

## OPACITY OF THE SOLAR CORE\*

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### ABSTRACT

The influence of individual heavy elements on the opacity of the solar core is investigated. Increases and decreases in abundances are considered quantitatively. The recently revised photospheric iron abundance and the possible effects of diffusion are included. Together, these two changes may increase the opacity at the solar center by almost 50 percent relative to that obtained with a Lambert-Warner photospheric composition.

A discussion of the uncertainty in the physical theory is given. Improved opacities are provided in a form that is convenient for solar evolutionary calculations.

For opacity considerations, the conditions within main-sequence Population I models having radiative cores are similar to those of the Sun, and the discussion here is relevant for such stars.

### I. INTRODUCTION

Recent work (e.g., Garz *et al.* 1969*a, b*) has shown that some previously accepted heavy-element abundances for the solar photosphere are in substantial error. In addition to changing the accepted abundances, this result emphasizes the possibility of error in such determinations and suggests that the latest values should not be accepted as completely accurate. Further, in constructing solar models and examining solar evolution, it is the abundances below the photosphere that are of primary importance through their effect on the Rosseland opacity.<sup>1</sup> For example, the calculated neutrino flux for the Davis, Harmer, and Hoffman (1968) experiment is quite sensitive to the value of the opacity near the solar center (Bahcall, Bahcall, and Ulrich 1969; Shaviv 1969; Watson 1969*c*). Of course, photospheric abundances, even if known exactly, are not necessarily the same as those of the core. For example, estimates suggest that diffusion may have reduced the surface abundances of many elements heavier than helium by as much as 50 percent (Aller and Chapman 1960; Delcroix and Grevesse 1968).

The purpose of this paper is to examine, under solar-core conditions, individual opacity contributions of the various heavy elements and to determine the effect of changes in their abundances. This is done by explicit calculation of the opacities which are obtained when the abundances of representative elements are altered. Although the physical theory of opacity processes on which these calculations are based may well contain errors, the relative importance of the various elements is expected to be much less sensitive to revisions in theory than is the numerical value of the total opacity. The effect of uncertainty in physical theory is considered in § V. Opacities for solar calculations which contain recent improvements are provided in § IV.

The physical conditions within Population I main-sequence models having radiative cores are similar, for opacity considerations, to those of the Sun. Hence, the work of this paper also indicates the opacity uncertainties in such stars.

### II. CALCULATION OF THE OPACITIES

Details of the physical theories and calculational techniques used in the computations have been described elsewhere (Watson 1969*a, b*, 1970).

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<sup>1</sup> In the remainder of this paper, "opacity" is understood to mean the Rosseland radiative opacity.

With the exception of the abundance of argon, the solar photospheric abundances of Lambert and Warner (Lambert 1967, 1968; Lambert and Warner 1968*a, b*; Warner 1968) are taken as the "basis" heavy-element (nuclear charge greater than 2) composition from which variations are made. The universal abundance of argon (e.g., Allen 1955) is used. The total heavy-element mass fraction  $Z$  that is used is then 0.0149. For simplicity only two helium abundances, those of  $Y = 0.22$  and  $Y = 0.60$  (where  $Y =$  helium mass fraction), are considered in the computations. These are assumed to represent roughly the helium abundance of the zero-age solar composition and the helium abundance at the center of the present Sun. Linear interpolation is adequate for extending these results to other values of  $Y$ .

The opacities in the figures are calculated for the set of temperatures and densities thought to exist in the present Sun. During the main-sequence phase the central temperature of the Sun has only increased from about  $14 \times 10^6$  ° to about  $16 \times 10^6$  ° K. More important for the purposes here is that, at a given temperature, the density has

TABLE 1  
RELATION BETWEEN SOLAR MASS FRACTION,  
TEMPERATURE, AND DENSITY USED  
IN OUR CALCULATIONS

Mass Fraction	Temperature ( $\times 10^6$ ° K)	Density (g cm $^{-3}$ )	Hydrogen Mass Fraction
0.00 .....	15.7	158	0.35
0.10 .....	12.8	83	0.59
0.20 .....	11.3	59	0.63
0.30 .....	10.0	46	0.67
0.60 .....	7.0	18	0.77
0.95 .....	3.0	0.74	0.77

NOTE.—The distribution of hydrogen employed in Fig. 3 is also shown. These are assumed to approximate conditions in the present Sun (e.g., Iben 1967).

changed by less than about 10 percent (e.g., Strömgren 1965). Opacities for a single set of temperature-density points are thus a satisfactory indicator of the relative importance of various heavy elements for the main-sequence phase of solar evolution.

### III. RESULTS

All computations are made at the temperatures and densities shown in Table 1. The investigations are made by excluding certain elements, and by increasing their abundances by various multiplicative factors. Thus, since percentage changes are considered, the results are dependent upon the assumed "basis" abundances.

#### a) Decreases in Abundances

These results are not given explicitly because decreases in the abundance of any single heavy element have little influence on the total solar opacity. At the points in Table 1 above  $3 \times 10^6$  ° K, each heavy element contributes a few percent or less. Of course, away from the solar center the total contribution is substantial since there are ten or so such elements. For example, a curve in Figure 3 shows the fraction of the opacity due to electron scattering and free-free absorption for the composition containing the revised iron abundance. This is roughly the contribution of H and He. The increase given in Figure 2 resulting from the revised iron abundance can be employed in conjunction with

Figure 3 to estimate the opacity due to heavy elements in the Lambert-Warner composition.

At temperatures of  $3 \times 10^6$  °K a few elements—O, Ne, Mg, and Si—each provide contributions of 10–30 percent. Oxygen is the most important of these.

#### b) *Increases in Abundances*

The effect of increasing, one at a time, the abundances of certain representative elements by factors of 5 and 10 is illustrated in Figures 1 and 2, respectively. Increases in iron are seen to be very important at the solar center. With the exception of the inner  $0.05 M_{\odot}$  and the outer  $0.2 M_{\odot}$ , silicon is the most important heavy element in the Lambert-Warner composition. At temperatures of a few million degrees K, increases in oxygen cause the greatest enhancement of the opacity. Only four representative elements have been investigated explicitly; however, the above conclusions are assumed to indicate approximately the importances of the various nuclear-charge species when allowance is made for the different relative abundances.

The influence of the revised photospheric iron abundance (Garz *et al.* 1969*a, b*) is illustrated by the points representing the iron increases in Figure 2. This change has been discussed in detail elsewhere (Watson 1969*c*). If maximum estimates of diffusion (Delcroix and Grevesse 1968) are correct, a more likely heavy-element composition at the solar core is obtained by increasing the Lambert-Warner abundance of iron by a factor of 20 and that of all other heavy elements by a factor of 2. The influence of these changes is shown in Figure 3. Clearly they are quite substantial; the enhancement at the center is almost 50 percent.

#### IV. SUGGESTED SOLAR OPACITIES FOR MODEL CALCULATIONS

In order to minimize possible errors in interpolation, we have used the approximate constancy of the ratio  $\rho/T_6^3$  (where  $\rho$  is the density in grams per cubic centimeter and  $T_6$  is the temperature in millions of degrees K) throughout the region below the solar convective envelope. This procedure also decreases the required opacity computations. Opacities are obtained for temperatures and densities at which  $\rho/T_6^3$  approximates that of the zero-age models and for those at which it is similar to that in present solar models. This is done for helium mass fractions that are representative of the zero-age abundance and the present abundance at the solar center. The heavy-element composition is that of Lambert and Warner with iron increased by a factor of 10. Autoionization lines and correlations in electron scattering are included. These opacities are presented in Table 2.

Below the minimum temperature given in the tables, the increase in iron can be regarded as an increase in the total heavy-element abundance. Existing tables can then be used (e.g., Watson 1970).

#### V. ERRORS IN THE PHYSICAL THEORY

Substantial uncertainties are known to remain in the treatment of line and bound-free absorption (e.g., Carson *et al.* 1968; Carson and Hollingsworth 1968). Lines are particularly difficult to treat because a knowledge of their energies, splittings, pressure broadenings, and transition probabilities is necessary. Under some conditions the autoionization breadth is also desirable (Watson 1969*a*). The bound-free contribution is sensitive to the energies of the edges and to the splitting of the atomic states. In investigations of these problems not only do the usual atomic-structure difficulties exist which are associated with the many bound electrons, but also the influence of the free electrons and ions can be significant.

In Figure 3 the fractional contribution due to relatively well-understood processes—electron scattering and free-free absorption—is shown as a function of mass fraction. A

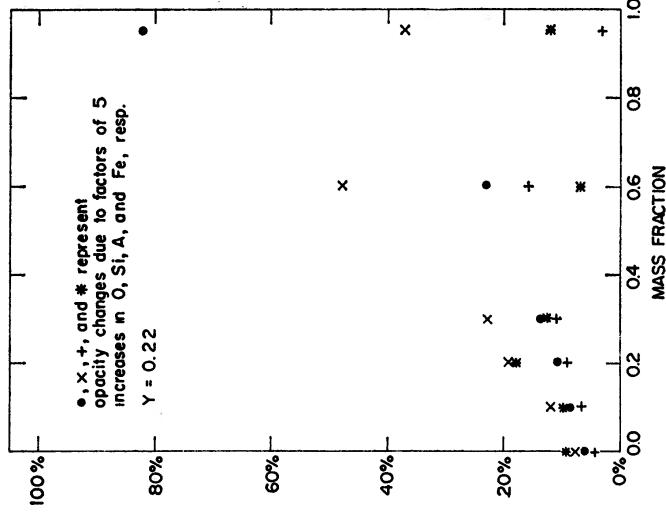


FIG. 1

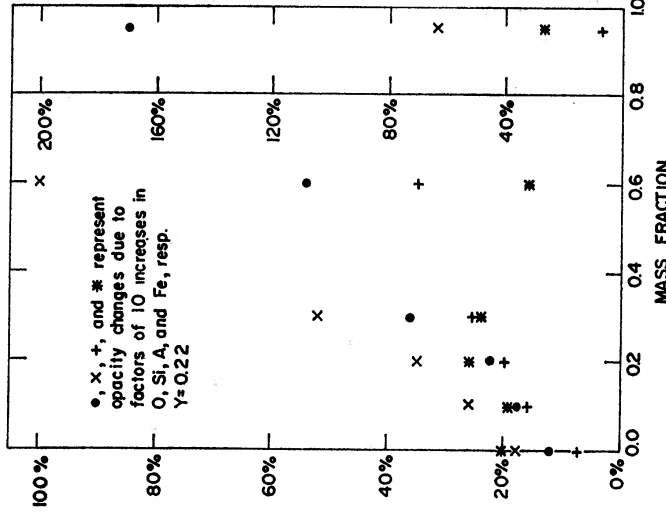


FIG. 2

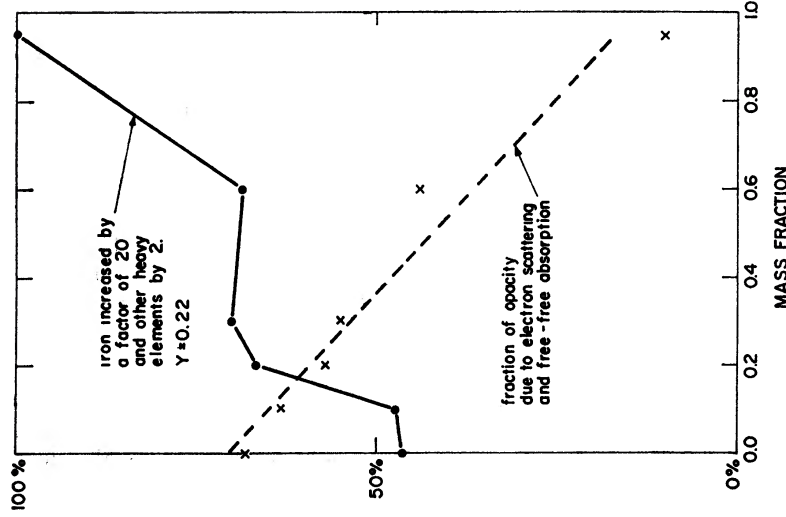


FIG. 3

FIG. 1.—Percentage increases in opacity obtained by increasing, one at a time, the abundances of representative heavy elements in the Lambert-Warner composition by a factor of 5, versus solar mass fraction. Helium mass fraction  $Y = .022$ . Relation between mass fraction, density, and temperature is given in Table 1.

FIG. 2.—Same as Fig. 1, except that the abundances here are increased by a factor of 10.

FIG. 3.—Increase in opacity obtained when, in the Lambert-Warner composition, the iron abundance is increased by a factor of 20 and that of all other heavy elements by a factor of 2, versus solar mass fraction. Helium mass fraction = 0.22. Also shown is the contribution of electron scattering and free-free absorption to the opacity for a Lambert-Warner composition in which iron is increased by a factor of 10. Hydrogen abundance versus mass fraction as given in Table 1 is used in the linear interpolations between the two computed values of  $Y$  in order to obtain these points.

reasonable estimate for the maximum uncertainty in the remainder, that due to lines and bound-free absorption, is about a factor of 2 in regions away from the solar center. Near the center, highly ionized iron is the primary contributor, and the accuracy is expected to be somewhat better.

VI. CONCLUSIONS

Near the solar center, the simple Kramers's law which expresses the bound-free opacity as a linear function of the *total* heavy-element abundance *Z* is not correct. The

TABLE 2  
ROSSELAND OPACITY AS A FUNCTION OF TEMPERATURE  
FOR SOLAR EVOLUTIONARY CALCULATIONS  
A. HELIUM MASS FRACTION = 0.22

$T_0$ ( $\times 10^6$ ° K)	$(\rho/T_0^3) = 0.0350$		$(\rho/T_0^3) = 0.0407$	
	$\rho$ (g cm $^{-3}$ )	$\kappa_R$ (cm $^2$ g $^{-1}$ )	$\rho$ (g cm $^{-3}$ )	$\kappa_R$ (cm $^2$ g $^{-1}$ )
15.7.....	136	1.08	158	1.15
12.8.....	74.0	1.24	86.0	1.30
11.3.....	51.0	1.54	59.0	1.61
10.0.....	35.0	1.75	41.0	1.83
7.0.....	12.1	2.68	14.0	2.82
4.5.....	3.20	6.11	3.72	6.31
3.0.....	0.945	14.6	1.10	15.0
1.8.....	0.205	43.3	0.240	46.1
1.0.....	0.0350	85.3	0.041	92.3

B. HELIUM MASS FRACTION = 0.60

$T_0$ ( $\times 10^6$ ° K)	$(\rho/T_0^3) = 0.0350$		$(\rho/T_0^3) = 0.0407$	
	$\rho$ (g cm $^{-3}$ )	$\kappa_R$ (cm $^2$ g $^{-1}$ )	$\rho$ (g cm $^{-3}$ )	$\kappa_R$ (cm $^2$ g $^{-1}$ )
15.7.....	136	.95	158	1.00
12.8.....	74.0	1.22	86.0	1.27
11.3.....	51.0	1.41	59.0	1.47
10.0.....	35.0	1.59	41.0	1.67
7.0.....	12.1	2.45	14.0	2.57
4.5.....	3.20	5.59	3.72	5.81
3.0.....	0.945	13.3	1.10	13.7
1.8.....	0.205	38.9	0.240	41.5
1.0.....	0.0350	72.2	0.041	78.2

NOTE.—Composition for heavy elements is that of Lambert and Warner with the iron abundance increased by a factor of 10.

factor of 10 increase in the iron abundance is a 10 percent increase in *Z*; it causes a factor of 2 or so enhancement of the bound-free opacity. This is because it is primarily the heavier elements that have bound electrons and edges of sufficient energy to be important.

Considered together, the error estimates of the previous section and the results of the investigation by Bahcall *et al.* (1969) with regard to the opacity sensitivity of the solar neutrino flux indicate that the flux discrepancy (e.g., Davis *et al.* 1968; Iben 1969) is unlikely to be resolved, for radiative-core models, by uncertainties in opacity alone.

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