela.

1772 Feb. 21.0		1805 Dec. 25.0	1812 Oct. 9.0
1772 Feb. 17.675		1806 Jan. 2.3990	1812 Sept. 27.8230
213.362		218.1036	218.1351
260.222	...	253.3533	253.3952
17.055		13.5913	13.5851
0.99038		0.907159	0.904652
0.72588		0.745871	0.746272
1819 June 15.0	1826 Mar. 30.0	1832 Dec. 3.0	1839 Aug. 9.0
1819 June 24.1142	1826 Mar. 18.9498	1832 Nov. 26.6152	1839 July 23.1635
218.2789	218.2894	221.6864	221.7910
253.2738	253.2573	249.9436	249.8298
13.5458	13.5640	13.2181	13.2080
0.909014	0.902430	0.879072	0.878359
0.745355	0.746575	0.751299	0.751422
Primary component		Secondary component	
1846 Jan. 24.0	1852 Sept. 29.0	1846 Jan. 24.0	1852 Sept. 29.0
1846 Feb. 11.4942	1852 Sept. 23.5432	1846 Feb. 11.5782	1852 Sept. 24.2212
223.0866	223.2176	223.0902	223.2198
247.4156	247.2805	247.4117	247.2778
12.5773	12.5507	12.5786	12.5520
0.856440	0.860594	0.856460	0.860625
0.756599	0.755828	0.756608	0.755879

TABLE VI. Predicted elements for P/Biela.*

1972 Jan. 6.6828 255.1283 212.8486 7.5946 0.828460 0.766254	1971 Dec. 21.7172 255.0788 212.7650 7.6070 0.824640 0.766974	1971 Dec. 15.7010 254.9394 212.8265 7.6291 0.823120 0.767265	1971 Dec. 24.4471 254.8425 212.9160 7.6217 0.824605 0.767387
1852 (0.19)	1859 (0.05)	1866 (0.12)	1872 (0.06)
1972 Jan. 12.8492 254.7655 213.0848 7.6085 0.828448 0.766555	1972 Jan. 28.7476 254.7469 213.1732 7.5848 0.832038 0.765886	1971 Oct. 3.6268 254.3153 213.0741 7.7381 0.811188 0.769171	1971 Aug. 6.3819 252.8219 214.3712 7.9986 0.804324 0.770092
1879 (0.19)	1899 (0.11)	1905	1912
1971 Aug. 15.7665 253.0147 214.1870 7.9647 0.805162 0.770045	1971 July 29.4522 252.6080 214.5412 8.0289 0.803592 0.770106	1971 July 27.6975 252.6069 214.5163 8.0229 0.803636 0.769996	1971 July 28.2689 252.5861 214.5131 8.0219 0.803601 0.770004
1919	1938	1958	continuing

* The elements are all for the epoch 1971 Nov. 25.0. The year at the foot of each set shows when the nongravitational forces were assumed to cease, while the figures in parentheses give the resulting minimum geocentric distance in 1971 (in astronomical units).

such solutions are shown in Table IV. It will be noted that we obtain the best fits by supposing that the secondary component had a nongravitational secular *deceleration*, comparable to or possibly even larger than the secular acceleration of the primary. The uncertainty in the absolute minimum distance of 30 000 km is probably on the order of 10 000 km, and while the mutual perturbations of the two components are clearly negligible at such distances, any conclusions could be strongly influenced if there were any lingering physical connection between the components—several observers indeed suggesting that a faint luminous bridge connected the two comets, even in 1852. But the indications are that the splitting took place between early 1842 and mid-1843, during which time the comet would have been some 6 A.U. from the Sun and more than 1 A.U. from Jupiter; it was at aphelion in November 1842. The secondary seems to have left the primary essentially in a sunward direction at a speed on the order of 1 m sec⁻¹.

The motion of P/Biela—the primary component, anyway—is really very uniform during the whole interval 1806–1852. The comet was also observed in 1772, although the positions obtained then are very rough. Extrapolation of Orbit I back to 1772 gives a perihelion time some 3.5 days too early. To fit the 1772 observations tolerably well, we assumed that A_1 and A_2 instantaneously decreased (reckoning backward) to 0.12 of their subsequent values shortly before the 1806 return. We should note that the comet passed only 0.38 A.U. from Jupiter in 1794.

In Table V we give various osculating elements during the interval 1772–1852. The 1772 elements were obtained by the procedure just mentioned. The elements for 1806–1819 are from Orbit I, those for 1826–1846

(primary) from Orbit II, and those for 1852 (primary) from Orbit IV. The elements for the secondary component correspond to the -1.0 solution in Table IV.

As is well known, P/Biela has not been observed since 1852. On the other hand, the only occasion when serious searches were made for it was in 1866. The prediction supplied for that return seems to have been reliable, conditions were favorable, and searches were extensive. Assuming that the primary component, at any rate, had not completely disintegrated, we conclude that it had faded considerably and suggest that it may have become asteroidal. (This question will be discussed further in Sec. IV.) If so, the nongravitational effects would evidently have ceased. If they had ceased suddenly, at the time of the unfavorable 1859 perihelion passage, it so happens that the comet will approach to a distance of 0.05 A.U. from the Earth in November 1971, this being the closest approach since November 1805. In Table VI we give several sets of current elements for the comet's primary component, these being derived from Orbit II but on the supposition that the nongravitational effects ceased abruptly near perihelion in the years stated. The corresponding minimum geocentric distances at the current return are indicated, and it will be noticed that the comet will come close to the Earth even if we assume that the nongravitational forces were still acting at the end of the nineteenth century. Another reason for favoring the idea that these forces ceased earlier is that if they continued to act after 1879, or indeed if they are still acting, the comet should have come within 0.03 A.U. of the Earth in 1938, and there would presumably have been a good chance of rediscovering it accidentally. Some of the solutions would have brought the comet near the Earth also in 1951, and it is largely because

TABLE VII. P/Brosen.

1846 Mar. 5.0	1851 Oct. 5.0	1857 Mar. 27.0	1862 Oct. 27.0
1846 Feb. 25.8671	1851 Sept. 24.6201	1857 Mar. 29.7458	1862 Oct. 13.6049
13.7978	13.8621	14.0187	14.0802
104.1248	104.0711	103.0530	103.0067
30.9166	30.8803	29.7994	29.7927
0.650112	0.651368	0.620510	0.619923
0.793086	0.792806	0.801727	0.801879
1868 Apr. 18.0	1873 Oct. 9.0	1879 Apr. 1.0	
1868 Apr. 17.9217	1873 Oct. 10.9822	1879 Mar. 31.0345	
14.8253	14.8612	14.9348	
102.3295	102.3052	102.2793	
29.3711	29.4072	29.3845	
0.597025	0.593793	0.589846	
0.807977	0.808825	0.809820	

of the differential perturbations then, coupled with differential perturbations by Jupiter in 1960, that there is an apparent discontinuity between the elements corresponding to cessation of forces in 1899 and cessation in 1905.

P/Brosen

The following nongravitational parameters were derived from the last three observed returns of this comet (1868–1873–1879):

$$10^5 A_1 = +1.68 \pm 0.11$$

$$10^6 A_2 = +2.2388 \pm 0.0025 \quad \text{Orbit I (1868–1879).}$$

By nineteenth-century standards the fit is good, the mean residual of the 127 observations used being 2.61 arc sec. Corresponding representations have also been made from the other two combinations of three consecutive apparitions, 1857–1868–1873 and 1846–1857–1868, the nongravitational parameters being

$$10^5 A_1 = +1.16 \pm 0.12$$

$$10^6 A_2 = -1.4739 \pm 0.0017 \quad \text{Orbit II (1857–1873)}$$

$$10^5 A_1 = +0.19 \pm 0.14$$

$$10^6 A_2 = -1.1643 \pm 0.0009 \quad \text{Orbit III (1846–1868).}$$

As with P/Pons–Winnecke and P/Faye, there again seems to have been a change of sign of A_2 , but the forces involved here are much larger (considering the small perihelion distance), and this time the latest value of A_2 (Orbit I) is numerically larger than before. Furthermore, from 1846 to 1873, A_2 was increasing numerically, suggesting that the change of sign arose rather suddenly between 1873 and 1879. A further orbit was therefore fitted to the observations at the four apparitions 1846–1873, and it included a solution for B_2 :

$$\left. \begin{aligned} 10^5 A_1 &= +0.32 \pm 0.01 \\ 10^6 A_2 &= -1.1545 \pm 0.0005 \end{aligned} \right\} \text{Epoch 1857 Mar. 27.0 E.T.}$$

$$B_2 = -0.7435 \pm 0.0037 \quad \text{Orbit IV (1846–1873).}$$

The osculating elements 1846–1873 corresponding to Orbit IV are listed in Table VII. As is to be expected, however, Orbit IV yields large residuals in 1879; they amount to 5000 arc sec, and the perihelion time correction $\Delta T = +0.8$ day. The 1879 elements in Table VII are thus from Orbit I.

The value of A_2 in Orbit I corresponds to an approximate effective inverse-square value A_2' (see Paper II) of $+6.7 \times 10^{-6}$. This is larger than the results obtained for any other periodic comet (see Paper III, Table IX). The situation is reminiscent of that with P/Perrine–Mrkos, an orbit representing (but not very well) the three most recent returns giving an effective $A_2' = -20 \times 10^{-6}$, some 8 times larger than it had been earlier (see Paper III). The P/Perrine–Mrkos case did not involve a change of sign, and we attributed the difficulty to the close approach to Jupiter in 1959. P/Brosen was far from Jupiter during the 1873–1879 revolution, and in fact the only approaches to Jupiter during the whole span covered by the observations were relatively moderate: to 1.3 A.U. in 1866 and 0.83 A.U. in 1854. The situation is thus more comparable with that of P/Giacobini–Zinner, where Yeomans (1971) found an effective $A_2' = +5.5 \times 10^{-6}$ for an orbit fitted to the three most recent returns, while earlier in the century the motion had been much more regular, with A_2' some 3 or 4 times smaller. He was able to narrow down the anomaly to the 1959–1966 revolution, and no approach to Jupiter was involved.

These findings for P/Giacobini–Zinner and P/Brosen show that we were hasty to assume that the close approach to Jupiter was completely responsible for the large change in the orbit of P/Perrine–Mrkos; in any case, until the latter comet has been reobserved in 1975, we cannot exclude the possibility that it was affected during the 1962–1968 revolution, and it is likewise impossible to say whether the change in the motion of P/Schaumasse (see Paper II) occurred during the 1935–1943 revolution (which involved a close approach to Jupiter) or during the 1927–1935 revolution.

Extensive searches were made for P/Brosen at its favorable returns in 1890 and 1901—photographically

TABLE VIII. Predicted elements for P/Brorsen.*

1973 Feb. 22.9728	1973 Feb. 27.3433	1973 Mar. 3.5262
18.0416	18.0910	18.1411
98.1827	98.1285	98.0710
24.0883	23.9906	23.8847
0.532712	0.532384	0.531957
0.828481	0.828586	0.828721

* The elements are all for the epoch 1973 Feb. 7.0.

on the latter occasion. As with P/Biela, we suggest that if the comet still existed then, it had become asteroidal, the nongravitational effects having ceased. For simplicity we assume that these effects stopped in 1879, and in Table VIII we give three sets of elements for the comet's next predicted return. The first set is from a direct gravitational fit to the observations in 1873 and 1879, and it agrees closely with the earlier prediction (Marsden 1963). The second set starts from the 1879 elements in Table VII, while the starting elements for the third differ from those of the second by the same amounts that these differ from those of the first. Indications are that the 1973 return will be favorable, but we cannot exclude the possibility that the comet will be outside the range given by these predictions.

P/Tempel-Swift

The first three apparitions of this comet (1869-1870, 1880-1881, 1891-1892) were discussed in great detail by Maubant (1914), who established that there was a significant secular acceleration. His calculation was based on 12 normal places, and there were large systematic trends, amounting to 20 arc sec and more, in his residuals. Our own calculation, based on 19 selected observations at these same apparitions, confirms the secular acceleration but also yields systematic residuals of up to 20 arc sec; the nongravitational parameters were

$$10^5 A_1 = -1.38 \pm 0.84$$

$$10^6 A_2 = -4.602 \pm 0.011 \quad \text{Orbit I (1869-1892).}$$

Recent calculations on the orbit of this comet have also been made by Schrutka (1971), who confirms the secular acceleration and severe difficulties with the residuals. An orbit determined by us from his normal places at the same three apparitions gives a result very similar to Orbit I, the nongravitational parameters being

$$10^5 A_1 = -0.80 \pm 0.86$$

$$10^6 A_2 = -4.615 \pm 0.008 \quad \text{Orbit II (1869-1892).}$$

As yet another independent check, several more observations were collected from the literature. The orbit solution again yields residuals of some 20 arc sec, the nongravitational parameters this time being

$$10^5 A_1 = -2.94 \pm 0.57$$

$$10^6 A_2 = -4.589 \pm 0.005 \quad \text{Orbit III (1869-1892).}$$

The following nongravitational parameters are from an orbit fitted to a few selected observations in 1880-1881, 1891-1892, and 1908, and they also give residuals of some 20 arc sec:

$$10^5 A_1 = +0.92 \pm 0.39$$

$$10^6 A_2 = -4.158 \pm 0.028 \quad \text{Orbit IV (1880-1908).}$$

Comparison of these results suggests that A_2 is numerically decreasing with time, and the following orbit was obtained from observations at all four apparitions of the comet, a solution being made also for B_2 :

$$\left. \begin{array}{l} 10^5 A_1 = +0.41 \pm 0.39 \\ 10^6 A_2 = -4.590 \pm 0.011 \end{array} \right\} \text{Epoch 1880 Nov. 21.0 E.T.}$$

$$B_2 = +0.210 \pm 0.015 \quad \text{Orbit V (1869-1908).}$$

TABLE IX. P/Tempel-Swift.

1869 Dec. 9.0	1875 June 11.0	1880 Nov. 21.0	1886 May 14.0
1869 Nov. 19.3053	1875 May 15.4765	1880 Nov. 8.4970	1886 May 11.5493
106.1236	106.0871	106.1431	106.2358
297.9818	297.9906	297.9039	297.8420
5.4050	5.4027	5.4029	5.4010
1.063102	1.067548	1.067202	1.072645
0.658121	0.657045	0.657333	0.656047
1891 Nov. 4.0	1897 June 5.0	1903 Feb. 15.0	1908 Sept. 16.0
1891 Nov. 17.8659	1897 June 6.6026	1903 Jan. 27.8998	1908 Oct. 5.0749
106.6905	106.9417	113.3243	113.4992
297.3893	297.2026	291.1868	291.0609
5.3936	5.3938	5.4445	5.4453
1.086420	1.089735	1.150622	1.152561
0.652988	0.652378	0.638485	0.638120

TABLE X. Approximate elements for P/Tempel-Swift.

<i>T</i>	ω_{1950}	Ω_{1950}	i_{1950}	q (A.U.)	e	Period (yr)	Minimum distance from Jupiter (A.U.)
1897 June 6.6	106°9	297°2	5°4	1.090	0.652	5.55	
1903 Jan. 27.9	113.3	291.2	5.4	1.151	0.638	5.68	1.16, 1899 May
1908 Oct. 5.1	113.5	291.1	5.4	1.153	0.638	5.68	
1914 July 22.6	139.3	265.1	7.1	1.235	0.619	5.83	0.61, 1911 May
1920 May 24.2	139.4	265.0	7.1	1.236	0.618	5.83	
1926 Apr. 28.5	154.4	249.4	10.5	1.325	0.596	5.94	0.50, 1923 July
1932 Apr. 11.2	154.6	249.3	10.5	1.327	0.596	5.95	
1938 May 18.3	162.0	241.5	13.3	1.487	0.558	6.18	0.53, 1935 Apr.
1944 July 27.9	162.2	241.4	13.3	1.490	0.558	6.19	
1950 Nov. 10.0	163.6	240.2	13.5	1.572	0.541	6.35	1.33, 1946 Nov.
1957 Mar. 21.5	163.8	240.1	13.5	1.578	0.540	6.36	
1963 Aug. 4.4	163.8	240.0	13.5	1.594	0.537	6.40	
1969 Dec. 29.9	163.9	240.0	13.4	1.602	0.536	6.41	
1976 May 25.8	163.8	240.0	13.5	1.600	0.536	6.41	
1982 Oct. 22.0	163.7	240.0	13.4	1.605	0.535	6.42	

Orbit V also leaves systematic residuals of 20 arc sec, and it seems to be impossible to remove them. It is not improbable that they are due to observational problems associated with the fact that the comet was very diffuse in appearance and close to the Earth during each of its first three apparitions, the least distances being 0.25, 0.13, and 0.24 A.U., respectively.

Table IX gives osculating elements from Orbit V for epochs near all the perihelion passages 1869–1908. The comet's subsequent motion is complicated by a series of close approaches to Jupiter, with the comet slowly passing through the 2:1 mean-motion resonance. Approximate values of some of the orbital elements (again derived from Orbit V), and data concerning the approaches to Jupiter are listed in Table X.

More precise figures for 1969, together with an ephemeris, have been published elsewhere (Marsden 1969b). The comet was not recovered then, quite probably because of its extreme faintness, although the unknown character of the nongravitational effects since 1908 obviously introduces some uncertainty into current predictions.

III. RATES OF MASS LOSS FROM COMETS

As was first suggested by Whipple (1950), the observed nongravitational effects on a comet can be interpreted in terms of the loss of volatile substances from the comet. A general formula derived by the second author (Sekanina 1969) gives the product of the relative mass-loss rate $\Delta M/M$ and an anisotropy coefficient λ ($0 \leq \lambda \leq 1$) that describes the distribution of the vaporization rate of the ice over the nuclear surface. The coefficient λ would be zero if the vaporization were perfectly isotropic (although in this case there would be no observable nongravitational effects); it would be unity if the material were vaporized from a limited area and in a specific direction; and it would equal 0.44 if the rate of vaporization were proportional to the effective insolation. The actual value of λ is very

uncertain, but we should expect it to be related to the amplitude of the temperature variations between the sunlit and the dark hemispheres of the nucleus. The existence of jets and streamers in the atmospheres of many comets suggests that λ may often be relatively high. Assuming that the thermal velocity of the vaporized material is constant at 0.6 km sec⁻¹ and that the radial component of the force is small compared with the transverse component, we have been able to estimate that $\lambda \approx 0.3$ for P/Encke (Sekanina 1971b).

The numerical results are summarized in Table XI, which gives for each comet and the specified time spans the average perihelion and aphelion distances q , Q , the minimum relative mass-loss rate $\lambda(\Delta M/M)$, and the exposure rate W , which is the amount of solar energy incident vertically per square centimeter of the nuclear surface per revolution. At small distances from the Sun the exposure rate basically determines the rate of sublimation from a purely icy nucleus. If all the heat is spent on sublimation, and if the effects of rotation and reflectivity are taken into account, the upper limit for the thickness ΔR of the surface layer sublimated out per revolution is given by

$$\Delta R = \frac{W(1-A_0)}{4L\rho}, \quad (1)$$

where A_0 is the Bond albedo for the solar flux, ρ is the density of the volatiles, and L is their vaporization rate per gram. For H₂O solids, $L=650$ cal g⁻¹, $A_0=0.7$, so that ΔR is of the order of several meters per revolution, depending on the bulk density and the degree of contamination by dust particles.

IV. LONG-TERM VARIATIONS IN THE NONGRAVITATIONAL EFFECTS

Determination of Numerical Values of B_2

In Paper II we remarked on the dangers of solving for the time-variation factor B_2 of the transverse non-

TABLE XI. Mass-loss data for short-period comets.

Comet	Date	q (A.U.)	Q (A.U.)	$\lambda(\Delta M/M)$ (% rev ⁻¹)	W (10 ⁶ cal cm ⁻² rev ⁻¹)
Encke	1800	0.34	4.1	0.34*	1.32
	1900	0.34	4.1	0.12*	1.32
	1967	0.34	4.1	0.03*	1.32
Honda-Mrkos-Pajdušáková	1954	0.56	5.4	(0.1)	1.03
	Brorsen	1857	0.62	5.6	(0.2)
Pons-Winnecke	1858-1875	0.78	5.5	(0.08)	0.89
Biela	1805-1832	0.88	6.2	0.5	0.84
Giacobini-Zinner	1900	0.93	6.0	0.08	0.82
	1946	1.00	6.0	0.1	0.80
Tempel-Swift	1880	1.07	5.2	(0.07)	0.78
Schaumasse	1911-1928	1.17	6.8	(0.2)	0.73
	1943-1960	1.20	6.9	0.1	0.73
	Tempel 2	1873-1915	1.35	4.7	0.02
d'Arrest	1930-1967	1.36	4.7	(0.01)	0.72
	1943	1.39	5.7	0.03	0.70
Borrelly	1905-1968	1.42	5.9	0.04	0.69
Forbes	1961	1.54	5.4	(0.09)	0.67
Faye	1843-1881	1.71	5.9	0.2	0.64
	1910-1947	1.64	5.9	0.1	0.65
	1932-1969	1.63	6.0	0.08	0.65
	Wirtanen	1947-1967	1.62	5.5	(0.05)
Daniel	1937-1964	1.56	5.6	0.2	0.66
Arend	1951-1967	1.83	6.0	(0.02)	0.62
Schwassmann-Wachmann 2	1961	2.16	4.8	0.1	0.61
Whipple	1948	2.45	5.2	0.01	0.57

* These are the actual values of $\Delta M/M$.

gravitational component, mainly because of its high correlation with A_2 . However, for several of the comets we have considered, B_2 does give useful information, and an arc long enough for B_2 to become significant often enables A_1 to be determined more reliably, too. On the other hand, there are comets, particularly some of those discussed in the present paper, where B_2 is obviously a very unsatisfactory quantity.

Our exponential expression for the time variation clearly fails if the transverse component changes sign, and we now have three cases—P/Pons-Winnecke, P/Faye, and P/Brorsen—where this has occurred. There is nothing sacred about the exponential form, of course, and we could easily modify our expression for the nongravitational forces so that a sign change can be handled.

Much more serious is the problem of the large and apparently sudden changes in the motions of some comets, particularly now that we know these anomalies are not necessarily connected with close approaches to Jupiter. We have four clear-cut examples of this phenomenon: P/Schaumasse, P/Perrine-Mrkos, P/Giacobini-Zinner, and P/Brorsen. For two of these comets, values of B_2 have in fact been published that correspond to fits covering the times when these effective discontinuities have taken place: From the observations of P/Schaumasse since 1927 we found (Paper II) $B_2 = +1.5$, while Yeomans (1971) in his representation of the motion of P/Giacobini-Zinner

since 1939 obtained $B_2 = -3.7$. Both these solutions gave satisfactory residuals during the apparitions considered, but since they failed miserably at representing the observations at other apparitions, they were quickly dismissed as spurious. In each case the solution involved only four apparitions, and while it is often possible in principle to determine B_2 from such a span, it is now evident that the results can be very misleading. Until (and unless) the values of B_2 obtained for P/Forbes (+1.2) and P/Honda-Mrkos-Pajdušáková (-0.4) are confirmed by observations at a fifth apparition (presumably in 1974 for each comet), it might be more prudent for us to class these comets with those known to have experienced sudden anomalies. We can attribute the value $B_2 = -0.7$ derived from the first four apparitions of P/Brorsen to another jump, and it also seems probable that P/Biela experienced a jump between 1772 and 1806 (in addition to the one that affected at least the secondary component when that comet split). For convenience, we list the seven "erratic" comets in Table XII; and we shall discuss possible reasons for their behavior in Sec. V.

Meanwhile, to set the record straight, we list in Table XIII the comets for which B_2 has been reasonably well behaved for several revolutions. We have included the cases where *small* B_2 values (i.e., between -0.2 and +0.2) have been determined from four apparitions, for if jumps are involved, they cannot have been of much significance.

TABLE XII. Erratic comets.

Comet	Interval during which jump occurred	Number of revolutions between first and last apparitions
Giacobini-Zinner	1959-1965	10
Biela	1772-1805; also presumably at split in 1842-1843	12
Brorsen	1873-1879; also before 1873	6
Perrine-Mrkos	1955-1968	11
Schaumasse	1927-1943	6
Honda-Mrkos-Pajdušáková(?)	1948-1969	4
Forbes(?)	1929-1961	5

Physical Interpretation of B_2

In Papers II and III of this series, the first author suggested that the fact that B_2 seemed invariably to be positive implied that as a cometary nucleus evolved, it exposed surface material that became progressively less volatile; eventually, the nucleus would become an asteroidal object, comets like P/Arend-Rigaux and P/Neujmin 1, which look almost asteroidal and experience no nongravitational effects in their motions, apparently representing transition objects.

The second author (Sekanina 1969, 1971b, c), in attempting to interpret B_2 in physical terms, investigated two specific models for a cometary nucleus:

(i) A core-mantle model, consisting of a porous meteoric matrix with snows of volatile substances (primarily H_2O) embedded in it, surrounded by an envelope of free volatiles and loose dust particles.

(ii) A coreless model, composed entirely of free volatiles and loose dust particles.

Each model gradually shrinks under the effects of solar radiation, except that in the first case the decrease in radius ceases when the envelope has been removed. The core persists, releasing by diffusion and desorption mechanisms the volatile materials contained in its

subsurface layers, until we are left with an inert, meteoric matrix. A cometary nucleus consisting of a dismantled core would indeed have a positive B_2 coefficient, and this coefficient would progressively increase with time, as is observed for P/Encke. Assuming that there is a reasonably significant core (i.e., one at least some hundreds of meters in radius, or two or more orders of magnitude larger than the thickness of mantle sublimated out during one revolution), B_2 would be near zero throughout the mantle phase, reaching a minimum value not more negative than -0.01 some hundreds of revolutions before the date of dismantling. On the other hand, for the coreless model, slightly negative B_2 would be the rule, and it would become progressively more negative as the radius decreased. The nuclei of the last three comets listed in Table XIII might thus be considered as dismantled cores, while those of the first two comets are probably to be associated with the coreless model.

Although the coreless model leads to negative B_2 values, it is very unlikely that a typical short-period comet could be observed to have a B_2 as negative as -0.4 . Such a comet would be faint, having a nuclear radius not much larger than 50 m, and it would have only about 10 revolutions still to make before its dissolution. Furthermore, its rate of mass loss would amount to some 30% per revolution, which is considerably larger than all the figures given in Table XI—unless λ is unreasonably small. This suggests that it is indeed proper for us to suppose that the B_2 formally derived for P/Honda-Mrkos-Pajdušáková is spurious, or else λ could not be more than about 0.003.

In practice, it is certainly possible that a so-called coreless nucleus might have a tiny core. The most negative value of B_2 would be attained only a few revolutions before dismantling, and if that value is denoted by B , for a typical short-period comet the radius of the core would not be larger than $-10/B$ meters. Hence, any B_2 more negative than -0.1 implies that the core (if there is one) must be smaller than 100 m in radius. This can perhaps be expected to be the upper limit for the radius of any asteroidal remnant of P/Biela, and if the albedo is 0.1, it corresponds to an absolute magnitude of about 22. If dismantling—and

TABLE XIII. Comets with well-behaved B_2 parameters.

Comet	Time interval	B_2	Number of apparitions involved
Giacobini-Zinner	1900-1946	-0.1^a	6
Biela	1805-1852	slightly negative ^a	5
Borrelly	1904-1968	0.0	8
Daniel	1937-1964	zero or slightly positive	4
Wirtanen	1948-1968		4
Whipple	1933-1964		5
d'Arrest	1923-1964		4
Tempel-Swift	1869-1908		4
Schwassmann-Wachmann 2	1929-1968	$+0.3$	7
Tempel 2	1873-1967	$+0.4$	14
Encke	1927-1967	$+0.8^b$	12

^a Possibly becoming more negative with time.

^b Becoming more positive with time.

cessation of nongravitational effects—took place in 1859, the comet would have a maximum apparent brightness of 17 mag on its forthcoming approach to the Earth. It is probably a vain hope that P/Biela can be recovered, but in view of the unique circumstances we should make the most of the opportunity, and appropriate ephemerides have been published (Marsden 1971).

By means of the suggested models, therefore, it is possible to explain all the B_2 values listed in Table XIII. It is in fact possible to explain larger positive values (such as that tentatively derived for P/Forbes), but we run into difficulties with values more negative than about -0.2 . Whenever we have formally obtained a more negative value, it has been based on only four apparitions, and if we have been able to extend the calculations to more apparitions, the result has not been confirmed. Thus, we are clearly dealing with a different phenomenon.

The Erratic Comets

Comparison of Tables XII and XIII reveals immediately that two of the five comets we have classed as definitely erratic (P/Giacobini-Zinner and P/Biela) have had, during their quiescent phases, slightly negative values of B_2 . We can probably also include P/Schaumasse: the undisturbed periods for this comet span only three apparitions each, so values of B_2 cannot be obtained directly, but the orbit fitted to the 1943–1960 apparitions gives a numerically larger value of A_2 than that fitted to 1911–1928 (as was noted in Paper II), and from the two figures we can infer that the “true” value of B_2 is about -0.1 . There are not enough observed returns for us to establish the quiescent B_2 values for P/Brorsen and P/Perrine-Mrkos (as well as for the two possible members of the group). The three well-defined cases, however, suggest that we can tentatively identify the erratic comets with the coreless comets.

Two of the erratic comets have, of course, been lost for about a century (and these same two objects seem to have experienced jumps on more than one occasion). As for the physical appearance of the others, we note that P/Honda-Mrkos-Pajdušáková is characteristically an extraordinarily diffuse object whose brightness increases enormously with decreasing heliocentric distance: Even under the very favorable circumstances at its 1969 return, this comet was completely unobservable on its preperihelion arc at more than 1.0 A.U. from the Sun, and it has never been followed beyond a heliocentric distance of 1.3 A.U. after perihelion. P/Perrine-Mrkos is also a very diffuse object that would never have been rediscovered in 1955 if it had not had a total brightness some 7 mag brighter than could reasonably have been expected. There is no obvious difference in general appearance between the three remaining erratic comets and the majority of the “core-mantle” comets of Table XIII. Of the

latter, there may be some significance in the perfectly stellar appearance of P/Encke at heliocentric distances in excess of about 1.5 A.U., although the diffuseness and perihelion asymmetry of the “nuclear” brightness of P/d'Arrest more resembles the behavior of P/Honda-Mrkos-Pajdušáková and P/Perrine-Mrkos. We point out that, except for the uncertain case of P/Forbes, all the erratic comets have (or had at some time during the intervals in which the jumps took place) perihelion distances of less than 1.2 A.U.

Among the erratic comets, spectroscopic data are available only for P/Giacobini-Zinner (Swings 1965), P/Brorsen (Konkoly 1879), and P/Schaumasse (Swings and Haser 1956). The spectrum of P/Giacobini-Zinner is noted for its extremely strong continuum, and a moderate continuum has been observed in the other cases (bearing in mind that the spectra of P/Brorsen were necessarily visual). In the light of the convincing conclusion by Delsemme and Miller (1971) that continuous spectra could be due to scattering of light by flakes of clathrates, in addition to scattering by meteoric material, the existence of a continuum does not provide important information unless we know the variation in intensity with distance from the nucleus. Lack of a continuum, however, is significant and favors liberation of volatiles by desorption rather than by free vaporization: in this category we have the outstanding example of P/Encke, and the only other two comets with non-negative B_2 (Table XIII) for which spectra are available (P/Tempel 2 and P/d'Arrest) do not seem to have particularly strong continua either (Bobrovnikoff 1925; Roemer *et al.* 1970). Evidence from the meteor streams associated with P/Giacobini-Zinner and P/Encke tends to confirm the differences in the spectroscopic data (Swings 1965).

In spite of the generally small perihelion distances of the erratic comets, there is no particular evidence that the discontinuities have occurred in the vicinity of the sun and thus are somehow connected with typical cometary activity stimulated by appreciable insolation. On the contrary, the splitting of P/Biela took place beyond the orbit of Jupiter—and much too far from Jupiter itself for any possible tidal effects from that planet to be responsible. Other sources of internal cometary energy do not seem to be workable either. In particular, the radioactive heating mechanism discussed by Whipple and Stefanik (1966), although applicable at large heliocentric distances, is supposed to affect new comets, not short-period ones. Rotational instability, which has occasionally been mentioned as a potential splitting mechanism, does not generate a translatory impulse on the primary nucleus (if much more massive than the satellite) and thus cannot produce a discontinuity in its motion; but the primary nucleus must lose some of its angular momentum, and this could change the character of the regular nongravitational effects. The satellite nucleus will receive a translatory impulse and rotational momentum, both

of which compensate for the loss of angular momentum by the primary. The calculations on P/Biela, however, suggest that the motion of the secondary nucleus was secularly *decelerated*, while the presplit nucleus had a secular *acceleration*, and the implied change in rotational sense is not compatible with breakup due to rotational instability. As a result, it seems to us that the observed jumps and occasional breakups of comets may have an external cause, specifically impacts from smaller objects.

V. EFFECTS OF COLLISIONS BETWEEN COMETS AND INTERPLANETARY BOULDERS

General Remarks

Harwit (1967, 1968) has postulated the existence of a dense cloud of interplanetary boulders moving in low-eccentricity orbits and presumably of asteroidal origin. He has shown that the cloud of these roughly 10-meter-sized objects is consistent with our present knowledge from impact rates on the Earth and Moon, "spontaneously" split comets, and other evidence. We suggest that impacts from such boulders provide a plausible explanation for the behavior of erratic comets generally, whether there is splitting or merely what seems to be a discontinuity in the motion.

The impulse on the motion of a comet is typically of the order of 1 m sec^{-1} . In Sec. III of this paper we obtained a separation velocity of 0.7 to 1.4 m sec^{-1} for the components of P/Biela, while slightly larger values were derived for the impulses on P/Perrine-Mrkos and P/Schaumasse on the assumption that these anomalies took place in the vicinity of Jupiter (Paper III). For the latter comets we are unable to locate the actual points where the jumps occurred, but the true impulses are not likely to differ from our assumed figures by as much as an order of magnitude.

In the remainder of this section we shall discuss the impact hypothesis as it concerns the following points:

- (i) the size of the jumps in terms of the mass ratio of a cometary nucleus and an impacting boulder;
- (ii) the loss of mass from a comet as the result of impact;
- (iii) the relation between jumps and cometary splitting;
- (iv) the spatial concentration of boulders and the rate of impacts on cometary nuclei;
- (v) the reason why jumps are observed only for some particular comets;
- (vi) the possible nature of the boulders.

The Impact Mechanism

A hypervelocity impact has the features of a typical explosion. A shock wave starts spreading through the target medium from the point of impact, and a crater is formed.

The projection J along the normal to the surface of the momentum of the material expelled from the crater has been calculated by Stanyukovich (1961) from his model of crater formation:

$$J = (1 - \xi)T \left(\frac{2A}{Q_0} \cos^3 \phi_0 \right)^{\frac{1}{2}} (1 - \cos^{\frac{1}{2}} \phi_0), \quad (2)$$

and the mass expelled from the crater is

$$\Delta M = \frac{A(1 - \xi)T}{4Q_0} \sin^2 \phi_0 \cos \phi_0, \quad (3)$$

where T is the projectile's explosive energy; A is the mass of the deformed medium in terms of the mass of the explosive; ϕ_0 is the cone angle, which is related to the diameter/depth ratio of the crater (Shoemaker 1963; Baldwin 1965; Bouška 1968); Q_0 is the calorific content of the explosive, or the energy necessary for crushing 1 g of the target medium; and ξ is a coefficient accounting for energy losses.

Let M be the comet's mass, m the projectile's mass, ΔV the impulsive increment in the comet's velocity, and w the impact velocity. If a is the semimajor axis of the cometary orbit (in astronomical units), and e its eccentricity, and if we assume that the boulders move in circular orbits in the comet's orbital plane, the mean value of w (in km sec^{-1}), averaged by integration over the comet's revolution period, is

$$w = 42.1a^{-\frac{1}{2}} \left(1 - \frac{2\mathbf{K}}{\pi}(1 - e)^{\frac{1}{2}} \right)^{\frac{1}{2}}, \quad (4)$$

where \mathbf{K} is the complete elliptical integral of the first kind, with modulus $(2e)^{\frac{1}{2}}(1+e)^{-\frac{1}{2}}$. Figure 1 is a q, Q diagram for the seven erratic comets, and the solid lines represent constant values of w . For these particular comets, w varies between 7 and 14 km sec^{-1} . Conservation of momentum then gives

$$\frac{M}{m} = (1 - \xi) \frac{w^2}{\Delta V} \left(\frac{A}{2Q_0} \cos^3 \phi_0 \right)^{\frac{1}{2}} (1 - \cos^{\frac{1}{2}} \phi_0), \quad (5)$$

$$\frac{\Delta M}{M} = \frac{1}{4} \Delta V \left(\frac{A}{2Q_0} \right)^{\frac{1}{2}} \frac{\sin^2 \phi_0}{\cos^{\frac{1}{2}} \phi_0 (1 - \cos^{\frac{1}{2}} \phi_0)}. \quad (6)$$

Impact mechanics considered by Marcus (1969) and Dohnanyi (1971), and based largely on laboratory hypervelocity impact experiments by D. E. Gault, H. J. Moore, and their associates, give results comparable with Stanyukovich's model. The mechanical properties of the target are much more critical than are those of the projectile.

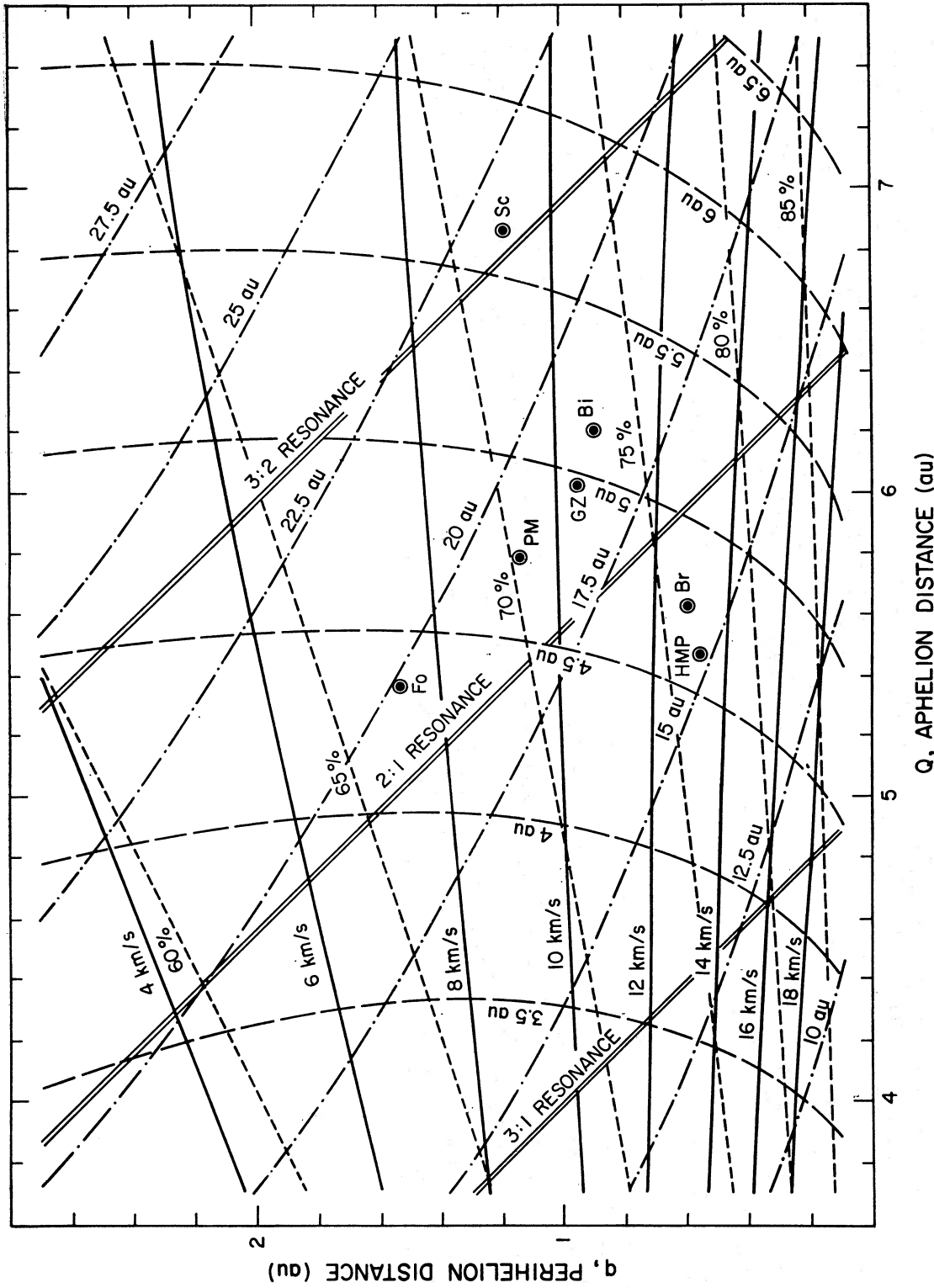


FIG. 1. A q, Q diagram for the erratic comets (Fo = Forbes, PM = Perrine-Mrkos, Sc = Schaumasse, GZ = Giacobini-Zinner, Bi = Biela, Br = Brorsen, HMP = Honda-Mrkos-Pajdušáková). *Solid lines:* mean collision velocity w between comet and boulder in circular, direct, and coplanar orbit. *Long dashes:* critical heliocentric distance r_0 below which the comet would be accelerated and beyond which it would be decelerated due to an impact by a boulder. *Short dashes:* probability P that the comet will be accelerated assuming a uniform distribution of boulders. *Dashed-and-dotted lines:* length of comet's orbital ellipse. *Double solid lines:* mean-motion resonances with Jupiter.

Impact Phenomena on Comets

For earthy materials, $A(1-\xi)/Q_0 \simeq 2.5 \times 10^{-8} \text{ sec}^2 \text{ cm}^{-2}$ (Shoemaker *et al.* 1961), while for more fragile and porous cometary materials we can estimate $A/Q_0 \simeq 10^{-7} \text{ sec}^2 \text{ cm}^{-2}$. For cometary snowballs the ratio must be still higher, perhaps by 1 to 2 orders of magnitude. Donn (1963) has shown that in interplanetary space an accreted snowball would have a density not exceeding 0.3 g cm^{-3} . Delsemme (1971) has recently given very strong arguments for the dominant role of water snows in comets.

The presence of meteoric particles introduces some uncertainty. The bulk density of the meteoroids associated with P/Giacobini-Zinner may be an order of magnitude smaller than that of the snows (Whipple 1963), and we may tentatively assume that the loose particles in the coreless nucleus do not increase the mean density, which we shall take to be 0.2 g cm^{-3} . The low tensile strength of conglomerate ice also suggests high A/Q_0 for a snowball nucleus. Accepting $A/Q_0 \simeq 10^{-5.5} \text{ sec}^2 \text{ cm}^{-2}$, $1-\xi \simeq 1$, $w = 10 \text{ km sec}^{-1}$, $\cos \phi_0 \simeq 0.4$, we find, from Eqs. (5) and (6),

$$\frac{M}{m} = \frac{10^6}{\Delta V}, \quad \frac{\Delta M}{M} = 0.1 \Delta V, \quad (7)$$

ΔV being measured in meters per second.

The zone in which nonlinear deformations take place must be much larger than the crater itself. Stanyukovich's (1961) list of conventional materials shows that in this zone the quantity A is 1 to 3 orders of magnitude greater than in the crater. Indeed, according to Nuckolls (Johnson *et al.* 1959), during the Rainier underground nuclear explosion rock was fractured to a distance of twice the radius of the crater.

It is therefore quite plausible to conclude that after a boulder impact cracks, fissures, and grooves extend over a volume of the nucleus almost 10 times that of the crater. Collision between a coreless comet and a boulder 10^6 times less massive could cause ruptures to spread through almost the whole nucleus. Splitting or complete disintegration of the comet would not be surprising. If the nucleus does in fact have a small core, the shock wave may be absorbed more easily or deflected and the disruption prevented. However, the mechanical bonds of the nuclear materials must be weakened to the extent that a comet could not survive many more such collisions. In this connection it may be significant that the two erratic comets that have apparently experienced at least two impacts are lost.

Rates of Impact

Photometric observations do not indicate any obvious systematic difference in brightness between comets presumed to be coreless and those whose nuclei might be dismantled cores. For a nuclear core, however, the

reflectivity would be that of the meteoric material, and it is reasonable to accept a Bond albedo of $\simeq 0.1$ and a geometric albedo of 0.15. On the other hand, a surface covered by H_2O snow would typically have a Bond albedo of 0.7 to 0.8 and a geometric albedo of 0.50. For grains of methane clathrate, Wenger's crude experimental data suggest an even higher Bond albedo (Delsemme and Miller 1971). Furthermore, for a nuclear core, we should presumably accept the "asteroidal" phase coefficient $\beta \simeq 0.03 \text{ mag deg}^{-1}$. A snow surface, on the other hand, should have, for small phase angles, $\beta \simeq 0.002 \text{ mag deg}^{-1}$ (Veverka 1970), and this could typically alter the derived absolute brightness by 0.5 to 1.0 mag.

The results by Roemer (1966) suggest that a typical coreless nucleus (for a short-period comet) might have a nuclear radius $R = 0.5 \text{ km}$, and with $\rho = 0.2 \text{ g cm}^{-3}$, its mass would be only 10^{14} g . It follows from Eq. (7) that an impulse such as those observed ($\Delta V \simeq 1 \text{ m sec}^{-1}$) could be produced by a collision with a boulder of mass 10^8 g . The differential effects of albedo and phase suggest that for a typical dismantled core we should take instead $R = 1.5 \text{ km}$ (cf. also Sekanina 1971c), and with $\rho = 1.5 \text{ g cm}^{-3}$, we obtain $M = 2 \times 10^{16} \text{ g}$, or 200 times greater than the mass of an average cometary snowball. An impulse of 1 m sec^{-1} on a cometary core would require the colliding boulder to have a mass of 10^{11} g .

We shall consider now the expected rate of impact by interplanetary boulders on a short-period comet. If s is the spatial mass density of the boulders and m their average mass, then the average number of revolutions made by the comet between two consecutive impacts is

$$N = m/sU, \quad (8)$$

where $U = \pi R^2 \Lambda$ is the volume of space swept out by the cometary nucleus during one revolution, Λ being the length of the orbital ellipse. Lines of constant Λ are shown by the dashed-and-dotted curves in Fig. 1, a typical value being 20 A.U. Accepting Harwit's (1967) result, $s \simeq 2 \times 10^{-18} \text{ g cm}^{-3}$, and taking $m = 10^8 \text{ g}$, we find that for our typical coreless comet, $N \simeq 20$ revolutions. On the other hand, in order to produce the same impulse on a typical core comet, we should have to take $m = 10^{11} \text{ g}$ and hence obtain $N > 2000$ revolutions.

The statistics on the observed frequency of discontinuities are extremely meager, but from the data summarized in Table XII for the erratic comets, we may deduce that N is at least in order-of-magnitude agreement with the figure derived above for coreless comets but absolutely incompatible with the solution for core comets. Of course, boulders of mass 10^8 g will collide with all comets alike, but the resulting velocity change on a core comet will not exceed about 0.001 m sec^{-1} . More detailed analysis of cometary motions could perhaps reveal impulses as small as 0.1 m sec^{-1} ,

but it would be impossible to separate smaller impulses from the "continuous" nongravitational effects.

Possible Nature of the Boulders

It can be shown that the impact of a boulder effectively produces on the comet's motion an acceleration or a deceleration according as to whether the heliocentric distance is less or greater than a critical value r_0 , given by

$$r_0 = \frac{9p}{4\theta^2} \sin^2 \left[\frac{1}{3} (\pi - \sin^{-1} \theta) \right], \quad (9)$$

where p is the comet's orbital semilatus rectum and

$$\theta = \frac{3\sqrt{6}}{8} (1 - e^2)^{\frac{1}{2}}. \quad (10)$$

For a uniform distribution of boulders, the probability P that a comet would be effectively accelerated would then be equal to the ratio of the orbital arc inside r_0 to the total orbital length. The critical distance r_0 and the probability P are plotted in Fig. 1, and we may note that $P > 65\%$ for all seven erratic comets.

For the comets that have made only four appearances, it is generally not possible to tell whether an impulse produced an effective acceleration or an effective deceleration; this depends on when the impulse occurred. It is evident, however, that P/Perrine-Mrkos and P/Schaumasse experienced significant effective accelerations, while P/Giacobini-Zinner and P/Brorsen (during its final revolution, anyway) were considerably decelerated. P/Biela seems to have had a large decelerative impulse during the eighteenth century, while its primary component was very slightly accelerated around the time of the split. From the short list of erratic comets available at present, we may tentatively conclude that impulsive decelerations are no less frequent than impulsive accelerations, which might be an indication that the spatial density of boulders increases with increasing heliocentric distance, to and somewhat beyond the orbit of Jupiter. (As already noted, it is now clear that P/Biela split at a heliocentric distance of 6 A.U.)

If this conclusion is correct, it suggests that the boulders do not originate from the minor planets. The extensive Palomar-Leiden Survey (van Houten *et al.* 1970) has confirmed that there is no evidence for the existence of minor planets moving in low-eccentricity orbits beyond the orbit of Jupiter. It may be dangerous to extrapolate down to tiny fragments, but there is little reason for supposing that the distribution of the latter would be radically different.

As an alternative, we should like to suggest that the boulders are cometary. If their density is 1.5 g cm^{-3} , their average diameter would be 5 m, and if only 0.2

g cm^{-3} , it would be 10 m. It is well known that many of the short-period comets have been ejected in from (and sometimes back out to) orbits of low eccentricity having semimajor axes comparable to that of Jupiter. By studying the evolution of typical orbits, Havnes (1970) has demonstrated the probable existence of a considerable number of comets in low-eccentricity, low-inclination orbits with their perihelia near the orbit of Jupiter; also see Kresák (1971). There is again the problem of extrapolation down to meter-sized objects, and much more remains to be investigated concerning the long-term evolution of the orbits of these comets. In any case, assumption of cometary, as opposed to planetary, origin of the boulders removes—at least qualitatively—the difficulty Harwit encountered in providing a mechanism by which the supply of boulders could be replenished in order to compensate for their self-destruction through mutual collisions.

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