

that mixing is only important at low altitudes in the lower corona. It should be stressed that other interpretations are possible and that mixing may actually dominate to much higher altitudes.

In the rest of this study, possible composition changes through radial diffusion are examined. The multi-component diffusion equations for plasmas have been used, and solutions have been obtained for various assumptions.

DELACHE (1967) has recently published a most comprehensive study of the lower corona that includes a detailed treatment of the effects of diffusion. The trends found here lend support to some of his conclusions although the differences in approach, parameters, and equations are such that detailed comparisons are difficult. The diffusion equation that he has used is for a rare species in a predominantly hydrogen plasma. In this equation, the coefficients are independent of abundances and can lead to unstable solutions. By using abundance-dependent coefficients, these solutions become stable; however, they also indicate a preference for the use of multi-component equations with abundance-dependent coefficients. Delache, on the other hand, has treated each charge species separately with detailed consideration of production and loss. Only the average charge at each temperature has been considered here. This procedure has been partially justified through a study of the change in the solutions with variation of the charge of the heavy ions. The results indicate that enhancements increase with increasing charge and that average charges that are much smaller than probable charges appear to be sufficient to give the trends that have been obtained.

Delache has also considered the possibility that thermal diffusion may introduce instability of stratification by driving heavy ions towards higher temperatures and altitudes. This has been treated in a slightly different manner here.

2. Mixing

Since non-radiative transport of heat and transport of particles in gases are related processes, information on the presence or absence of mixing can be inferred from an examination of the thermal gradient. ATHAY (1966), DUPREE and GOLDBERG (1967), and POTTASCH (1967b) have reported that this gradient in the lower corona appears to be consistent with a constant conductive flux of heat (i.e. $T^{5/2} \partial T / \partial r$ appears to be constant) between 10^5 and 10^6 K. This result may be interpreted in the following way. The principal source of heat is above 10^6 K and the principal heat sink is below 10^5 K for this region (see discussion, KUPERUS and ATHAY, 1967.) Magnetic fields do not affect heat transport so that they are small or are mainly radial. Heat is transported mainly through collisional conduction so that transport through turbulence is small.

A plausible inference from this interpretation is that particle transport by turbulence is also small so that diffusion is likely to be important between 10^5 and 10^6 K.

Other interpretations are also possible of which a few are considered here. The magnetic field may be weak and heat transport by turbulence may also have a coefficient that varies as $T^{5/2}$. This implies a rapid variation of turbulent transport with T

but may be possible. The magnetic field may have an appreciable horizontal component so that radial heat transport is altered from the $T^{5/2}$ dependence. Turbulent transport may be able to compensate so that the $T^{5/2}$ dependence is restored. This would seem to require a fortuitous set of conditions but may be possible. Heat sources and sinks may be distributed in such a way that turbulence may compensate to give the appearance of a conductive atmosphere.

On the other hand, the analyses may be consistent with considerable transport of heat through turbulence.

The neutral atmosphere of the earth has been examined for possible analogies in the processes of mixing and diffusion although conditions are quite different in the lower solar corona. Below 100 km both meteor trails (GREENHOW, 1959) and sodium vapor trails (BLAMONT and DE JAGER, 1962) show lateral growth that is characteristic of turbulence. Above 100–110 km, BLAMONT (1963) and BLAMONT and DE JAGER (1962) have found that the growth of sodium vapor trails can be explained by diffusion alone. Blamont indicates that although the altitude of the transition may vary, the transition from turbulent to diffusive regions occurs rather abruptly in 0.1–1 km.

Composition measurements of N_2 and Ar appear to be in qualitative agreement with the sodium vapor results. Above some altitude that is below 110 km (MEADOWS-REED and SMITH, 1964) or 120 km (HEDIN and NIER, 1966) the Ar-to- N_2 ratio appears to be below sea level values and to change with altitude in a manner that is characteristic of diffusive equilibrium.

Perhaps this transition from turbulent to laminar flow (LANDAU and LIFSHITZ, 1959; BLAMONT, 1963; KELLOGG, 1964) can be understood qualitatively in terms of rates of mixing and diffusion. The diffusion rate is proportional to the diffusion coefficient which varies approximately as T^a/n ($0.5 < a < 1$), where n is the number density. Since T increase and n decreases with altitude near the transition, diffusion rates increase with altitude. In the laminar flow region, disturbances on the flow that can produce eddies that will give mixing rates comparable to the local diffusion rates tend to decay. The Reynolds number, R , for these eddies is less than the critical Reynolds number, R_c . The kinematic viscosity, which appears in the denominator of R , varies approximately as the diffusion coefficient so that it tends to decrease R with altitude. At lower altitudes where R just exceeds R_c , periodic flow of finite amplitude may appear for the eddy mode of interest. At slightly larger R at slightly lower altitudes, disturbances on the flow tend to grow to such an extent that mixing rates exceed diffusion rates. The abrupt transition may then be due to (i) the rapid increase in diffusion rates with altitude; (ii) the tendency for the kinematic viscosity to produce a rapid decrease in R with altitude; (iii) the rather abrupt change from growth to decay of disturbances with change in the Reynolds number.

Such a transition from turbulent to laminar flow may or may not exist in the lower corona. Such parameters as temperature, temperature gradient, winds, and wind shear are or may be very different. At the earth's transition, the molecules are mainly neutral so that ions and electrons probably play a minor role; in the lower

corona, which is almost fully ionized, neutrals are probably very rare. The state of the gas is important since magnetic fields can have such large effects on transport properties of plasmas.

Unfortunately, as BILLINGS (1966) has commented in his review, there is a scarcity of definite information on magnetic fields in the corona although they have been postulated on numerous occasions. Lacking knowledge of coronal magnetic field, we have proceeded under the assumption that there may be times and locations where fields are of such a nature that they do not affect mixing or diffusion. Magnetic field with a substantial radial component will probably permit radial diffusion and tend to inhibit mixing.

In analogy with the earth's neutral atmosphere, a transition between mixing and diffusive regions has been assumed. Since diffusion rates and kinematic viscosity increase rapidly with altitude as on the earth, the lower region is likely to be mixed and the upper region diffusive. For simplicity, the transition has been assumed to be very abrupt as it appears to be in our atmosphere.

Some temperatures at which this transition may occur have been estimated by assuming that the critical Reynolds number in the corona may be scaled from the one at the earth's transition. The critical Reynolds number is LV/ν where L is the eddy size for rapid mixing, V is the eddy velocity, and ν is the kinematic viscosity. For ν at the earth's transition the 0°C value of CHAPMAN and COWLING (1961) for air has been used: $\nu = 3.4 \times 10^{18}/n (\text{cm}^2/\text{sec})$, where n is the number density. At 105 km, n is approximately $4.5 \times 10^{12} \text{ cm}^{-3}$. For ν in the lower corona, BRAGINSKII's (1957, 1958) result as given by SPITZER (1962) has been used assuming photospheric composition and coronal temperatures. For the particle density, the constant pressure approximation has been used (see discussion by POTTASCH, 1967b), so that $n = n_0 T_0/T \sim 6 \times 10^{14}/T$. In the lower solar corona, the approximate value of ν of $2 \times 10^{-7} T^{7/2} (\text{cm}^2/\text{sec})$ has been used. The velocity in the Reynolds number has been scaled by $(T/m)^{1/2}$ where m is the average molecular or ionic mass. If the L in the Reynolds number is scaled by solar and earth radii, the T of the transition from turbulent to diffusive regions is at approximately $5 \times 10^4 \text{ K}$. If L is scaled by local density scale heights, $kT/\bar{m}g$, where \bar{m} is the average particle mass, the temperature of the transition is at approximately $0.9 \times 10^5 \text{ K}$.

This latter value is near the 10^5 K value that is the lower limit from conductive heat flux inferences. In the calculations below, 10^5 K has been used for the boundary between turbulent and diffusive regions. Sufficient mixing to give photospheric composition has been assumed to this temperature; diffusion only has been assumed to occur above.

3. Temperature Gradient and Other Parameters

Diffusion in plasmas depends on such parameters as the temperature gradient and the masses and charges of the various ions. These and other parameters are discussed here.

The lower corona appears to be a region where the temperature rises rapidly

with altitude to a maximum value, T_{\max} , and then remains constant or decreases slowly with altitude (BILLINGS, 1966). The thermal gradient above 10^5 K has been obtained from the assumption that heat is transported down by ordinary heat conduction. For the lower corona, the main heat source is assumed to be near T_{\max} and the main sink below $T_0 \equiv 10^5$ K. The thermal gradient is given by

$$\frac{\partial T}{\partial r} = \left(\frac{T_0}{T} \right)^{5/2} \left(\frac{\partial T}{\partial r} \right)_0, \quad (1)$$

where $(\partial T/\partial r)_0$ is the gradient at T_0 . Values of this latter gradient may be approximately obtained by specifying that T shall attain coronal temperatures at some altitude.

BILLINGS (1966) finds indications from line widths that T_{\max} is reached within 20000 to 30000 km of the photosphere. He cites other evidence that indicates that T_{\max} is attained at 10^5 to 2×10^5 km above the photosphere.

Equation (1) may be integrated to give:

$$T = T_0 \left[1 + \frac{7}{2} \frac{1}{T_0} \left(\frac{\partial T}{\partial r} \right)_0 (r - r_0) \right]^{2/7}, \quad (2)$$

where r_0 is the radial distance at which $T = T_0$. Figure 1 shows Equation (2) evaluated for $(\partial T/\partial r)_0 = 0.1, 0.2, 0.5$ and 1 K/cm. A wide range of T_{\max} values for both ranges of T_{\max} altitudes mentioned by Billings are represented by these choices of $(\partial T/\partial r)_0$.

Because of the rising temperature from T_0 to T_{\max} , it seems likely that heat sources should exceed sinks between T_0 and T_{\max} . If this is so, less heat is conducted down with increasing altitude; then, thermal gradients may be less at altitudes above

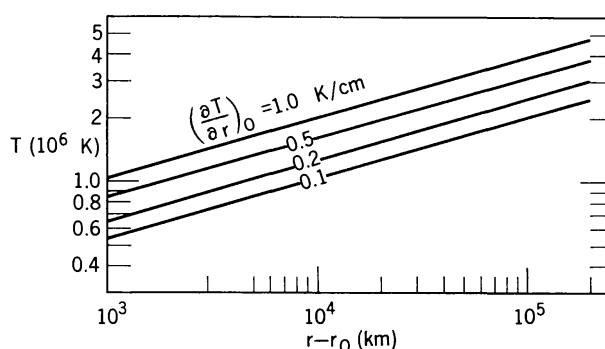


Fig. 1. Temperature as a function of altitude above r_0 where $T = 10^5$ K for a conductive lower corona. Various values of the thermal gradient at r_0 are indicated.

r_0 for a given $(\partial T/\partial r)_0$. On the other hand, if thermal gradients above r_0 are smaller, T_{\max} values for coronal temperature would also be smaller for a given $(\partial T/\partial r)_0$. No attempt has been made to find the thermal structure or the range of thermal structures that might actually exist in the lower corona. Instead, thermal gradients that are consistent with the assumptions and appear likely to cover a range of possible thermal gradients have been used.

Diffusion, especially thermal diffusion, depends on both the mass and charge of each ion species. The mass dependence is relatively much smaller and changes in mass with T are at most a few electron masses. For these reasons, a single mass has been used for all of the isotopes of each element at all T .

Both H and He, the most plentiful elements at the base of the assumed diffusion region, are likely to be stripped of electron throughout the assumed diffusion region (THOMAS and ATHAY, 1961). For other elements that have been considered, C, N, O, Ne, Si, Ca, and Fe, complete stripping is unlikely and the ionic charge is dependent on T . This dependence is illustrated in Figure 2, where the results of ALLEN and

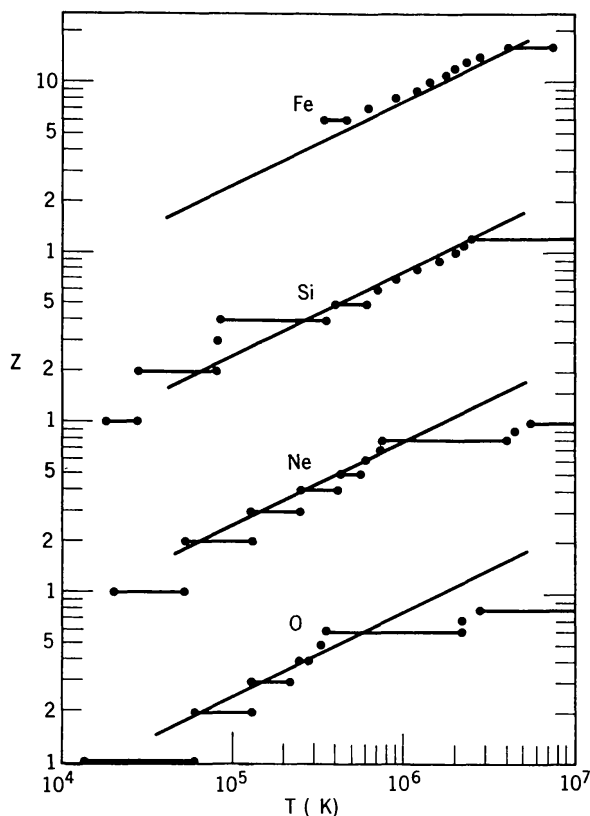


Fig. 2. Probable ionic charges from ALLEN and DUPREE (1967). Bars give temperature ranges where over 1/2 of an element has a particular charge. Dots give temperatures where a species is most abundant. The comparison straight line is $Z = 2.5(T/10^5)^{1/2}$.

DUPREE (1967) are plotted. These results include dielectronic recombination and auto-ionization. In this figure, the bars show ranges of T where more than one-half of an element has a particular charge. The dots give the T where a particular charge is most abundant.

Because these ions are not fully stripped of electrons, collision between them and the more energetic H^+ , He^{++} , and electrons will not be strictly Coulomb (CHAPMAN and COWLING, 1961; ALLIS, 1960; BURGERS, 1960). Some of these collisions are inelastic and result in excitation or ionization. It is likely that many more are elastic. Since the force law for these collisions are not Coulomb, diffusion can be modified

from what might be expected if they were strictly Coulomb. In thermal diffusion, for example, heavy species tend to migrate toward high temperatures for the Coulomb force law but toward low temperatures if the force law is like that for neutrals. It is quite possible that core collisions are similar to those between neutral particles.

The relative effectiveness of momentum transfer due to core and Coulomb encounters has been estimated through the ratio of average collision cross-sections. There are unfortunately few experimental or theoretical results on which to model collisions involving e , H^+ , and He^{++} with the cores of the heavy ions. The assumption has been made that such collisions are similar to those between charged species and neutral atoms for which data and theory exist (McDANIEL, 1964). At very low electron energies, elastic cross-sections with neutral atoms can be quite large. However, since electrons gain sufficient energy when approaching ions with core electrons, gas kinetic cross-sections appear to be appropriate and have been assumed. For collisions of singly charged ions in their parent gases, cross-sections that are about twice gas kinetic cross-sections appear appropriate from mobility studies. Since symmetry effects such as charge transfer are unlikely in collisions of H^+ and He^{++} with core electrons of heavy ions, gas-kinetic cross-sections have also been assumed for these collisions.

In the thermal diffusion of neutral atoms, the rigid elastic sphere model gives the largest effect; this model has been used for the core collisions. With an encounter distance, σ_{12} , for this model, the ratio of average collision cross-sections, which have been integrated over all impact parameters and relative velocities, is then (CHAPMAN and COWLING, 1961):

$$\frac{[\Omega_{12}^{(1)}(1)]_{r.e.s.}}{[\Omega_{12}^{(1)}(1)]_{Coul}} = \frac{\frac{\sigma_{12}^2}{2}(2kT)^{1/2}}{Z_1^2 Z_2^2 e^4 (2kT)^{-3/2} \ln \Lambda},$$

where Λ is the ratio of the Debye length to the minimum impact parameter for Coulomb collisions.

For the lower corona, $\ln \Lambda \approx 20$ appears to be appropriate. The subscript 1 is for e , H^+ , and He^{++} ; the subscript 2 is for the heavy elements with cores.

Data from ionic radii in crystals give radii that are quite near $1 \times 10^{-8} Z_2^{-1/2}$ (cm) for elements heavier than Ne. The ionic radii of C, N, and O appear to be about a factor of 3 smaller. An encounter distance for e , H^+ , and He^{++} with cores of $\sigma_{12} = 2 \times 10^{-8} Z_2^{-1/2}$ (cm) has been assumed. In Figure 2, a solid curve that is $Z_2 = 2.5 \sqrt{(T/10^5)}$ has been included for comparison purposes. Since the four elements have probable charges that are quite near this curve, this value of ionic charge has been assumed. With these estimates the ratio of average collision cross-sections is:

$$\frac{[\Omega_{12}^{(1)}(1)]_{r.e.s.}}{[\Omega_{12}^{(1)}(1)]_{Coul}} \approx \frac{0.92 \times 10^{-2} \sqrt{T/10^5}}{Z_1^2}.$$

For electrons the rigid elastic sphere cross-section should be modified since the attraction between the ion and the electron will cause more electrons to collide with

the core than would occur with neutral atoms. From trajectories and angular momentum conservation, the increases in cross-section of electrons striking the core has been estimated to be a factor of 3. It thus appears that core collision cross-sections are about 3% of the Coulomb one at $T=10^5\text{K}$ and 10% at $T=10^6\text{K}$ for electrons.

For H^+ and He^{++} , consideration is given to the repulsion that they experience when approaching heavy ions with cores. Let $(3/2)kT_1$ be the kinetic energy of the light ion at large distances from the heavy ion and let this energy be just sufficient for it to come in contact with the core. We may equate this energy to the Coulomb energy at contact

$$\frac{3}{2}kT_1 = \frac{Z_1 Z_2 e^2}{\sigma_{12}}.$$

Using the values of σ_{12} and Z_2 above, the following is obtained:

$$T_1/T = 2.2Z_1(T/10^5)^{1/2}.$$

Where $T=10^5\text{K}$, all light ions whose energy is greater than $2.2Z_1$ times the average energy are estimated to be able to collide with the core. This is very small for He^{++} and about 3% for protons; the fraction of protons that can collide with the core decreases quite rapidly with T .

Although more detailed calculations may be necessary, these considerations seem to indicate that core collisions are not very important in the lower corona unless cross-sections are much larger than the assumed gas-kinetic ones.

The approximation has been made that diffusion is given by essentially Coulomb collisions. For simplicity, a single average charge has been assumed for each element at any particular T . To illustrate the dependence of diffusion on ionic charge, values of the latter that are less than those given in Figure 2 have also been used. For elements other than H and He, ionic charges, $Z=Z_0(T/10^5)^{1/2}$, $Z_0(T/10^5)^{1/4}$, and Z_0 have been used with Z_0 ranging from 1 to 2.5.

The relative composition that has been assumed at the transition is shown in column 3 of Table I. Column 2 is taken from the photospheric composition of GOLDBERG *et al.* (1960).

Of considerable interest are estimates of times required to establish diffusive equilibrium in the lower corona. Using CHAPMAN's (1958) theory, SEATON (1964) has estimated 3 days to establish radial diffusive equilibrium. Using Chapman's theory but with the parameters that have been used here, a few days have been estimated for e, H^+ , and He^{++} to reach equilibrium.

Because these times are not too long, diffusive equilibrium has been assumed for the calculations below.

4. Diffusion

From moments of the Boltzmann equation, BURGERS (1960) has developed multi-component diffusion equations for a strictly Coulomb plasma. These equations have

TABLE I
Relative abundances

Element	logN (phot.)	logN (adopted)
H	12.00	12.00
He		11.00
C	8.72	8.7
N	7.98	8.0
O	8.96	9.0
Ne		8.0
Na	6.30	
Mg	7.40	
Al	6.20	
Si	7.50	8.0
S	7.30	
Ca	6.15	6.3
Fe	6.57	6.6

been used here. For plasmas with two ion species where the heavier species has a small concentration, the approximate equation that results from Burgers' equations appears to be identical to the one developed by ALLER and CHAPMAN (1960) except for a small difference in one of the coefficients (CHAPMAN, 1962). Although H^+ is predominant at the base of the assumed diffusion region, the approximate equations have been found to be inappropriate since composition changes that are calculated with them are so large that the heavy species are no longer rare. When two element equations are generalized for all concentrations of each species, the results again indicate the need for multi-component equations since calculated composition changes are sufficiently large that interaction between initially rare species becomes important.

Let the subscript, s , refer to charged particles with 1 for electrons and 2, 3, 4... for the elements. Let m_s , Z_s , and n_s be masses, charges, and number densities. The one-dimensional form of Burgers' diffusion equation is then:

$$\frac{\partial}{\partial r} p_s - \frac{\rho_s}{\rho} \frac{\partial}{\partial r} p - n_s Z_s e \mathbf{E} = \sum_t w_t W'_{st} + \frac{5}{2} k \frac{\partial T}{\partial r} \sum_t n_t H'_{st}, \quad (3)$$

where $p_s = n_s kT$; $\rho_s = n_s m_s$; $\rho = \sum \rho_s$; $p = \sum p_s$; e = absolute value of the electron charge; \mathbf{E} = electric field; w_t = diffusion velocities relative to the mean mass flow velocity; and W'_{st} and H'_{st} are elements of matrices.

Following the procedures of ALLER and CHAPMAN (1960) and BURGERS (1960), $e\mathbf{E}$ has been obtained from the electron equation and this has been substituted into the equation for the elements. The resulting equation is then:

$$\left. \begin{aligned} \frac{\partial}{\partial r} \ln n_s T - Z_s \frac{\partial}{\partial r} \ln \left(1 + \frac{1}{Z} \right) = & \frac{5}{9} \sum (H_{sh} + Z_s H_{1h}) \frac{\partial \ln T}{\partial r} \\ & + \frac{(m_s - Z_s \bar{m} + Z_s m_1)}{\bar{m}} \frac{\partial \ln p}{\partial r} + W_s + Z_s W_1, \end{aligned} \right\} \quad (4)$$

where

$$Z = \frac{\sum (1 - \delta_{1i}) n_i Z_i}{\sum (1 - \delta_{1i}) n_i},$$

$$\bar{m} = \frac{\sum n_i m_i}{\sum n_i},$$

$$H_{sh} = \sum_k \Omega_{sk} \frac{Q_{hk}^*}{Q},$$

$$\Omega_{sk} = \frac{Z_s^2 m_s^{3/2}}{(m_s + m_k)^{3/2} m_k} \left[-1 + \frac{\delta_{sk} 2^{3/2}}{n_s Z_s^2} \sum_i n_i Z_i^2 \left(\frac{m_i}{m_s + m_i} \right)^{3/2} \right],$$

$$Q_{hk} = \frac{Z_h^2 m_h^{3/2}}{(m_h + m_k)^{5/2}} \left[1 - \frac{\delta_{hk} 2^{5/2}}{27 n_h Z_h^2} \sum_i n_i Z_i^2 \left(\frac{m_i}{m_h + m_i} \right)^{3/2} \left(\frac{m_h}{m_h + m_i} \right) \left(16 + 30 \frac{m_h}{m_i} + 13 \frac{m_i}{m_h} \right) \right],$$

Q is the determinant of Q_{hk} , Q_{hk}^* is the minor of Q_{hk} , δ_{hk} is the Kroneker delta function,

$$W_s = \frac{\frac{4}{3} \sqrt{2\pi} e^4 \ln A}{(kT)^{5/2}} \sum_t n_t Z_t^2 Z_s^2 \left(\frac{m_s m_t}{m_s + m_t} \right)^{1/2} (w_t - w_s).$$

The summations are over electrons and all elements under consideration. The Ω_{sk} and Q_{hk} have been derived from Burgers' similar symbols but are not identical to them since a number of cancellations have been used.

In Equation (4), the term, W_s , is equivalent to the product of the inverse of the ordinary diffusion coefficient (1st approximation) and the difference between drift velocities in binary diffusion. The evaluation of $W_s + Z_s W_1$ for solar wind fluxes through the lower corona gives values that are small compared to the other terms. For this reason, this term has been neglected in the calculations below.

Using

$$\frac{\partial}{\partial r} \ln p = - \frac{\bar{m}g}{kT},$$

and initial conditions and parameters, Equation (4) has been solved on a computer. To check the computer code, three procedures were used in addition to tests for stability, convergence, and reverse integration. First, two element solutions were obtained. These were in agreement with results from the two element equations. In the second check, calculations of electron densities which do not use H_{ij} , were compared with the summation of $n_i Z_i$ over the elements, where H_{ij} are important, and found to be in agreement. In the third check, the derivatives of the densities of small concentration elements were compared with the ones calculated from the approximate two element formulae where one light species is predominant. Approximate checks were also possible where two light elements had large concentrations.

5. Diffusion Results

The results of the integration of Equations (4) are most conveniently displayed when log-log paper is used to plot $n_s(T)$. Although actual densities have been used, only relative changes have been calculated so that other normalizations may be obtained by moving all curves the same distance along the ordinate.

For most of these calculations, a thermal gradient that corresponds to $(\partial T/\partial r)_0 = 1 \text{ K/cm}$ at $T = 10^5 \text{ K}$ has been used. In Figure 3, equilibrium solutions for the multi-component diffusion equations are shown for $Z=1$ for the heavy elements. The e, H,

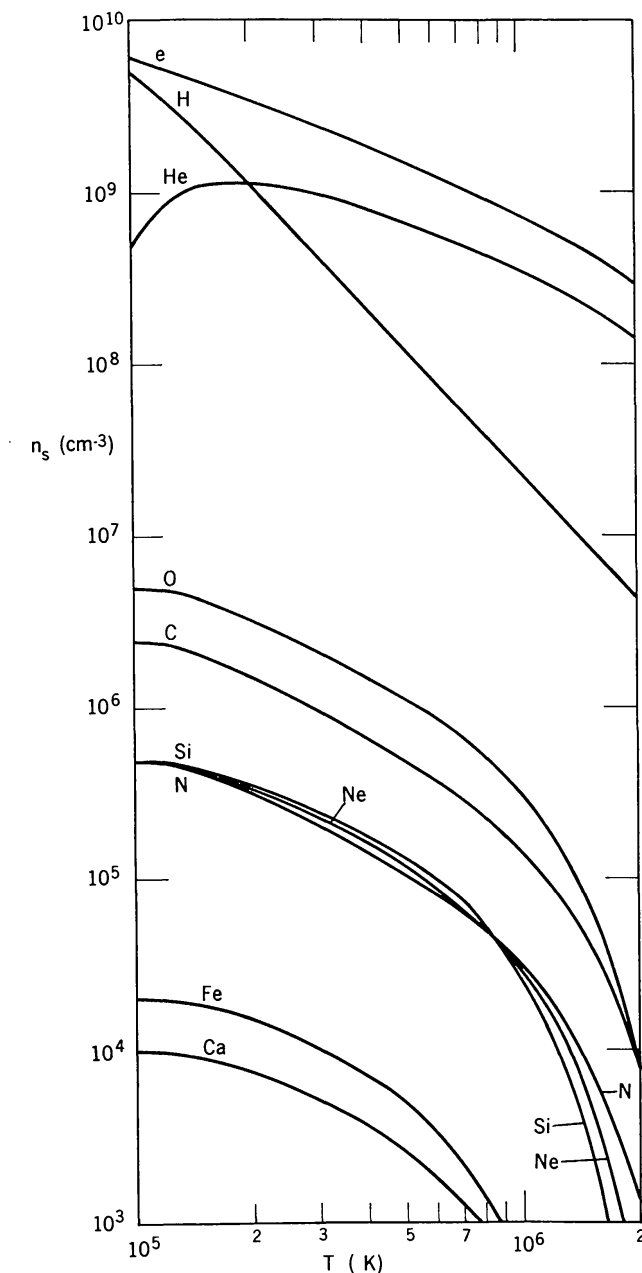


Fig. 3. Density of elements as a function of temperature for the low value of $Z=1$ for C, N, O, Ne, Si, Ca, and Fe.

and He densities are almost identical to results obtained from two element diffusion equations so that their distributions appear to be almost independent of the presence of the other elements. Because these other elements remain rare, their distributions appear to be independent of each other and dependent mainly on e , H , and He .

Let us consider oxygen as typical of these heavier elements. Below 7×10^5 K, thermal diffusion is the main contributor to the distribution; above 7×10^5 K pressure diffusion becomes increasingly important so that the density decreases more and more rapidly. This can be understood through the behavior of the thermal and pressure gradient since $(1/T) \partial T / \partial r$ varies as $T^{-7/2}$ while $(1/p) \partial p / \partial r$ varies as T^{-1} .

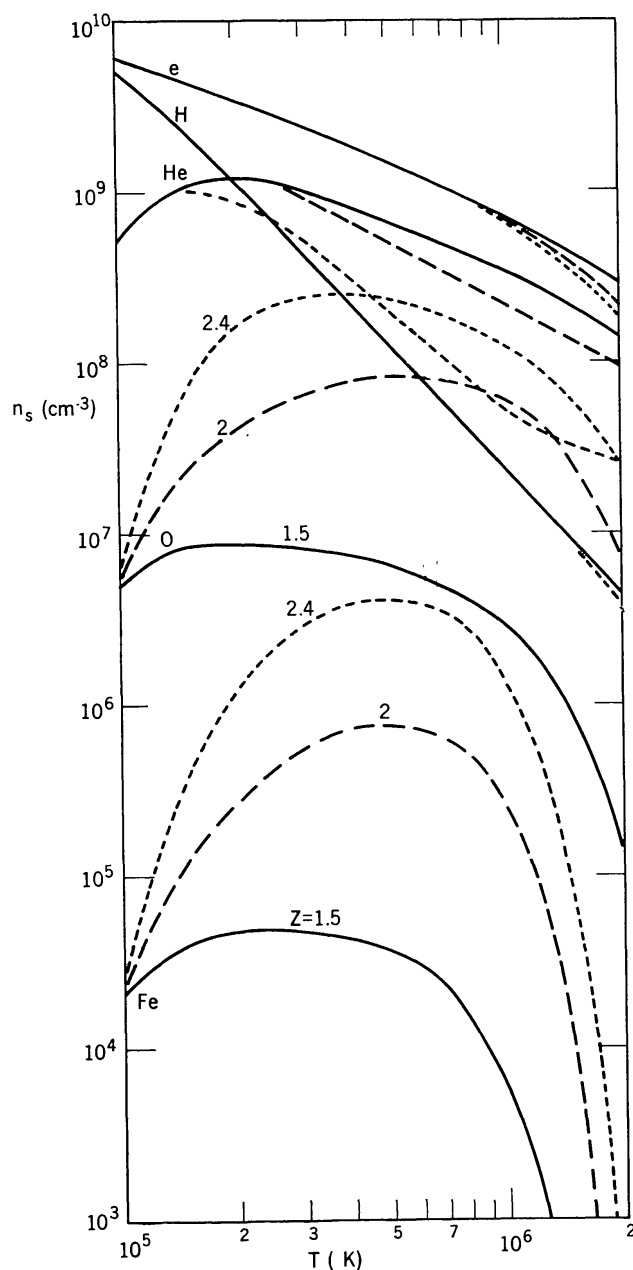


Fig. 4. Composition changes with temperature as a function of assumed values of Z for the heavy elements.

The distributions of the heavy elements show slight increases in thermal diffusion with increasing m_i and the onset at lower temperatures of the importance of pressure diffusion with increasing m_i .

The effect of changing the charge of the heavy elements is shown in Figure 4 where results due to constant values of $Z=1.5$, 2, and 2.4 have been plotted. Although the other heavy elements were included in the calculations, only O and Fe have been plotted. The large changes in composition and its dependence on Z is quite apparent.

Figure 5 shows changes in composition when $Z=2.5(T/10^5)^{1/2}$ which is quite near the full probable charges on the heavy ions. This charge is inappropriate for C, N, O, and Ne above about 10^6 K but has been used in this form for convenience and also

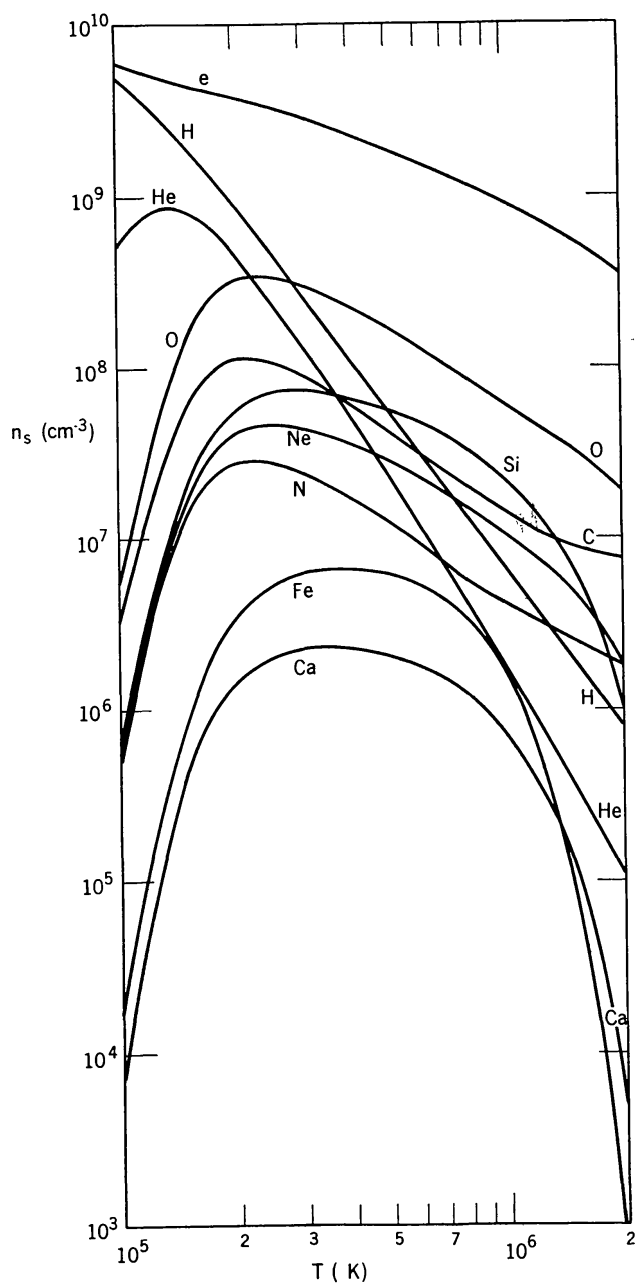


Fig. 5. Composition changes with temperature for $Z=2.5(T/10^5)^{1/2}$ for the heavy elements.

because these light elements showed little tendency to decrease sharply in abundance above 10^6 K when much lower charges were used (See O curve for $Z=2.4$ in Figure 4). Although these calculated composition changes are large, they (except for He) are not as large as those obtained through the use of two element equations with H as one of the elements.

In Figure 6 results for $Z=1.2(T/10^5)^{1/4}$ are plotted for $(\partial T/\partial r)_0$ values of 0.5, 1, and 2 K/cm. Again other heavy elements have been included in the calculations, but only O and Fe have been plotted. At lower T , changes in the thermal gradient produce little change in composition; at higher T , larger thermal gradients tend to extend enhancements of heavy elements to higher T .

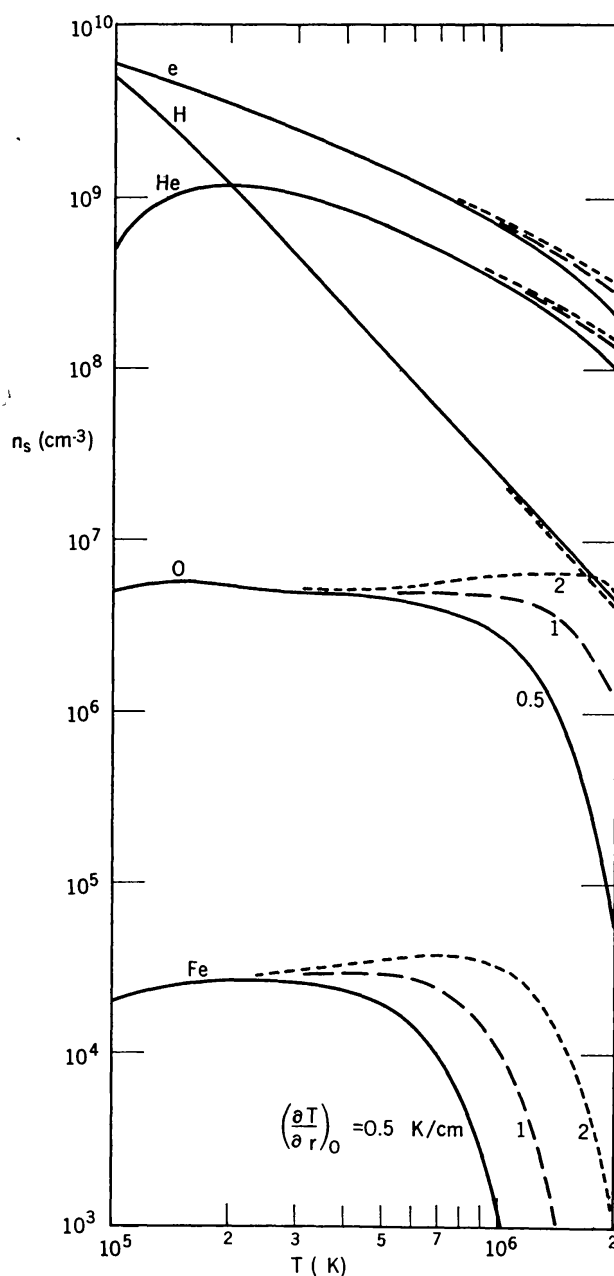


Fig. 6. Composition changes with temperature for $Z=1.2(T/10^5)^{1/4}$ for the heavy elements and for three values of the thermal gradient at 10^5 K.

The results in Figure 7 are similar to those of Figure 5 in that the full probable charges on the ions have been used. Again, of the heavy elements, only O and Fe are plotted although the calculation included the other elements. The elements C, N, O, Si, and Ne have a lower charge of $2.3(T/10^5)^{1/2}$ while Ca and Fe have $Z=2.5(T/10^5)^{1/2}$. The dashed curve has the same $(\partial T/\partial r)_0$ as Figure 5. Although the O distribution is relatively unchanged, Fe is enhanced up to 10 times over results shown in Figure 5. This again illustrates the sensitivity of diffusive composition changes to the ionic charge. The solid curves of Figure 7 are for lower thermal gradients corresponding to $(\partial T/\partial r)_0=0.1$ K/cm. This illustrates the sensitivity of composition

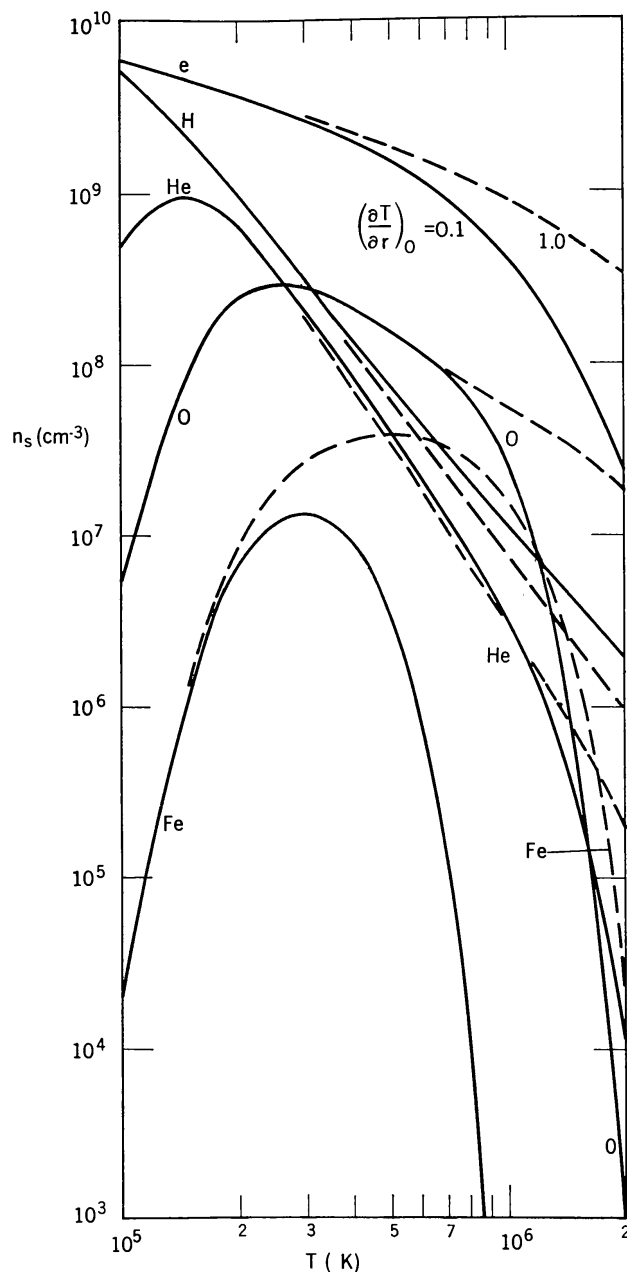


Fig. 7. Composition changes with temperature for $Z=2.3(T/10^5)^{1/2}$ for C, N, O, Ne, and Si and for $Z=2.5(T/10^5)^{1/2}$ for Ca and Fe. The solid line is for a $(\partial T/\partial r)_0=0.1$ K/cm. The dashed line for $(\partial T/\partial r)_0=1$ K/cm.

changes to the thermal gradient when nearly full charges of the ions are used and the necessity for a large enough thermal gradient if diffusion is to account for appreciable concentrations of Fe above 10^6 K.

6. Discussion

If pressure diffusion only were important, a large depletion of the heavy elements would have been anticipated. If thermal diffusion were to change this result to produce either a constant composition or enhancement of heavy elements relative to electrons, it might be expected to do so through the depletion of the lighter elements. This expectation appears to be confirmed in part by these calculations since hydrogen shows much depletion that is only slightly dependent on such parameters as the charge on the heavy elements and the thermal gradient. This depletion is not necessarily in contradiction with solar wind measurements, which give H^+ as the most abundant ion. This is because the composition can change at higher altitudes where the thermal gradient is zero or small since pressure diffusion appears capable of producing large changes. These calculations have not been extended to these regions since the transport equations that have been used do not apply at higher altitudes if composition changes occur as has been indicated. The reason for this is that the electric field in a plasma under gravity where heavier ions predominate is sufficient to accelerate H to the extent that transport equations that takes into account the 'runaway' phenomenon appear to be necessary (BURGERS, 1960).

Helium composition changes show only a slight dependence on the thermal gradient but a very large dependence on the charge of the heavier elements. When this charge is small (Figures 3, 4, and 6), He becomes the dominant element at higher altitudes and temperatures. It appears to change composition as the heavier and more highly charged species in a plasma with H alone. When the charge of the heavy elements is increased, He tends to act more and more like H in that it shows a decrease in abundance while the heavier elements show large enhancements.

With small charges, thermal diffusion appears capable of giving either very little enhancement or factors of 10 or 20 enhancements of the heavy elements relative to electrons (Figures 3 and 6). However, unless core collisions are much more effective than considerations above suggest, there seems to be little to justify the use of such small ionic charges. On the other hand, when probable ionic charges are used (Figures 5 and 7) enhancements are much larger than any that have been reported.

Some possible reasons for the large discrepancy may be one or a combination of the following:

- (1) The altitude of the mixing-diffusion transition may be quite different from the assumed one. There may also be changes in this altitude with location and time.

- (2) The altitude range of the transition may be much broader than has been assumed with variations with location and time.

(3) The thermal gradient may have different values at different locations and times. There may also be different maximum temperatures.

(4) Diffusive equilibrium may require much longer times if magnetic fields are strong enough and are not vertical. Regions of the lower corona may be far from equilibrium for this and other reasons. Horizontal diffusion may also produce large changes.

Suppose that diffusion as shown in Figures 5 and 7 should occur over certain regions of the lower corona. An interesting question arises, namely, will the change in composition with altitude bring about instability of stratification so that mixing or convection will occur? This question has been examined from only the simplest of considerations. Radiation, ionization changes, rotation, and magnetic fields have not been considered. From Archimedes' principle, the following condition for stability of stratification may be derived:

$$-\left(\frac{1}{T} \frac{\partial T}{\partial r}\right)_{\text{ad}} - \frac{1}{\bar{m}} \frac{\partial \bar{m}}{\partial r} + \frac{1}{T} \frac{\partial T}{\partial r} > 0 \quad \text{stable} \\ < 0 \quad \text{unstable,}$$

where the first term is due to changes in temperature of coronal gas under adiabatic displacement. This term is small compared to the others in the lower corona so it has been neglected.

The calculations indicate that thermal diffusion is powerful enough to satisfy the condition of unstable stratification. This is shown in Figure 8 for results from Figure 5. The slope of the straight line gives the condition for stable stratification.

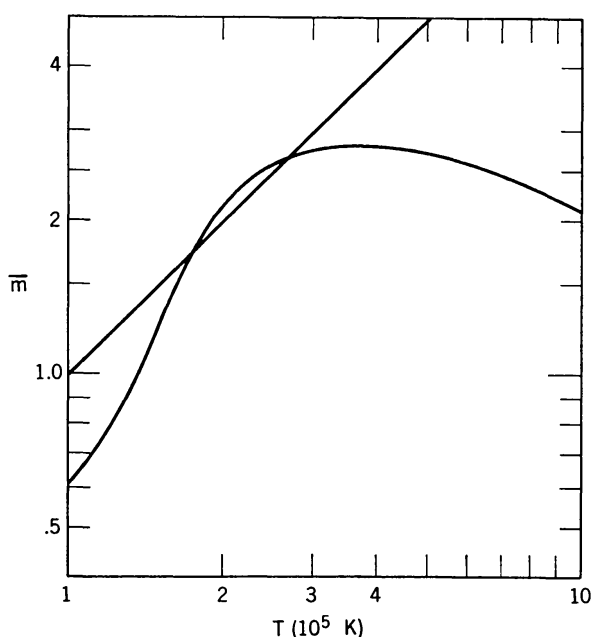


Fig. 8. Average mass, \bar{m} , as a function of temperature for the results of Figure 5. The slope of the straight line gives the approximate condition for stable stratification. When the slope of the \bar{m} curve exceeds this slope, convection may occur.

More positive slopes in the \bar{m} curve give temperatures where mixing or convection may occur. Thus between 10^5 K and 2.3×10^5 K, unstable stratification is possible for the conditions for Figure 5. Since composition changes with temperature are relatively independent of thermal gradients at these temperatures, this result would also hold for a large range of thermal gradients.

Because of the possibility that viscosity may inhibit convection, an approximation to the Rayleigh number has been evaluated. This number is $\alpha g \beta d^4 / \nu \kappa$ where α is the coefficient of thermal expansion, g is the acceleration due to gravity, $\beta = |\partial T / \partial r|$, d is the thickness of the unstable layer, ν is the kinematic viscosity, and κ is the thermal diffusivity. Because the average mass gradient gives instability while the thermal gradient gives stability, the value

$$\beta = \left| \frac{\partial T}{\partial r} - \frac{T}{\bar{m}} \frac{\partial \bar{m}}{\partial r} \right|$$

has been used in the approximation. This approximate Rayleigh number was found to exceed the critical Rayleigh number, $R_c = 635$, for small thermal gradients but to be smaller than R_c for larger thermal gradients. The critical gradient corresponds to those given by $(\partial T / \partial r)_0 \approx 0.1$ K/cm. The dependence on the thermal gradient is due to variations in the thickness of the unstable layer with the thermal gradient.

If these calculations have applicability, they indicate the absence of convection for the larger thermal gradients and convection for the smaller thermal gradients. In the latter situation, the average mass gradient may remain near the one that just gives stability since mixing tends to restore stable stratification.

There are two trends for heavy elements that are given by these studies: (i) diffusion appears capable of producing large composition changes if conditions permit; (ii) the composition at high temperatures can have a large dependence on the thermal gradient if diffusion is important.

The latter trend may assist in the understanding of some solar wind composition results. HUNDHAUSEN *et al.* (1967) and BAME *et al.* (1968) have reported that the He/H and the He/O ratio appears to be variable. If diffusion is to contribute to the explanation of this result, it may do so through changes in composition of the corona due to variations in the thermal gradient with location and time.

This same trend may also modify the necessity to explain enhanced forbidden line emission in hot regions through increased electron densities (ZIRIN, 1966). It may be that concentration increases also contribute to the enhanced emission.

Although diffusion may not produce as drastic changes as has been indicated, it may be partially operative to give significant composition changes. Some consequences, although speculative, may modify present interpretations or indicate that diffusion is or is not of importance. One would be the possible modification of radio temperatures of the corona since the Z that is presently used in the absorption coefficient may require change if diffusion were to change the composition enough. Composition changes may also modify and increase the temperature that is obtained from pressure scale heights through changes in the average mass.

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