

CIRCULAR POLARIZATION OF THERMAL RADIATION IN A MAGNETIC FIELD*

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ABSTRACT

It is argued that light emission of electronic origin from any thermal source in a magnetic field is characterized by a spectrally diffuse circular polarization along the B field, even when the emission is devoid of spectral structure, i.e., is blackbody-like. A model "gray-body" radiating system in a B field is shown to emit with a fractional circular polarization given to first order in B by $q(\omega) \sim -(eB/m)/\omega$, where ω is the optical frequency. For $B = 10^6$ gauss, $q \sim 10^{-4}$ at visible wavelengths. The polarization is right-handed for B along the light k -vector. The effect has implications for the detection of magnetic fields in condensed stars (such as white dwarfs) and other stellar objects, and also for plasma and solid-state physics. Results of a preliminary laboratory experiment, which has verified the reality of the effect, are also summarized.

I. INTRODUCTION

The possibility has been raised that magnetic fields of 10^6 – 10^{16} gauss or more may exist in some white dwarfs and neutron stars (Woltjer 1964; Ostriker and Hartwick 1968). Since condensed stars tend to show little spectral structure, optical detection of the fields in such stars by discrete-line Zeeman structure (Babcock 1958) appears not to be feasible. This circumstance raises the question of whether a magnetic field imposes a more general feature on radiation from a star, or from any thermal light source, which persists even in the total absence of spectroscopic structure, in which case it should be generally detectable.

In this paper we will argue that a spectrally diffuse circular polarization must accompany any emission of electronic origin from a thermal source in a magnetic field. The mechanism is illustrated in terms of a model gray-body-like radiator, which by definition has a totally structureless spectrum, i.e., one of constant spectral emissivity. Departure from the constant-emissivity assumption in real bodies, as from spectroscopic structure, would impose an additional pattern of circular polarization which would add algebraically to the fundamental gray-body term. The effect predicted may in reality be more or less modified by internal radiative transfer, and by temperature and field gradients in a star. We will refer to a laboratory experiment which has shown the existence of the basic effect, revealing a circular-polarization fraction whose sign and approximate magnitude are in conformity with the simple theory.

The Zeeman components of a spectral line, radiated along the magnetic field direction, are circularly polarized. There are equally many left- and right-hand components. If the lines are severely and uniformly broadened, to widths $\Delta\omega = \Gamma$ much greater than the Zeeman frequency Ω , a single broad line results. It is well known that such a broadened Zeeman manifold retains a "hidden" fractional circular polarization $q(\omega)$, of magnitude $|\theta_{\max}| \sim \Omega/\Gamma$ and of a spectral pattern resembling the derivative of the broad line. This indeed underlies the magnetograph technique for extracting Zeeman splittings. If all other Zeeman-split transitions are similarly broadened, and if further the general broadening is carried to the point of extreme overlapping among all lines in the spectrum,

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we might guess the net polarization to cancel uniformly to zero—since the plus and minus peaks (right- and left-hand polarizations) of the various $q(\omega)$ patterns of unrelated transitions would overlap randomly. But this surmise ignores (a) the fact that all the *normal* Zeeman splittings (spin-independent) are of one sign, i.e., same wavelength ordering of left- and right-hand components, and (b) a subtle difference between the oscillator strengths of different Zeeman components, unimportant and unnoticed in ordinary spectroscopy. Both these features are contained in the model calculation that follows, in which a net polarization is shown to survive.

II. GRAY-BODY MODEL: HARMONIC OSCILLATORS

As the prototype radiating system we take a collection of electronic harmonic oscillators. The number of oscillators per unit frequency range $\Delta\omega$ is taken as constant. Such a system has the special property of *constant spectral emissivity* $e(\omega)$, meaning the following: Suppose all the oscillators are sustained in thermal equilibrium at uniform temperature T , through nonradiative interactions with a thermostat (which, for example, might be the store of mechanical kinetic energy of the atoms in a gas) (see Fig. 1). We assume

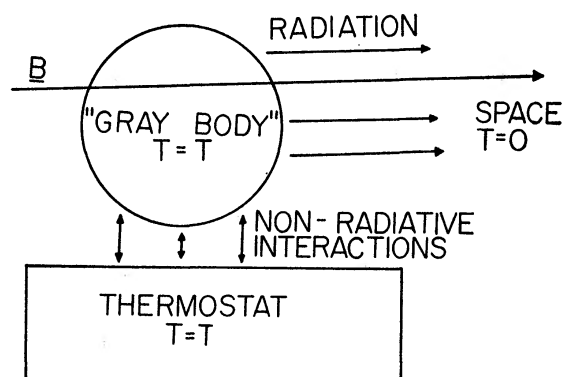


FIG. 1.—Gray-body model for the magneto emission discussed in the text of §§ II and III

the oscillators to radiate into cold space, a nonequilibrium process. We further suppose the mean free path for an emitted photon to be large compared with the dimensions of the system, so that photon exchange is unimportant, and that the oscillators radiate independently. Now the radiation curve from this system, a “gray body,” happens to be identical with the Planck law:

$$P(\omega) = \frac{A\omega^3}{e^{\hbar\omega/kT} - 1}. \quad (1)$$

This feature is in a sense accidental, since the Planck law itself is an equilibrium property of a photon gas and has no connection with the properties of matter. The gray body has also the complementary special feature of constant spectral absorptivity: an absorption spectrum taken through a section of the material is merely a horizontal straight line. *The gray body is thus a point of reference for discussing the “hidden” Zeeman structure at issue here.*

A remark is demanded about our omission of internal radiative transfer. Real bodies of interest are more often quite opaque. However, the net emission from a surface layer is at least partly a direct emission from the oscillators into the surrounding cold space, and to this extent the external radiation reflects properties of the material oscillators. Further comment on this question will be made in § IV.

To discuss the effect of a uniform magnetic field B , say along a z -axis, we may use

three-dimensional harmonic-oscillator wave functions in the spherical-coordinate representation (n, l, m) , having orientational degeneracies $(2l + 1)$; or alternatively we may use cylindrical-coordinate functions. The Hamiltonian (without radiation-field terms) is

$$\mathfrak{H} = \frac{1}{2m} (\mathbf{P} - e\mathbf{A})^2 + \frac{1}{2}kr^2. \quad (2)$$

In the gauge $\mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B}$, the magnetic terms of equation (2) are the $-\mathbf{u} \cdot \mathbf{B}$ term $-(e/m)\mathbf{p} \cdot \mathbf{A} = -(eB/2m)L_z$; and $(e^2B^2/8m)\rho^2$. Since $r^2 = \rho^2 + z^2$, the wave equation separates in cylindrical coordinates and is exactly soluble to all orders in B . The solutions are familiar in other contexts, e.g., in a model for diamagnetism of metals (Van Vleck 1932, p. 353). Thermal-statistical aspects are then dealt with by taking sums over these states. In our context, optical emission, we must take sums in which the selection rules $\Delta m = +1$ and -1 are included.

For a simpler physical picture, rather than do the complete quantum-mechanical calculation we will break the problem into limiting regions (a) quantum and (b) classical cases, in which $\hbar\omega \gg kT$ and $\hbar\omega \ll kT$, respectively. In region *a*, the sum over states is trivial, while in region *b* we have classical statistical mechanics. In region *a* we discuss only effects first order in B , appropriate except in the extreme situation $\mu B \gg kT$. In region *b*, it is easy to give the exact result, and to show how the polarization behaves down to $\omega = 0$ in the model.

Since the term $-(e/2m)\mathbf{p} \cdot \mathbf{A} = -\mathbf{u} \cdot \mathbf{B}$ in equation (2) symmetrically Zeeman-splits all states (nl) into $(2l + 1)$ equally spaced levels, it also symmetrically splits the spectroscopic transition frequencies. Therefore, to first order in B , the number of radiative transitions per unit frequency interval $dn/d\omega$ (density of equivalent classical oscillators) *remains constant* and independent of both ω and B . In higher order this is not so, as will be seen in § IIb.

The Zeeman frequency will be denoted $\Omega \equiv eB/(2m)$.

a) Quantum Limit

Here equation (1) becomes

$$P(\omega) \simeq A'\omega^3 e^{-\hbar\omega/kT}. \quad (3)$$

Significant radiation comes only from the transition from the first excited state, a *p*-state, to the ground *s*-state. The ($m = \pm 1$) *p*-states radiate along B (*z*-axis). At any ω , radiation is due to two transitions, one each from those two oscillators which with $B = 0$ had frequencies $\omega - \Omega$ and $\omega + \Omega$; these emit right- and left-hand circular components, respectively (see Fig. 2). To keep track of signs we work with oscillating positrons $e = +|e|$; and right-hand means rotating from $+x$ toward $+y$.

The spontaneous emission is equivalent to stimulated emission due to the zero-point electromagnetic fields; a Golden Rule gives the rate for electric-dipole radiation by a single oscillator, as:

$$p(\omega) = (\hbar\nu)w = (\hbar\nu) \frac{2\pi}{\hbar} |\langle u | e\mathbf{E} \cdot \mathbf{r} | l \rangle|^2 \rho(\hbar\omega) N_u, \quad (4a)$$

where u and l are the initial (upper) and final (lower) oscillator states and N_u is the upper-state population. Since $\langle E^2 \rangle \propto \frac{1}{2}\hbar\omega$ (energy of one radiation-field normal mode) and $\rho(\hbar\omega) \propto \omega^2$ (density of field modes), we have

$$p(\omega) = a\omega^4 |\langle u | x_a | l \rangle|^2 N_u, \quad (4b)$$

where x_β means x, y , etc. In our case, as shown in more detail in the Appendix,

$$p_\pm(\omega) = a\omega^4 |\langle n, 1, \pm 1 | x \mp iy | 000 \rangle|^2 e^{-\hbar\omega/kT}. \quad (5)$$

Here $\hbar\omega$ is the transition energy, in the presence of B , of some oscillator from the $m = +1$ or -1 excited state to the ground s -state. Such an oscillator has intrinsic frequency (with $B = 0$) $\omega + \Omega$ or $\omega - \Omega$ in the respective cases. Now since the B field does not change the wave function in first order, the squared matrix element is merely a constant times $(\omega \pm \Omega)^{-1}$; thus, to this order,

$$p_{\pm}(\omega) = \frac{a'\omega^4}{\omega \pm \Omega} e^{-\hbar\omega/kT} \simeq p(\omega) \left(1 \mp \frac{\Omega}{\omega}\right). \quad (6)$$

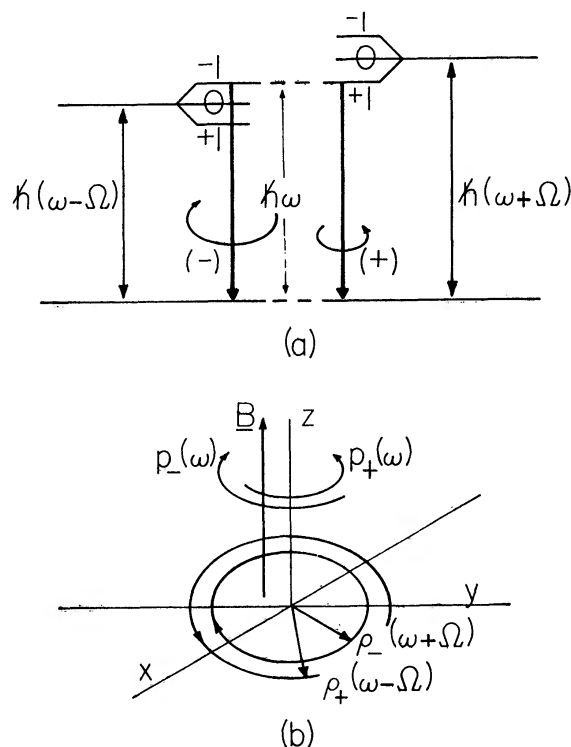


FIG. 2.—Scheme for the computations of § II. In (a), the quantum limit, we show the ground s -state and first excited p -state of two of the isotropic harmonic oscillators in the gray-body distribution, of frequencies (with $B = 0$) $\omega - \Omega$ and $\omega + \Omega$, respectively. In (b), the classical limit, we show the two circularly polarized components of the general orbit $r(t)$ of a harmonic-oscillator particle, in presence of a B field, projected in the (x, y) -plane. (The particle is a positron, for the sake of sign conventions.)

It follows that $P(\omega)$ splits into two branches, with right- and left-hand (+ and -) circular polarizations, respectively. The net fractional polarization is

$$q(\omega) \equiv \frac{P_+(\omega) - P_-(\omega)}{P_+(\omega) + P_-(\omega)} \simeq -\frac{\Omega}{\omega} = -\frac{eB}{2m\omega}. \quad (7)$$

This is left-handed for positive e , thus right-handed for electrons.

The mechanism behind this simple effect is the “rigidity” of the eigenfunctions against magnetic perturbation. Only a Zeeman precession of the wave functions about z occurs. The total oscillator strength of the ± 1 doublet is conserved but is transferred unequally to the Zeeman components: The frequency of the $(-)$ one goes up, but its matrix element remains constant; thus its oscillator strength (proportional in general to $|\langle u|x|l \rangle|^2 \omega$, where ω is the actual emitting frequency) is relatively enhanced.

b) *Classical Limit*

In this limit equation (1) is

$$P(\omega) \cong A'' \omega^2 kT. \quad (8)$$

Since $\mathbf{p} = m\mathbf{v} + e\mathbf{A}$, the Hamiltonian (eq. [2]) can be written as

$$\mathfrak{H} = \frac{1}{2}K\mathbf{r}^2 + \frac{1}{2}m\mathbf{v}^2 \equiv \mathfrak{H}^*(\mathbf{r}, \mathbf{v}). \quad (2a)$$

We call this $\mathfrak{H}^*$, following Van Vleck (1932, p. 97), remembering that $m\mathbf{v}$ is not the canonical momentum and does not account for the magnetic force. But it correctly gives the total energy of the oscillator for any $(\mathbf{r}, \mathbf{v})$, that energy available for mechanical interaction (e.g., by collisions) with the thermostat. Though the canonical ensemble density $\exp(\mathfrak{H}^*/kT)$ in $(\mathbf{r}, \mathbf{v})$ -space is independent of B to all orders, as is evident, averages involving *accelerations* may depend on B , since these involve the Lorentz force. Thus the character of the radiation (proportional to the acceleration squared) may be influenced by B .

The equation of motion $m\partial^2\mathbf{r}/\partial t^2 = -k\mathbf{r} + e(\partial\mathbf{r}/\partial t) \times \mathbf{B}$ is elementary. The z -part (for B along z) is disposed of by writing $\mathbf{r}(t) = z(t)\mathbf{k}_1 + \boldsymbol{\varrho}(t)$, where $\mathbf{k}_1$ is a unit vector along z and $\boldsymbol{\varrho}(t)$ is in the (x, y) -plane. With $\omega^2 = K/m$ and (as before) $\Omega = eB/2m$, we have

$$d^2\boldsymbol{\varrho}/dt^2 = -\omega^2\boldsymbol{\varrho} + 2\Omega d\boldsymbol{\varrho}/dt \times \mathbf{k}_1.$$

This has two complementary solutions

$$\boldsymbol{\varrho}_{\pm}(t) = \rho_{\pm}[i \cos(\omega_{\mp}t + \phi_{\pm}) + j \sin(\omega_{\mp}t + \phi_{\pm})]$$

which are right- and left-hand *circular* orbits, of constant radii $\rho_{\pm}$, respectively. The frequencies are (the $+$ orbit having lower frequency since we assume $e = +|e|$):

$$\omega_{\mp} = (\omega^2 + \Omega^2)^{1/2} \mp \Omega. \quad (9)$$

Generally $\boldsymbol{\varrho}(t) = \boldsymbol{\varrho}_+(t) + \boldsymbol{\varrho}_-(t)$; this is an ellipse Larmor-precessing about z . Since equation (2a) splits into z and xy parts, equipartition of thermal energy gives $\langle \mathfrak{H}_{xy}^* \rangle = 2kT$. This may further be partitioned: Noting that $d\boldsymbol{\varrho}_{\pm}/dt = \pm\omega_{\mp}\mathbf{k}_1 \times \boldsymbol{\varrho}_{\pm}$, we can easily show that

$$\mathfrak{H}_{xy}^* = \frac{1}{2}m(\omega^2 + \omega_-^2)\rho_+^2 + \frac{1}{2}m(\omega^2 + \omega_+^2)\rho_-^2 \equiv \mathfrak{H}^{**}(\rho_+, \rho_-, \theta_+, \theta_-). \quad (10)$$

We have made a second coordinate transformation, i.e., $(x, y, dx/dt, dy/dt) \rightarrow (\rho_+, \rho_-, \theta_+, \theta_-)$; the $\theta_{\pm}$ are the circular-orbit phase angles $(\omega_{\mp}t + \phi_{\pm})$. (The Jacobian of this transformation is just $4\omega^2\rho_+\rho_-$.) From the structure of $\mathfrak{H}_{xy}^*$ (separate ρ_+ and ρ_- terms) it follows that

$$(\omega^2 + \omega_-^2)\langle \rho_+^2 \rangle = (\omega^2 + \omega_+^2)\langle \rho_-^2 \rangle = 2kT/m. \quad (11)$$

Now, electric-dipole emission is proportional to $a^2\omega^4$, where a is the oscillator amplitude. For the right-hand circularly polarized radiation due to the ρ_+ component of an oscillator of “intrinsic” (i.e., $[K/m]^{1/2}$) frequency ω we have, by using equation (11),

$$p_+(\omega_-) = a\langle \rho_+^2 \rangle \omega_-^4 = a \frac{2kT}{m} \frac{\omega_-^4}{\omega^2 + \omega_-^2}.$$

Now using $\omega^2 = \omega_-^2 + 2\Omega\omega_-$ from equation (9), and finally changing the designations by replacing ω_- with ω , we obtain:

$$p_+(\omega) = a \frac{kT}{m} \frac{\omega^3}{\omega + \Omega} = p(\omega) \frac{\omega}{\omega + \Omega}. \quad (12a)$$

This (+) emission of frequency ω is actually from an oscillator with $B = 0$ frequency equal to $(\omega^2 + 2\Omega\omega)^{1/2}$.

Analogously,

$$p_-(\omega) = p(\omega)\omega/(\omega - \Omega). \quad (12b)$$

Since to first order in $B \propto \Omega$ the spectral density of both (+) and (−) oscillating orbits is constant, we have a fractional circular polarization

$$q(\omega) \cong -\Omega/\omega \quad (|\Omega/\omega| \ll 1), \quad (13)$$

identical with the quantum result (7).

To find the exact classical result when Ω/ω is not small, we note that the net polarization is not determined strictly by comparing the strengths $p_{\pm}(\omega)$ of single-oscillator emissions at ω . Changes in the densities $dn_{\pm}/d\omega$ must be included, i.e., in the effective numbers of oscillators emitting (+) and (−) radiation, respectively, within unit $\Delta\omega$ of ω . From equation (9), notice that in the interval $0 \leq \omega < 2\Omega$ there are no ω_+ values at all, and therefore no ρ_- orbits ($dn_-/d\omega = 0$) in that region, wherein simply $q(\omega) = 1$. To compute $dn_{\pm}/d\omega$ we use the device of writing $\omega(n) = n\delta$, where δ is an infinitesimal fixed interval and $\omega(n)$ is the intrinsic frequency $(K_n/m)^{1/2}$ of the n th oscillator. To find $dn_{\pm}/d\omega$, we have, from equation (9), $[\omega(n)]^2 = \omega_n^2 - 2\Omega\omega(n) = n^2\delta^2 - 2\Omega\omega$, or $n_{\pm}(\omega) = (\omega^2 \pm 2\Omega\omega)^{1/2}/\delta$, from which:

$$\frac{dn_+}{d\omega} = \frac{\omega + \Omega}{(\omega^2 + 2\Omega\omega)^{1/2}}; \quad \text{similarly} \quad \frac{dn_-}{d\omega} = \frac{\omega - \Omega}{(\omega^2 - 2\Omega\omega)^{1/2}}, \quad (14)$$

the latter being valid only for $\omega > 2\Omega$, since $dn_-/d\omega = 0$ for $0 < \omega \leq 2\Omega$.

The net radiation of either type is then

$$P_{\pm}(\omega) = p_{\pm}(\omega)dn_{\pm}/d\omega. \quad (15)$$

The polarization may now be computed from equations (12a), (12b), and (14); and we find after some algebra:

$$q(\omega) = \frac{P_+(\omega) - P_-(\omega)}{P_+(\omega) + P_-(\omega)} = (s^2 - 1)^{1/2} - s \quad (s \geq 1), \quad (16)$$

$$q(\omega) = +1 \quad (0 < s < 1),$$

where $s = \omega/(2\Omega)$. When $s \gg 1$, $q(s) \cong -1/(2s) = -\Omega/\omega$, agreeing with equation (13) as well as equation (7).

The magnetic circular polarization of our “gray-body” emitter composed of electronic harmonic oscillators is displayed in Figure 3.

III. FREE-ELECTRON (BREMSSTRAHLUNG) MODELS

Any radiative or absorptive effect present in a harmonic-oscillator model must have a more general counterpart. In fact, the first-order result $q(\omega) \cong -\Omega/\omega$ comes out of at least one other model of an independent-electron radiator except for small positive-integer factors, as we now show.

Bremsstrahlung is significant for thermal radiation from an ionized gas or plasma, and from incandescent solids, particularly metals. We deal with semielastic electron-atom collisions—i.e., unbound- to unbound-state collisions—and only in the classical and nonrelativistic limits. In a weak B field, the case becomes that of magnetically perturbed bremsstrahlung. (This is distinct from synchrotron and cyclotron radiation; in those, the acceleration causing emission is due to the Lorentz force itself, collisions being unimportant.) The cyclotron period π/Ω is taken as long compared with the mean intercollision interval τ ; the model is illustrated in Figure 4.

Without a B field, the classical path $\mathbf{r}(t)$ of an electron is a random walk with constant velocity between collisions (see Fig. 4, *a*). For our purposes, a useful way to describe the radiation is by Fourier-analyzing $\mathbf{r}(t)$:

$$\mathbf{r}(t) = \sum_{\omega} (ix_{\omega} + jy_{\omega})e^{i\omega t} + kz(t). \quad (17)$$

The x_{ω} , y_{ω} are effective electric dipoles radiating at frequency ω , and the spectrum depends on the free-path and velocity distributions.

Imposing a B field does not change the electron thermal-velocity distribution in an $(\mathbf{r}, \mathbf{v})$ -ensemble space, as noted in § I*b* above; also, to first order the mean free path is not affected by B . In an $x'y'$ frame rotating at the cyclotron frequency $\Omega' = 2\Omega = eB/m$ about z , as in Figure 4, *b*, the ensemble $\mathbf{r}(t)$ with B present is then the *same* as in the stationary frame with $B = 0$. The expansion (17) is the same in the rotating frame with B applied, but with i, j replaced by rotating vectors i', j' . A representative linear dipole in the rotating frame, e.g., $i'r_{\omega} \cos(\omega t)$, where $r_{\omega} \sim x_{\omega}, y_{\omega}$, decomposes in the stationary frame into two contrarotating circular components radiating at frequencies $\omega \pm \Omega'$, respectively. These components are of equal amplitudes; *but they have different emissive*

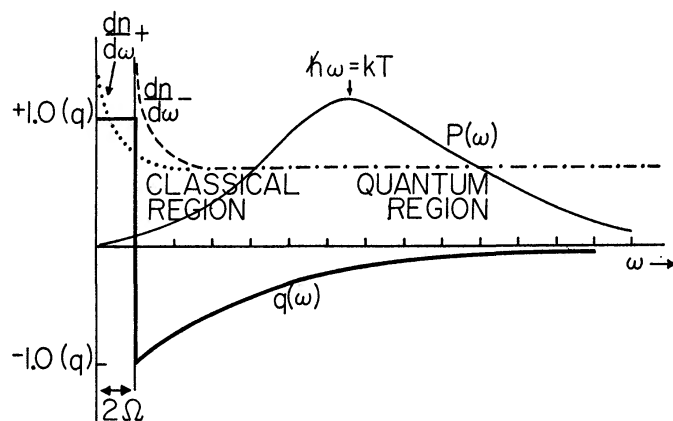


FIG. 3.—The magnetoemission $q(\omega)$, i.e., the fractional circular polarization, from the model of § II of the text. Shown also are the effective spectral emissivities $dn_{+}/d\omega$ and $dn_{-}/d\omega$ for right- and left-circularly polarized emission; and the gray-body (Planck-like) radiation curve $P(\omega)$. The singular behavior of $q(\omega)$ near $\omega = 0$ probably has only qualitative resemblance to the magnetoemission at low frequencies (e.g., radio emission) from real bodies.

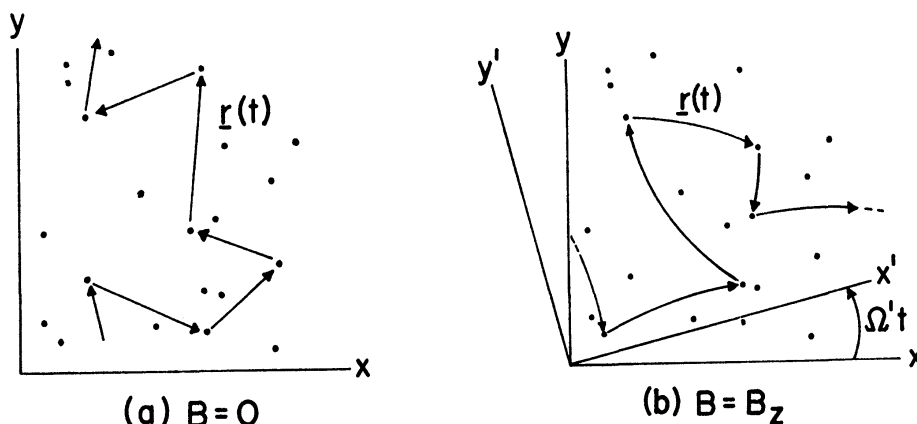


FIG. 4.—Free-electron model of § III of the text—bremsstrahlung in the presence of a weak magnetic field.

strengths, since the latter are now proportional to $(\omega \pm \Omega')^4$, respectively, in accord with the general ω^4 dependence of the power radiated by an electric dipole.

The velocity component $v_x(t) = dx(t)/dt$ is merely a sequence of statistically independent rectangular pulses of widths $\sim \tau$; the power spectrum of this is $\langle |V_x(\omega)|^2 \rangle \propto 1/[1 + (\omega\tau)^2]$. The power spectrum of the equivalent dipole moments $x_\omega = V_x(\omega)/(i\omega)$ is therefore

$$\langle |r_\omega|^2 \rangle \propto \frac{1}{\omega^2(1 + \omega^2\tau^2)}.$$

The power in the two circularly polarized emissions seen in the stationary frame is then

$$p_\pm(\omega) \propto r_{\omega \pm \Omega'}^2 \omega^4 \propto \frac{\omega^4}{(\omega \pm \Omega')^2[1 + (\omega \pm \Omega')^2\tau^2]}; \quad (18)$$

and to first order the fractional circular polarization is

$$q(\omega) = -\frac{2\Omega'}{\omega} \frac{1 + 2\omega^2\tau^2}{1 + \omega^2\tau^2}. \quad (19)$$

There are two cases, viz., (a) $\omega\tau \gg 1$, as for optical-frequency emission from a dilute ionized gas, and (b) $\omega\tau \ll 1$, pertaining, for example, to radiation from incandescent metals, in the radiofrequency to infrared range. In case (a) the spectrum is not "gray" but is frequency-independent, as is well known and as is seen in equation (18) (with Ω' set to 0). In case (b), $p(\omega) \propto \omega^2$, and the emission is in fact gray, as in equation (8).

In any event, $q(\omega)$ in equation (19) is similar to the harmonic-oscillator result; but it comes out 4 and 8 times larger in the two cases (b) and (a), respectively. These factors reflect in part the ratio 2 between the cyclotron and Larmor frequencies ($\Omega' = 2\Omega$). They also are related to the lack of a potential term in the statistical-mechanical partition of thermal energy in free-electron systems; compare with the development in § IIb.

IV. OPAQUE EMITTERS AND RADIATIVE EQUILIBRIUM

The above models apply literally only to an optically thin body—a case of somewhat restricted interest. To discuss magnetoemission from opaque objects, large in extent compared with photon mean free paths, we identify two situations.

a) *Thermal-Equilibrium Models*

In these models the body is maintained throughout in thermal equilibrium, e.g., at a single uniform temperature, by nonradiative processes (conduction, convection), and the surface radiates into surrounding cold space. The character of emission from a homogeneous, uniform-temperature body can be deduced from detailed balance. The medium properties are defined through a complex refractive index $n(\omega) = n_1(\omega) + in_2(\omega)$. In a B field, $n(\omega)$ becomes an axial tensor; but right- and left-circularly polarized radiations propagate along B , specifically, in terms of scalars $n_\pm(\omega, B)$. Now if the body is very dilute, such that $|n - 1| \ll 1$ for all ω , then the surface reflectivity vanishes; the body is a black absorber, thus also a black emitter. All spectroscopic and polarization features, including those associated with a B field, must vanish, to order $|n - 1|$. On the other hand, if the body is dense, such that $|n - 1|$ is appreciable, then surface reflectivity is finite and the spectral emissivities $e(\omega)$ or $e_\pm(\omega)$ depend on $n(\omega)$ or $n_\pm(\omega, B)$. In this case $e(\omega)$ is always less than unity, but it may be constant (gray body). The magnetoemission fraction $q(\omega)$ is then essentially the same as that of a thin section of the medium, reduced at most by a fraction of the order of unity. This case is exemplified by incandescent solid metals (Kemp, Swedlund, and Evans 1970). In other language, the condition for appreciable q can be written $\omega < \omega_p$, where ω_p is the plasma frequency, $\omega_p^2 = Ne^2/m\epsilon_0$, N being the electron density.

In some white-dwarf stars, it is speculated that surface densities may be as high as one or two orders of magnitude less than in terrestrial solids (Van Horn 1970), with presumably a rather well-defined boundary. Magnetoemission in that event would necessarily exist (given the B fields), since ω_p would be as high as a visible-light frequency.

b) Energy-Source Models

In these models there is an energy source within the body, and there is a spatial temperature gradient controlled by radiative transfer—e.g., a radial gradient as in a star. If the surface region of the body is heated only by radiative transfer from inside, conclusions drawn from thermal-equilibrium models as in § IVa are not trustworthy. We do not as yet have a model for magnetoemission in this case, but we can make some qualitative remarks. Thinking in terms of a B field locally normal to the surface, we can distinguish between transverse interactions, within a surface layer, versus radial interactions. Due to the intrinsic magnetoemissivity of the medium, radiation from any hot inner region must always have a net helicity. This tends to be canceled by the converse magnetoabsorption of cool outer layers. But the magnetoemission carries a net angular-momentum flux; the integral of $p(\omega)\hbar q(\omega)$ for gray radiation does not vanish, as is easily shown. For identical cancellation of the net surface magnetoemission in the steady state, outer layers would have to absorb this angular-momentum flux continuously—a process requiring a backward transfer of angular momentum, between layers, by *nonradiative* processes (convection, etc.). Barring perfect cancellation, the magnetic sublevels of outer layers take on non-Boltzmann steady-state populations, compensating for the magnetoabsorption of the cool layers in such a way that the net emission has a circular polarization. Yet with regard to frequency, rather than polarization, radiative exchange can still result in a net blackbody emission spectrum, since angular momentum is not involved in equilibration of the energy levels if the magnetic splittings are ignored. It would thus appear that under certain conditions a genuinely “blackbody” magnetoemission is possible.

V. EXPERIMENTAL AND OBSERVATIONAL ASPECTS

A laboratory verification of the reality of this magnetoemission, which has implications for solid-state physics and other fields as well as for astrophysics, has been made (Kemp, Swedlund, and Evans 1970). I refer to that paper for details. Briefly, the experiment consisted of placing various thermal light sources in a laboratory magnet field, and looking for partial circular polarization in light emitted along B . For $B = 10^4$ gauss, the expected gray body q is some few parts in 10^6 for visible light; although in certain related work (referenced in the paper) on magnetorefectance in metals, ellipticities down to 10^{-7} have been measured. Three types of objects were studied: incandescent metals, solid insulators, and an oxyacetylene flame, variously at temperatures 1000° – 1500° C. Detection was at $1.5\ \mu\text{m}$ in the near-infrared, also in the visible in the flame. All metals showed magnetoemissive q -values essentially as predicted in the present paper, though rather smaller. Insulators and oxide layers gave little or no signals—an interesting null result which says something about the nature of incandescence in such solids (see paper). Signals from the flame, which incidentally approximates the “optically thin” case, were of the magnitude of the gray-body q but negative in sign, interpreted as due to a broad and predominant “spectroscopic” contribution from the C_2 bands, proportional to the derivative of the electronic emissivity.

An astronomical search for broad-band magnetoemission is in progress at the University of Oregon's Pine Mountain Observatory, using the 24-inch reflector. The instrumentation is based on a photoelastic modulator, analogous to the arrangement in the above laboratory experiments except that observation so far has been done only in the visible (~ 4000 – $6000\ \text{\AA}$) with a photomultiplier. In brighter objects, apparent magnitude less than 10, an estimated q down to 10^{-4} and probably 10^{-5} can be detected,

without undue spurious effects. The main project is a survey of selected white-dwarf stars from the tabulation of Eggen and Greenstein (1965). The search for strong B fields in the DA dwarfs, in particular, has been somewhat narrowed by recent null results from H-line spectroscopic studies (Angel and Landstreet 1970; Preston 1970), indicating no cases of $B \geq 10^6$ gauss in the stars covered. Our own work on the DA star W1346 shows that any broad-band q (~ 5000 Å) is surely less than 2×10^{-3} . We concentrate now on the DC dwarfs (no spectral lines) and on one or two with only very broad lines or bands; these are perhaps more intriguing for several reasons. Though stars of this type accessible to us are faint ($m \geq 13$), long-time signal integration may permit us to discern $q \gtrsim 10^{-4}$. If we allow for the possibility (see Angel and Landstreet 1970) that the long-time average fields may be negligible owing to rapid rotation, periods as short as minutes must be searched for by statistical methods.

Simpler programs are to look for broad-band magnetoemission in magnetic Ap stars and in sunspots. This should corroborate existing magnetograph data, but it might well provide some new information. In view of the ω^{-1} dependence of the gray body q , infrared observations in these cases would be most interesting. The writer is designing circular polarization apparatus for the 4- and 10- μ m regions. One may speculate that the effective B fields seen in such measurements will differ from those causing narrow-line Zeeman splitting. For example, the infrared emission could come partly from deeper surface layers, where B might be larger. Such extension is not possible with the magnetograph method, which is confined to spectral regions (and to objects) with favorable spectral lines.

Note added in proof.—Recently, strong circular polarization has been detected in the optical emission of a white-dwarf star. Preliminary wavelength dependence of the polarization is suggestive of the $-\Omega/\omega$ gray-body magnetoemission formula, though perhaps modified by other structure (see Kemp *et al.*, 1970, *Ap. J. [Letters]*, **161**, L77; and papers in preparation).

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APPENDIX

ELECTRIC-DIPOLE TRANSITIONS IN A B FIELD

That the electric-dipole transition moment in the presence of a B field is just the matrix element of $(x \pm iy)$ is a key point in § IIa and needs elaboration. Consider the Hamiltonian

$$\mathfrak{H} = V(r) + \frac{1}{2m} (p - eA_0 - eA_1)^2,$$

where A_0 and A_1 are vector potentials of the B field and the radiation field, respectively, and can be taken as

$$A_0 = -\frac{1}{2}\mathbf{r} \times \mathbf{B}, \quad A_1 = \frac{E}{\omega} [(i + ij)e^{-i\omega t} + \text{compl. conj.}].$$

Here A_1 represents, specifically, a right-hand circularly polarized wave with electric field E . For the selection rule $\Delta M = +1$, only the $e^{-i\omega t}$ part of A_1 causes transitions; if only this part is kept, the Hamiltonian to first order in B is

$$\mathfrak{H} = \underbrace{V(r) + \frac{1}{2m} p^2 - \Omega L_z}_{\mathfrak{H}_0} - \underbrace{\frac{eE}{\omega} \left[\frac{1}{m} (p_x + ip_y) - \Omega(x + iy) \right]}_{\mathfrak{H}_1} e^{-i\omega t}.$$

Now using $dx/dt = (i/\hbar)[\mathfrak{S}_0, x] = (1/m)p_x + \Omega y$, and $dy/dt = (i/\hbar)[\mathfrak{S}_0, y] = (1/m)p_y - \Omega x$, it is easily shown that

$$\mathfrak{S}_1 = -\frac{eE}{\omega} e^{-i\omega t}(dx/dt + idy/dt),$$

the terms $\pm i\Omega(x + iy)$ cancelling. But for a transition between states a and b , of energies $\hbar\omega_a$ and $\hbar\omega_b$, respectively, such that $\omega_a - \omega_b = \omega$, we have:

$$\langle a | dx/dt | b \rangle = \frac{d}{dt} \int \psi_a^* x \psi_b = i\omega \langle a | x | b \rangle$$

and analogously for dy/dt . Thus,

$$\langle a | \mathfrak{S}_1 | b \rangle = eE \langle a | x + iy | b \rangle,$$

which justifies use of the intuitively reasonable "rotating dipole moment" in equation (5).

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