Mon. Not. R. astr. Soc. (1970) **147**, 279–286.

GENERATION OF MAGNETIC FIELDS IN THE RADIATION ERA

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(Received 1969 October 7)

SUMMARY

It is shown that magnetic fields are generated during the radiation era of the early Universe in regions that have rotation. These fields are weak compared with the present intensity of the galactic magnetic field and therefore must be amplified as the Galaxy forms and evolves.

1. INTRODUCTION

The origin of galactic magnetic fields is an interesting and perplexing problem, as shown by Hoyle (1958). Our knowledge in this subject is so small that most work on the formation and evolution of galaxies either ignores or takes for granted the existence of widespread magnetic fields. One possible theory (Zel'dovich 1965; Thorne 1967) is that galactic fields are fossil remnants of a primordial field that originated with the Universe. This theory, however, evades the problem of origin at the cost of complicating the initial conditions in cosmology. Methods of generating seed fields have been proposed by Biermann (1950) and Browne (1968), but owing to the immense inductance of a galaxy the fields grow extremely slowly and remain relatively weak even after 10^{10} years (Harrison 1969).

In this discussion it is proposed that magnetic fields are generated in proto-galaxies*. We adopt the view that the 3 °K background radiation (Penzias & Wilson 1965; Dicke *et al.* 1965) is a relic of a hot and dense early Universe (Alpher & Herman 1948; Gamow 1948a,b). Also we assume that proto-galaxies already exist in the primordial fireball and possess angular momentum.† With these assumptions it can be shown that magnetic fields are generated in proto-galaxies during the radiation era. The generating mechanism depends on the transfer of angular momentum between the ion and electron–photon gases in an expanding radiation dominated plasma. Although the magnetic fields are weak this method of generation has the advantage that it is relatively rapid. Proto-galaxies therefore emerge from the radiation era already possessing large-scale seed fields. The amplification of these initially acquired fields occurs in the subsequent stages of galactic evolution, and this aspect of the subject will be discussed in a later paper.

An elementary description of the generating mechanism is the following. As a simple model of an eddy in the fireball we consider a spherical region of radius r, uniformly rotating, and containing radiation of uniform density ρ_γ, and

* 'Proto-galaxy' here means a rudimentary configuration in the early Universe that evolves eventually into a galaxy.

† The origin of the angular momentum is discussed elsewhere (Harrison 1970).

matter of uniform density ρ consisting of ions and non-relativistic electrons. As the eddy expands both ρr^3 and $\rho_\gamma r^4$ remain constant. Let ω and ω_γ be the angular velocities of matter and radiation; then in the absence of interactions the angular momenta proportional to $\rho\omega r^5$ and $\rho_\gamma \omega_\gamma r^5$ are separately conserved. Hence

$$\omega \propto r^{-2}, \qquad \omega_\gamma \propto r^{-1}, \tag{1a, b}$$

and as the eddy expands radiation spins down more slowly than matter. The next step is to restore the interactions. The Thomson cross-section is much larger for electrons than for ions and the electrons tend therefore to be dragged along by the photon gas. In the radiation era of $\rho_\gamma > \rho$ the electron–photon coupling is relatively tight and in effect there are only two fluids: a positively charged ion gas of density ρ and a negatively charged photon gas of density ρ_γ. The electron mass is small and the electron–photon gas will therefore still obey (1b). The difference in the angular velocities of the ion and electron–photon gases in an expanding eddy generates a magnetic field which is shown to be approximately

$$\mathbf{B} = -2(m_H c/e)\boldsymbol{\omega} = -2 \cdot 1 \times 10^{-4}\, \boldsymbol{\omega} \text{ Gauss}, \tag{2}$$

in the case of a hydrogen plasma. The resistive drag on the ion gas moving through the electron–photon gas is generally small compared with the effect of the induced electric field.

Our treatment supposes that ions are non-relativistic in order that ρr^3 is a constant. The temperature must therefore satisfy $kT < m_H c^2$. However when $kT \gtrsim m_e c^2$ (where m_e is the electron mass) there is copious lepton-pair production and the lepton–nucleon interactions effectively lock together the nucleon and lepton–photon gases. Thus the generation of a magnetic field begins at the commencement of the radiation era when the temperature drops below

$$m_e c^2/k = 6 \times 10^9\,°\mathrm{K}$$

and the electrons are therefore non-relativistic. The generation ceases when the radiation density is low enough for the electron–ion interactions to dominate over the electron–photon interactions. For our purpose this occurs at the end of the radiation era when $\rho \sim \rho_\gamma$ at a temperature $T \sim 3 \times 10^3\,°\mathrm{K}$; recombination is now appreciable (Peebles 1965, 1968; Weyman 1966; Novikov & Zel'dovich 1967), and the radiation decouples from the electrons. It turns out that the final intensity of the magnetic field is almost independent of when the radiation era ceases. In the temperature range $6 \times 10^9 > T > 3 \times 10^3\,°\mathrm{K}$ the densities are roughly $10^4 > \rho_\gamma > 10^{-20}$, $10^{-2} > \rho > 10^{-20}$ g cm^{-3}, and the age of the Universe is $1 > t > 10^{12}$ s.

The maximum field intensity in a proto-galaxy at the end of the radiation era is estimated as follows. The parameter

$$\chi_\gamma = 3\omega_\gamma^2 / 8\pi G \rho_\gamma, \tag{3}$$

is a measure of the anisotropy in the rotating region. Because $\omega_\gamma \propto r^{-1}$, $\rho_\gamma \propto r^{-4}$, it follows $\chi_\gamma \propto r^2$ and the anisotropy increases with expansion. But χ_γ cannot greatly exceed unity for otherwise the rotating region develops into a pancake configuration, and either breaks up or is slowed down by viscous damping in the fireball. We assume therefore that the maximum value χ_γ can attain is of order unity. The age of the Universe in its early stages is $t = (3/32\,\pi G \rho_\gamma)^{1/2}$, and hence $\omega_\gamma t = \frac{1}{2}\chi_\gamma^{1/2} \lesssim \frac{1}{2}$. At the end of the radiation era $t \sim 10^{12}$ s, and therefore $\omega \lesssim 10^{-12}\ \mathrm{s}^{-1}$ and the maximum field strength from (2) is $B \sim 10^{-16}$ Gauss.

In the following we derive the result (2) for isotropic expansion and show that it also holds for anisotropic expansion. The subject of viscous damping by photon diffusion is briefly discussed, and some comments are made on the problem of amplifying weak seed fields.

2. ISOTROPIC EXPANSION

We assume the rotating region is homogeneous (i.e., it satisfies the cosmological postulate), and consider a subregion sufficiently small that the expansion and rotation velocities are small compared with c. The radius of the sub-region is thus small in comparison with the Hubble distance. For the moment we assume the expansion is uniform.

In the weak field approximation the equation of motion of the ions is

$$\frac{d\mathbf{V}}{dt} = \alpha(\mathbf{E}+\mathbf{V}\times\mathbf{B}-\sigma^{-1}\mathbf{j})-\nabla\phi, \tag{4}$$

where $\alpha = e/mc$ and e and m are their charge and mass; $\mathbf{E}$ and $\mathbf{B}$ are the electromagnetic fields, $\mathbf{j}$ is the electric current, ϕ is the gravitational potential, and σ is the conductivity ($\alpha\sigma^{-1}\mathbf{j}$ is the rate of momentum transfer per unit mass to the electrons). Because of the assumed homogeneity there are no pressure gradients. For the velocity:

$$\mathbf{V} = \mathbf{u}+\mathbf{v}, \qquad \mathbf{u} = \mathbf{r}\dot{R}/R, \tag{5a, b}$$

where $\mathbf{u}$ is the radial velocity of uniform expansion, $\mathbf{v}$ the rotational velocity, $\mathbf{r}$ is the comoving position vector, and $R(t)$ is the expansion parameter (also $\dot{R} = dR/dt$). It follows

$$\nabla\times\mathbf{V} = \nabla\times\mathbf{v} = \boldsymbol{\zeta}, \tag{6}$$

where $\boldsymbol{\zeta}$ is the vorticity, and

$$\nabla.\mathbf{V} = \nabla.\mathbf{u} = -\dot{\rho}/\rho \tag{7}$$

for an ion density of ρ. With Maxwell's equations we find the curl of (4) is

$$\frac{d}{dt}\,[R^2(\boldsymbol{\zeta}+\alpha\mathbf{B})] = \frac{\alpha}{4\pi\sigma}\,R^2\nabla^2\mathbf{B}, \tag{8}$$

where $\nabla.\mathbf{u} = 3\dot{R}/R$, $\mathbf{a}.\nabla\mathbf{v} = 0$, $\mathbf{a}.\nabla\mathbf{u} = a\dot{R}/R$, and $\mathbf{a}$ stands for either $\boldsymbol{\zeta}$ or $\mathbf{B}$. In a radiation dominated plasma the electron mass is effectively $m\rho_\gamma/\rho$ and is larger than m, and the conductivity is therefore reduced by a factor $(m_e/m)^{1/2}$. In spite of the increased resistivity it can be shown that the ∇^2 term in (8) is negligible for proto-galaxies.* Hence, integrating (8) we obtain

$$\boldsymbol{\zeta}+\alpha\mathbf{B} = (R_1/R)^2\boldsymbol{\zeta}_1, \tag{9}$$

where $\boldsymbol{\zeta}_1$ is the initial vorticity at the commencement of the radiation era when the magnetic field is zero. This equation, with R constant, is the basis of the Einstein–de Haas effect (Einstein 1916; de Haas 1916).

* The decay of the field in a time $4\pi\sigma\lambda^2 \sim 10^{12}$ s is small for wavelengths λ embracing material masses greater than 10^{17} g.

The gravitational and inertial forces acting on the electrons are relatively small and for the electron gas

$$\mathbf{E}+\mathbf{V}_\gamma \times \mathbf{B} = \sigma^{-1}\mathbf{j}+(\alpha\rho)^{-1}\,\mathbf{P}_{e\gamma}, \tag{10}$$

where $\mathbf{P}_{e\gamma} = 4c\sigma_T\rho\rho_\gamma(\mathbf{v}_\gamma-\mathbf{v}_e)/3m_H$ is the rate of momentum transfer from photons to electrons per unit volume and σ_T is the Thomson cross-section (see Pauli 1958; Peebles 1967). We note that $P_{e\gamma}\sim\rho vR/\dot{R}$ from (8), (11) and (14) below with $\zeta R\sim\zeta_\gamma R\sim$ constant, and therefore

$$\frac{v_\gamma-v_e}{v}\sim\frac{m_H R}{\rho_\gamma\sigma_T c\dot{R}}\sim\frac{G^{1/2}m_H}{\rho_\gamma{}^{1/2}\sigma_T c}.$$

Thus $(v_\gamma-v_e)/v$ changes from 10^{-16} to 10^{-4} in the radiation era and justifies our use of V_γ in place of V_e in (10). From the curl of (10) we have

$$\alpha\rho d(R^2\mathbf{B})/dt = -R^2\nabla\times\mathbf{P}_{e\gamma}, \tag{11}$$

on neglecting the $\nabla^2 B$ term.

The electron–photon collisions ensure that the radiation field has fluid-like properties, and for non-relativistic fluid velocities the equation of motion of the radiation is

$$(\rho_\gamma+c^{-2}p_\gamma)\frac{d\mathbf{V}_\gamma}{dt}+c^{-2}\mathbf{V}_\gamma\frac{dp_\gamma}{dt} = -(\rho_\gamma+c^{-2}p_\gamma)\nabla\phi-\mathbf{P}_{e\gamma}, \tag{12}$$

and for the equation of continuity:

$$\nabla.\mathbf{V}_\gamma = \nabla.\mathbf{u} = -(\rho_\gamma+c^{-2}p_\gamma)^{-1}\,d\rho_\gamma/dt, \tag{13}$$

where $p_\gamma = \frac{1}{3}\rho_\gamma c^2$. Hence the curl of (12) gives

$$4\rho_\gamma\,d(R\boldsymbol{\zeta}_\gamma)/dt = -3R\nabla\times\mathbf{P}_{e\gamma} \tag{14}$$

for isotropic expansion, and $\rho_\gamma R^4$ = constant from (13).

By combining (8), (11) and (14) and integrating, we find

$$R^5(4\rho_\gamma\boldsymbol{\zeta}_\gamma+3\rho\boldsymbol{\zeta}) = R_1{}^5(4\rho_{\gamma 1}+3\rho_1)\,\boldsymbol{\zeta}_1. \tag{15}$$

Equations (9) and (15) determine the magnetic field. Since $\rho_{\gamma 1}\gg\rho_1$, it follows

$$\alpha\mathbf{B} = (R_1/R)\boldsymbol{\zeta}_\gamma-\boldsymbol{\zeta}, \tag{16}$$

and because $(4\pi\rho e/m_H c)(\boldsymbol{\zeta}_\gamma-\boldsymbol{\zeta}) = -\nabla^2\mathbf{B}$:

$$\alpha(\mathbf{B}-\mu_1{}^2\nabla_R{}^2\mathbf{B}) = -(1-R_1/R)\boldsymbol{\zeta}, \tag{17}$$

where $\mu = (m_H{}^2c^2/4\pi\rho e^2)^{1/2}\simeq 3\times10^{-5}\,\rho^{-1/2}$ cm is the collision-free penetration depth, and $\nabla_R{}^2 = (R/R_1)^2\nabla^2$ is the Laplacian in comoving coordinates. In our case the $\nabla_R{}^2\mathbf{B}$ term in (17) is negligibly small and therefore

$$\mathbf{B} = -(m_H c/e)(1-T/T_1)\boldsymbol{\zeta}, \tag{18}$$

since RT = constant.

At the end of the radiation era $T/T_1\sim10^{-6}$ and therefore proto-galaxies have magnetic fields given by (2). In the case of non-uniform rotation the additional terms in the equations, such as $\mathbf{B}.\nabla\mathbf{v}$, generate a toroidal field of $B_\phi\sim B\omega t\lesssim B$.

3. ANISOTROPIC EXPANSION

We assume uniform density and rotation and show that the magnetic field is unaffected by anisotropic expansion. Let $R_{\parallel}$, $R_{\perp}$ be the expansion parameters parallel, perpendicular to the angular velocity. The equation of continuity is

$$(\rho+pc^{-2})(\dot{R}_{\parallel}/R_{\parallel}+2\dot{R}_{\perp}/R_{\perp})+d\rho/dt = 0, \tag{19}$$

or $\rho R_{\parallel}R_{\perp}^2$, $\rho_\gamma(R_{\parallel}R_{\perp}^2)^{4/3}$ are constants. With $\mathbf{a}.\nabla\mathbf{V} = \mathbf{a}\dot{R}_{\parallel}/R_{\parallel}$, the ion vorticity equation (8) is now

$$d\,[R_{\perp}^2(\boldsymbol{\zeta}+\alpha\mathbf{B})]/dt = 0, \tag{20}$$

thus giving in place of (9)

$$\boldsymbol{\zeta}+\alpha\mathbf{B} = (R_{\perp 1}/R_{\perp})^2\,\boldsymbol{\zeta}_1. \tag{21}$$

Also, the curl of the electron equation (10) is

$$\alpha\rho d(R_{\perp}^2\mathbf{B})/dt = -R_{\perp}^2\nabla\times\mathbf{P}_{e\gamma}, \tag{22}$$

and the radiation vorticity equation becomes

$$4\rho_\gamma d(\boldsymbol{\zeta}_\gamma R_{\perp}^{4/3}/R_{\parallel}^{1/3})/dt = -3(R_{\perp}^{4/3}/R_{\parallel}^{1/3})\nabla\times\mathbf{P}_{e\gamma}. \tag{23}$$

Eliminating $\mathbf{P}_{e\gamma}$ from the last two equations and integrating, we find

$$R_{\parallel}R_{\perp}^4(4\rho_\gamma\boldsymbol{\zeta}_\gamma+3\rho\boldsymbol{\zeta}) = R_{\parallel 1}R_{\perp 1}^4(4\rho_{\gamma 1}+3\rho_1)\boldsymbol{\zeta}_1. \tag{24}$$

From (21) and (24) we obtain

$$\mathbf{B} = -(m_Hc/e)\boldsymbol{\zeta}[1-(R_{\parallel 1}R_{\perp 1}^2/R_{\parallel}R_{\perp}^2)^{1/3}]. \tag{25}$$

But because $(R_{\parallel}R_{\perp}^2)^{1/3}T$ = constant this last equation is identical with (18).

4. VISCOUS DAMPING AND THE BRAKING FIELD

Viscous damping, due to the diffusion of photons between the rotating and non-rotating regions, begins to be important as the radius of the rotating region approaches the Hubble distance. For simplicity, we shall consider a later stage when the radius is small compared with the Hubble distance; viscous damping is then more important and this is not therefore an unreasonable approximation. We assume also that the density is everywhere uniform.

Equation (12) must now include an additional term $\eta\nabla^2\mathbf{V}_\gamma$, where

$$\eta = 4\rho_\gamma m_Hc/15\rho\sigma_T$$

(Thomas 1930) is the radiative viscosity. The contribution of the material mass to the equation of motion is small and in place of (14) we have

$$d(R\boldsymbol{\zeta}_\gamma)/dt = (m_Hc/5\rho\sigma_T)\nabla^2R\boldsymbol{\zeta}_\gamma. \tag{26}$$

Let

$$\nabla_R{}^2\boldsymbol{\zeta}_k+k^2\,\boldsymbol{\zeta}_k = 0, \tag{27}$$

then from (26) and $R\propto t^{1/2}$, it follows

$$R\boldsymbol{\zeta}_k(\mathbf{r},t) = \mathrm{R}_c\boldsymbol{\zeta}_k(\mathbf{r})\,e^{-f(t)} \tag{28a}$$

where

$$f(t) = 2m_Hck^2t^{3/2}/15\rho_c\sigma_Tt_c^{1/2}, \tag{28b}$$

and t_c is the epoch at which radiation decouples. For the lowest mode the material mass is $M \sim 4\pi\rho_c/3k^3$, and this mode damps to e^{-1} when

$$M_D \sim \frac{4\pi}{3\rho_c^{1/2}} \left(\frac{2m_H c t_c}{15\sigma_T}\right)^{3/2} \sim 10^{11} M_\odot. \tag{29}$$

All masses greater than M_D survive unscathed but lesser masses have greatly diminished rotation. The value of M_D is similar to Silk's (1968) result for the damping of density inhomogeneities by photon diffusion.

Viscous dissipation does not entirely destroy the magnetic field. Suppose the vorticity is reduced to zero at some time $t_2 > t_1$. Then according to (9) the magnetic field is

$$\alpha \mathbf{B}_2 = (R_1/R_2)^2 \zeta_1. \tag{30a}$$

But $\mathbf{B}R^2$ is constant during the subsequent expansion, and so

$$\alpha \mathbf{B}_F = (R_1/R)^2 \zeta_1, \tag{30b}$$

where the subscript F denotes Field's (1968, private communication) 'braking field'. We have approximately $\zeta_1 R_1 = \zeta R$, and therefore

$$\mathbf{B}_F = -(T/T_1)\,\mathbf{B}, \tag{31}$$

where $\mathbf{B}$ is the field generated in the absence of viscous damping. The braking field in the damped region is therefore weaker, and of opposite sign, than in the undamped regions.

5. DISCUSSION

We notice that if the radiation era contains regions of matter and antimatter (Harrison 1967, 1968), which later evolve into galaxies and antigalaxies, the seed field—and presumably the final amplified field—is oppositely directed in the antigalaxies.

Furthermore, cosmological models possessing rotation must also generate cosmic magnetic fields. From the observed anisotropy (Partridge & Wilkinson 1967) of the microwave radiation, Hawking (1969) estimates that the present angular velocity of the Universe is at most $\omega \sim 10^{-3}\,(tz)^{-1} \sim 10^{-21}\,z^{-1}\,\mathrm{s}^{-1}$, where z is the redshift since the radiation was last scattered. Both B and ω now vary as R^{-2}, and therefore $B \sim 10^{-4}\,\omega$ Gauss from (2), or $B \sim 10^{-25}\,z^{-1}$ Gauss. The value of z depends on the thermal history of the intergalactic medium and lies in the range $10 < z < 10^3$, and the cosmic magnetic field is therefore extremely weak.

We have shown that proto-galaxies with rotation emerge from the radiation era with seed magnetic fields. These fields are weak and for a radiation era of moderately uniform density have a maximum value of 10^{-16} Gauss. The main question that arises is whether such a proto-galactic seed field can be amplified during the various stages of galactic evolution to a final intensity of a few micro-Gauss. According to the collapse model of Eggen, Lynden-Bell & Sandage (1962) and Partridge & Peebles (1967a, b) the proto-Galaxy continues to expand after the radiation era and then commences to collapse at roughly $t \sim 10^8$ year. The initial density is of order $10^{-20}\,\mathrm{g\,cm^{-3}}$ at the end of the radiation era and the final density of the Galaxy after collapse is of order $10^{-23}\,\mathrm{g\,cm^{-3}}$. The maximum intensity of the field therefore drops to 10^{-18} Gauss. If the field is squeezed entirely into the disc by gas settling out of the halo the intensity may be increased by as much as two orders of magnitude and we are back to 10^{-16} Gauss. The field is predominantly radial, and differential rotation will generate an azimuthal field of $B_\phi \sim 10^{-16}\,\Omega t$, where Ω is a typical angular velocity of the Galaxy. The maximum present azimuthal field strength is thus of order 10^{-14} Gauss, and falls a long way short of the required value.

We are forced to the conclusion that either proto-galaxies have fields of the order 10^{-8} Gauss at the end of the radiation era, and the seed field generated by the method we have described is unimportant, or the early stages of galaxy formation are fundamentally different from what has been previously supposed. Before reaching a decision the following points should be noted. The seed field generated by rotation in the radiation era, although weak, has the advantage that it is created rapidly by the control of the entire material angular momentum of the Galaxy, and it is difficult to see how a stronger proto-galactic seed field can be generated in the relatively short time available. Also, the direction of **B** given by (2) is such that differential rotation, with the inner regions spinning more rapidly, generates azimuthal components above and below the galactic plane in the general directions observed by Morris & Berge (1964).

It is possible that our present tentative views on the early stages of galaxy formation are in need of revision, and the amplification of the galactic field to its present intensity may serve as a vital clue. One possibility (Harrison 1970) is that proto-galaxies in the radiation era have densities much greater than the mean density. It can then be shown that the toroidal field is more intense and is sufficient to account for the present magnetic field in the galactic disc.

ACKNOWLEDGMENTS

This research was supported in part by the National Science Foundation.

I am greatly indebted to L. M. Ozernoi for drawing my attention to the work of Ozernoi & Chernin (1967, 1968a, b) on rotation and viscous damping in the radiation dominated era. The calculations in Section 4 on viscous damping are similar to those of Ozernoi and Chernin.

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