1971AJ.....76..932V

THE ASTRONOMICAL JOURNAL VOLUME 76, NUMBER 9 NOVEMBER 1971

Trigonometric Parallaxes Determined with the Yerkes Observatory 40-inch Refractor. I. Methods of Observation, Measurement, and Reduction, and the First Results

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(Received 7 June 1971)

The first results of the new Yerkes Observatory 40-inch refractor parallax program are presented. The program has implemented many of the suggestions made by Vasilevskis to increase the accuracy of trigonometric parallaxes. Based on an analysis of 142 reference-star relative parallaxes in seven different fields, the accuracy of the new Yerkes parallaxes is found to be approximately 4 times greater than that of previous Yerkes determinations based on a similar amount of plate material. The external error of the parallaxes has been evaluated from the distribution of reference-star relative parallaxes and found to be approximately one-third greater than the internal error.

IN 1966 a new parallax program with the Yerkes Observatory 40-inch refractor was started. The principal goals were to explore and develop more accurate and efficient methods of observation, measurement, and reduction, and in the process to determine accurate trigonometric parallaxes for a number of faint stars. The 60 stars selected for this program have an average visual magnitude of $V\sim14.4$ and are divided between white dwarfs and large proper-motion late-type stars.

I. OBSERVATIONS

Since the mean visual magnitude for the stars on the parallax program is $V\sim14.4$, 30-min exposures are usually necessary, except for the brighter objects where 5 to 20-min exposures are used. As a general rule, only one exposure per plate is made, except for exposure times less than 10 min where two exposures per plate are usually made. Kodak 103a-G spectroscopic emulsions were used in combination with a Schott GG14 "A" filter for all exposures.

In order to maximize the measured parallactic shift, the observations are restricted to those dates when the parallax factor is near a maximum. The average total parallax factor for the stars reported here is 0.940. In order to achieve this large value it was usually necessary to observe several hours from the meridian. Correction for the effects of differential color refraction introduced by observing off the meridian will be discussed in Sec. IVB.

The camera used for all the plates discussed here is a manually controlled double-slide plate holder constructed in 1929. Due to the use of a nonflat filter installed in 1957, large scale changes, dependent on the camera position angle, are introduced. The effects of this filter on the accuracy of the parallaxes are discussed in Sec. IVA.

During 1966 and 1967, the observations were made by D. H. DeVorkin, B. F. Jones, W. Schierer, D. G. York, and myself. Since January 1968 nearly all of the observations have been made by E. U. Vilkki.

Except for the first few plates taken in early 1966, all plates have been developed vertically in tanks. In October 1967, a nitrogen-burst agitation system was installed which improved the uniformity of development and consistency of the results (van Altena 1971b). Three different developers have been used: D-19, Metol Sulphite, and MWP-2. The majority of the plates under discussion were developed in Metol Sulphite, while MWP-2 is the developer currently used. After September 1966, all plates were rinsed in Perma Wash, a hypo neutralizer, which reduces the total washing time to 6 minutes, including the Perma Wash rinse. The shortened time in solution prevents excessive swelling and separation of the emulsion from the glass plate, shortens the drying time considerably, and as a result probably reduces the shifting of the emulsion. All plates have been dried vertically in the darkroom, which is thermostatically controlled at 68±1 °F at less than 50% relative humidity.

II. SELECTION OF THE REFERENCE STARS

As was noted previously, the observations are usually made far from the meridian. It is therefore necessary to evaluate the effects of differential color refraction and apply corrections to the measured image positions. Vasilevskis (1966) suggested that the color correction could be evaluated by selecting stars with a large range of color as reference stars and solving explicitly for the color term. He also proposed that the magnitude equation be solved for by choosing reference stars with a range in brightness rather than all being approximately of the same magnitude as the parallax star. For the 40-inch refractor, the magnitude equation is typically $0\overset{''}{.}10$/mag. To solve for the above parameters, a larger than usual number of reference stars are necessary. As a preliminary choice, all stars judged measurable on the poorest acceptable plate of the series were identified. Image diameter measures of these stars were then made on the best plate of the series and on the blue and red prints of the Palomar Observatory Sky Survey prints. From these data a color–magnitude diagram was prepared and a selection of up to 24 stars with a large range in color and magnitude, consistent with a uniform dis-

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tribution over the 150-mm diameter circle centered on the parallax star, were chosen as reference stars.

III. MEASUREMENTS

Nearly all the plates were measured by E. Vilkki on the Ridell-Spotz single-screw measuring machine, which has been digitized to 0.5 μ and equipped with a photoelectric image bisector. These modifications are described in more detail in another publication (van Altena 1970). The external accuracy of a single bisection on fair quality images when the photoelectric image bisector is used is ± 1.0 to 1.4 μ m.e., while the repeatability is ± 0.5 μ m.e. Tests showed that a combination of making two settings on an image and measurement in both direct and reverse positions increased the accuracy of a position by 10% to 15%. Since this slightly higher accuracy was achieved only after approximately tripling the measuring time, all plates were measured only in the direct position with one bisection per image.

IV. PLATE-CONSTANT REDUCTIONS

After correcting the measured coordinates for the errors of the measuring machine, it was necessary to correct for the effects of the nonflat filter and for differential color refraction.

A. Corrections for the Filter

Inteferograms of the two sides of the GG14 "A" filter show that, although there is very steep curvature of different shape on either side, the surfaces are moderately smooth. Measurements in x and y of the image displacements due to the nonflat filter were made with a small telescope on a grid of about 50 positions in the central 150-mm by 200-mm area of the filter. The averaged x and y errors are plotted in Fig. 1. A previous determination by van Altena and Monnier (1968) of the x error curve is in good agreement with the present results. The error introduced by using a mean curve for each coordinate is approximately 2 μ, and can therefore be considered as a small second-order effect. It is clear that this filter introduces a significant scale change when compared with a flat filter. Removal of these filter errors from the measured coordinates resulted in a maximum change of an individual reference star parallax of $0''.0002$, and is therefore insignificant. The parallax of the central star was not changed. Deviations from the mean curves may introduce a small amount of error into the reference star parallaxes in the outer regions of the plate, however the error should be less than $0''.0002$.

B. Corrections for Differential Color Refraction

Due to the broad spectral passband of the 40-inch filter–plate combination (Kodak 103a-G+Schott GG14 "A", λλ 4900–5900 Å) and because nearly all of the morning observations are made east of the meridian and the evening observations west of the meridian, it is necessary to correct the x and y measurements for the effects of differential color refraction.

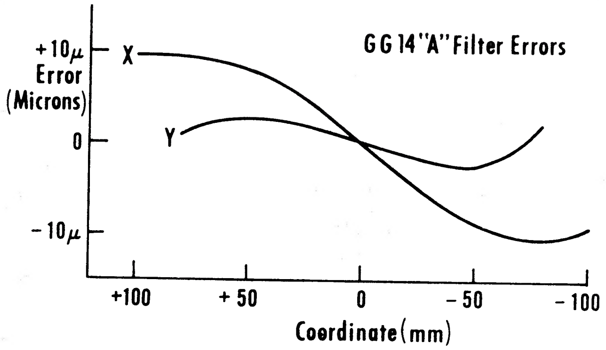

FIG. 1. The average X error due to the filter as a function of the X coordinate, and the average Y error as a function of the Y coordinate. The *ordinate* is in microns and the *abcissa* in millimeters. A flat filter would be represented by a horizontal line.

As mentioned earlier, Vasilevskis (1966) had proposed that the color coefficient be determined for each plate by adopting reference stars with a large range in color index and explicitly solving for the color dependence. Unfortunately, the scatter in the reference star residuals made it impossible to adequately determine the color coefficient for each plate and therefore other methods were adopted to determine the necessary correction. Vasilevskis (1971) has also found that it is better to apply the color correction before the plate solutions.

The color correction has been evaluated in two different ways: first, by computing the effective wavelength as a function of spectral type for our passband, and second, by directly determining the color coefficient from observations of a star cluster over a large range of air masses. Calculations by J. Hofslund for saturated images yielded a color coefficient of $-0''.013$ per magnitude of $B-V$ color index for a zenith distance of 45°. In addition, observations of the open cluster NGC 6866 were made by Hofslund from dusk through dawn on one night. Hoag *et al.* (1961) have published photoelectric and photographic determinations of the $B-V$ color index for a large number of stars in the region of this cluster and list stars with a range of 1.6 in $B-V$. Nineteen of these stars were selected for measurement on nine plates. Least-squares solutions for the color coefficient at a zenith distance of 45° gave $-0''.0133 \pm 0''.0013$ (m.e.) per magnitude of $B-V$. The agreement is very good and a final value of $-0''.013$ has been adopted for all other reductions.

The refraction coefficient used for correction of the measured coordinates on all plates taken with the 40-inch refractor is

$$R''=(55.7390-0.013(B-V))\cdot\tan z-0.081\cdot\tan^2 z. \qquad (1)$$

All measured coordinates have been reduced to outside the atmosphere.

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TABLE I. Results for the parallax stars.

NO	NAME	α(1950)	δ(1950)	V	B-V	U-B	Sp	π_r + m.e.	C1	C2	$\delta\pi_{abs}$	π_{abs}	M_V + m.e.
1	G175-34A	$4^h26^m50^s$	+58°53′3	11.10	+1.62	+1.14	dM4	+0″1649 ±0″0053	-0″0112	-0.75	+0″0018	+0″170	+12.25 ±0.06
2	G175-34B	4 26 50	+58 53.3	12.45	+0.33	-0.49	DC	+0.1719 ±0.0056	-0.0112	+0.54	+0.0018	+0.170	+13.60 ±0.06
	mean							+0.1681 ±0.0038			+0.0018	+0.170	
3	G165-47	14 04 49	+38 51.7	14.4	+1.6			+0.0396 ±0.0095	-0.0063	-0.73	+0.0033	+0.043	+12.6 ±0.66
4	L1274-23	16 22 57	+15 48.4	13.48	+1.42	+1.09		+0.0233 ±0.0047	-0.0055	-0.66	+0.0017	+0.025	+10.47 ±0.61
5	LP9-231	17 56 38	+82 44.2	14.36	+0.37	-0.51	DAs	+0.0608 ±0.0052	-0.0117	+0.54	+0.0019	+0.063	+13.36 ±0.24
6	G142-52	19 44 29	+11 58.4	14.31	+1.48	+1.17		+0.0235 ±0.0080	-0.0076	-0.61	+0.0015	+0.025	+11.30 ±0.95
7	G144-25	20 38 11	+15 18.8	13.41	+1.72	+1.29	dM4-5	+0.1003 ±0.0068	-0.0077	-0.99	+0.0018	+0.102	+13.45 ±0.19
8	W1037	22 26 16	+5 33.9	14.19	+1.42	+1.22	sdK6	+0.0205 ±0.0062	-0.0062	-0.55	+0.0028	+0.023	+11.00 ±0.76

NO	NAME	DATES	μ_x + m.e.	μ_y + m.e.	μ + m.e.	θ + m.e.	m.e. 1	m.e. 1(π)	Im	Npl
1	G175-34A	1966.8-69.8	+1″2946 ±0″0047	-2″0388 ±0″0062	2″4150 ±0″0077	147°58 ±0°12	±0″0280	±0″0356	54	27
2	G175-34B	1966.8-69.8	+1.3457 ±0.0086	-1.9380 ±0.0055	2.3593 ±0.0102	145.22 ±0.17	±0.0300	±0.0382	48	24
	mean	1966.8-69.8	+1.3063 ±0.0041	-1.9824 ±0.0041	2.3740 ±0.0057	146.61 ±0.09	±0.0290	±0.0369		
3	G165-47	1966.5-69.5	+0.3610 ±0.0156	-0.9968 ±0.0085	1.0601 ±0.0177	160.09 ±0.65	±0.0346	±0.0342	17	17
4	L1274-23	1966.6-69.6	+0.1292 ±0.0059	-1.2015 ±0.0039	1.2084 ±0.0070	173.86 ±0.23	±0.0271	±0.0278	40	22
5	LP9-231	1966.3-68.6	-1.2953 ±0.0065	+3.3700 ±0.0056	3.6103 ±0.0085	338.97 ±0.09	±0.0326	±0.0333	46	42
6	G142-52	1966.5-69.4	-0.5220 ±0.0083	-1.3885 ±0.0069	1.4833 ±0.0107	200.60 ±0.29	±0.0289	±0.0300	20	19
7	G144-25	1966.5-69.4	+1.3223 ±0.0071	+0.6621 ±0.0044	1.4788 ±0.0083	63.40 ±0.22	±0.0273	±0.0296	24	24
8	W1037	1966.5-69.5	+0.5061 ±0.0052	-1.5518 ±0.0038	1.6322 ±0.0064	161.93 ±0.15	±0.0238	±0.0232	19	17

Since differential color refraction spuriously increases the measured relative parallax for a star that is redder than the reference frame, an incorrect color index will likewise introduce a spurious measured parallax when corrections are made according to Eq. (1). To provide a means for correcting for this error, I have run all parallax and plate-constant solutions twice, once using the color index of the parallax star as given in Table I, and once using the color index plus 1.00. The difference in the measured relative parallax between the two solutions then allows us to correct the published parallaxes should more accurate colors become available for the parallax or reference stars. Using the coefficients $C1$ and $C2$ given in Table I for each field we may correct the parallaxes with the following equation:

$$\pi\text{ corrected}=\pi\text{ published}+C1\cdot[(B-V)_{\pi}-\langle B-V\rangle_{\text{ref.}}+C2]. \quad (2)$$

C. The Plate Constants and Parallax Solutions

An average standard plate was formed by reducing all plates to a plate near the mean epoch by the method of least squares using the three usual linear unknowns and the magnitude equation. Using this new standard plate, all plates were again reduced using the same four unknowns and the x and y residuals from this solution were then plotted versus both coordinates, $x\cdot y$, coma, magnitude, and color on the IBM 1132 printer to check for possible systematic trends. In a few cases, a quadratic term in the coordinates was obvious and a solution was run which included the quadratic terms. No physical cause has been found for these occasional quadratic terms. Once the plate solutions are completed, the parallax star is reduced using the best plate constants for that plate. A single combined solution for the parallax and proper-motion components was then run for each reference star and the parallax star. These first-order proper motions were then used to remove the effects of proper motion in the reference-star positions and the plate-constant solutions were run again. In most cases the improvement in the individual plate solutions was minor, but in cases where a moderately large reference-star proper motion was encountered, a significant improvement was achieved. In the case of Ross 640 (van Altena 1971a), where the plates extended over a period of about 15 years, the weights of the early- and late-epoch plates were increased by factors of from 10 to 15 after removal of the reference-star proper motions. New solutions were now run for the parallax and proper motion and the residuals plotted versus the date of exposure, parallax factor, and the refraction coefficient for each star. The iterative approach used here is a simplification of one suggested by Eichhorn and Jeffrys (1971).

It should be noted that the reference-star parallaxes are computed with respect to a reference frame which includes that star. In principle, one should eliminate each star from the reference frame, repeat the plate-constant solutions without it, and then solve for the parallax. However, since there are an average of 20 reference stars per field, the effect of leaving the reference star in the plate-constant solution is negligible.

V. THE CORRECTION TO ABSOLUTE PARALLAX

The correction to absolute parallax has been estimated in two different ways. Murray and Clube (1970) give the mean secular parallax as a function of the average magnitude of the reference frame and its galactic latitude. On the other hand, since there is an average of 20 reference stars for each field, one can derive a reasonably accurate proper-motion dispersion for each field and compare it with the expected velocity dispersion at the apparent magnitude and galactic latitude of the reference frame. For this calculation, I have utilized an interstellar reddening at the galactic pole of $E(B-V)=0.04$ (Sturch 1967), a visual absorption of $A_V=0.8$ mag/kpc, and a derived galactic scale height of 150 pc for the absorbing layer. The velocity ellipsoid, vertex, and scale height as a function of spectral type are as given by Allen (1963) in his Secs.

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118 and 119 and the spectrum–luminosity distribution of stars is given by Allen (1963) in his Sec. 117. In general the two methods gave very similar results as is shown in Table II and an average value was adopted.

Since the nonrandom selection of reference stars vitiates, to a certain extent, the use of a statistical correction to absolute, and since it is possible to accurately correct for differential color refraction analytically, I plan to discontinue the selection of reference stars according to their color. It is not felt that the corrections to absolute derived here are seriously in error.

VI. RESULTS FOR THE PARALLAX STARS

The results for the individual parallax stars are given in Table I. The columns labeled *C*1 and *C*2 give the coefficients to be used for correcting the relative parallaxes in this table and also in Table IV, if better values of the $B-V$ color index should become available. The corrected parallax may be obtained from Eq. (2). The column labeled $\delta\pi_{\mathrm{abs}}$ gives the correction to absolute parallax, while the mean error of the absolute magnitude is derived from the external error of the parallax as discussed in Sec. VIII. In the lower half of Table I, the unit weight errors refer to a single average quality image on one plate. The column mel(π) is the error of the parallax reduced to unit weight as discussed in Sec. VII, while the columns labeled *Im* and *Npl* are respectively, the number of images used in the parallax solution and the number of plates.

In those cases where other published values exist, I have combined them to form a weighted mean parallax in Table III. The weights used are external as estimated for the new Yerkes parallaxes in Sec. VIII, and for the astrometric reflector (N) by Strand and Riddle (1970). In the case of LP9-231 there is no way at the present time to assign an external error for the Pulkowa (P) (Kiselev and Sumzina 1970) or the 48-inch Schmidt (L) (Luyten 1967) parallaxes, so all four errors have been arbitrarily increased by one-third. An inspection of Table III shows that the agreement between the new Yerkes and the astrometric reflector parallaxes is good in all cases.

The individual stars will now be discussed.

(*Nos. 1 and 2*) *G175-34:* This double star is also known as Stein 2051 and G175-34B is EG 180. A preliminary parallax of +0″.19 was published by van de Kamp *et al.* (1966). The system consists of a late M-type dwarf and a white dwarf with a continuous spectrum, classified by Eggen and Greenstein (1967) as dM4 and DC. The colors given by EG place the M star 0.18 mag above the main sequence after correcting for the ultraviolet excess as outlined by Gliese (1969). The white-dwarf component lies 0.07 mag below the white-dwarf calibration given by van Altena (1969). The Index Catalogue of Double Stars (Jeffers and van den Bos 1963) lists the separation and position angle in 1908 as 6″.8 and 145°, while measurements from the Yerkes plates yield average values of 6″.719 and 85°.84 from 54 images on 27 plates. To within the accuracy of the relative motions of the two components, the motion is approximately circular. Assuming that the orbit is circular and that the semimajor axes are 6″.72, I find the radius of the orbit to be 40 A.U., the period to be 368 yr, and the sum of the masses to be 0.46*M*. If one assumes a mass of 0.20 for the dM4 component, then the provisional mass of the white dwarf is about 0.30, a value that seems rather small. Continued observations of this interesting system should yield an accurate determination of the mass for the DC component.

Table II. Reference-star mean magnitudes and corrections to absolute parallax.

NO	NAME	<v>	<b-v>	$\delta\pi_{\mathrm{sec}}$	$\delta\pi_{\mathrm{stat}}$	$<\delta\pi>$
1	G175-34	12.5	+0.85	+0″.0016	+0″.0019	+0″.0018
3	G165-47	14.2	+0.87	+0.0032	+0.0034	+0.0033
4	L1274-23	13.5	+0.76	+0.0022	+0.0012	+0.0017
5	LP9-231	13.8	+0.90	+0.0017	+0.0020	+0.0019
6	G142-52	14.4	+0.87	+0.0011	+0.0018	+0.0015
7	G144-25	13.4	+0.73	+0.0015	+0.0022	+0.0018
8	W1037	13.6	+0.87	+0.0028	+0.0028	+0.0028

(*No. 3*) *G165-47:* This is a main-sequence M-type dwarf to within the parallax and the color estimate by Giclas *et al.* (1964).

(*No. 4*) *L1274-23:* This star is also known as LFT1274 and G138-25. The photoelectric colors from Priser (1970) and the mean parallax from Table III place this star on the main sequence to within the error of the parallax after correcting for the ultraviolet excess.

(*No. 5*) *LP9-231:* This large-proper-motion white dwarf is also known as vAC-216, G259-21, and EG199. It was discovered by Luyten (1965) and van Altena (1966), and proposed as a pygmy star by Luyten (1965). However, spectra and colors by van Altena (1966) and a preliminary parallax by Riddle and Klugh (1967) showed that it was a normal high-velocity DAs white dwarf. The weighted mean parallax from Table III and the average colors from van Altena (1966) and Eggen and Sandage (1967) place LP9-231 0.28±0.24 mag above the lower white-dwarf sequence.

Table III. Average parallaxes and absolute magnitudes.

NO	NAME	π_r	W	π_{abs}	m.e.	M_V	m.e.
4	L1274-23	+0″.023 (Yk)	20	+0″.022	±0″.006	10.19	±0.59
		+0.014 (N)	10				
		+0.020	30				
5	LP9-231	+0.061 (Yk)	20	+0.063	±0.005	13.36	±0.17
		+0.057 (N)	20				
		+0.106 (L)	1				
		+0.075 (P)	4				
		+0.061	45				
6	G142-52	+0.024 (Yk)	8	+0.023	±0.007	11.02	±0.66
		+0.020 (N)	10				
		+0.022	18				
7	G144-25	+0.100 (Yk)	12	+0.102	±0.007	13.45	±0.15
		+0.099 (N)	10				
		+0.100	22				

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TABLE IV. Reference-star positions, magnitudes, parallaxes, and proper motions.

Field	N	X	Y	D$_{40}$	b-v	π_{rel} +	me	μx +	me	μy +	me	mel	Im
G175-34	1	-11.9419	-1.7793	250	0.3	-0.0016	41	-0.0101	40	0.0047	42	277	62
	2	-9.7647	1.6276	350	0.7	0.0159	29	0.0098	34	-0.0168	25	197	62
	3	-9.0782	7.1458	250	0.5	-0.0059	32	0.0012	34	0.0055	30	212	58
	4	-8.9083	-2.1478	200	1.1	0.0046	46	0.0015	46	0.0135	46	306	62
	5	-5.1926	2.1481	300	1.2	-0.0102	34	-0.0039	40	0.0075	28	227	62
	6	-4.6118	-7.4514	250	0.6	-0.0142	31	0.0371	30	-0.0257	33	212	62
	7	-4.5167	-11.8047	225	0.4	0.0140	36	-0.0029	40	0.0000	33	244	62
	8	-4.3937	2.3361	200	0.8	0.0013	34	-0.0002	33	-0.0211	35	229	62
	9	-2.3000	4.3905	275	0.3	-0.0041	32	-0.0057	35	0.0000	29	214	62
	10	-1.2797	-2.8452	200	1.5	0.0107	50	-0.0064	48	0.0139	50	327	60
	11	0.3504	11.9446	200	0.3	-0.0038	41	-0.0105	41	0.0021	41	273	62
	12	1.6004	-9.0402	175	1.3	-0.0163	41	-0.0136	43	0.0063	39	274	62
	13	1.6461	3.2727	350	1.2	-0.0092	43	-0.0126	44	0.0118	43	290	62
	14	2.9532	3.7006	225	1.7	0.0052	38	0.0036	39	0.0055	37	256	62
	15	4.1506	-8.8426	200	0.8	0.0038	43	-0.0020	51	-0.0001	34	284	61
	16	4.5661	8.7786	175	0.6	0.0027	40	0.0097	39	0.0025	42	268	61
	17	5.6533	-10.6039	300	1.1	-0.0024	30	0.0037	29	0.0066	31	203	62
	18	7.7126	-0.7138	350	0.5	0.0068	33	-0.0001	30	0.0105	36	223	61
	19	8.1851	-3.7837	275	1.1	-0.0020	32	0.0013	37	-0.0075	28	217	62
	20	8.4041	-0.2925	225	0.9	0.0054	35	-0.0007	37	0.0002	34	238	62
	21	9.4590	-8.2267	325	0.3	-0.0040	28	-0.0076	31	0.0002	25	187	62
	22	10.0080	2.2866	200	1.1	-0.0013	34	-0.0014	43	-0.0073	25	232	62
	23	11.5870	6.1722	275	1.6	0.0015	34	0.0045	36	-0.0107	33	228	62
	24	12.3212	1.7637	250	0.5	0.0033	35	0.0134	38	-0.0009	32	232	61
G165-47	1	-10.4451	5.4625	200	0.8	0.0020	81	-0.0154	171	-0.0301	51	286	16
	2	-10.0342	11.3080	175	0.5	-0.0024	80	0.0023	169	0.0244	47	272	15
	3	-2.1484	-6.2602	175	0.8	-0.0032	77	0.0001	63	-0.0269	92	263	16
	4	-1.7392	-12.7505	200	0.8	-0.0047	51	-0.0014	82	0.0225	46	184	17
	5	1.4256	-5.5694	200	0.8	0.0095	61	0.0189	104	-0.0021	51	222	17
	6	4.6983	-3.5709	200	1.1	-0.0065	81	0.0259	125	0.0239	78	295	17
	7	5.8411	4.5314	175	0.8	0.0036	71	0.0027	115	0.0200	66	260	17
	8	6.4288	-9.3214	175	1.1	0.0052	94	-0.0284	144	-0.0011	100	330	16
	9	9.5931	0.0899	175	1.1	-0.0063	72	-0.0137	124	-0.0321	57	254	16
L1274-23	1	-11.0584	7.5010	275	0.7	0.0015	46	-0.0028	50	-0.0080	47	272	40
	2	-10.9701	-1.5191	250	1.0	0.0019	37	-0.0010	46	0.0059	33	219	42
	3	-9.4815	9.6504	300	0.7	-0.0009	38	0.0036	38	-0.0027	42	226	41
	4	-9.5756	-6.8780	250	0.9	-0.0002	35	-0.0008	41	0.0111	33	200	37
	5	-4.8283	-4.1491	250	0.8	0.0015	48	0.0062	53	-0.0133	52	290	43
	6	-4.6804	-1.5297	250	0.7	0.0047	49	-0.0016	56	0.0116	48	288	40
	7	-2.6763	-9.8922	275	0.7	-0.0032	23	-0.0002	25	-0.0113	26	144	43
	8	-2.5226	3.5458	300	0.7	-0.0138	32	-0.0016	41	-0.0032	30	197	43
	9	-2.0619	12.8860	250	0.8	-0.0005	20	0.0000	21	0.0039	21	118	40
	10	2.1614	-6.4842	225	0.9	0.0000	58	-0.0046	60	-0.0003	56	316	35
	11	4.3889	-3.2342	275	0.9	0.0043	46	0.0061	54	-0.0040	48	277	41
	12	4.8882	8.9800	350	0.6	0.0128	36	-0.0020	45	0.0234	38	203	41
	13	5.9817	0.8158	300	0.8	0.0004	34	-0.0016	43	-0.0024	31	205	42
	14	8.7985	-6.9579	275	0.7	0.0007	27	0.0044	32	0.0056	28	166	42
	15	9.8382	8.1052	325	0.7	-0.0069	21	0.0035	26	-0.0166	19	129	43
	16	10.1880	-6.4941	275	0.7	-0.0007	30	-0.0054	37	0.0064	31	178	39
LP9-231	1	-10.8313	2.7231	250	0.7	0.0041	42	0.0007	53	-0.0099	43	258	45
	2	-9.3785	-3.8058	200	0.5	0.0048	39	0.0077	53	0.0003	36	241	46
	3	-8.4519	7.1443	225	1.4	-0.0070	35	0.0104	42	0.0066	42	213	42
	4	-7.7694	-8.9736	250	1.3	0.0026	25	-0.0121	32	0.0086	26	156	46
	5	-7.0809	-6.4675	275	1.9	-0.0080	35	0.0022	50	-0.0005	29	216	46
	6	-5.8559	-0.7702	325	0.3	0.0068	37	-0.0003	43	-0.0085	42	228	46
	7	-4.3017	2.2486	225	0.3	0.0088	50	0.0026	62	0.0164	51	304	45
	8	-2.9772	10.2852	350	0.9	0.0000	28	-0.0219	34	0.0109	30	173	46
	9	-1.7912	1.3369	275	1.3	-0.0063	38	0.0125	56	-0.0291	33	237	45
	10	-1.0211	-3.1949	350	1.9	-0.0097	27	0.0051	30	0.0012	33	170	46
	11	-0.7977	-4.4386	200	1.3	-0.0125	44	-0.0092	59	0.0045	43	271	45
	12	-0.2978	3.2239	225	0.8	-0.0068	43	-0.0160	54	0.0020	46	264	45
	13	0.5934	-7.6516	250	0.1	0.0038	41	-0.0062	51	0.0042	47	242	41
	14	2.5088	-12.2889	350	1.5	0.0008	28	0.0039	34	-0.0002	31	177	46
	15	3.0563	3.5238	200	0.7	-0.0009	48	-0.0096	63	-0.0130	49	297	45
	16	3.7551	-5.0575	175	0.7	0.0086	54	0.0072	71	0.0019	51	320	43
	17	5.1587	11.0755	250	0.3	0.0024	28	0.0183	38	-0.0107	26	175	45
	18	5.4929	-1.1595	350	1.3	0.0054	36	0.0046	42	0.0069	40	218	44
	19	5.7906	-9.0355	250	0.7	0.0020	31	-0.0029	32	-0.0104	40	194	45
	20	5.7492	6.2521	250	0.3	0.0063	41	0.0054	46	0.0138	49	255	46
	21	7.3271	1.6672	200	1.3	0.0107	42	0.0048	51	0.0151	45	253	44
	22	8.5200	-8.4691	175	0.7	-0.0025	47	0.0012	57	-0.0060	54	284	44
	23	10.7850	6.1284	200	0.6	-0.0100	41	-0.0092	55	-0.0114	41	234	39
	24	12.1080	-2.4398	200	1.1	-0.0041	43	-0.0074	60	0.0086	31	236	39
G142-52	1	-11.3153	5.9214	250	0.9	-0.0047	50	0.0006	61	-0.0067	23	173	18
	2	-8.0142	-3.2499	275	1.0	0.0158	57	-0.0180	56	0.0003	46	207	19
	3	-6.5707	2.2953	300	1.2	0.0040	77	0.0039	77	0.0083	60	277	19
	4	-5.7571	4.7497	300	1.0	0.0100	82	-0.0071	89	0.0051	55	285	18
	5	-5.9559	-8.7821	275	0.2	-0.0014	64	0.0117	84	0.0022	54	212	17
	6	-3.6463	2.0704	300	1.4	-0.0152	75	0.0062	59	0.0009	69	270	19
	7	-3.6822	-4.9208	275	1.0	-0.0068	65	0.0230	65	-0.0026	50	234	19
	8	-3.1887	-12.9406	275	0.5	-0.0045	53	-0.0064	59	0.0009	33	181	18
	9	-2.3138	-6.5246	300	0.5	-0.0106	58	0.0104	69	0.0045	34	210	19
	10	-1.5828	-2.4870	275	0.9	-0.0015	55	-0.0106	57	-0.0102	41	198	19
	11	-0.1611	2.5129	275	0.7	0.0185	86	-0.0142	104	-0.0105	49	292	17
	12	-0.8914	7.6359	250	1.3	-0.0192	85	0.0202	78	0.0085	71	306	19
	13	1.1402	2.7075	275	1.0	-0.0065	80	0.0097	99	0.0013	40	286	19
	14	1.2180	11.2747	275	0.2	0.0117	59	-0.0144	87	0.0012	41	197	17
	15	2.5106	12.9008	300	1.2	-0.0120	38	0.0100	44	0.0019	24	138	19
	16	2.6143	0.3113	300	1.0	0.0040	75	-0.0177	98	0.0057	29	269	19
	17	3.0067	9.6267	325	1.1	0.0130	49	-0.0132	58	-0.0102	29	177	19
	18	3.8389	-5.4727	325	0.9	-0.0040	54	-0.0001	66	-0.0070	29	193	19
	19	5.2013	-9.7413	275	0.5	-0.0008	83	0.0124	77	-0.0061	69	297	19
	20	5.4746	-5.6367	325	0.9	0.0114	53	-0.0050	54	0.0087	40	192	19
	21	5.4934	-11.9962	275	1.0	0.0095	45	-0.0113	41	0.0012	36	158	18
	22	7.8470	2.6027	350	0.9	-0.0049	60	0.0113	69	-0.0045	38	217	19
	23	8.2978	4.4480	250	1.0	0.0130	50	-0.0096	47	0.0012	41	180	19
	24	10.3491	-1.2950	300	0.5	-0.0160	58	0.0091	60	0.0070	50	202	18
G144-25	1	-10.8544	1.5848	275	0.0	-0.0095	48	0.0015	52	-0.0031	30	195	24
	2	-9.6780	-4.1910	225	0.0	-0.0028	48	-0.0040	52	0.0033	30	194	24
	3	-8.3803	-6.3181	250	1.2	0.0001	52	0.0133	61	0.0194	25	208	24
	4	-8.4691	3.4357	275	1.2	-0.0110	45	0.0037	47	0.0020	30	183	24
	5	-8.0977	-3.3628	275	1.6	0.0022	53	-0.0134	45	0.0013	42	190	22
	6	-6.1486	-8.3186	250	0.4	0.0152	46	-0.0035	43	0.0009	35	187	24
	7	-4.1306	0.7859	225	0.8	0.0042	67	-0.0017	69	0.0024	45	268	24
	8	-3.1721	4.4301	250	0.8	-0.0022	65	-0.0020	70	0.0035	40	262	24
	9	-2.1189	-9.9661	225	1.2	-0.0090	53	0.0205	58	-0.0186	31	213	24
	10	-0.7429	-2.1088	250	1.2	-0.0003	45	0.0149	45	-0.0087	32	180	23
	11	-0.5040	-5.7959	275	0.8	-0.0046	37	0.0015	34	0.0113	28	149	24
	12	0.0393	6.4604	275	0.4	0.0091	50	0.0005	46	-0.0145	39	203	24
	13	1.5986	9.3439	225	0.4	0.0056	55	-0.0053	53	0.0004	40	220	24
	14	3.0617	-1.3713	225	1.2	0.0065	50	-0.0215	42	0.0013	40	193	23
	15	3.2244	5.4252	250	1.2	0.0069	54	-0.0068	52	-0.0141	38	209	23
	16	4.6996	-6.5196	275	0.4	0.0091	47	-0.0051	51	-0.0107	28	189	24
	17	6.1584	-11.3495	250	0.8	0.0021	43	-0.0142	42	-0.0002	31	173	24
	18	6.2488	1.4101	225	1.2	-0.0040	44	-0.0135	45	-0.0040	31	179	24
	19	6.4456	5.1926	250	0.4	0.0031	39	0.0004	41	0.0070	27	158	23
	20	8.3835	-4.0849	250	0.8	0.0010	53	0.0140	52	0.0052	38	213	24
	21	10.1852	-6.5895	250	0.8	-0.0044	50	-0.0032	46	-0.0046	39	203	24
	22	10.4852	-2.9683	225	0.0	-0.0050	45	-0.0015	46	-0.0006	31	183	24
	23	12.3942	4.6709	250	0.0	-0.0076	51	0.0242	52	0.0080	35	208	24
	24	13.0155	1.1181	250	0.8	-0.0030	42	-0.0027	38	0.0120	33	166	22
W1037	1	-9.2671	4.4331	325	1.1	-0.0015	62	-0.0133	50	-0.0011	40	234	19
	2	-8.1487	-4.2905	325	0.3	0.0010	66	0.0032	59	-0.0029	37	257	20
	3	-7.8935	8.5676	225	0.3	0.0040	68	0.0144	50	-0.0238	48	266	20
	4	-7.2043	9.2785	350	0.3	-0.0013	49	-0.0022	43	0.0026	27	190	20
	5	-6.4354	11.4411	375	1.1	0.0005	70	-0.0077	47	0.0044	52	271	20
	6	-5.5225	8.6265	200	0.3	-0.0158	63	0.0111	52	0.0078	42	232	18
	7	-5.3463	1.8175	250	0.7	0.0032	72	0.0066	70	0.0150	32	279	20
	8	-3.7878	8.8652	300	1.1	0.0128	75	-0.0125	64	0.0119	43	289	20
	9	-0.6174	6.2553	375	1.1	-0.0057	55	0.0051	43	-0.0023	36	213	20
	10	-0.2596	0.0671	350	1.1	-0.0044	60	-0.0009	52	-0.0158	34	233	20
	11	2.0160	-10.9838	250	1.1	-0.0300	71	0.0163	64	0.0236	37	275	20
	12	4.1896	-9.9906	225	0.7	0.0109	89	-0.0231	81	-0.0163	48	346	20
	13	4.2170	6.2133	250	1.8	-0.0002	76	0.0046	68	0.0095	42	296	20
	14	6.0135	-0.3713	275	0.7	0.0169	73	0.0120	62	-0.0109	43	275	19
	15	6.5691	2.4458	350	0.7	0.0137	66	-0.0054	48	0.0031	46	251	19
	16	6.8683	-7.6498	225	1.5	0.0002	76	-0.0131	66	-0.0108	43	293	20
	17	7.4907	-5.4445	225	1.5	0.0180	59	0.0212	47	0.0271	37	227	19
	18	8.1690	0.0605	275	1.1	0.0043	53	-0.0082	46	-0.0188	30	205	20
	19	9.7674	1.2014	200	0.3	0.0057	100	-0.0346	88	-0.0138	62	369	18
	20	11.4723	5.8121	250	1.1	-0.0161	88	-0.0005	76	0.0067	51	340	20
	21	12.5239	2.9244	300	0.3	-0.0158	66	0.0222	51	0.0050	44	255	20

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(*No. 6*) *G142-52:* Both the Yerkes and the astrometric reflector parallax, when combined with the photoelectric photometry of Priser (1970) place G142-52 1.65 mag below the main sequence; this star has no ultraviolet excess.

(*No. 7*) *G144-25:* The mean parallax in Table III when combined with the photometry of Priser (1970) places G144-25 0.53±0.15 mag below the main sequence after correcting for the ultraviolet excess. The spectral type is dM4–5 (Pesch, private communication).

(*No. 8*) *W1037:* This star is also known as LFT1718 and G18-51. A 100-inch reflector parallax by van Maanen (1923) of $+0\overset{''}{.}054$ indicated that this star was about 4 mag subluminous. However, since reference stars as far as 9 arc min from the optical axis were used, Strand (1962) considers it unreliable and it is therefore not included in Table III. The Yerkes parallax, when combined with the photometry of Eggen (1968) places W1037 2.2 mag below the main sequence after correcting for the ultraviolet excess. Eggen (1968) has measured the $R-I$ color index and finds it to be very peculiar, placing W1037 some 2 mag below the usual $R-I$ relation when the new parallax is used. The spectral type was determined by Joy (1947) to be sdK6.

VII. COMPARISON OF ERRORS WITH OTHER PUBLISHED MATERIAL

Since one of the principal goals of the parallax program was to try to increase the accuracy of trigonometric parallaxes, it is of interest to compare the errors obtained here with previous values derived with the Yerkes Observatory 40-inch refractor and with other current published material. The comparison may be made in three separate ways: first, from a direct comparison of the parallaxes; second, from a comparison of the unit weight errors of the parallax solutions; and third, from a comparison of the error of the parallax reduced to unit weight.

(i) Only four comparisons are available, hence the results may not be too reliable. The stars for which published parallaxes are available are: LP9-231, G138-25, G142-52, and G144-25. The ratios of $\pi(\mathrm{Yk})-\pi$ (mean published) to the vectorially added errors are respectively, 0.00, +0.44, +1.50, and +0.14. Except for the observation that the agreement is very good, no other conclusions are warranted due to the small number of stars in common.

(ii) The unit weight error of the parallax solution is a good measure of the average positional accuracy on the average plate. Before a comparison with other published material can be made, it is necessary to place the new Yerkes unit weight errors on the Sproul system defined by van de Kamp (1945). Unit weight for a new Yerkes error is defined by a single average quality (fair) image, while for the Sproul system unit weight is defined by two good images on one plate. The weight for a single fair image is $W=0.5$, therefore the Yerkes unit weight error must be scaled by $W^{\frac{1}{2}}=0.7$, yielding an average value of $\pm0\overset{''}{.}016$ m.e. for 142 reference-star parallaxes in Table IV. The corresponding average value for the last two series of Yerkes parallaxes (Strand and Hall 1951; Strand and Riddle 1969) is $\pm0\overset{''}{.}046$ m.e. for 49 parallaxes. A new Yerkes position is apparently 2.9 times as accurate as an old one of the same weight. The average unit weight error of the first 100 astrometric reflector parallaxes (Riddle 1970) is $\pm0\overset{''}{.}015$ m.e., which implies that a new Yerkes position is 0.94 times as accurate as one from the astrometric reflector.

(iii) Since we are concerned here primarily with the determination of parallaxes, the most relevant measure of the accuracy is the error of the parallax reduced to unit weight, which is defined as the error of the final parallax times $(N-M)^{\frac{1}{2}}$, where N is the number of plates and M is the number of unknowns used in the least-squares solution for the parallax. For the new Yerkes data, $M=5$, while for the older Yerkes data and for the astrometric reflector data, $M=3$. The average error of the parallax reduced to unit weight for 142 Yerkes reference-star parallaxes is $\pm0\overset{''}{.}017$ m.e., while that for the last two series of Yerkes parallaxes is $\pm0\overset{''}{.}066$ m.e. and that for the astrometric reflector is $\pm0\overset{''}{.}024$ m.e. A new Yerkes position derived for the purpose of determining a trigonometric parallax is apparently 3.9 times as accurate as an old Yerkes position and 1.41 times as accurate as a comparable astrometric reflector position of the same weight. The principal difference between the unit weight plate error and the unit weight parallax error is that the average parallax factor enters in the latter value, illustrating the importance of observing at the maximum parallax factor. These results are in good agreement with those found in a preliminary report on the Yerkes Observatory parallax program (van Altena 1971c).

VIII. THE EXTERNAL ACCURACY OF THE YERKES PARALLAXES

The reference-star relative parallaxes, proper motions, positions, image diameters, and colors are given in Table IV. In Table IV, the X, Y coordinates are aligned approximately with the Equatorial System and are in minutes of arc with the parallax star located at $X=Y=0.0$, the image diameters are given in microns, the color index in magnitudes, the mean errors in arc seconds per 10^4 yr, and the unit weight plate error in arc seconds per 10^5 yr. The column labeled *Im* gives the number of images from which the quantities in Table IV were determined. The purpose of listing these values is to provide a means of evaluating the external accuracy of the parallaxes. If all the reference stars were located at the same distance from the Sun, then the observed dispersion in the reference-star parallaxes

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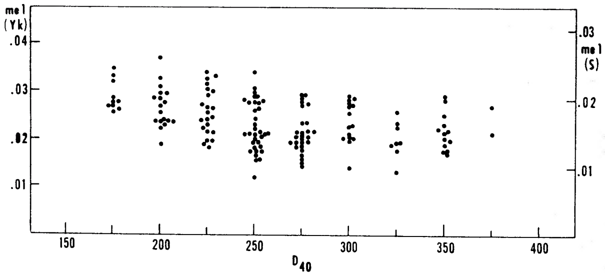

FIG. 2. A plot of the unit weight errors from the parallax solution as a function of the image diameter in microns as measured on the best plate of a given series. The *left-hand scale* gives the unit weight errors on the new Yerkes system, while the *right-hand scale* gives the corresponding unit weight errors on the Sproul system.

would give us the external accuracy of the parallaxes directly. There are, however, two effects which will increase the observed dispersion; the real distance distribution of the stars, and the corrections for differential color refraction, since the measured colors have mean errors of approximately 0.25 mag. These two effects can be easily evaluated. For the mid-galactic latitudes at an average reference-star magnitude of $V \sim 13.6$, I find a dispersion in relative parallaxes of $\pm 0\overset{''}{.}0022$ m.e., while for the color effect I find a dispersion of $\pm 0\overset{''}{.}0021$ m.e. The combined effect is $\pm 0\overset{''}{.}0030$ m.e. The dispersion of the 142 reference-star parallaxes is $\pm 0\overset{''}{.}0080$ m.e., which when decreased for the distance and color effects yields $\pm 0\overset{''}{.}0074$, while the average error of the relative parallaxes is $\pm 0\overset{''}{.}0054$ m.e. The ratio of the external error to the internal error is then 1.37. A second way of evaluating this ratio is to compare the average unit weight error of the plate-constant solutions with the average unit weight error of the parallax solutions. The average unit weight error (for one fair quality image on one plate) for the plate solutions in the x coordinate is $\pm 0\overset{''}{.}0294$ m.e., while that for the parallax solutions is $\pm 0\overset{''}{.}0234$ m.e. The ratio of the external to the internal error is 1.26, or when averaged with the previous result is 1.32. The implication is that my computed internal errors should be increased by about one-third to give external errors. This figure is not definitive, but probably lies between 25% and 50%.

In Fig. 2 I have plotted the unit weight error of the parallax solutions as a function of the image diameter, as measured usually on the best plate of each series. The errors are clearly a function of the brightness of the star on the plate. A diameter of 250 μ corresponds to a saturated image in average seeing of 2.5 sec of arc. The average unit weight error for 104 saturated images is $\pm 0\overset{''}{.}0215$ m.e., on the new Yerkes system, or $0\overset{''}{.}0150$ m.e. on the Sproul system (van de Kamp 1945).

IX. DISCUSSION

It was stated earlier that one of the major goals of the new Yerkes parallax program was to obtain large increases in the accuracy of trigonometric parallaxes. This has been achieved, but it is also clear that further substantial increases in the accuracy are still possible. An inspection of the residuals from the plate-constant and parallax solutions shows that our more recent plates are more accurate than the plates taken in 1966 and 1967 when the observing and developing techniques lacked homogeneity. There has also been a sharp increase in the quality of the plates since we began routine observations with the new automatic guiding camera in January 1971. The GG14 "A" filter discussed earlier undoubtedly introduces small local image position errors in addition to the large scale change. The maximum image position error introduced by the new filter in the automatic guiding camera is $\pm 0.2\,\mu$. While our inexpensive photoelectric image bisector has worked very well, the external accuracy of our measuring system is still about 40% poorer than that obtainable with current automatic measuring machines. In addition, Vasilevskis (1969) has shown, in a preliminary report on the Lick Observatory parallax program, that the Lick parallax error reduced to unit weight is about 30% smaller than that obtained here and about 2 times smaller than that obtained with the astrometric reflector. If further results substantiate the preliminary results, then this is a further indication that future Yerkes parallaxes can be made more accurate.

X. CONCLUSION

In conclusion, the major goals of this project have been largely satisfied although it is clear that there still remain many areas where significant improvements can be made. The proposals of Vasilevskis (1966), that a substantially larger number of reference stars be used in the least-squares solutions, that an explicit solution of the magnitude equation be made, that the average reference-star magnitude be made much fainter, and that observations be made off the meridian in order to maximize the parallax factor, have been successfully implemented on a conventional refractor with a long history of parallax work. When the above procedures are used along with photoelectric image bisection, the use of a hyponeutralizer, and a careful evaluation of the reference-star residuals, an increase in the accuracy of the parallaxes of a factor of 4 is obtained for a comparable amount of plate material from the same telescope but measured and reduced using older techniques.

The use of a large number of reference stars has also allowed us to investigate the external error of the parallaxes without the need for a direct comparison with parallaxes determined elsewhere. While the latter test is still the conclusive test, it seems unlikely that the external errors derived from the reference-star parallaxes will be significantly altered.

ACKNOWLEDGMENTS

I would like to thank Dr. S. Vasilevskis for many helpful discussions during the past years on the subject

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