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The Mass of Jupiter from the Motion of (76) Freia

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In Hill's 1873 list of minor planets of the Hecuba group suitable for determination of the mass of Jupiter, (76) Freia has the largest coefficient for the long-period term in longitude. Observations consisted of 15 photographic plates taken at Heidelberg Königstuhl Observatory (1912–1933) and remeasured at the U. S. Naval Observatory, several plates remeasured at Lowell and Goethe Link Observatories, and 79 plates taken at the Naval Observatory (1947–1971). In addition, 43 visual observations taken at Berlin, Vienna, and Leipzig Observatories (1864–1893) were reduced to the same catalogue system as the photographic observations. Variational equations were integrated for the differential correction process. The derived reciprocal mass of Jupiter is 1047.366±0.007 (m.e.).

In 1873, Hill suggested a group of minor planets which would be useful in determining the mass of Jupiter because of the dynamical property that their mean motions are nearly commensurable with Jupiter's mean motion in the ratio 2:1, producing a single dominant term of long period and large amplitude in the representation of the longitude.

Recently, Klepczynski (1970) compiled a similar list. In preparing this list, the coefficient of the long-period resonant term was evaluated for the numbered minor planets close to the 2:1, 3:2, and 4:3 commensurabilities. Those minor planets whose coefficients exhibited a change of 1ʺ.5 or more when a change of 0.003% in the disturbing mass of Jupiter was made, were considered as suitable objects to use in an analysis to improve the adopted value of the mass of Jupiter. Only one minor planet, (76) Freia, was common to both lists. Hence, the sensitivity of Freia's orbit to a small change in the mass of Jupiter is greater than that for any of those minor planets from the Hill list which have been used up to now (Fiala 1968; O'Handley 1969; Zielenbach 1969; Klepczynski 1969; Doggett 1971).

Unfortunately, since Freia was discovered in 1862, a set of observations which covers the full 149-year cycle of the long-period perturbation is not yet in existence. There is a series of photographic observations from 1947–1971 taken at the U. S. Naval Observatory, and a series taken at Königstuhl Observatory extending from 1912–1933. By combining these two data sets with some of the early visual observations, a good observational data set which covers 75% of the long-period term was obtained.

In summary, because more than 50% of the long period was covered and because of the greater sensitivity, we felt that significant results could be obtained from this investigation. The compilation of a highly accurate, though somewhat select, data set reinforced this conclusion.

I. VISUAL OBSERVATIONS

Before 1912, only visual observations were made of this minor planet. They had to be included to obtain the longest arc possible, in order to cover more of the long-period term; but in view of known large errors in observations of this type, the series of observations were selected, rather than being all-inclusive, and reductions were made with utmost rigor. Only the three observatories at Berlin, Leipzig, and Vienna made regular rather than sporadic observations of Freia from the time of discovery in 1862 until 1893.

The observations, given as apparent positions, were reduced to mean positions for the beginning of the Besselian year using Besselian Day Numbers computed according to the precepts of the contemporary issues of the *Berliner Jahrbuch.* To keep the spirit of the investigation by having a homogeneous set of observations, catalogue corrections were computed to put the observations on the FK4 system. This was done by (i) identifying the reference star for the differential measurement in the Yale Zone Catalogue, (ii) recomputing the position for the epoch of the observation, and (iii) adding to the position of the minor planet the difference between the recomputed and the published positions plus a tabular zone correction to the FK4. The mean position was taken to 1950.0 by rigorous precession formulas, using Newcomb's precession. The final position was then on the same theoretical base as the photographic observations. For details of the procedure, see Fiala (1968).

Although 57 observations were processed, only 43 were retained in the final solutions. The others showed very large discrepancies when compared with the provisional orbit. The scatter of the 43 retained was much larger than that of the photographic observations.

II. PHOTOGRAPHIC OBSERVATIONS

Since 1950, the Astrometry and Astrophysics Division of the U. S. Naval Observatory, as part of its minor planet observing program, has regularly observed Freia with a 15-inch astrograph. These observations have recently been remeasured using the automatic measuring machine of the USNO and reduced by Mrs. B. Mintz and Dr. R. S. Harrington (1971). This series consists of 70 observations extending from 1950–1971 and includes 14 oppositions.

In order to augment these observations, a search of old USNO plates was made and a letter requesting aid in locating old plates with Freia on them was sent to 11 observatories. The search of old USNO plates yielded nine additional observations, made with a 10-inch astrograph, covering the oppositions of 1947, 1948, and 1949.

Dr. F. K. Edmondson of the Goethe Link Observatory, Dr. H. L. Giclas of the Lowell Observatory, and Dr. H. Elsaesser of the Heidelberg-Königstuhl Observatory sent favorable replies to the letter. The Goethe-Link Observatory remeasured plates to furnish two observations from the 1961 opposition. The Lowell Observatory remeasured plates to furnish three observations from the 1930 opposition and two from the 1931 opposition. All new measurements of the minor planet and comparison stars were sent to the USNO for reduction.

Since the Königstuhl Observatory had a rather large number of plates, it was thought best to lend them for remeasurement; they were subsequently hand transferred to the USNO, where they were measured on the Mann measuring engine. These plates yielded 16 observations of the oppositions of 1912, 1915, 1918, 1919, 1921, 1924, 1931, 1933, and 1958.

The measures of all photographic observations were reduced utilizing a solution by the method of least squares for three linear and three quadratic terms of the plate constants. For most plates, 12 comparison stars were selected for measurement so that stars which had poor positions or proper motions or were poorly measured could be eliminated from the solution.

As a result of these efforts, we obtained a series of photographic observations extending from 1912 to 1971 which were uniformly treated and reduced using the same catalogue for reference stars (SAO Catalogue, 1966). The SAO Catalogue was chosen because it is the most practical source for positions of faint stars which have already been put on the system of the FK4 Catalogue by the application of zone corrections. However, not all plates were measured on the same engine, as had been the original plan.

III. PROCEDURE AND RESULTS

Ephemerides of Freia's positions and velocity were obtained by direct numerical integration of the second-order differential equations in a heliocentric equatorial rectangular reference frame defined by the mean equator and equinox of 1950.0. In the calculation of perturbations, Mercury's motion was neglected and its mass was added to the Sun's. Coordinates for Venus and the Earth–Moon barycenter were taken from the Herget tabulations (1953, 1955). Positions for Mars were supplied by Duncombe (1964). For the five outer

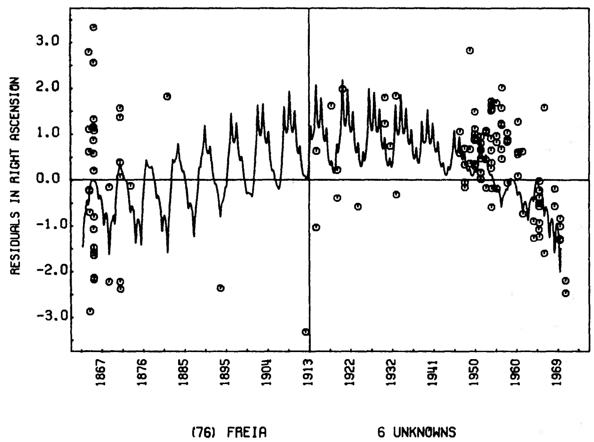

FIG. 1. The *circles* represent the residuals in right ascension (seconds of arc) of the observations from 1862 to 1971 after a solution for six unknowns in which the initial ephemeris of (76) Freia was based on a disturbing mass of Jupiter of 1047.355. The *solid curve* represents the difference in right ascension between two ephemerides which have been fit to observations and which utilized disturbing masses of Jupiter of 1047.390 and 1047.355, representing an error of 0.003%.

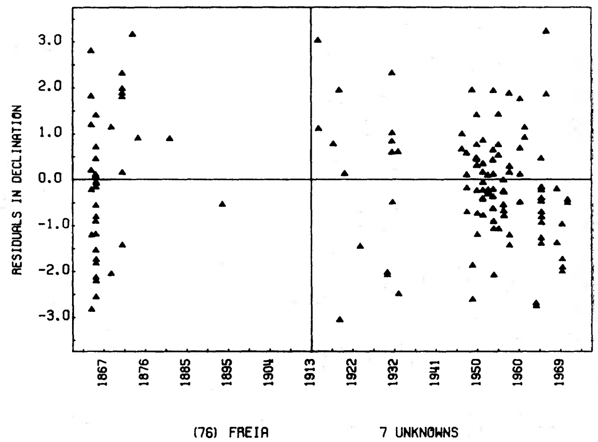

FIG. 3. The *triangles* represent the residuals in declination (seconds of arc) of the observations from 1862 to 1971 after a solution for seven unknowns in which the initial ephemeris of (76) Freia was based on a disturbing mass of Jupiter of 1047.355.

planets, the integrations of Eckert, Brouwer and Clemence (1951) were used. The system of planetary masses was that of the I. A. U.

In addition to Jupiter's mass, the quantities differentially corrected were the rectangular coordinate and velocity components at epoch. The partial derivatives may be obtained by integration, as in the method explained by Herget (1968). The necessary expressions for constructing the integrand matrix of the 21 partial derivatives are exact, possess symmetry, involve no coordinate transformations, utilize quantities available directly from the integration for position, and yield equations which may be handled by the same double quadrature formulas as for the motion itself. Using procedures described by Janiczek (1968), all integrations were constructed forward and backward at a 5-day interval from the epoch of osculation, J.E.D. 2420000.5 (20 August 1913). The effects of tenth differences were retained.

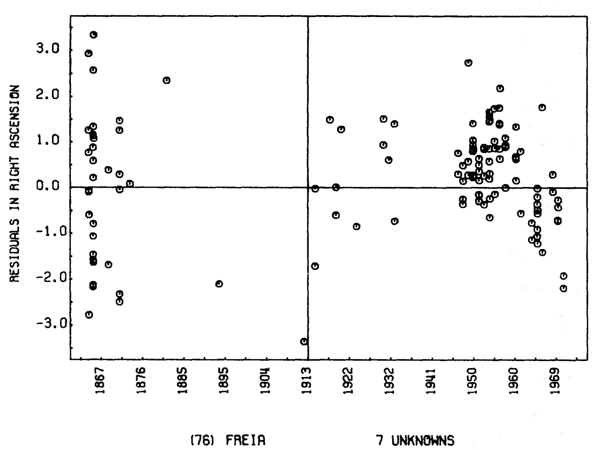

FIG. 2. The *circles* represent the residuals in right ascension (seconds of arc) of the observations from 1862 to 1971 after a solution for seven unknowns in which the initial ephemeris of (76) Freia was based on a disturbing mass of Jupiter of 1047.355.

Reduction of the planet's position to geocenter was accomplished by adding to the heliocentric positions of Freia those solar coordinates found by evaluation of the algorithm forming the basis for Newcomb's *Tables of the Sun*, and hence the *American Ephemeris*. Further reduction was made by application of corrections for geocentric parallax, elliptic aberration, and planetary aberration (using the integrated velocity).

During the first part of the work, two distinct orbits were fit to observations which had been supplied by the Minor Planet Center at the University of Cincinnati. The first case was based on the I.A.U. reciprocal Jovian mass 1047.355. As a check upon the first, the second case incorporated the arbitrary value 1047.390. After convergence of the six orbital elements in each case, the correction to Jupiter's mass was introduced and both cases solved for simultaneous corrections to elements and mass. Agreement of the results from the two cases was exact for all corrected quantities. Figure 1 illustrates the residuals $\Delta\alpha \cos\delta$ from the six-unknown fit to observations using the reciprocal mass 1047.355. The solid curve represents the difference in right ascension between the two ephemerides and indicates that the behavior of residuals during the last 25 years is due in large part to perturbations by Jupiter.

At the same time as the above process for testing the adequacy of methods and computer programs was concluded, remeasurement and rigorous reduction of older plates and visual observations together with a scrutiny of the modern positions was completed. Utilizing this new set of observations, the solution was repeated, and a new integration and differential correction were performed. Transformation of position and velocity from the final solution gives the following

TABLE I. Results from the studies of nine minor planets.

Minor planet	Reciprocal mass	Mean error	Year	Investigator
65 Cybele	1047.387	±0.004	1967	O'Handley
57 Mnemosyne	1047.356	±0.004	1968	Fiala
48 Doris	1047.340	±0.024	1968	Zielenbach
10 Hygeia	1047.351	±0.006	1969	Klepczynski
24 Themis	1047.359	±0.010	1969	Klepczynski
31 Euphrosyne	1047.372	±0.006	1969	Klepczynski
52 Europa	1047.337	±0.027	1969	Klepczynski
49 Pales	1047.340	±0.013	1969	Doggett
76 Freia	1047.366	±0.007	1971	Klepczynski; Janiczek; Fiala

elliptic elements, referring to the ecliptic:

$$a = 3.41123513 \text{ A.U.}$$
$$e = 0.17357884$$
$$M_0 = 342°54'20''.44$$
$$i = 2\ \ 2\ 57.55$$
$$\omega = 235\ \ 7\ 11.31$$
$$\Omega = 212\ \ 9\ 23.17$$

The mass of Jupiter, simultaneously found, is 1/1047.366 ±0.007 (m.e.). Figures 2 and 3 show the residuals after solution in $\Delta\alpha \cos\delta$ and $\Delta\delta$, respectively.

IV. DISCUSSION

The scatter of residuals after solution was not as small as expected, nor was the amplitude of the long-period term reduced as much as expected. A solution using only the photographic observations of 1947–1971 does flatten out the residual scatter in that span, as shown in Fig. 4, indicating that the perturbing effect by a change in the mass of Jupiter is probably real; yet the correlations among variables get so large that one may not put much faith in the numbers from this solution.

It appears that the limit has been reached in the accuracy with which the mass of Jupiter can be determined utilizing currently available observations of minor planets, in the sense that the mean errors of unit weight (and the mean residuals) seem to have reached a lower limit; unfortunately, the results from the studies of nine minor planets do not agree as well as anticipated (see Table I). It has turned out that the effects of correcting the mass constant of Jupiter were nearly lost in the noise of the observations, even after the effort to account for systematic reduction and catalogue errors; this is especially true of the visual observations, which have a major role in these studies.

It is our opinion, based on our own separate studies and the work of Pierce (1971), that continued photographic observations of these nine and other minor planets would increase the precision with which their orbits are known, perhaps allowing detection of even smaller corrections to other constants, e.g., equator and equinox. The observations must be taken in a uniform way at frequent intervals, for distribution and homogeneity are as important as number (Brouwer 1941).

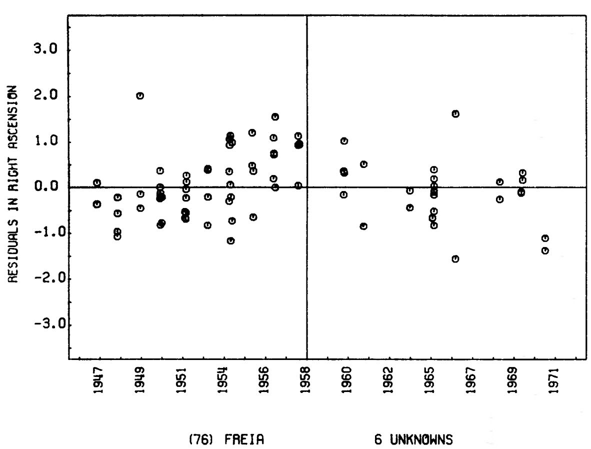

FIG. 4. The *circles* represent the residuals in right ascension (seconds of arc) of the photographic observations made at the U. S. Naval Observatory from 1947 to 1971 after a solution for seven unknowns in which the initial ephemeris of (76) Freia was based on a disturbing mass of Jupiter of 1047.355.

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