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DATA FOR NEPTUNE FROM OCCULTATION OBSERVATIONS

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ABSTRACT

Photoelectric observations of the occultation of BD$-17°4388$ by Neptune on 1968 April 7 are analyzed. The observed occultation is produced by differential refraction of the starlight in Neptune's atmosphere, between about 400 and 500 km above the surface; at this height the molecular number density $N \simeq 10^{12}$–10^{13} cm^{-3}, compared with $N \simeq 10^{20}$ cm^{-3} at the base of the visible atmosphere. The atmospheric scale height is 28.9 ± 2.6 (p.e.) km, so the ratio T/μ of temperature to mean molecular weight is 38. The most likely atmospheric composition is then H_2 with a few percent CH_4.

The occultation layer defines an oblate spheroid of equatorial radius 25265 ± 36 (p.e.) km and axial ratio 0.9741 ± 0.0051. If the atmosphere is strictly isothermal, the equatorial radius of the planetary body is then 24753 ± 59 km. This corresponds to a mean density for Neptune of 1.666 ± 0.015 g cm^{-3} (which is close to the value for Uranus). Structure in the observed occultation curves suggests that the atmospheric temperature may increase inward; the mean density may then be larger than 1.666 g cm^{-3}, but is very unlikely to exceed 1.77 g cm^{-3}.

I. INTRODUCTION

The occultation of BD$-17°4388$ by Neptune on 1968 April 7 was observed photoelectrically at Mount Stromlo Observatory (IAU Circ. No. 2067, 1968) and at the Dodaira and Okayama stations of the Tokyo Astronomical Observatory (IAU Circ. No. 2068, 1968). Continuous tracings were obtained from two telescopes at Mount Stromlo and from one telescope at each of the Japanese stations.[1]

The behavior of an occultation light curve depends on the scale height of the planet's atmosphere, which in turn determines the ratio T/μ of the absolute temperature to the mean molecular weight. The timing of immersion and emersion from two or more observatories allows the determination of the planet's radius and, with known mass, also the mean density of the planet. Preliminary determinations of these data have been made by Osawa, Ichimura, and Shimizu (1968), Takenouchi, Tomita, and Hirayama (1968), Kovalevsky (1968), Taylor (1968), and Freeman and Lyngå (1969).

The present paper aims at a full discussion of the information to be derived from the photoelectric light curves with consistent time determination and with due regard to the refraction effects in the atmosphere of Neptune. We shall first derive the dimensions of Neptune and its position relative to the occulted star, then a model of the atmosphere, and finally the mean density of the planet itself.

II. DIMENSIONS AND POSITIONS

Previous determinations of Neptune's radius from occultation data show a larger scatter than the errors of timing could possibly give. This is due mainly to difficulties in

[1] We are grateful to Tokyo Observatory for communicating copies of the photoelectric tracings before publication.

estimating the times for various intensity levels in the very disturbed light curves. Figure 1 shows an example of a photoelectric light curve. The spikes (flares of light intensity) are caused in the atmosphere of Neptune, and they will be discussed below. Also, we shall derive later the theoretical light curve, which is marked by a dashed line in Figure 1. The times on which the present determination of radius is based correspond to 9 percent fading of initial starlight. This corresponds to $\phi_0/\phi = 1.1$ according to notations below. This light level is far enough in toward occultation for the light curves to have a reasonable gradient, and yet is outside the very disturbed part of the light curve. The estimated standard deviation associated with the $\phi_0/\phi = 1.1$ times is 2 seconds.

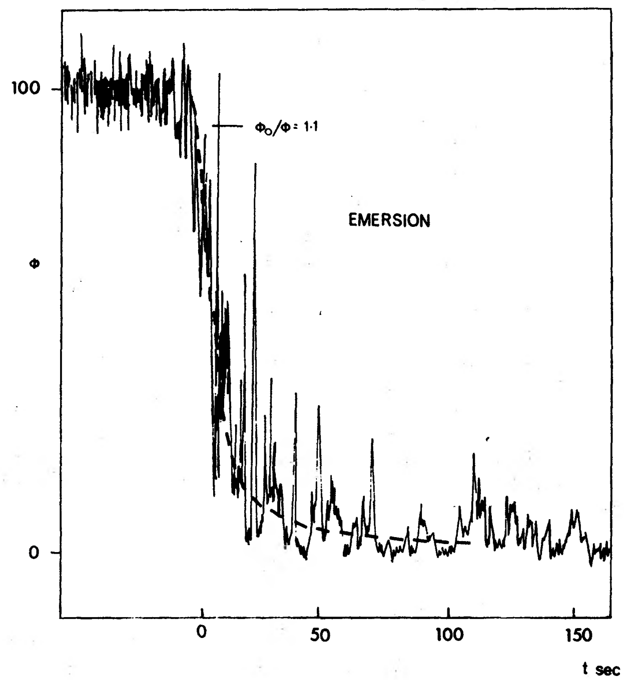

FIG. 1*a*.—Observed occultation curve (*V*-band, emersion, Mount Stromlo). ***Dashed line,*** theoretical occultation curve corresponding to a scale height of 28.9 km.

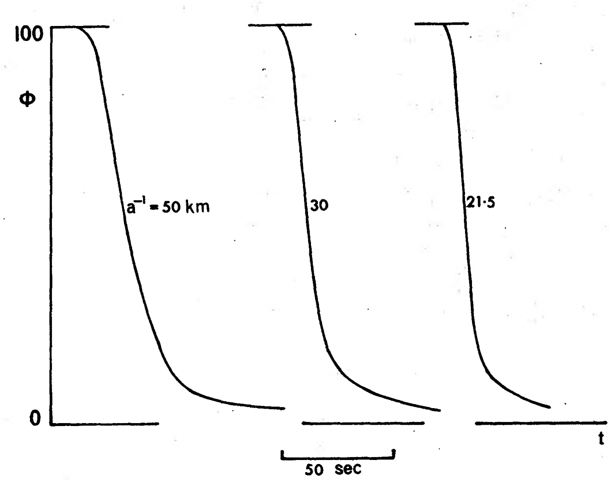

FIG. 1*b*.—Theoretical occultation curves with the same time and intensity scales as in Fig. 1*a* (a^{-1} is the scale height).

Let us assume that Neptune's motion relative to the star is that given by ephemeris. The position of Neptune at a given time and the position of the star are not known to the accuracy required. However, we are only concerned with the differences between the equatorial coordinates for the star and those for Neptune at an arbitrarily chosen time. All coordinates are referred to the position of Neptune on 1968 April 7 ($\alpha_{7.68}$, $\delta_{7.68}$), and the two unknown position differences are

$$X = \alpha_{\rm star} - \alpha_{7.68}$$

and

$$Y = \delta_{\rm star} - \delta_{7.68} .$$

Let us now assume that Neptune is spherical with radius R. This was in fact done in our preliminary solution (Freeman and Lyngå 1969). We have in that case three unknowns (X, Y, and R), while each event (immersion or emersion) from each observatory (Mount Stromlo, Dodaira, and Okayama) will furnish one equation. In practice it is convenient first to calculate, from the time of the event i, the coordinates of the relevant observatory, and the distance to Neptune, the differences

$$x_i = \alpha_i - \alpha_{7.68} ,$$

$$y_i = \delta_i - \delta_{7.68} ,$$

where (α_i, δ_i) are the topocentric coordinates of Neptune's center. The topocentric coordinates of the event point on the limb of Neptune are for every observation equal to the star's coordinates (refraction alters the star's apparent position by 0\".00013, which is negligible), so the apparent radius is given by

$$R^2 = (x_i - X)^2 + (y_i - Y)^2 , \qquad i = 1 , \ldots , 6 . \tag{1}$$

If we assume instead that Neptune is ellipsoidal with semiaxes a and b and small flattening $k = (a - b)/a$, then the radius vector at latitude ϕ has length

$$r = a - ak \sin^2 \phi , \tag{2}$$

and we have the following equations:

$$a^2(1 - k \sin^2 \phi_i)^2 = (x_i - X)^2 + (y_i - Y)^2 , \qquad i = 1 , \ldots , 6 . \tag{3}$$

In order to calculate the latitudes ϕ_i of the points of event on Neptune it is necessary to know the direction of Neptune's pole, the direction to the Earth, and the position angle ψ of the event points as seen from the Earth. Figure 2 shows that these data determine the latitude uniquely. The pole of Neptune is given by Gill and Gault (1968), the direction to the Earth is found from ephemeris, and the position angle is determined by using the preliminary solution for the spherical case. Table 1 gives the latitudes determined.

The system of equation (3) was solved by the method of least squares after a weighting had been introduced on the terms x_i and y_i, reflecting the fact that the main uncertainty lies in the time determination and that, since the motion in x is about 4 times that in y, y_i is much better determined than x_i. The results are presented in Table 2. We have also given the value of a corrected for gravitational deflection, which, if the mass determination by Gill and Gault (1968) is used, amounts to 0\".00356. The change in this deflection through the atmosphere is small and has been neglected in the discussion below.

It is of interest to compare our value of the flattening, 0.026, with the value 0.017[2]

[2] The equatorial radius given in Table 2 increases the dynamical estimate of the flattening to 0.020.

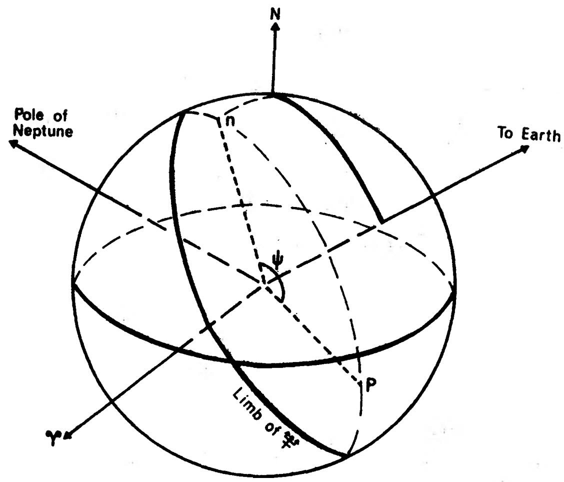

FIG. 2.—Equatorial system of coordinates translated to center of Neptune. The angle ψ defines the position of the event point P on the apparent limb of Neptune; n is the northernmost point on Neptune's limb as seen from Earth.

TABLE 1

DATA FOR OBSERVED EVENTS

	Mount Stromlo Observatory		Dodaira Observatory		Okayama Observatory	
Event	Immersion	Emersion	Immersion	Emersion	Immersion	Emersion
U.T. (for $\phi_0/\phi = 1.1$)..	$15^h56^m21^s$	$16^h36^m53^s$	$15^h56^m16^s$	$16^h41^m38^s$	$15^h56^m46^s$	$16^h42^m01^s$
Position angle on Neptune's limb (ψ)........	104°41′	225°45′	88°33′	241°48′	89°12′	241°13′
Latitude on Neptune (ϕ)........	−55°45′	−2°04′	−40°05′	+13°46′	−40°34′	+13°12′

TABLE 2

GEOMETRICAL DATA FOR NEPTUNE ($\phi_0/\phi = 1.1$), ELLIPSOIDAL SOLUTION

Parameter	Value	S.D.
Coordinates of star relative to Neptune:		
X..............................	+0″0087	0″0027
Y..............................	−0″4955	0″0046
Semimajor axis, derived (a)...........	1″1768	0″0025
Semimajor axis, corrected for gravitational deflection (a_c)..............	1″1794	...
Flattening, k........................	0.0259	0.0075

NOTE.—The standard deviations (S.D.) correspond to an estimated S.D. associated with the $\phi_0/\phi = 1.1$ times of 2 seconds.

expected by Brouwer and Clemence (1961) from dynamical considerations based on the motion of Triton's orbital plane. Our high value could indicate that the outer atmosphere of Neptune defines a more oblate spheroid than the planet itself, as is indeed the case for the Earth.

III. THE ISOTHERMAL ATMOSPHERE MODEL

Several authors (Fabry 1929; Baum and Code 1953; de Vaucouleurs and de Vaucouleurs 1961) have discussed the total light curve for a planet and a star which is being occulted, assuming that the spherical planet has a homogenous isothermal atmosphere of depth small compared with the planet's radius r_0. These conditions are reasonable approximations to those valid in the present case, and we shall first repeat those of the formulae derived by Baum and Code which will be employed later.

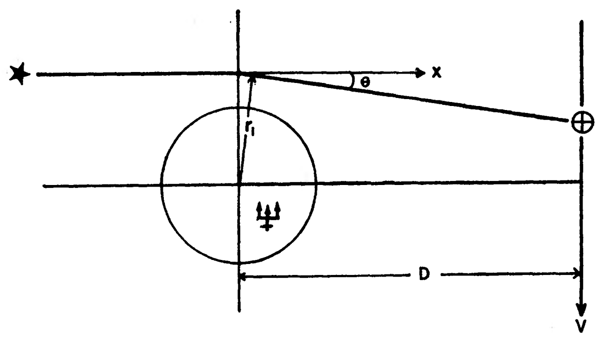

FIG. 3.—Influence of refraction on a light ray passing through the occultation layer of Neptune's atmosphere.

The refractive index is

$$n(r) - 1 = (n_0 - 1)e^{-a(r-r_0)}, \tag{4}$$

where the inverse scale height a is given by

$$a = \mu g/RT; \tag{5}$$

here μ is the mean molecular weight, g the surface gravity, R the gas constant, and T the temperature. It then follows that the angular refraction θ (see Fig. 3) is

$$\theta \simeq (2\pi r_0 a)^{1/2}(n_1 - 1), \tag{6}$$

where n_1 is the refractive index at the light ray's closest approach to the planet ($r = r_1$).

Now let ϕ be the *stellar* flux received at the telescope; ϕ_0 is its value in the absence of refraction. From the geometry of the refraction it follows that

$$\phi_0/\phi = 1 + aD\theta. \tag{7}$$

By differentiating equation (7) with respect to the time t and from the geometry it follows that

$$\phi_0/\phi + \log(\phi_0/\phi - 1) = aVt, \tag{8}$$

where V is the velocity of the telescope as shown in Figure 3.

Equation (8) is a family of theoretical occultation curves; fitting these curves to the observed occultation curve gives the inverse scale height a and, from equation (5), also the ratio T/μ for the atmosphere. Figure 1*a* shows an example of the occultation curves observed at Mount Stromlo; Figure 1*b*, on the same scale, shows some theoretical occultation curves derived from equation (8). The velocity V is 17.60 km sec^{-1}.

The observed curves show many sharp spikes. In deriving the scale height by fitting curves corresponding to equation (8) we have ignored these spikes. Their nature and our justification for ignoring them are discussed below. Our fit of the theoretical curves to all the available occultation curves gives

$$a = 0.0346 \pm 0.003 \text{ (p.e.) km}^{-1} ,$$

$$a^{-1} = 28.9 \pm 2.6 \text{ km.}$$

TABLE 3

QUANTITIES FOR NEPTUNE

Quantity	Value
Atmospheric scale height	$a^{-1}=28.9$ km
Earth-Neptune distance	D $=29.53$ a.u.
Neptune's approximate radius	r_0 $=25000$ km
Gravitational acceleration on Neptune's surface	g $=1088$ cm sec^{-2}

TABLE 4

MODELS FOR NEPTUNE'S ATMOSPHERE

Model	Source	Composition	μ	A
I	Herzberg (1952)	H_2:He=1:3, 1% CH_4	3.6	0.91
II	Urey (1959)	H_2: N_2=1:1, 1% CH_4	15	3.27
III	Kuiper (1949)	H_2, 1% CH_4	2.2	2.12
IV	See text	H_2, 3% CH_4	2.4	2.21

NOTE.—A is the molar refractivity and μ is the mean molecular weight.

IV. PROPERTIES OF NEPTUNE'S ATMOSPHERE

Spectroscopy of Neptune has led to several suggested compositions for its atmosphere. In this section we calculate some physical data required by these models. We also locate the occultation layer in the atmosphere. Table 3 lists quantities for Neptune used in this section.

Table 4 describes the four models of atmospheric composition for which we have calculated physical data below. In this table μ is the mean molecular weight, and the molar refractivity A is found from

$$A = 4 \times 10^{23} (n - 1)/N , \qquad (9)$$

where n is the refractive index and N the molecular number density. Model IV is chosen to investigate the effect of extra CH_4 on the model by Kuiper. In fact, Owen (1967) determined spectroscopically a mass load of 6 km atm methane (NTP) in Neptune's atmosphere with an estimated uncertainty of at least 35 percent; this mass load is somewhat larger than the value used by Kuiper (2.5 km atm).

a) Rayleigh Scattering

Let us first show that extinction through Rayleigh scattering has a negligible effect on the observed occultation curves. The extinction coefficient is

$$\beta = \frac{32\,\pi^3(n-1)^2}{3\,\lambda^4\,N} = 1.32 \times 10^{-4}\, A(n-1)\ \mathrm{cm}^{-1}\,, \tag{10}$$

where the wavelength λ is taken as 0.50 μ, intermediate between the Johnson B- and V-bands. The total extinction for a ray with (β_1, n_1) at its point of closest approach to Neptune is (Baum and Code 1953; see also Fig. 3)

$$\int_{-\infty}^{+\infty} \beta\, dx = \beta_1 \left(\frac{8r_0}{a}\right)^{1/2} = 3.18 \times 10^4\, A(n_1 - 1) \tag{11}$$

and from equation (6)

$$\int_{-\infty}^{+\infty} \beta\, dx = 2.93 \times 10^2\, A\, \theta\,. \tag{12}$$

TABLE 5

DATA FOR VARIOUS ATMOSPHERIC LAYERS

(See § IV*b* for definitions of symbols)

ϕ_0/ϕ	θ	n_1-1	N_{I}	N_{II}	N_{III}	N_{IV}
100	6.5×10^{-7}	8.9×10^{-9}	4.0×10^{15}	1.1×10^{15}	1.7×10^{15}	1.6×10^{15}
5	2.6×10^{-8}	3.6×10^{-10}	1.6×10^{14}	4.5×10^{13}	6.9×10^{13}	6.5×10^{13}
1.1	6.5×10^{-10}	8.9×10^{-12}	4.0×10^{12}	1.1×10^{12}	1.7×10^{12}	1.6×10^{12}
1.01	6.5×10^{-11}	8.9×10^{-13}	4.0×10^{11}	1.1×10^{11}	1.7×10^{11}	1.6×10^{11}

For example, when the stellar flux has been reduced by differential refraction to 1 percent of its unrefracted value, i.e., $\phi_0/\phi = 100$, then equation (7) gives an angular deviation of $\theta = 6.5 \times 10^{-7} = 0''.13$, and the total extinction for the four composition models is: model I, 0.0003 mag; model II, 0.0009 mag; model III, 0.0006 mag; model IV, 0.0006 mag.

b) Molecular Densities

We shall now evaluate for the four models the deviation θ, the refractive index n_1, and the molecular number density N corresponding to certain values of ϕ_0/ϕ; n_1 and N refer to the point of closest approach to the planet. These data are needed later, and are shown in Table 5.

c) The Visible Atmosphere

Before we can locate the occultation layer in the atmosphere we must discuss the visible atmosphere, by which we mean that part of Neptune's atmosphere which contributes significantly to the visible radiation from Neptune. From equation (10) the extinction coefficients for the four models are

$$(7.3 \times 10^{-9}, 9.5 \times 10^{-8}, 4.0 \times 10^{-8}, 4.3 \times 10^{-8})\ \mathrm{cm}^{-1}\,,$$

so optical depth unity is reached for a path length l of

$$l = (1370, 105, 250, 233) \text{ km atm (NTP)} . \quad (13)$$

In a similar calculation, Kuiper (1949) estimated that about 0.36 of this path length is responsible for the visible radiation; this accounts for the double path of the light in and out of the atmosphere and for multiple scattering. If we make the same assumption, then the upper

$$(499, 38, 91, 85) \text{ km atm (NTP)} \quad (14)$$

of the atmosphere produce the visible radiation.

For the gravitational acceleration $g = 1088$ cm sec^{-2}, the pressure at the base of this visible atmosphere is

$$P_b = (8.3, 2.7, 0.95, 0.97) \text{ atmospheres} . \quad (15)$$

Kuiper (1949) arbitrarily takes the upper edge of the visible atmosphere at a pressure of 0.125 atmospheres. Using this value and again assuming an isothermal atmosphere with a scale height of 28.9 km, we derive a thickness d of the visible atmosphere of

$$d = (121, 89, 59, 59) \text{ km} . \quad (16)$$

Finally, we derive the molecular number density N at the top and base of the visible atmosphere. From equation (5) we get $T/\mu = 38$, so that

$$T = (137, 570, 84, 91)\ ^\circ\text{K} . \quad (17)$$

From equations (15) and (17) the number density at the base of the visible atmosphere is

$$N_b = (4.7 \times 10^{20}, 3.5 \times 10^{19}, 8.3 \times 10^{19}, 7.8 \times 10^{19}) \text{ molecules cm}^{-3} , \quad (18)$$

and at the top of the visible atmosphere ($P = 0.125$ atm) we get

$$N_t = (7.1 \times 10^{18}, 1.6 \times 10^{18}, 1.1 \times 10^{19}, 1.0 \times 10^{19}) \text{ molecules cm}^{-3} . \quad (19)$$

d) Location of Occultation Layer

We can now calculate where in the atmosphere the occultation took place. We are particularly interested in what altitude corresponds to the level $\phi_0/\phi = 1.1$, as the radius derived above refers to this level. This altitude Δr relative to the base of the visible atmosphere is given by

$$a\Delta r = \ln (N_b/N_{\phi_0/\phi=1.1}) , \quad (20)$$

and, if the values of equation (18) and Table 5 are used,

$$\Delta r = (537, 499, 512, 512) \text{ km} . \quad (21)$$

Finally, we need to locate the base of the visible atmosphere. Urey (1959, model II) suggests $P = 6$ atm for the "cloud level," and Herzberg (1952, model I) gives $P = 8$ atm at the base of the visible atmosphere. These pressures are within 0.8 scale height = 23 km of the pressures given by equation (15), so we identify the base of the visible atmosphere with "the cloud level." When determining the mean density of the planet we shall also assume that the planet's surface is at this level within the accuracy of the atmospheric-model calculations.

Table 6 shows, for Kuiper's model (III), the altitudes Δr above the planet's surface of the occultation events and of the top of the visible atmosphere. The molecular densities, $N(r)$, are also given. We want to point out the very low molecular density in the occultation layer and the high altitude of this layer. It is then of some importance to correct for the thickness of the atmosphere in deriving a meaningful mean density for the planetary body.

TABLE 6

HEIGHTS AND NUMBER DENSITIES FOR KUIPER'S MODEL (III)
(See § IV*d* for definition of Δr)

ϕ_0/ϕ	Δr (km)	N (cm^{-3})	Remarks
1.1......	512	1.7×10^{12}	Reference level for geometrical data
5........	405	6.9×10^{13}	
....	59	1.1×10^{19}	Top of visible atmosphere ($p=\frac{1}{8}$ atm)
....	0	8.3×10^{19}	$\tau=1$

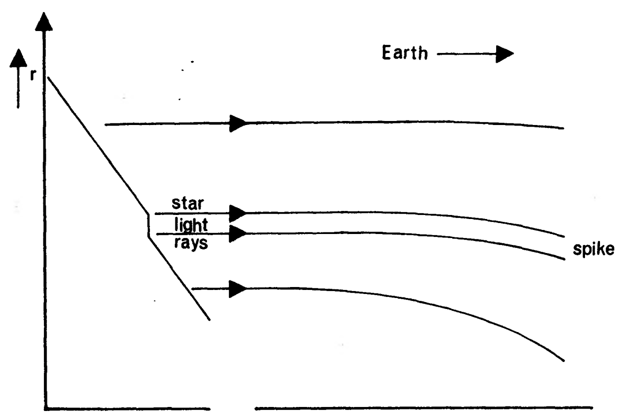

FIG. 4.—Schematic representation of the molecular number density $N(r)$. Light rays corresponding to a spike in occultation curve are shown.

e) The Spikes in the Occultation Curves

As is clear from Figure 1 the occultation curves show several very prominent spikes of short time duration. We ignored these in the determination of the scale height, and now discuss their nature to justify this procedure.

Because of the very low extinction in the occultation layer, the considerable altitude of this layer above Neptune's surface, and the rather irregular nature of the spikes, it seems most unlikely that these are produced by extinction or diffraction effects. We shall look for their cause within the differential refraction picture.

We have adopted what seems the simplest explanation, namely, that the spikes are produced in thin atmospheric layers for which $|dN/dr| < |dN/dr|_{\text{isothermal}}$. In such layers the refractive spreading is relatively low and the corresponding observed flux ϕ is relatively high. The principle is shown in Figure 4.

The extent of the layers can be estimated from the following observations:

1. Photometric and proper motion data for BD $-17°4388$ make it likely that this star is a K dwarf at a distance of about 10 pc; its angular diameter is then 7×10^{-4} seconds

of arc, which corresponds to 10 km at the distance of Neptune. Even if the star had been a giant this projected diameter would only have been 14 km.

2. A few spikes have heights corresponding to $\phi_0/\phi \leq 1.2$; i.e., more than about 80 percent of the unrefracted intensity.

3. From the observed intensity and duration of the spikes we can calculate the radial thickness of the layers in which they arise; this thickness appears to be between 10 and 15 km. The presence of intense spikes (2) means, by using (1), that some layers are *at least* 10 km thick.

4. Spikes appear on all available occultation curves. The distance between the two Mount Stromlo telescopes corresponds to about 200 m on Neptune's periphery; the agreement of spike times as observed with these two telescopes is one to one. The distance between the two Japanese telescopes corresponds to about 260 km on Neptune's periphery; there is only a weak correlation between these spike times. Between spike times observed in Japan and in Australia there is no correlation at all. The extent of the layers along the limb of Neptune is then of the order of 300 km.

5. Finally, we estimate the extent of the layers in the line of sight. Numerical integration shows that about three-quarters of the angular refraction given by equation (6) occurs for the part of the light ray with $|x| \lesssim 1000$ km (see Fig. 3). The presence of the intense spikes (2) then suggests that the line-of-sight extent of the layers is at least of the order of 1000 km.

To summarize: If the suggested origin of the spikes in atmospheric layers with $|dN/dr| < |dN/dr|_{\text{isothermal}}$ is correct, then these layers are about $\frac{1}{3}$ scale height thick (compared with the occultation-layer thickness of about 10 scale heights from $\phi_0/\phi =$ 1.01 to $\phi_0/\phi = 100$), and their extent parallel to Neptune's surface is at least of the order of 100–1000 km. The relatively small thickness of these layers means that we can regard the spikes as representing local deviations from a mean $N(r)$ distribution; our procedure of ignoring the spikes in the derivation of the scale height then seems reasonable.

We must point out, however, that the presence of layers with $|dN/dr| <$ $|dN/dr|_{\text{isothermal}}$ is not consistent with a strictly isothermal atmosphere in hydrostatic equilibrium and, in particular, implies that the atmospheric temperature increases toward the surface of the planet. From the equation of state for an ideal gas and from hydrostatic equilibrium it follows that small increments of temperature, density, and radius satisfy

$$\frac{\Delta T}{T} + \frac{\Delta N}{N} = -a\Delta r\,. \tag{22}$$

For example, for the layers producing the most intense spikes, $\Delta N/N$ is probably very small over the thickness Δr of the layer, so $\Delta T/T \simeq -a\Delta r$. But according to point (3) above, $a\Delta r \simeq \frac{1}{3}$ for such a layer, so that $\Delta T/T \simeq -\frac{1}{3}$. This crude example indicates that there may be significant deviations from the simple approximation of an isothermal atmosphere.

From the adiabatic equation of state we can calculate the adiabatic temperature gradient for hydrostatic equilibrium

$$(dT/dr)_{\text{ad}} = -\left(1 - \frac{1}{\gamma}\right)aT = -0.29\ aT \tag{23}$$

for a diatomic gas. For the intense-spike layers with $\Delta T/T \simeq -a\Delta r$ we find that $|dT/dr| > |dT/dr|_{\text{ad}}$, so these layers are probably convectively unstable.

In the next section we will use the results of Table 6 to derive the mean density of Neptune. Because the atmosphere is probably not strictly isothermal, the calculated height of the $\phi_0/\phi = 1.1$ level above the surface of Neptune, as given in Table 6, may be in error. We can estimate roughly the size of this error from a simple example. Let the temperature of the atmosphere be T_1 for $r > r_J$ and T_2 for $r_0 < r < r_J$ ($T_1 < T_2$), and let the occultation layer be above the discontinuity at r_J. Assume the atmosphere is homogeneous and in hydrostatic equilibrium. Then the difference between the real height of the occultation layer above r_0 and the height calculated on the assumption that the atmosphere is isothermal with temperature T_1 is $r_J(1 - T_1/T_2)$. For example, for $r_J = 200$ km and $T_1/T_2 = \frac{1}{2}$, this difference is 100 km. The occultation-layer height given in Table 6 is then probably a *lower limit*, and may be of order 100 km less than the real occultation-layer height.

We note that similar density fluctuations exist in the Earth's atmosphere; these have been reviewed by Lindblad (1968). It appears that there are density gradients smaller as well as larger than the isothermal gradient. In our case, the larger gradients would correspond to sharp dips in the observed occultation curve. A few such dips are present on the tracings; their number, however, is rather less than the number of spikes. These dips would partly compensate the inward temperature increases corresponding to the spikes.

V. CHOICE OF ATMOSPHERIC MODEL

a) *On the Presence of Clouds in Neptune's Atmosphere*

The presence of clouds in Neptune's atmosphere could give us a valuable test of the models suggested. Because of the small angular diameter of Neptune, the scintillations in the Earth's atmosphere make detailed studies of the surface of Neptune very difficult. However, a set of three drawings (Dollfus 1961) was obtained at Pic du Midi during three nights with very little air turbulence. These drawings show surface shadings which change their relative position and thus indicate the possibility of clouds. However, Kuiper (private communication 1969) has seen no such shadings during observations made under excellent conditions with the Mount Palomar 200-inch reflector and the McDonald 82-inch reflector.

We shall now investigate what effect the presence of methane or ammonium clouds would have on the physical model of the atmosphere. If the scale height of the atmosphere is H and an atmospheric component with molecular weight μ has a density ρ at a level over which the mass load is L, then

$$L \sim \frac{\rho H}{\mu};$$

or, if L is expressed in km atm, ρ in g cm^{-3}, and H in km,

$$\rho = 4.5 \times 10^{-5} \, \mu L/H . \tag{24}$$

Inserting

$$\rho = \frac{\mu}{RT} p$$

where p is the partial pressure of the atmospheric component in mm Hg, T is the temperature (° K), and R is the gas constant, we get

$$p/T = 2.80 \, L/H = 0.097 \, L \tag{25}$$

if we assume that, for Neptune, $H = 28.9$ km. The ratio between p and T thus depends only on the mass load L of the particular atmospheric component studied. Relations for different values of L are given in Figure 5 as straight lines. The curved lines in this figure are relations between temperature and pressure for saturation of methane and ammonium according to the Landolt-Börnstein (1960) tables.

The amount of methane in Neptune's atmosphere has been determined spectroscopically by Owen (1967). He points out that his value of 6 km atm has an uncertainty of more than 35 percent. Figure 5 shows that the corresponding saturation temperature for methane is 86° K and that this is rather insensitive to errors in L. This temperature is within the range of those given for models III and IV by equation (17). If a temperature gradient of the kind described above exists, then the temperature at the base of the visible atmosphere would be so high as to preclude the presence of methane clouds.

Ammonium would, on the other hand, be able to form clouds at the temperatures required. We have obtained plates from the 74-inch coudé spectrograph (10 Å mm^{-1}) of Neptune and Jupiter. In Jupiter's spectrum one can see strong lines of ammonium between 6400 and 6500 Å, but these are not visible in the Neptune spectrum. Also, for Uranus, Urey (1959) points out that the ammonium pressure at the cloud level is about 3×10^{-10} atm and that this is too low to support effective clouds.

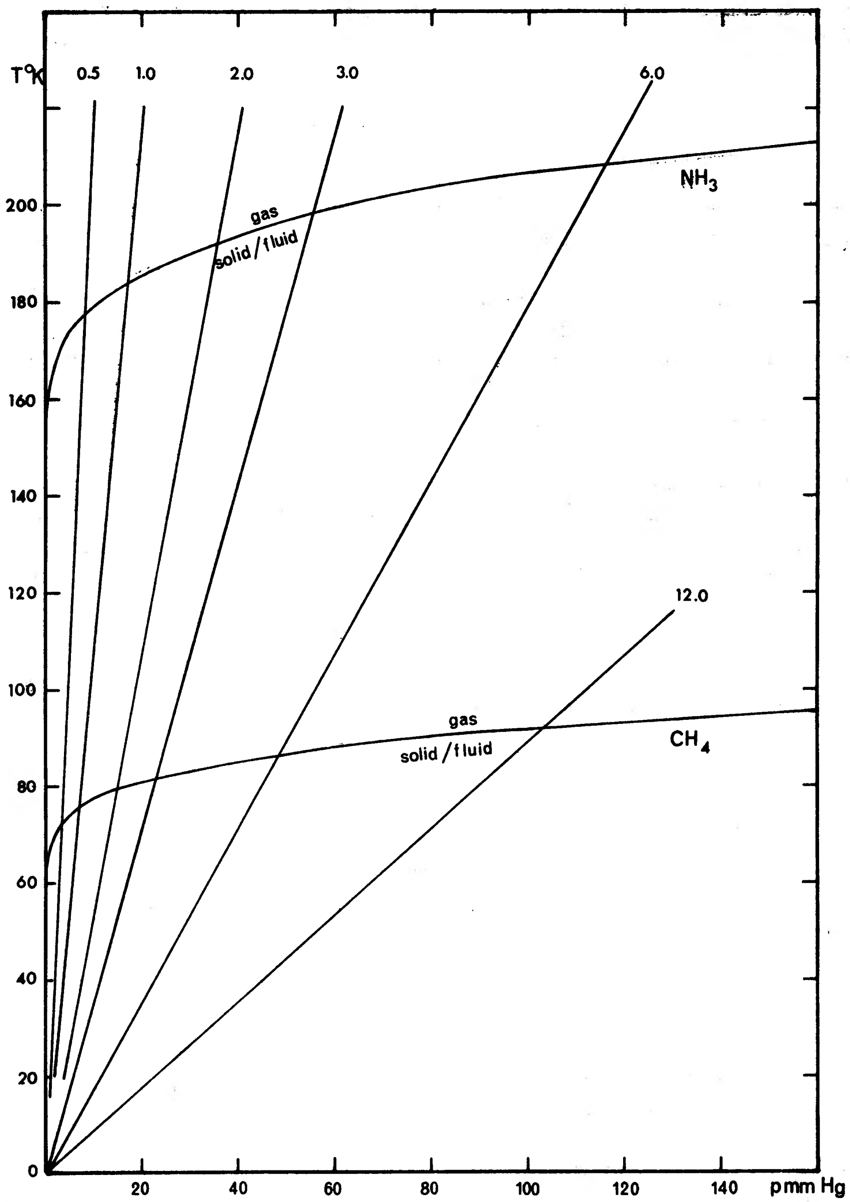

FIG. 5.—Relation between temperature and pressure for saturation of methane and ammonia. Straight lines represent equation (25) for values of the mass load (km atm) as denoted in the figure.

b) *Other Temperature Information*

The equilibrium temperature for Neptune is 54° K, and the radiometric temperature at 1.9 cm is 180° K (Kellermann and Pauliny-Toth 1966). Particularly in view of the low equilibrium temperature it seems to us that either model III (Kuiper 1949) or model IV (the Kuiper model with additional methane) is the most acceptable.

VI. MEAN DENSITY OF NEPTUNE

The geometrical data for Neptune are listed in Table 2. If we assume that the ephemeris distance, 29.53 a.u., and the solar parallax, 8″.794, are accurate, then the equatorial radius, corrected for gravitational deflection, is 25265 ± 36 (p.e.) km at the $\phi_0/\phi = 1.1$ level in the occultation layer.

If the atmosphere is strictly isothermal, then from equation (21) we can take the altitude of the $\phi_0/\phi = 1.1$ level to be 512 ± 47 (p.e.) km above the planetary surface; the probable error includes the uncertainties in the scale height and in locating the *base* of the visible atmosphere (§ IV*d*). To calculate the mean density of Neptune, we assume this altitude is independent of latitude. Neptune's mass is $(19296 \pm 9)^{-1}$ solar masses (Gill and Gault 1968), the solar mass is $(1.989 \pm 0.002) \times 10^{33}$ g (Allen 1963), the equatorial radius of the planetary body is 24753 ± 59 km, and the flattening is 0.0259 ± 0.0051 (p.e.) from Table 2. These data lead to a mean density of 1.666 ± 0.015 (p.e.) g cm^{-3}. This value is slightly higher than the value determined earlier (Freeman and Lyngå 1969) by assuming the planet to be spherical.

If, for reasons that we discussed previously, the altitude of the occultation layer is underestimated, we would get a slightly altered mean density. The maximum conceivable effect would double the altitude of the occultation layer, and this would increase the density to 1.77 g cm^{-3}. However, the correction, if present at all, is expected to be much smaller.

The fact that the dynamically determined flattening (Brouwer and Clemence 1961) is smaller than the presently determined one may be indicative of a higher temperature and consequently larger scale height close to the planet's equator. This would also mean a slightly higher mean density. We emphasize that no such variation in scale height is observed in the occultation layer although the observations cover a fair range in latitude.

We note that the density of Neptune derived above is nearly the same as the density of Uranus (1.60 g cm^{-3}) given by Allen (1963). Neptune and Uranus are known to have fairly similar atmospheric compositions (see Kuiper 1949); it seems likely that their internal compositions are also similar.

Since this paper was prepared, Kovalevsky and Link (1969) have discussed data from the Japanese occultation curves. Their procedure for deriving the atmospheric scale height does not take into account the nature of the spikes observed in the occultation curves, and therefore their values for the scale height are larger than that derived in § III of this paper. (This also applies to the scale-height determination by Osawa *et al.* 1968.)

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REFERENCES

Allen, C. W. 1963, *Astrophysical Quantities* (London: Athlone Press).
Baum, W. A., and Code, A. D. 1953, *A.J.*, **58**, 108.
Brouwer, D., and Clemence, G. M. 1961, *Planets and Satellites* (Chicago: University of Chicago Press), chap. 3.
Dollfus, A. 1961, in *Planets and Satellites*, ed. G. P. Kuiper and B. M. Middlehurst (Chicago: University of Chicago Press), pp. 534–571.

Fabry, Ch. 1929, *J. d. Obs.*, **12**, 1.
Freeman, K. C., and Lyngå, G. 1969, *Proc. Astr. Soc. Australia*, **1**, 203.
Gill, J. R., and Gault, B. L. 1968, *A.J.*, **73**, 895.
Herzberg, G. 1952, *Ap. J.*, **115**, 337.
Kellermann, K. I., and Pauliny-Toth, I. I. K. 1966, *A.J.*, **71**, 390.
Kovalevsky, J. 1968, *C.R.*, **267**, 594.
Kovalevsky, J., and Link, F. 1969, *Astr. and Ap.*, **2**, 398.
Kuiper, G. P. 1949, *The Atmospheres of the Earth and Planets* (Chicago: University of Chicago Press), p. 304.
Landolt-Börnstein, 1960, *Zahlenwerte und Funktionen*, Vol. II, Pt. 2a (Berlin: Springer-Verlag).
Lindblad, B. A. 1968, *Lund. Medd.*, No. 237.
Osawa, K., Ichimura, K., and Shimizu, M. 1968, *Tokyo Astr. Bull.*, Ser. II, No. 184.
Owen, T. 1967, *Icarus*, **6**, 108.
Takenouchi, T., Tomita, K., and Hirayama, T. 1968, *Tokyo Astr. Bull.*, Ser. II, No. 183.
Taylor, G. 1968, *Nature*, **219**, 474.
Urey, H. C. 1959, *Hbd. d. Phys.*, **52**, 363–418.
Vaucouleurs, G. de, and Vaucouleurs, A. de. 1961, *Houga Annales*, Vol. **3**, Part 1.