1970AJ.....75..589E

THE ASTRONOMICAL JOURNAL VOLUME 75, NUMBER 5 JUNE 1970

Photoelectric Measurement of Lunar Occultations. III. Lunar Limb Effects

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(Received 27 February 1970)

The problem of interpretation of occultation traces is reconsidered in the light of modern techniques of observation and computation, with special reference to the effects of irregularities on the lunar limb and their discrimination from effects due to intrinsic properties of occulted stars. General diffraction formulas are developed and several series of computed cases are considered. Errors of timing due to limb irregularities are investigated. It is concluded that the time deduced from the standard curve of best fit to the observed trace probably represents a timing appropriate to an average limb height taken over some 20 meters in the neighborhood of the point of geometrical occultation. Tests of somewhat limited applicability can be devised to identify cases where lunar limb irregularities have produced distortion of traces. One statistical test fails to reveal any definite indication of the presence of distortion in the traces obtained up to the fall of 1969. There remain some traces which show features which could be interpreted as due to distortion by limb effects. The use of two-color observations for the discrimination of limb effects from stellar effects is discussed. Necessary conditions for the interpretation of features of two-color observations, as due to duplicity or the presence of a sensible angular diameter, are stated.

I. INTRODUCTION

IT has been recognized for many years that irregularities on the lunar limb can produce significant distortions in the traces of stars occulted in their neighborhood. For example, Diercks and Hunger (1952) made a series of computations of the effects produced by elevations and depressions on the lunar limb, and investigated the influence of their dimensions, shape, and distance from the line of occultation. Calculations of this kind were then necessarily laborious. Diercks and Hunger were able to consider only a rather limited range of cases, and confined themselves essentially to the region of the principal maximum of the diffraction pattern. They pointed out, quite correctly, that the distortions of occultation curves produced by lunar limb irregularities could be seriously misleading and could simulate those due to a sensible stellar diameter. By implication they rule out the usefulness of the occultation method, though they did remark, quoting Jackson (1950) in the same sense, that the question whether distortion ought to be attributed to the lunar limb or to intrinsic properties in the star, might be resolved by making duplicate observations from different sites separated by as little as 100 m. This proposal retains its value in the modern context.

With modern equipment, much fainter stars can be observed so that photoelectric occultation observation can become a routine procedure. Improved time resolution of recorders, and filters of narrow width permit the detection of several fringes on the better occultation traces. Computers allow us readily to produce theoretical traces for a much greater variety of cases than those studied by Diercks and Hunger.

The central problem remains the same: how to decide in any given case whether the analysis of an occultation curve should attribute deviations from the standard form to the lunar limb or to the star itself. The problem affects all aspects of a program of observation of stellar occultations including timing, and the detection of duplicity. As an example, Fig. 1 shows two computed occultation curves for monochromatic light, one for the case of an elevated region of the lunar limb on the line of occultation of a single star, the other for the case of a double star occulted at a smooth limb. The similarity, certainly on the monotonic part of the curve and near the principal maximum, is remarkable. The differences only show up in the higher-order fringes and these will in practice become less pronounced when we include the effects of a finite bandwidth for response of the equipment. There is a good physical reason why the two curves should resemble each other closely. The double-star case is the combination of two occultation curves, one of which is delayed with respect to the other. In the case of the lunar limb irregularity the combination of the two components is not linear, but there are two components, one arising from the general limb, and one from the elevated portion, and we have chosen a case in which these contributions are comparable. As in the double-star case, one of these lags with respect to the other, and the resultant rather closely mimics the effect of stellar duplicity.

In practical cases the occultation curves will not have precisely the form associated with monochromatic light, but will suffer changes due to the bandwidth of the equipment employed (Paper I, Nather and Evans 1970). In any realistic comparison of theory and observation this factor cannot be omitted, but the effects are different for different telescopes and even for the same one if a variety of filters is employed. In discussing the distortion effects due to lunar limb irregularities we have to consider a great variety of cases even for monochromatic light. The situation is already complicated enough without introducing considerations of bandwidth and, for the present we endeavor to bring out some important general considerations which apply to the monochromatic case, while remembering that these may be modified in a variety of idiosyncratic ways

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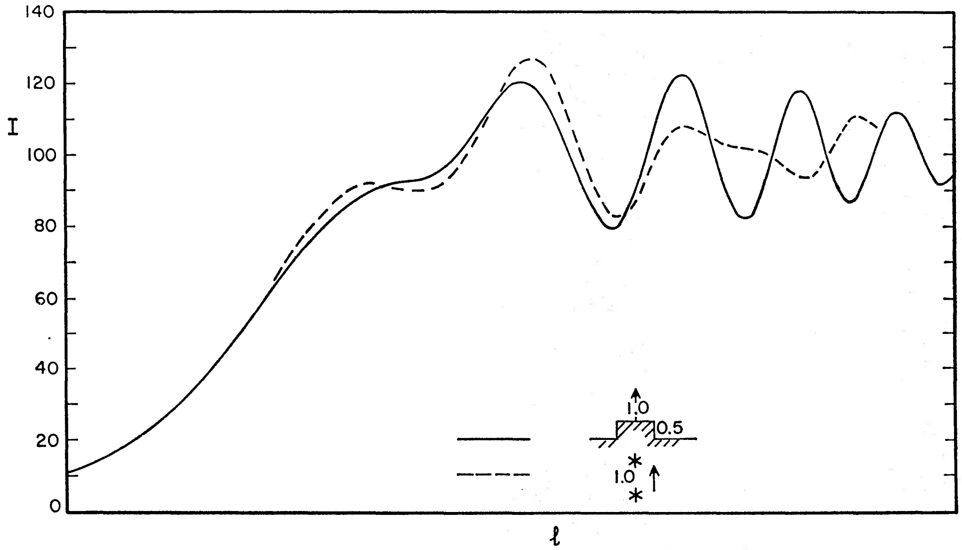

FIG. 1. Similarity of computed occultation traces for a double star and a lunar limb with an obstacle on it.

when they are applied to the practical conditions of observers.

One very important distinction which must be kept in mind is the following. Discussions such as that of Diercks and Hunger, and our own in later paragraphs, deal with situations that *can* arise. There is also the problem of the frequency with which such situations *do* arise. The former can be dealt with in strict mathematical terms. The latter depend on statistical arguments, and will only be settled finally when we have a great many occultation traces to study. If the lunar limb were regularly castellated into a series of five-meter blocks we should probably never get very much information of astrophysical value about occulted stars from studying their occultation traces. If on the other hand such features are rare, distortion will be an acceptable risk and we shall not go far wrong by assuming that the lunar limb is smooth.

In the context of the determination of large stellar diameters, arguments were put forward many years ago (Evans 1955) in support of the opinion that the lunar limb is rather smooth. This was done at a time when it was still possible to believe that lunar mountains were extremely steep and craggy, with slopes of 60 deg or more. No doubt such ideas, which were common in semipopular works, derived from the observed shapes of lunar shadows at the terminator and a failure to remove the vertical exaggeration produced by nearly grazing illumination. It was contended then that lunar slopes on the limb must be much more gentle, for, if they were not, we should encounter the following phenomenon, not merely near grazing incidence, but also for occultations not far removed from the vertex of the lunar motion with respect to the star background. Stars could disappear behind a pinnacle, or reappear in a crevasse, giving double disappearances at the same occultation whenever a slope occurred which was steeper than the inclination to the horizontal of the line of relative motion of the star. Based on some 30 or 40 occultations it was then remarked that no such phenomenon had ever been observed, and with the number of observations more than doubled, we can still make the same statement. Double disappearances where stars go in and out behind mountains only occur at truly grazing occultations. Thus, though the lunar *limb* is not perfectly smooth and level, it is probably not rough in any real sense of the word.

Considering the fact that the lunar *surface* is in many places distinctly rough, it is at first sight somewhat difficult to reconcile these two statements. To recapitulate the argument of the earlier paper, we imagine that we might make the lunar limb rough by planting a craggy mountain along the limb: but then libration would move it below the limb much of the time. If we then tried to preserve the roughness by putting a great many mountain ranges in the same area, we should find, as we can verify by looking at terrestrial landscapes, that the skyline is always much less rough than the individual element which composes the landscape. A chasm in a foreground range is usually filled in by a more distant elevation, and, as every amateur photographer knows we can only get a dramatic photograph of the Grand Canyon or the Matterhorn by carefully choosing our vantage point and shooting along an upward line of sight.

This argument is couched in very general terms, and is devoid of numbers: it appears to hold on almost any scale from the mountain to the sand castle. Only experience will tell whether our opinion is valid. The Apollo pictures and many of the Orbiter photographs have shown lunar skylines which are rather smooth, a term which includes gentle slopes and small elevations and depressions, in short, an absence of pinnacles and crevasses. Even these photographs are not compelling evidence, for we view the true limb from an infinite distance tangentially, and we alter the conditions of

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view by approaching it. We need a very high resolution for our purposes, since, when we considered occultations of Antares, we thought in terms of mountains 40 m high: now, for a general discussion of the diffraction phenomena, we need to think of mountains only 2 or 3 m high. Even the best Orbiter and Apollo data only just bring us into this size range.

After these preliminaries we turn to the general problem of diffraction at the lunar limb.

II. GENERAL DIFFRACTION THEORY

In Fig. 2, let N be the foot of the normal to the plane of the sky in the direction of the star, and D the lunar distance. Let the direction of relative motion of the lunar limb to the star be along MN, where $MN=l$. Take a fiducial line in the plane of the sky through M and normal to MN. Lunar limb irregularities are to be measured from this fiducial line as a function of a coordinate y measured along it. Suppose that from $y=k_n$ to $y=k_{n+1}$ the lunar limb is elevated above the fiducial line to a height h_n. To compute the effect at P from the unobscured area of sky we have to integrate the contributions from all areas of sky typified by the element dA, the integration being carried out for the whole sky to the right of the lunar limb. The computation is for monochromatic light of wavelength λ and, as is usual in this subject we work all dimensions in terms of the natural unit $(D\lambda/2)^{\frac{1}{2}}$. For $D=400\,000$ km, and $\lambda=5000$ Å, the natural unit is, in round figures, 10 m. In angular terms this corresponds to 0$''$.005, and, for the case of an occultation near the zenith occurring at the vertex of relative motion of the moon with respect to the star background, this corresponds to 10 msec of time. The precise relations between the different scales vary with circumstances, but it is useful to keep this general correspondence in mind.

We define the Fresnel integrals,

$$X(v)=\int_0^v \cos(\pi p^2/2)dp: \quad Y(v)=\int_0^v \sin(\pi p^2/2)dp,$$

and readily find that the intensity I at P is given by

$$I=\tfrac{1}{4}(F_1^2+F_2^2),$$

where

$$F_1=\sum_{n=-\infty}^{n=+\infty}\{[X(k_{n+1})-X(k_n)][\tfrac{1}{2}+X(l-h_n)] -[Y(k_{n+1})-Y(k_n)][\tfrac{1}{2}+Y(l-h_n)]\},$$

$$F_2=\sum_{n=-\infty}^{n=+\infty}\{[X(k_{n+1})-X(k_n)][\tfrac{1}{2}+Y(l-h_n)] +[Y(k_{n+1})-Y(k_n)][\tfrac{1}{2}+X(l-h_n)]\}. \quad (1)$$

If we wish to replace the step function describing the lunar limb by a continuous function, $H(y)$, the results

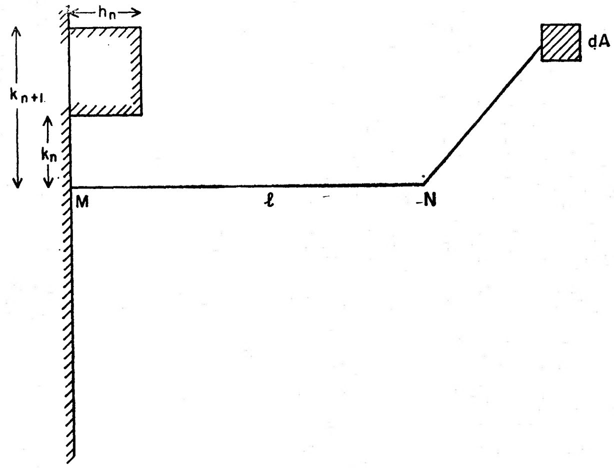

FIG. 2. Coordinates for computation of occultation traces.

of Eq. (1) are modified to

$$F_1=\int_{-\infty}^{+\infty}\{X(l-H)\cos(\pi y^2/2) -Y(l-H)\sin(\pi y^2/2)\}dy,$$

$$F_2=1+\int_{-\infty}^{+\infty}\{Y(l-H)\cos(\pi y^2/2) +X(l-H)\sin(\pi y^2/2)\}dy. \quad (2)$$

A few statements can be made about the general properties of these functions. If $H(y)=0$, then

$$F_1=X(l)-Y(l): \quad F_2=1+X(l)+Y(l)$$

which are the ordinary expressions for a smooth limb.

If $H(y)=h=\text{const}$, then

$$F_1=X(l-h)-Y(l-h): \quad F_2=1+X(l-h)+Y(l-h),$$

that is, elevating the whole limb merely shifts the curve, as it should.

The formulation of Eqs. (2) does not restrict us to the case of normal incidence on the lunar limb. If we write $H(y)=y\tan\theta$ we can rotate the coordinate axes, and find, for a smooth inclined limb,

$$F_1=\int_{-\infty}^{+\infty}\{X(l\cos\theta)\cos(\pi p^2/2) -Y(l\cos\theta)\sin(\pi p^2/2)\}dp =X(l\cos\theta)-Y(l\cos\theta),$$

so that the occultation curve changes only by a scale change in the abscissa. This result is useful, as has been previously remarked (Evans 1955) as a means of determining the average limb slope when the occultation takes place behind a smooth inclined limb. We have not yet satisfactorily defined the nature of the averaging

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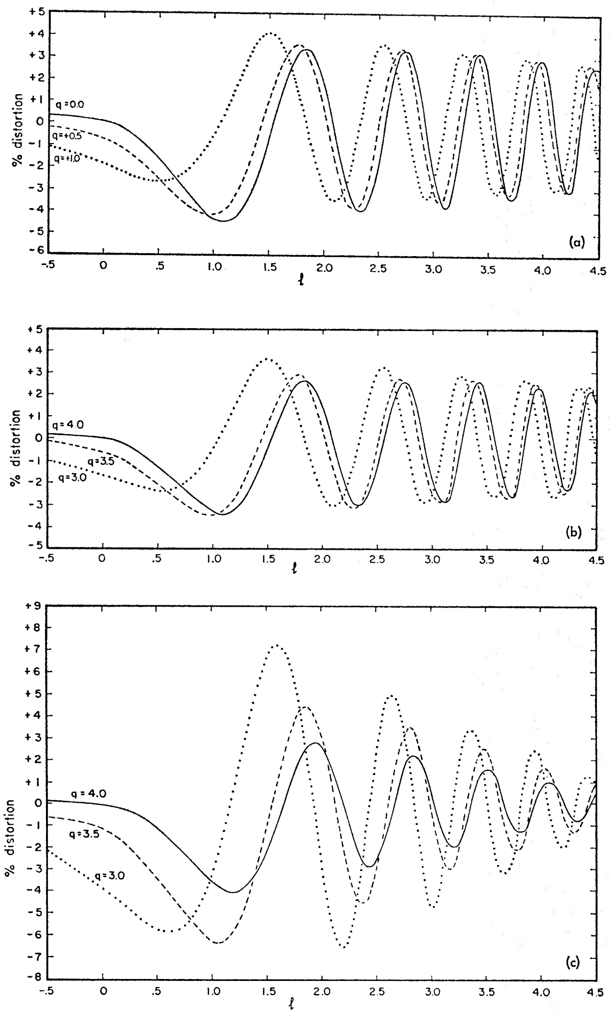

FIG. 3. (a) and (b) Percentage deviations from standard curve for obstacle 0.2×0.2 natural units at increasing off-line distances, q. (c) The same for an obstacle 0.4×0.4.

process, but it appears to take place over a short length of limb of the order of 25 to 50 m in length.

Diercks and Hunger's discussion showed that the distortion produced by a limb obstacle of given size was not strongly dependent on its shape, that it decreased as the obstacle was moved away from the line of occultation, and also decreased as this was reduced in size. More extensive computations suggest that the roughly linear field of results produced by them is an oversimplification. Shape is not very important provided one does not take a given area of obstacle in the form of a pinnacle or crevasse. In other respects the situation is distinctly worse than they thought: for example, they have no curves with the characteristics of our example in Fig. 1.

The real situation is extremely intractable mainly for the reason that any kind of generalization is difficult, and one can never be sure that even a wide variety of computational programs has not missed out some case of critical importance.

Consider the case of a single square elevation of dimensions $p \times p$, centered at a distance q from the line of geometrical occultation. Then we have

$$F_1 = X(l) - Y(l) - \{[X(l)-X(l-p)][X(q+p/2)-X(q-p/2)] - [Y(l)-Y(l-p)][Y(q+p/2)-Y(q-p/2)]\},$$

$$F_2 = 1 + X(l) + Y(l) - \{[Y(l)-Y(l-p)][X(q+p/2)-X(q-p/2)] + [X(l)-X(l-p)][Y(q+p/2)-Y(q-p/2)]\}. \quad (3)$$

These are exact expressions. It would appear from this that we can divide them into two parts, the first two or three terms as the case may be, corresponding to the standard case of occultation at a smooth limb, together with the expressions in curly brackets, which might be thought of as distortion terms, which clearly will tend to zero as p tends to zero. This line of approach might be useful if the distortion terms were always small. In fact, they are not, and when they are not small, the functions F_1 and F_2 can closely approximate to values which give the total occultation curve a different horizontal scale and zero point, in such a way that the deviations from the standard curve of best fit are quite small. It turns out that in many cases it is quite misleading to think of the expressions given by Eqs. (3) along these traditional lines.

Another difficulty of principle arises if we try to consider the case of one small obstacle on the lunar limb. Suppose we approximate to Eqs. (3) by taking in Eqs. (2) $H(y)=0$ everywhere except for some small range within which H has a small positive constant value. The traditional procedure will then be to replace the integrals by the product of the range of integration and the value of the integrand at some point within the range. This will produce from Eqs. (2) a form in which the distortion terms are proportional to the area of the obstacle, but having a magnitude independent of its distance off the line of occultation. The fallacy of this reasoning lies in the fact that the terms cos $(\pi y^2/2)$ and sin $(\pi y^2/2)$ in the integrands of Eqs. (2) have a spatial frequency which becomes indefinitely large the larger the value of y. The integrals do converge, but they converge for the reason, which Fresnel clearly well understood, that, no matter how small the range of integration, as the value of y about which $H(y)$ is nonzero increases, eventually this range will contain a number of complete cycles of these rapidly varying terms. The integration over each complete cycle will approximate to zero, and the integration over the designated range will tend to zero because the fraction of the range occupied by a partial cycle will decrease as the off-line distance of the obstacles increases. The effect of a small obstacle does decrease with increasing off-line distance, but, if the obstacle is a small one the rate of

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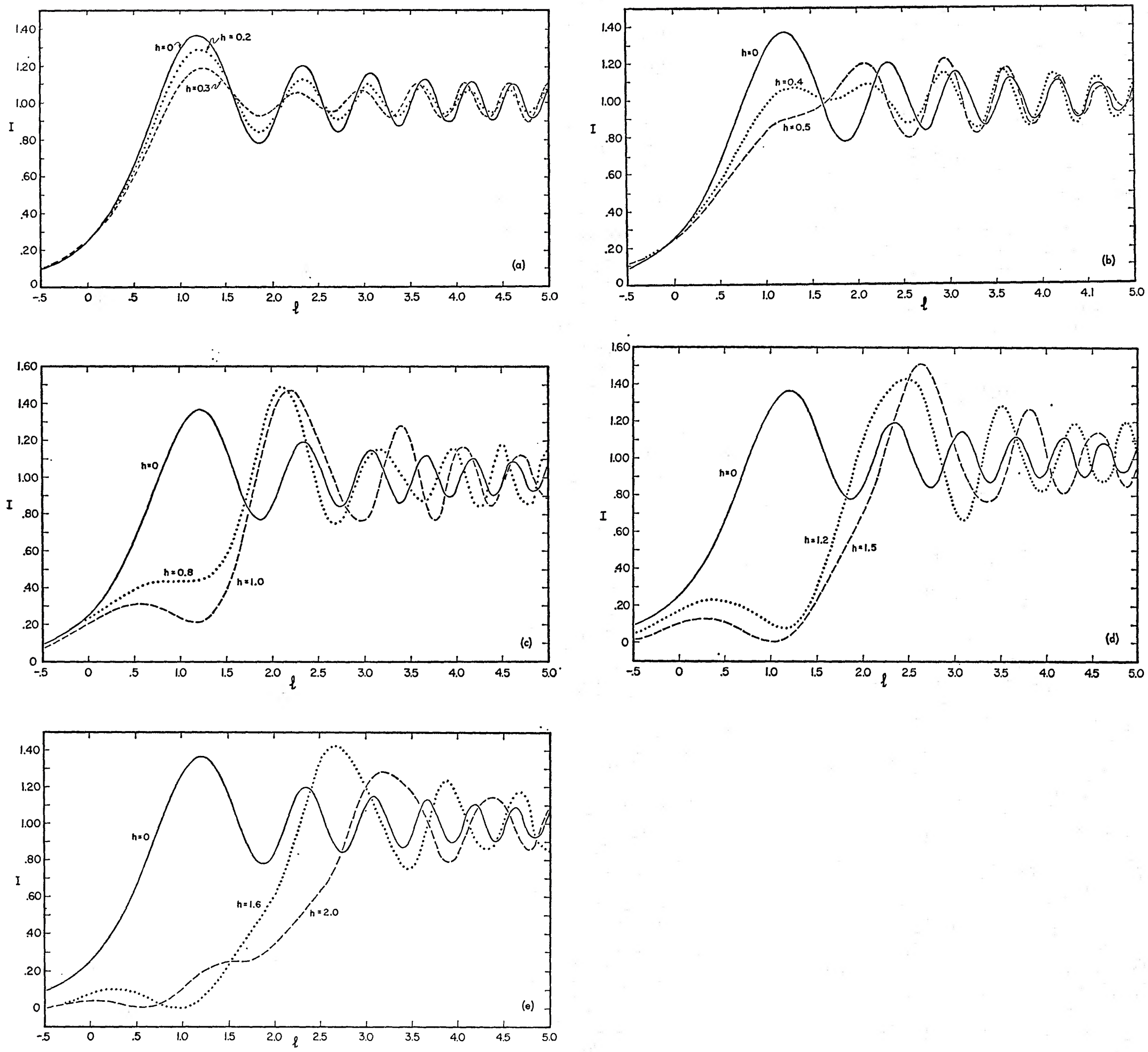

Fig. 4 (a)–(e). Computed traces compared with standard curve for obstacles of height *h*, width 2*h*, for increasing *h*, compared in each case with the standard curve.

decrease is very slow. This is well illustrated by the curves of Fig. 3 (a) and (b) which show the percentage distortion produced by a square obstacle measuring 0.2×0.2 natural units at different distances off from the line of occultation. Figure 3 (c) shows the percentage distortion for a similar obstacle measuring 0.4×0.4 natural units. The amount of distortion caused holds up longer for a small obstacle than a larger.

III. FURTHER DISCUSSION OF DISTORTION

Now consider the occultation curves for a series of obstacles with symmetrical normal incidence, of height *h* and width 2*h*. Computed occultation curves in this series are illustrated in Fig. 4. For values of *h* between zero and 0.3 the amplitudes of the fringes are reduced but no other serious change of form occurs. This imitates the effect of a sensible stellar diameter [Fig. 4(a)]. Distortion is most severe in the range of *h* from about 0.4 to about 0.8 natural units, when the similarity of the curves to double-star traces is most pronounced [Fig. 4(b)]. At larger dimensions, a feeble fringe develops within the geometrical shadow [Figs. 4 (c) and (d)], a feature which might be diagnostically valuable, and the amplitude of the ordinary fringes is enhanced. For still larger dimensions the curve shows diminished distortion but with a bodily displacement of amount about equal to the height of the limb feature [Fig. 4 (e)]. This again is what might be expected. When an irregularity has more than a certain size it becomes the limb.

For any series of computations we might attempt a verbal description of this sort, but it seems more useful to make plots of the characteristic points on the computed curves as defined by Fig. 5. Figures 6 (a)–(d), give examples of such plots derived from various series of computations. They give a useful condensed de-

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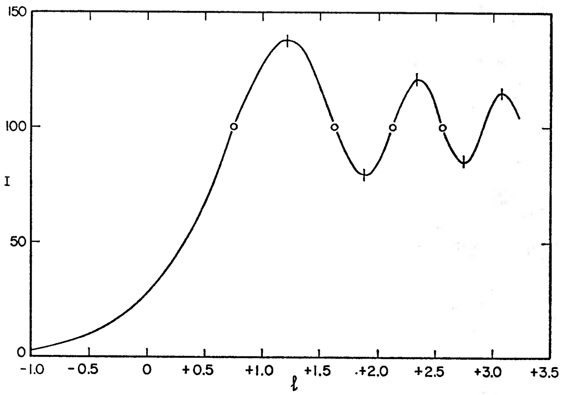

FIG. 5. Characteristic points for a trace, defined as heights and depths of first few maxima and minima, and values of l for 100% crossings.

scription of a series of occultation curves, and enable us to pick out a computed one which closely matches any given observational situation. Unfortunately, it will be clear enough by now that there is no hope of uniquely inferring the form of the lunar limb from the character of the occultation curve.

IV. INFERENCES FROM OBSERVED CURVES

From an occultation curve we normally expect to infer a time of occultation and, in most cases, a limb slope. The occultation time is determined in the best cases by fitting a standard curve (modified by appropriate bandwidth) to the observed one. To do this the computed curve has to be scaled horizontally, because actual occultations take place at inclined incidence on the limb. The scale factor computed from the predicted lunar motion may not match precisely the scale determined by best fit, and from this we infer a slope for the relevant region of the lunar limb. The process is dealt with in detail in a future paper in this series. A timing is inferred from the passage through the 25% level of the curve of best fit. If we see no fringes and think they are lost in seeing fluctuations, we will derive the occultation timing from the passage through the 25% level of the best smoothed empirical curve drawn through the observations. Fringes may be absent because the star has a sensible angular diameter, but one is normally well alerted to this type of exceptional case by other information available about the star in question. Fringes can also be lost, as one can see from Fig. 6 (c), for certain dimensions of limb irregularity. When the fringes cease to be well marked in the monochromatic computation they will have an even smaller amplitude when some bandwidth correction is applied. This may well be the explanation for Run No. 607 of the star BD +23° 1941 which shows a quite slow monotonic trace (Fig. 7) and no fringes, though the star is faint and the trace is noisy, and, with spectral type F8, there is no reasonable expectation of a sensible diameter.

In some observed cases we may find it difficult to get a good fit to the standard curve. For example, if we treat the computed curve of Fig. 8 as if it were an observed curve, we can fit a scaled standard curve to it.

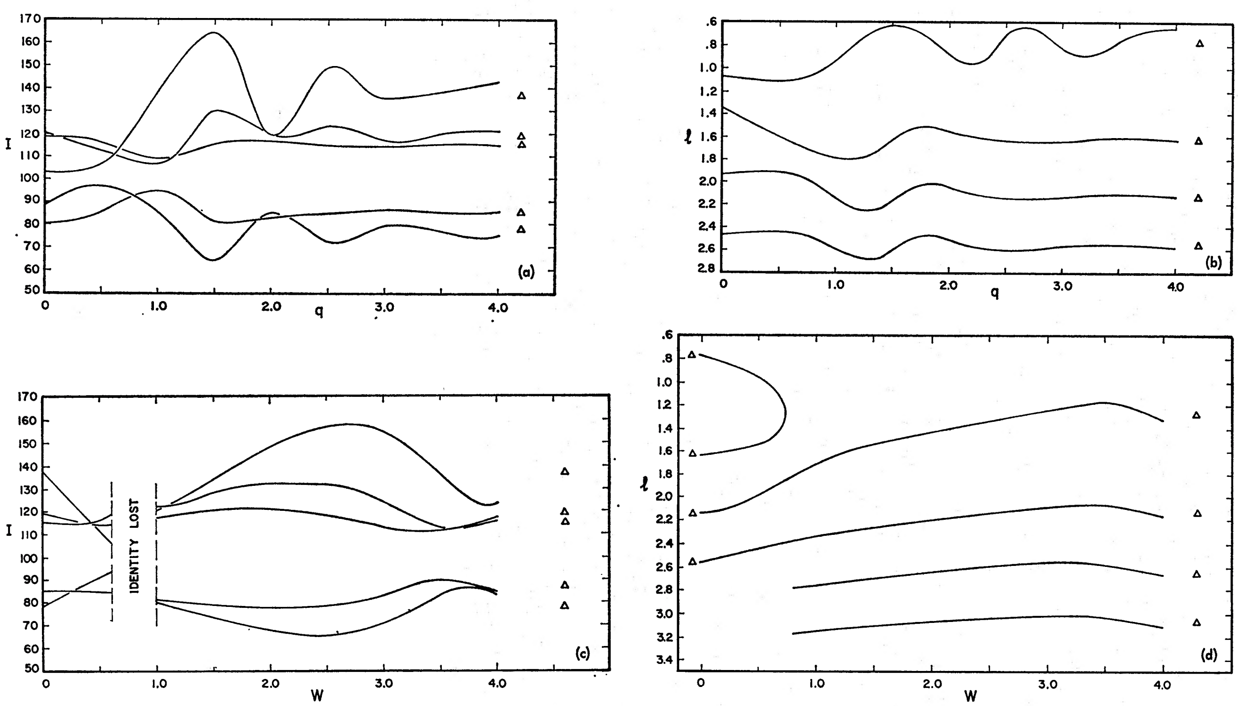

FIG. 6. (a) and (b) Characteristic points for obstacle 0.6 × 0.6 units at increasing off-line distance, q. (c) and (d) Characteristic points for obstacle of height 0.5 and increasing width W, centrally occulted. *Triangles* mark values for $W=0$ and $W=\infty$.

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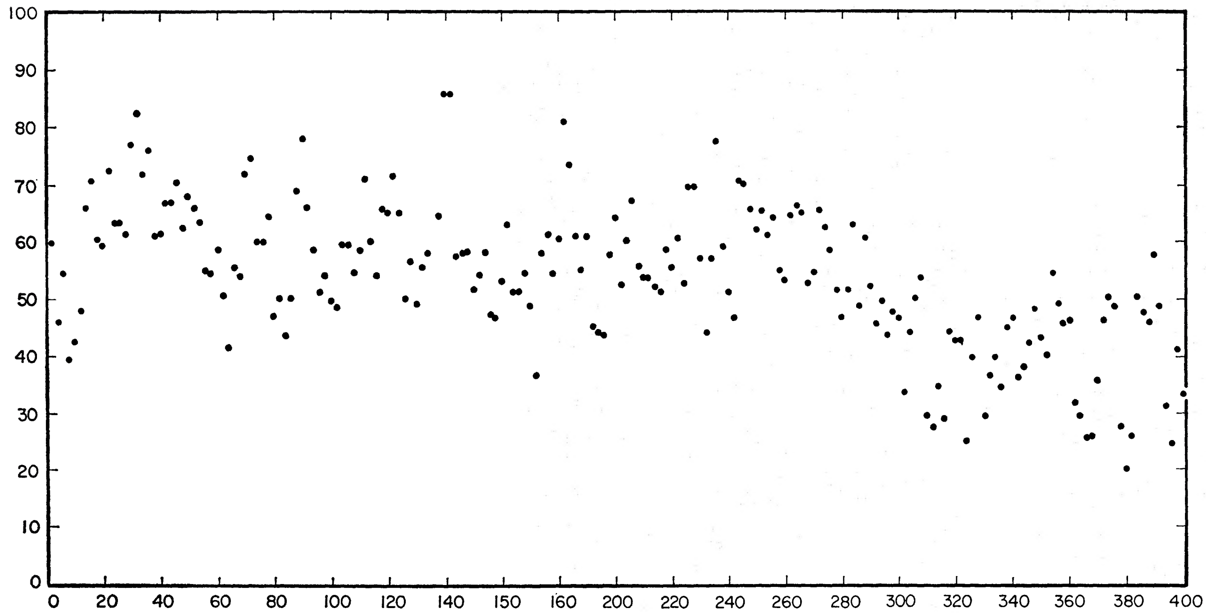

FIG. 7. Observed trace: Run No. 607 of BD +23° 1941.

We readily make a fit to the oscillatory portion of the curve, but the monotonic part of the curve fits badly, because this is deliberately chosen as a case in which the effect is very marked. The 25% passage of the fitted curve differs from the passage through the "observed" curve by 1.3 natural units, or 13 msec according to the rule-of-thumb scale conversion set out earlier in this paper. Depending on the way in which occultation curves are analyzed, it seems that we might run into errors large compared with timing accuracy and larger than we can happily accept.

However, we must ask ourselves what we mean by timing an occultation. We might define this as the instant when the astrometric position of the star coincides with the lunar limb, but if the limb is irregular, do we mean the summit of the obstacle, as we ought if it is large, or the smoothed limb ignoring the obstacle, as we might if it were small?

The answer seems to be that for this purpose the diffraction process in some sense averages the limb over a short distance and that what we can determine is the occultation at a limb which has been smoothed over some range. What precisely this range is may be hard to say, and the range may be different for different aspects of the phenomenon. For example, the curves of Fig. 4 show that for a large limb feature a curve essentially recovers the standard form displaced by the height of the feature only when this is rather more than 4.0 natural units (say 40 m) wide, while the arguments now to be advanced suggest that for timings the averaging may take place over a range of only about half this.

In Fig. 9(a) we plot the abscissa values associated with 25% passages on computed curves for obstacles of a given height h and increasing width W. The occultations take place normally and symmetrically on these features.

In all cases we find the odd result that the timing deduced from the 25% passage hardly changes until the feature is about 20 m wide. The time then jumps to a much higher value, which tends to return as the obstacle increases in width to the value corresponding to the whole limb being elevated by the height of the obstacle.

Now we can also estimate occultation times by fitting standard curves to the fringes and finding the position of the 25% point on the fitted curve. If distortion is not severe there will be no difference between these two estimates of the occultation time, the one the actual passage through the 25% level, the other the passage of the standard curve of best fit to the oscillations through the 25% level. If there is a significant difference it may be attributed to limb irregularity. For

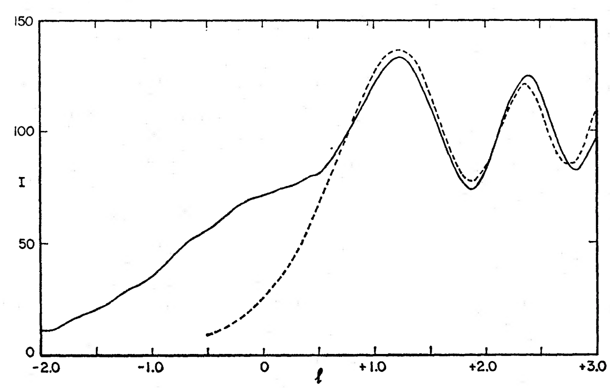

FIG. 8. Fit of standard curve to a computed curve distorted by obstacle of height 0.6 and width 1.2 units.

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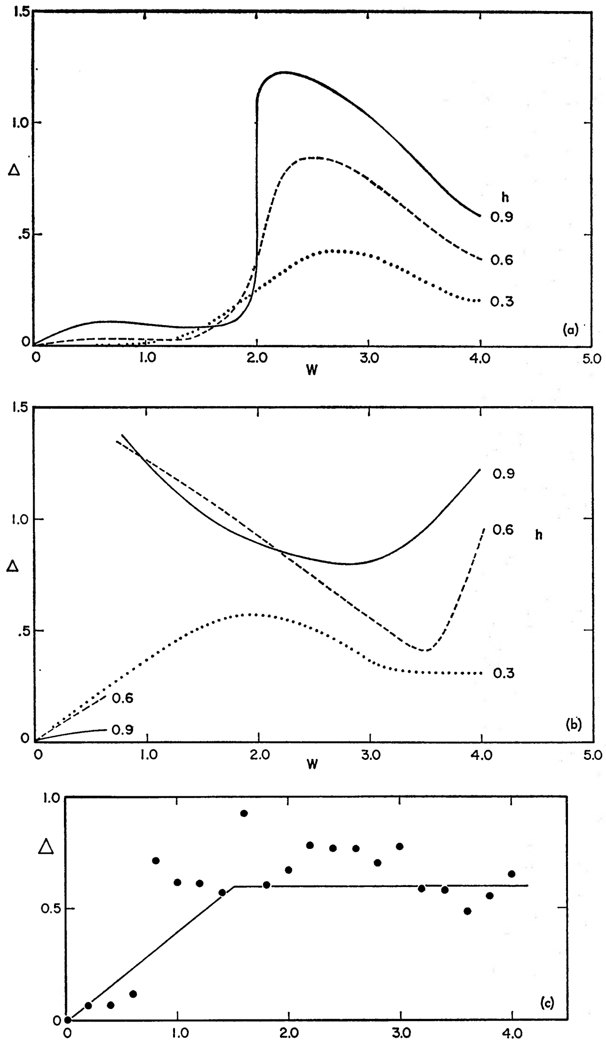

FIG. 9. (a) Timing errors in natural units for computed curves for obstacles of height h and width W. (b) Timing errors for standard curves fitted to fringes of computed curves for same obstacles. (c) Mean time for the actual and fitted curves refers to average limb over about 20 m for obstacle of height 0.6 units.

all the accuracy we need for the present discussion we can locate the 25% point on the fitted curve by stepping back from the second maximum to the first and on by an equal amount. On the standard curve the first two maxima occur near 1.23 and 2.35, with the 25% crossing at zero, so the stepping rule, which correctly adjusts the scale of the abscissa, should, in the standard case give 0.12 instead of zero, corresponding to an error of about 1 msec, which is quite close enough for the present discussion.

If we carry out this rough and ready fitting process we find the timings shown in Fig. 9(b). Taking the mean of these two estimates suggests the following empirical rule: If observed occultation curves can be well fitted by theoretical ones (with bandwidth included), then the time to be adopted is the crossing time both for the fitted curve and the smoothed observed curve. If cases occur where the fit to the oscillatory section is a bad fit to the monotonic section of the curve, the correct procedure would then seem to be to take the mean of the two time estimates, which, in any practical case are unlikely to differ by as much as 15 msec. The time then derived should be, within 2 or 3 msec of the true time of occultation, where the limb so defined is a running mean over a limb length of about 20 m [Fig. 9(c)]. How general this empirical procedure may be is hard to say. In all probability, for discrepancies between the oscillatory and monotonic portions of an observational curve which are not too severe, the time deduced from the curve of best fit over the whole curve will approximate to the value averaged in this way.

The curve of Fig. 8 shows that one effect of a limb obstacle can be to compress the spacing of the diffraction fringes as compared with the standard case and to elongate the monotonic portion of the curve. It will also be clear from Figs. 6 (b) and (d) that for the major proportion of the runs of computations considered, the spacing of the 100% crossings is rather well preserved and tends to be changed by compression (usually with preservation of the ratios) most for those cases where the elevation of obstacles near the point of occultation is high compared with their width or offset.

It was hoped that the spacing might indicate the average slope of a rough limb but this does not appear to be correct. From our observations we have used the spacing of fringes to deduce an average slope of a smooth limb in accordance with the result found in the general discussion of diffraction. Usually the scale change is small and would correspond to a slope of under 5°. The additional effects of limb roughness may introduce a scatter in the values found but are not likely to invalidate the general conclusions especially when the scale factors are near unity.

Any criterion which will enable us to estimate the roughness of the limb will be valuable. In particular we should, when sufficient observations are available, examine the incidence of scale changes of the fringe pattern for occultations near the vertex of relative motion of the moon with respect to the stars. Here compression can only be the result of limb roughness, while significant dilatation might be due either to lunar slopes or to roughness. However, smooth lunar slopes would have to be extremely steep to produce a significant effect and we may safely attribute any significant dilatation to roughness as well. Fortunately, the vertex of relative motion occurs over quite a large range of position angles near 90° under different conditions of lunar motion so that a considerable part of the east limb of the moon can be studied in this way from data for disappearances. Reappearances at the west limb will yield similar information when techniques for observation of these are sufficiently improved.

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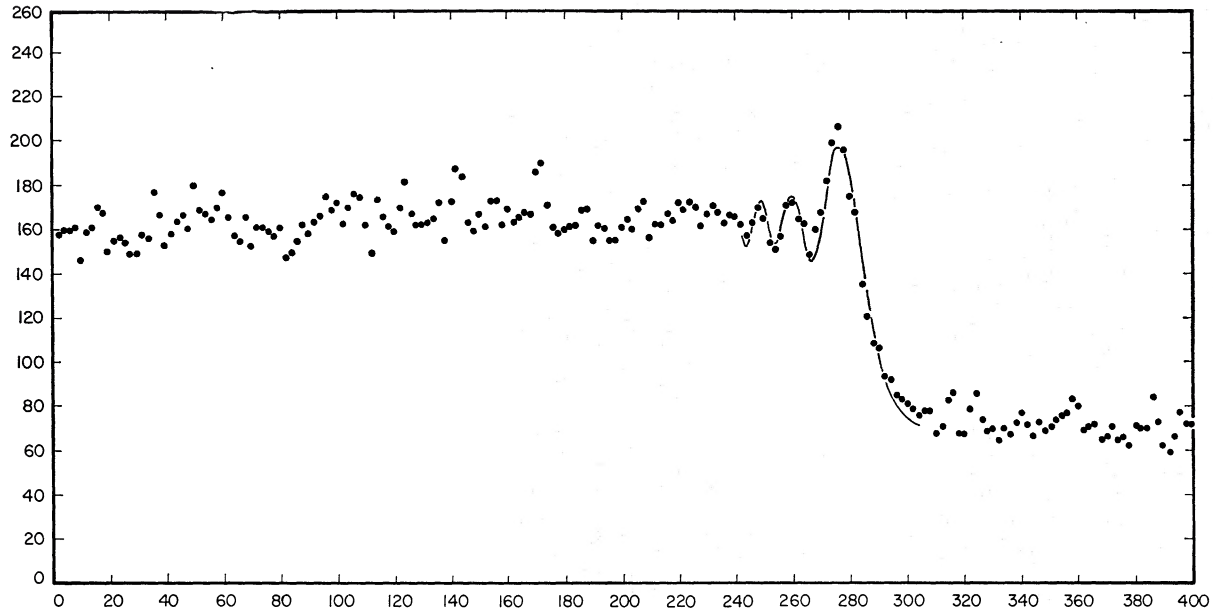

FIG. 10. Fit of computed curve to Run No. 406A of BD +28° 1154 which shows abnormally rapid fringe decrement interpretable as due to limb obstacle of height 0.5 and width 3.5 units.

A study of the plots in Fig. 6 (a) and (c) is of interest. The decrement of the amplitudes of the various fringes changes for a variety of limb obstacles, and, for example, a possible interpretation of Run No. 406A of BD +28° 1154 can be read off these diagrams and a reasonable fit (though with bandwidth neglected) can be secured (Fig. 10). What is not clear is the common denominator of the several different irregularities which might produce the same occultation curve.

All this discussion lies in the field of events which might happen. Is there anything to guide us as to what does happen? The only cause which can ever produce a noise-free occultation trace which anywhere surpasses the 137% level is limb irregularity [see Fig. 6 (a) and (c)]. Duplicity and sensible angular diameter can never do anything except reduce the maximum point on a noise-free curve to a value below the maximum for a standard curve. Now consider the maximum level attained on each of our traces. Some are very noisy, and we call these class c. The better sort, still with appreciable noise, go into class b; those for which the essential features are clearly discernible from noise go into class a. If we measure all our traces for the maximum value attained anywhere in the run, we can plot histograms for the three quality classes. Because we are plotting maxima, the effect of high-frequency noise will be to increase the level of the measured points above the true noise-free maxima.

It is to be expected that the maxima reached will decrease with improving quality. This is illustrated in Fig. 11 where, admittedly for rather few cases, we see that the maxima attained tend down toward the critical level of 137% as the quality improves. If a true intensity above 137% were the sole certain criterion of limb effects, then no case yet observed can be said certainly to demonstrate this. This does not prove that there are no cases in which irregularities are important, since we have no statistical way of assessing the probability of distortion in the one direction as compared with the other. If there are few or no features on the limb capable of producing distortion by increase, this does not imply that there are none producing distortion by decrease because, on the whole the latter

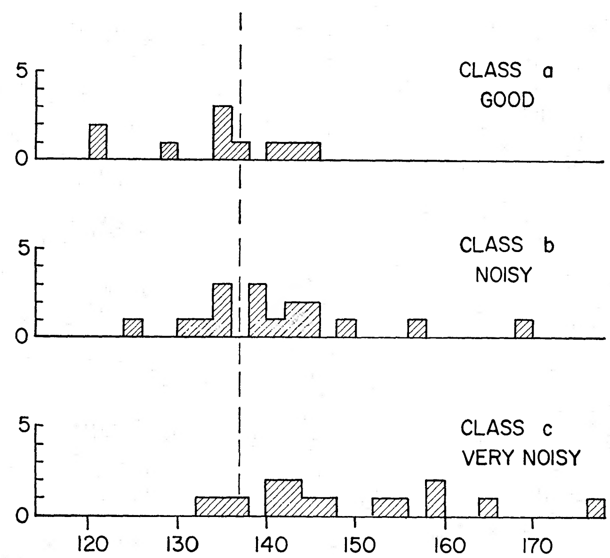

FIG. 11. Maximum percentage heights attained for observed traces of different qualities: Only limb irregularities can give noise-free readings in excess of 137%. With improving quality, maxima approach this figure.

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are smaller than the former. The result, so far as it goes, is none the less encouraging.

V. DUPLICITY

For double stars of comparable brightness and separations in the direction of the line of occultation which are large—say at least 0″.01—there is no doubt of the reality of duplicity [see Run No. 402 of BD +28° 1138 and No. 401 of −03° 3289, Nather and Evans (1970)] and it is unlikely that anyone would suggest that they are artifacts due to lunar limb effects. It is for close separations where the fringes from one component are superposed on those from the other that interpretation becomes uncertain. This is the area where limb irregularities might be invoked as an alternative explanation, and with no little justification as is shown in Fig. 1. Some possible forms are extremely strange in appearance (Fig. 12).

VI. DIAMETERS

The effect of sensible angular diameters on occultation curves has been computed too often to demand extended discussion here (Williams 1939; Evans 1957; Paper I) to mention only a few references. For small diameters, in the range from zero to 0″.005, the amplitude of the oscillations decreases. In the range near 0″.010, the monotonic part of the curve extends, and the oscillations become less pronounced. Near the largest diameters known (about 0″.040), the curve consists almost entirely of a smooth transition from zero to 100% and the only evidence of diffraction is a slight overshoot of two or three percent. Nearly all these effects can be produced by lunar limb irregularities, though probably not those of the large diameter curve if pinnacles and crevasses are ruled out. Reduction of fringe amplitude and extension of the monotonic portion certainly are possibilities. If the curve is produced by a star of large diameter, limb darkening of the star is not a factor of primary importance in determining the curve form, and a diameter can immediately be found as corresponding to twice the abscissa range from the 20% to the 80% levels. In all analyses the diameters are produced in the natural units, which have to be converted to angular measure, so that for monochromatic observations the inferred diameter expressed in natural units will be different for different wavelengths.

VII. TWO-COLOR OBSERVATIONS

Diercks and Hunger's proposal for duplicate observations from neighboring sites remains valuable even if the sites are not close together. Indeed, for close binary stars the sites should be preferably some hundreds of miles apart, so that a complete solution for separation and position angle can be derived.

A proposal made by R. Edward Nather promises to be of outstanding significance in the resolution of these problems implemented either alone or in addition to that of Diercks and Hunger. This is to make simultaneous observations at the same place in two different colors. The effects of limb irregularities are all expressed in natural units, and the change of wavelength, particularly if the observations are made in narrow bands, alters the relation between physical dimensions and natural units. If for instance the adopted wavelengths were 6000 and 4000 Å, i.e., in the ratio 3:2, the size of the natural units would be nearly in the ratio 4:5 and a given limb irregularity would have smaller dimensions in natural units at the longer wavelength than the shorter. Since it is noticeable that when distortion is most severe it is most sensitive to obstacle size, the

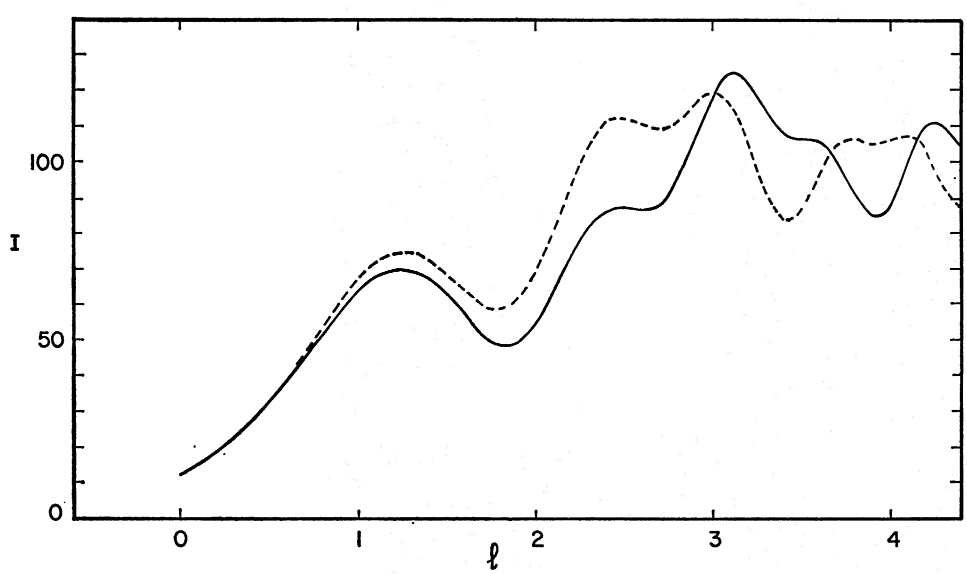

FIG. 12. Computed double-star traces for pairs of equal magnitude and separations of 1.6 and 2.0 natural units.

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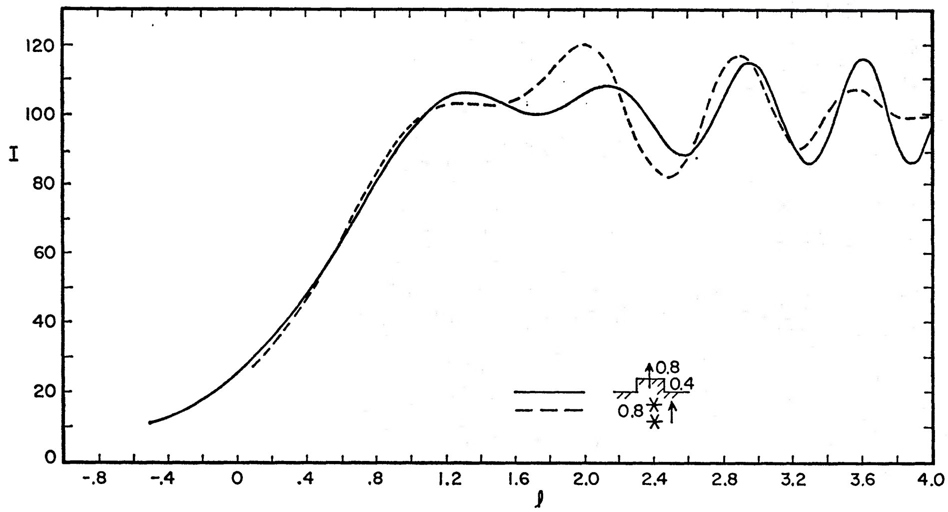

FIG. 13. Transform of Fig. 1 to a new wavelength.

biggest change would be produced by a change of wavelength just where it is most needed. [See Fig. 4 (a)–(e), where the computed pairs of curves are related as above.]

The matched cases of Fig. 1 do not transform in the same way with change of wavelength, as we show in Fig. 13 where the double-star components are taken to be of the same luminosity again at the second wavelength. The reason for this is basically that duplicity involves only dimensions along the line of occultation, whereas the diffraction phenomena produced by a limb obstacle involve dimensions both along the limb and transverse to it.

It is not to be supposed that the relative luminosity of two stellar components will be the same at two different wavelengths, and so the necessary condition for postulating that a given pair of traces are to be accounted for as due to a double star is the following: if each is analyzed on the assumption of duplicity, then the timings of the components must be the same in the two cases within about 2 to 3 msec, which represents about the goodness of fit which can be obtained.

If in the case of angular diameters, a necessary condition for attribution of observed effects to the star is that the diameter in natural units shall vary inversely as the square root of the wavelength, provided that effects such as possession of prominences or other irregularities of form showing a dependence on wavelength within the bands used, can be reasonably excluded.

VIII. CONCLUSION

This closer analysis of the effects of limb irregularities shows a situation more complicated than that envisaged by Diercks and Hunger, and much more difficult to resolve. A few pointers have been found which should help in discrimination, and further study and observational experience may reveal more. The opinion is offered that the whole problem may turn out to be of rather minor importance in the interpretation of occultation traces. Undoubtedly the occultation method, practiced routinely, does reveal binary stars of a closeness undetectable by other means. The identification of still closer cases represents a challenge to further observation and theoretical discussion. The most hopeful approach is the increase of opportunities for duplicate observation by different observers, and, most of all the development of observations in more than one color.

ACKNOWLEDGMENTS

The topics discussed in this paper have been found to be very intractable because the variety of conditions encountered makes it almost impossible to make any generalization to which there are not important exceptions. The degree of coherence which has been achieved is largely due to the numerous discussions with R. Edward Nather and the facility with which computational cases have been produced is almost entirely due to the programming skill of M. M. McCants. I am extremely grateful to both of them.

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