

## SHORT-PERIOD VARIABILITY OF B, A, AND F STARS IV. VARIABILITY IN THE LOWER HERTZSPRUNG GAP\*

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### ABSTRACT

The lower instability strip extends from the main sequence upward to the RR Lyrae region. Hot and cool borders were established which intersect the main sequence at A4 and F2. In the instability strip, more than 20 percent of the stars show regular variability larger than 0.010 mag. These variables have normal masses and show a wide range in metal abundances and rotational velocities. Metal abundance and binary characteristics appear to have no influence on the incidence of variability. Pulsation is found to be strongly related to rotational velocity,  $v \sin i$ . The variables of small amplitudes share the high rotational velocity of the nonvariables, while the incidence of larger amplitudes depends critically on low rotational velocity. The mean rotational velocity of the variables 2 magnitudes above the main sequence is less than 0.3 that of similar nonvariables. This effect decreases as the main sequence is approached.

A period-luminosity-color relation is established, with an average deviation of less than 0.3 mag in  $M_v$ . The observed coefficients of  $\log P$  and  $\log (T_e/T_{e\odot})$  agree adequately with the predicted relation  $M_v = -2.5 \log P - 10 \log (T_e/T_{e\odot}) - 1.25 \log M + \text{constant}$  which assumes a variable  $Q$ -value. Metal abundance and rotation have no effect on the period. The pulsation constant  $Q$  is found to vary slightly along the instability strip, and a preliminary value of  $Q = 0.029 \pm 0.007$  is derived, indicating that the variables pulsate in the fundamental and/or first overtone. It is proposed that the period differences among classical Cepheids,  $\delta$  Sct stars, RR Lyrae stars, and dwarf Cepheids are caused not only by luminosity differences but also by large mass differences.

### I. INTRODUCTION

In Paper III (Breger 1969) the known data for over 300 field stars situated in the lower Hertzsprung gap in the H-R diagram and examined for variability were collected. These data can be analyzed to determine the nature of this variability and to compare the observable parameters of the variables and nonvariables, since the number of known  $\delta$  Sct variables is now large enough to derive meaningful relationships. Several variables found in galactic clusters (unpublished) as well as partially unpublished observations by Millis (1967) have also been included in the discussion.

The term *nonvariable* has been applied to all stars for which no variability could be found. A nonvariable can actually be a variable star if (i) its amplitude is too small to be detected ( $A_v \lesssim 0.010$  mag for the survey), (ii) its period is too long and its amplitude is too small to produce a detectable variation during the time tested (e.g.,  $P = 1$  day,  $A_v = 0.05$  mag), (iii) it experiences a beat phenomenon resulting in little variability during the time of observation, or (iv) it varies over a number of years and then stops pulsation. Consequently, the *actual* constancy of any star *inside* the instability strip has not been assumed in the discussions.

### II. THE POSITION OF THE LOWER INSTABILITY STRIP IN THE H-R DIAGRAM

Figure 1 shows the position of the known  $\delta$  Sct variables and constant stars in and near the lower instability strip ( $M_v > 0.0$ ). It must be noted that here the proportion

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of variables to constant stars is too large, since many  $\delta$  Sct variables have been detected (possibly by accident) without determining a corresponding number of constant stars (as in a properly conducted survey). The H-R diagram of the  $\delta$  Sct survey in Paper III presumably shows a more realistic proportion.

Several conclusions about the instability strip can be drawn immediately. Two boundaries are readily apparent. The existence of these boundaries should not imply that pulsation takes place inside these borders only. Small-amplitude, possibly semiregular, pulsation may be possible even outside the instability strip, but only inside these borders pulsation regular, strong, and common enough to be easily detectable.

i) A cool border exists at F2 ( $b - y = 0.21$ ) on the main sequence and at F6 near luminosity class III ( $b - y = 0.25$ ). Both larger-amplitude as well as smaller-amplitude variables are situated near this border. Two of the small-amplitude variables (HR 7501 and HR 8666, a suspected variable) are on the cool border. The previously variable, but

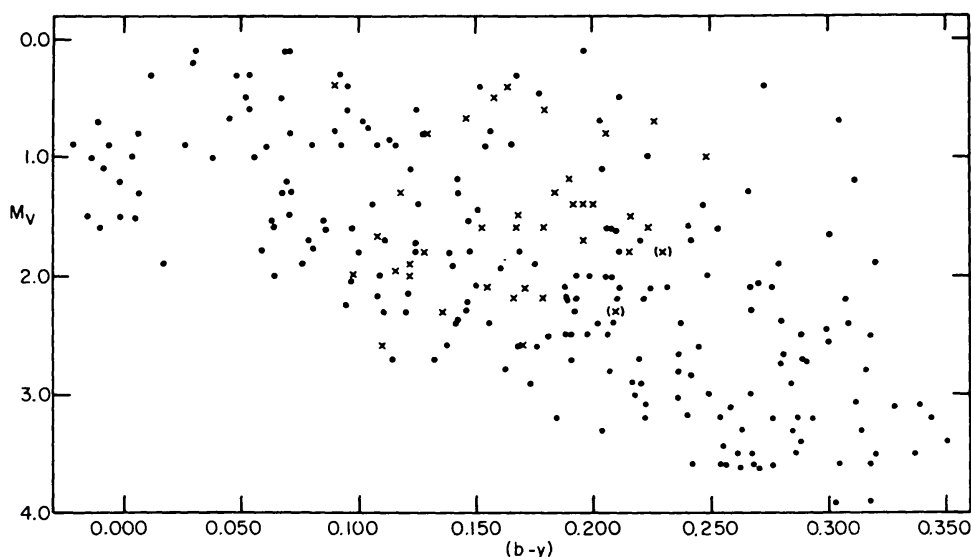


FIG. 1.—H-R diagram for all known variable (*crosses*) and nonvariable (*dots*) stars with known *uvby* photometry and fainter than  $M_v = 0.0$ .

now constant, HR 8130 ( $\tau$  Cyg) is situated near the bottom end of the cool border. This suggests that no clear-cut border exists along the instability strip. As the borders are approached, the amplitudes of the variables become smaller until no more variables can be detected observationally. It is interesting to note that the cool border of the instability strip is located at a much higher temperature than that of the onset of deep hydrogen convection zones (which should destroy pulsation) and that of the abrupt change of stellar rotation near the main sequence (at  $b - y = 0.285$  [Wilson 1966; Demarque and Roeder 1967; Kraft 1967]).

ii) No variables are found hotter than A4 ( $b - y = 0.09$ ). Were it not for the A4 III variable HR 432, a slope in the H-R diagram similar to that of the cool border could be drawn. The variability of HR 432 is well established. In addition to the light variations, the velocity of the K-line varies by 7 km sec<sup>-1</sup> with a similar period. This excludes the possibility of a W UMa-type eclipsing system. The long period of 0.16 days also is characteristic for such a luminous  $\delta$  Sct variable.

Struve (1955) has speculated on the existence of a blue type of variable star, the hypothetical Maia sequence. Two examples would be the suspected variable  $\gamma$  UMi (HR 5735, A3 II-III) and Maia (B8) in the Pleiades cluster. Both stars lie to the left of the present border of the instability strip and were reobserved, but no light variation

could be found. Only on one night did the observations of  $\gamma$  UMi show a large scatter,  $s = 0.008$  mag, which lasted for a period of 35 min. These observations, however, are extremely suspect because of the presence of a very variable air mass on that night. During 3.5 and 1.4 hours of observing with better photometric conditions on two other nights, no variability could be found ( $s = 0.002$  mag). Furthermore, in the  $\delta$  Sct survey discussed in Paper III, more than forty field stars hotter than  $b - y = 0.090$  were tested for light variability, and no variability could be found. On the other hand, several rapidly rotating late B stars are known to be unstable and to eject shells. Instabilities of this nature can cause spectroscopic as well as small light changes. This type of instability, however, has to be distinguished from regular pulsation. Hence, there is very little evidence so far for the existence of pulsation in normal stars outside the late A and early F stars.

iii) The probability that a star situated in the instability strip is a variable increases with increasing luminosity. Near the center of the instability strip (where maximum pulsation takes place), the probability increases from about 20 percent near the main sequence to 45 percent for the more luminous stars. However, because of the small number of variables found in the survey, this result is not statistically significant. However, many of the more luminous stars also tend to have larger amplitudes.

Several variables are found on the main sequence, e.g., the variables in the Hyades, Praesepe, and Coma clusters. Main-sequence variables have been predicted theoretically by Kippenhahn (1965), who found that pulsational instabilities could arise with periods of about 1 hour at  $T_e = 7500^\circ$ . Both the period and the effective temperature are thus confirmed by observation.

The question arises of whether pulsation on the main sequence is age dependent and whether pulsation can occur in young main-sequence stars. We may even ask whether a star contracting toward the main sequence to become a B or early A star can experience regular pulsation during its short stay in the instability strip. The Pleiades Cluster has a well-developed main sequence between A4 and F2 and has an approximate age of  $10^8$  years (Mendoza 1956), as compared with about  $5 \times 10^8$  years for the Hyades Cluster. A search for variables in the Pleiades Cluster and NGC 2264 is presently in progress.

### III. THE MASS PROBLEM

Recently, Danziger and Dickens (1967), Danziger and Kuhi (1966), and Kuhi and Danziger (1967) have postulated from spectrophotometric scans that many  $\delta$  Sct stars have less than one-half the mass that would be inferred from their positions in the H-R diagram. On the other hand, these conclusions have been challenged by Bessell (1967a) on the basis of an incorrect calibration of the absolute energy distribution of Vega and underestimated line blanketing. Furthermore, inspection of the original measurements indicates that the adoption of a mean extinction for Mount Wilson (as used by the cited authors) *can* lead to the derivation of faulty masses.

It is believed here that the masses of  $\delta$  Sct stars are normal for their position in the H-R diagram for the following reasons:

i) The positions in the H-R diagram of  $\delta$  Sct variables in open clusters such as the Hyades, Praesepe, and Coma are similar to those of the constant stars, indicating that at least the cluster  $\delta$  Sct variables have normal masses.

ii) At a given absolute magnitude, undermassive stars would have a lower gravity and hence a larger Balmer jump. This would make the calibration between size of the Balmer jump, color, strength of the  $H\beta$  line, and absolute magnitude as determined from nonvariable stars invalid. In Paper III it was shown that this calibration is valid (Fig. 2).

iii) Even more important: Because of the uncertainties involved in comparing spectrophotometric measurements with model atmospheres to obtain effective temperatures and surface gravities and consequently the mass, it is extremely important to investigate

both constant *and* variable stars before a mass difference between them can be regarded as certain. This was not done by the above investigators. Preliminary results by Breger and Kuhl using the new calibration of the absolute energy distribution of Vega by Hayes (1967), as well as new blanketing corrections determined individually for each star, show essentially no mass difference between the variables and nonvariables.

#### IV. THE EFFECT OF ABUNDANCE AND BINARY NATURE ON PULSATION

Since both variables and constant stars are present in the lower instability strip, the physical properties of these two groups can be compared to determine the effect on the incidence of pulsation. In the absence of adequate theoretical predictions, one would hope to find possibly physical differences between the two groups. If these two groups are actually the same, i.e., if all the stars inside the instability strip actually pulsate, then

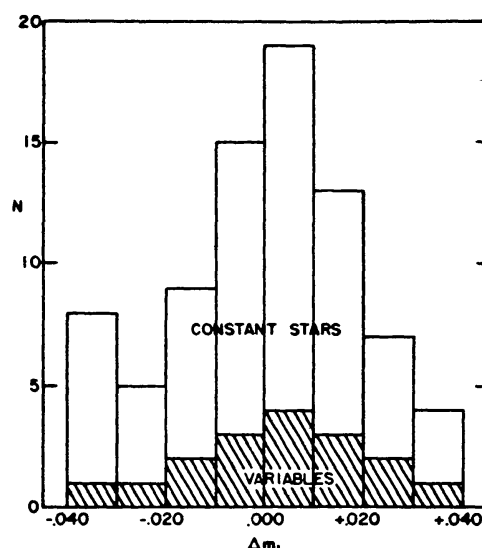


FIG. 2.—Dependence of variability on the metal-abundance parameter  $\Delta m_1$

such a comparison would determine the effects of the various parameters on the amplitude of pulsation.

The helium content cannot be measured directly for these stars. We can, however, rule out differing helium abundances as the main factors governing variability of stars in the lower Hertzsprung gap. Certain galactic clusters such as Praesepe and Coma, which are thought to be chemically homogeneous, contain both variable and nonvariable stars in this region.

Because several previously known  $\delta$  Sct variables have Fe/H ratios higher than that of the Sun, it has been speculated that high metal abundance could cause pulsation. Furthermore, the low rotational velocities of the previously known  $\delta$  Sct variables suggested a connection among the metallic-line A stars, which have both high abundances and low rotational velocities and are found in the same region of the H-R diagram. The blanketing parameter of the *uvby* system,  $\Delta m_1$ , has been shown to be primarily a measure of metal abundance for A and F stars. In order to avoid any bias in the selection of stars, only variable and nonvariables as determined in the variability survey discussed in Paper III were used for comparison. The result is shown in Figure 2. The percentage of variables in each  $\Delta m_1$  interval is essentially constant. Hence, differences in metal

abundance have no effect on the incidence of pulsation, at least within the narrow range of abundances found in the A and F stars close to the Sun.

Furthermore, a connection between Am stars and  $\delta$  Sct variables appears unlikely. During the  $\delta$  Sct survey, at least twelve Am stars were tested for variability. At least eight of these lie inside the instability strip, and only one Am star was found to be variable (HR 114). The incidence of variability among Am stars is therefore similar to that of normal stars. Hence, there should be neither an exclusion effect nor a connection between the two types of phenomena.

Preston's discovery that the variable  $\delta$  Del is a close, double-lined spectroscopic binary raised the question of whether the  $\delta$  Sct phenomenon could be binary induced. From a simultaneous photometric and spectroscopic study of the variable 14 Aur (HR 1706, binary 3<sup>d</sup>8), Chevalier, Perrin, and Le Contel (1968) have suggested that pulsation of this star is nonradial and is influenced by the close companion.

Available data on the binary nature of most  $\delta$  Sct variables are very limited. In Paper III, a personal estimate of the probability of a variable  $\gamma$ -velocity (defined as the velocity of the center of the star) for most variables has been given. Only three stars are definitely close binary systems in which interactions could occur. Two of these, HR 1706 and  $\delta$  Del, have similar unusual spectra (Cowley 1968). For five stars now known to be variables (HR 21, 2107, 4715, 5960, and 8494), Abt (1965) could find no indication of a sufficiently variable radial velocity to conclude the existence of companions. Furthermore, an equal proportion of variables and nonvariables in the  $\delta$  Sct survey are listed as spectroscopic binaries (often only suspected) or have variable radial velocities according to the *Bright Star Catalogue* (Hoffleit 1964). Inspection of the available radial-velocity data for the only pulsating Am star found so far (HR 114) shows no radial-velocity variations (apart from a small pulsation amplitude of 5 km sec<sup>-1</sup>). Furthermore, the star  $\delta$  Sct has been investigated in detail to find a possible companion, with negative results (see Paper III). So far, the limited data suggest that it is unlikely that the variability of *all*  $\delta$  Sct stars has been induced by observationally detectable companions.

A simple view would be that pulsation can take place in most stars in the instability strip. Variables with close companions do not have to pulsate, but if they do, their pulsation might be modified by the interaction of the companion. Binaries such as  $\delta$  Del and HR 1706 would probably pulsate even if they were not binaries. This view appears reasonable by the further argument of Abt (1965), who investigated a large number of normal A stars and found a conspicuous absence of close binary systems. Close binary systems, on the other hand, often contain a brighter metallic-line A star (Abt 1961*a*). In the  $\delta$  Sct region, 20 percent of the stars inside the instability strip are found to be variable. Because of the large number of detected variables with amplitudes close to the lower limit of detection, the actual percentage of variables should be much larger. The variability survey has not found any connection between variability and Am characteristics. This implies that the large majority of  $\delta$  Sct stars cannot be close binary systems. We conclude that the pulsation in the Scuti region is not caused by tidal interaction from observationally detectable companions.

#### V. THE EFFECT OF ROTATION ON PULSATION

Preston (1965) has suggested that stellar rotation might inhibit pulsation and that rapid rotation and pulsation might be mutually exclusive among the Cepheid-like variables in the Hertzsprung gap. The subsequent discovery of fast-rotating  $\delta$  Sct stars (Millis 1966*a*, 1966*b*; Breger 1966) of small amplitude has shown that the relation between pulsation and rotation is complex. According to Preston, it could be the differential rotation, pole to equator, that would be expected to accompany expansion and contraction as the damping mechanism that inhibits the pulsation of rapidly rotating stars. There are two ways of investigating the effect of rotation on the pulsation of  $\delta$  Sct vari-

ables: (i) to study the effect of rotation on the amplitudes and (ii) to check whether there exist any systematic differences in rotational velocities between variables and nonvariables in the instability strip.

The first method has been attempted by Dickens (1968). The effect of rotation on the star can be expressed quantitatively by the ratio of centrifugal force to gravity,  $V^2/Rg$ . It was shown by Dickens that, if three amplitude groupings were formed, larger amplitudes should have lower values of  $V^2/Rg$ . However, his values used for the gravities, in our opinion, are incorrect (§ III). Since the values of  $Rg$  for  $\delta$  Sct variables vary much less than the values of  $V^2$ , we have confined the following discussion to  $V$  only.

More seriously, Dickens has neglected a possibly important third parameter—luminosity (or period, because of the period-luminosity relation). Variables of higher amplitude also tend to be more luminous; in fact, the ratio of amplitude to period is approximately constant for the three amplitude groupings used by Dickens.

Clearly, the observational data deal with three parameters. This is also evident when the effect of rotation on the incidence of observed pulsation is examined as a function of luminosity. The rotational velocities of the variables were compared with those of nonvariable stars inside the instability strip. (Cluster stars were also included under “all

TABLE 1  
ROTATIONAL VELOCITIES FOR VARIABLES AND NONVARIABLES

| $\Delta M_v$<br>$= M_v(MS) - M_v(*)$<br>(mag) | Typical<br>Amplitude<br>(mag) |           | $\langle v \sin i \rangle_{\text{var}}$<br>(km sec <sup>-1</sup> ) | <i>N</i> | $\langle v \sin i \rangle_{\text{const}}$<br>(km sec <sup>-1</sup> ) | <i>N</i> | $\langle v \sin i \rangle_{\text{var}}$<br>$\langle v \sin i \rangle_{\text{const}}$ |
|-----------------------------------------------|-------------------------------|-----------|--------------------------------------------------------------------|----------|----------------------------------------------------------------------|----------|--------------------------------------------------------------------------------------|
| $\leq 1.0 \dots\dots$                         | 0.02                          | Survey    | $153 \pm 14$                                                       | 5        | $118 \pm 9$                                                          | 13       | $1.30 \pm .15$                                                                       |
|                                               |                               | All stars | $137 \pm 10$                                                       | 12       | $102 \pm 7$                                                          | 30       | $1.34 \pm .14$                                                                       |
| $1.0 < \Delta M_v \leq 2.0 \dots\dots$        | 0.05                          | Survey    | $89 \pm 11$                                                        | 5        | $116 \pm 24$                                                         | 7        | $0.77 \pm .25$                                                                       |
|                                               |                               | All stars | $66 \pm 8$                                                         | 14       | $118 \pm 13$                                                         | 17       | $0.56 \pm .09$                                                                       |
| $2.0 < \Delta M_v \dots\dots\dots$            | 0.09                          | Survey    | $27 \pm 18$                                                        | 2        | $162 \pm 4$                                                          | 3        | $0.17 \pm .11$                                                                       |
|                                               |                               | All stars | $38 \pm 10$                                                        | 4        | $140 \pm 18$                                                         | 6        | $0.27 \pm .08$                                                                       |

stars.”) For a constant  $b - y$ , a parameter  $\Delta M_v = M_v$  (main sequence)  $- M_v$  (star) was defined. Mean values with probable errors are shown in Table 1. We notice immediately that at high luminosity the occurrence of pulsation critically depends on low rotational velocity. This effect decreases as we approach the main sequence, and rotation apparently ceases to be important on the main sequence. The *t*-test was applied to these results, which were found to be extremely significant.

The available data are insufficient to study the  $(v \sin i)$ -distribution of both the variables and the nonvariables as a function of luminosity. We can nevertheless attempt to answer the question of whether the correlation of rotation and pulsation is luminosity dependent.

We first notice that the small-amplitude variables ( $A_v \sim 0.02$  mag) have large rotational velocities similar to those of nonvariable stars and that this is independent of luminosity. Some examples are HR 432, HR 515, and HD 24550. The star HR 515 is listed as being of spectral type A7n, indicating wide lines and high rotational velocity; for HR 432, we have measured  $v \sin i = 110$  km sec<sup>-1</sup>; and Dickens (1967) reports 100 km sec for HD 24550. The star 4 CVn, however, is an example of a large-amplitude star with a high rotational velocity (85 km sec<sup>-1</sup>).

An amplitude-rotation diagram plotted for variables has been divided into three period/magnitude groups (Fig. 3). Within a large scatter, the amplitude-rotation relation appears to be the same for two groups, while the third one does not contradict it. This would mean that the damping effect of the rotation on amplitude is not luminosity dependent.

On the other hand, it is presently not clear why so many low-rotation stars near the main sequence do not pulsate, while nearly all the luminous slow rotators have large amplitudes.

We also note that the average rotational velocity of the twelve known variables near the main sequence is higher than that of the thirty constant stars. This result, however, is not statistically significant, since the probability of this happening in a random distribution is 17 percent.

To summarize, rotational velocity  $v \sin i$  has been found to be extremely significant in the sense that large-amplitude pulsation depends critically on low rotational velocities. The small-amplitude variables have rotational velocities very similar to those of the

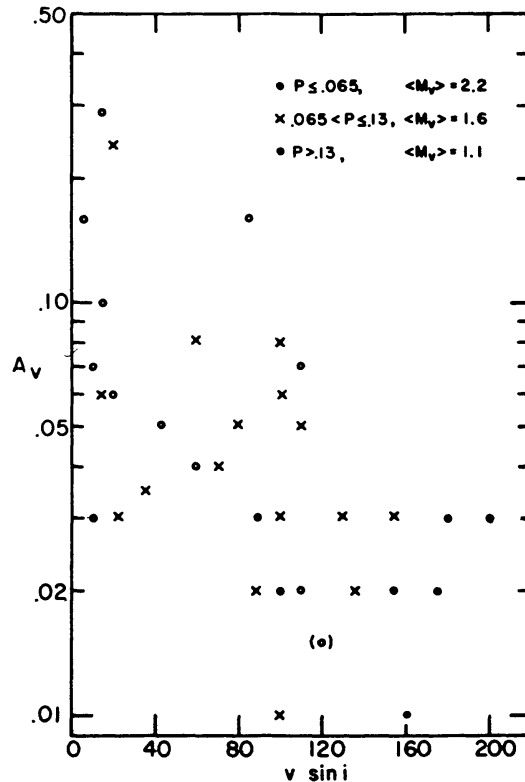


FIG. 3.—Distribution of amplitude and rotational velocity for the  $\delta$  Sct variables. Amplitudes are in mag, while  $v \sin i$  is measured in  $\text{km sec}^{-1}$ .

nonvariables. There are indications that the effect of rotation on pulsation may be dependent on the luminosity of the star.

#### VI. THE PERIOD-LUMINOSITY-COLOR RELATION AND $Q$ -VALUES

The existence of period-luminosity and period-luminosity-color relations (hereinafter called P-L and P-L-C relations, respectively) has been demonstrated for the older Population I Cepheids and  $\beta$  Cep stars, while for RR Lyrae stars, in several clusters P-L-C relations are being discovered (Roberts and Sandage 1955; Geyer 1968). Because the  $\delta$  Sct stars are bright and are closer to the main sequence than the other variables, they lend themselves well to an analysis of the relation between period, luminosity, rotation, temperature, metal abundance, and other factors. On the other hand,  $\delta$  Sct variables have a much smaller range in absolute magnitude and color than do, for example, the Population I Cepheids.

Let  $M_{\text{bol}}$  be the absolute bolometric magnitude,  $P$  the period of pulsation,  $T_e$  the

effective temperature,  $R$  the radius of the star, and  $\mathfrak{M}$  the mass. Consider the following basic relationships:

$$L \propto R^2 T_e^4, \quad (1)$$

$$\mathfrak{M} \propto \rho R^3, \quad (2)$$

$$P\sqrt{(\rho/\rho_\odot)} = Q. \quad (3)$$

Variations in the internal structure of the variables are likely, and hence the constancy of the pulsation constant  $Q$  has not been assumed.

From the three equations above, we obtain

$$\begin{aligned} M_{\text{bol}} - M_{\text{bol}(\odot)} &= -3.3 \log P - 10 \log (T_e/T_{e\odot}) \\ &+ 3.3 \log Q - 1.7 \log (\mathfrak{M}/\mathfrak{M}_\odot). \end{aligned} \quad (4)$$

The bolometric corrections relative to the Sun should be small for  $\delta$  Sct stars; hence, they will be ignored in the following discussion. Should these equations be used to compute masses for these variables, then the bolometric corrections for the hotter variables must be known, since the computed mass is extremely sensitive to the absolute magnitude.

Equation (4) shows that the period is by far the most important parameter on the right-hand side of the equation. Although the effective temperature enters with a coefficient of  $-10$ , the value of  $\log (T_e/T_{e\odot})$  does not change markedly from the red to the blue side of the instability strip.

When equation (4) was applied to the observational data and normal masses were assumed, the differences between the *predicted* and *measured* absolute magnitudes were still correlated with period, showing that the real coefficient of  $\log P$  is not as predicted by equation (4). This could be caused by (i) very large mass loss in many of the variables, which is unlikely in the light of above arguments, and/or (ii)  $Q$  not being constant. If the  $\delta$  Sct stars pulsate in different overtones or in a mixture of overtones,  $Q$  cannot be constant. However, no separation into two distinct groups (e.g., pulsators in the fundamental or the first harmonic) can be made. However, for a particular mode of pulsation,  $Q$  is constant only if the assumption of homology holds. Even if equation (4) does not fit the observed slope of  $\log P$  because of the influence of other parameters, it can still be used to calculate values of  $Q$  to see if and how  $Q$  varies. The calculated value of  $Q$  for any particular star is sensitive to presently inaccurate values of  $T_e$ ,  $\mathfrak{M}$ , and bolometric corrections. Therefore, only a preliminary mean value of  $Q = 0.029 \pm 0.007$  days (mean absolute error per *single* star) was obtained. This is consistent with the view of a slightly variable  $Q$  and the idea that the  $\delta$  Sct stars do not pulsate in the third or fourth overtone (as has previously been proposed) but in the first overtone and/or possibly in the fundamental. This mean  $Q$ -value is consistent with  $Q$ -values derived from ratios of fundamental and first-overtone periods in many short-period variables (Christy 1966a). Fernie (1965) has already proposed a variable  $Q$  (even within a particular class of variables). From a discussion of classical Cepheids and  $\beta$  Cep stars, he suggests a relation connecting the period, radius, and mass:

$$P \propto R^2 \mathfrak{M}^{-0.5}. \quad (5)$$

If we use this relation rather than equation (3), equation (4) will be replaced by

$$M_v = -2.5 \log P - 10 \log (T_e/T_{e\odot}) - 1.25 \log (\mathfrak{M}/\mathfrak{M}_\odot) + a, \quad (6)$$

where  $a$  is a constant.

Since the absolute mass determination by means of spectrophotometric scans or pulsational properties cannot determine masses to a precision of a few tenths of solar

mass, there is no way to test the reality of the term  $-1.25 \log (\mathcal{M}/\mathcal{M}_{\odot})$ . On the basis of evolutionary tracks (Iben 1967) and mass determinations from visual binaries, we expect the mass of the variables to be a function of luminosity. Since the observations do not include mass measurements, we ask: What is the expected systematic effect on the coefficients of  $\log P$  and  $\log (T_e/T_{e\odot})$  if the mass term were neglected? If we eliminate mass completely by means of a reasonable mass-luminosity relation, the coefficients change only by about 10 percent.

We shall now turn to the *observational* data and investigate the function  $\log P = f(M_v, T_e, \text{metal abundance}, v \sin i)$ .

*a) Period, Luminosity, and Temperature*

The observed relations between period and luminosity as well as between period and color are shown in Figures 4 and 5, respectively. Correlations clearly exist. The straight

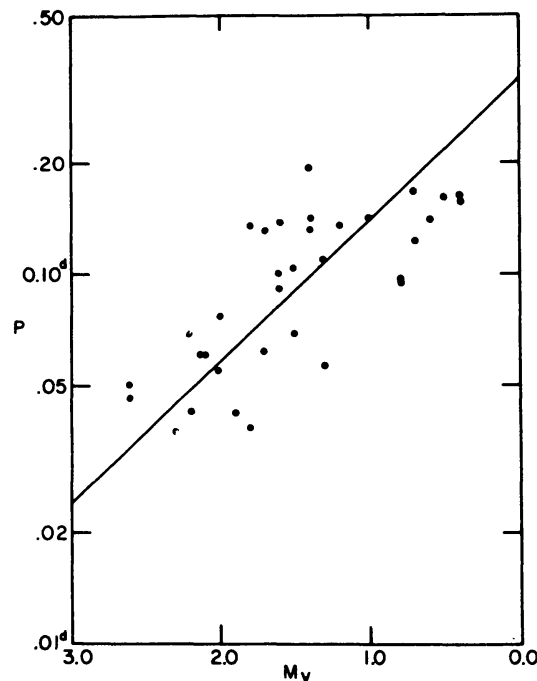


FIG. 4.—Observed P-L relations for  $\delta$  Sct stars of well-determined absolute magnitudes. The straight line has been computed by least squares.

line in Figure 4 (representing a P-L relation) has been computed by least squares after the uncertainties in both the period and the absolute magnitude were considered. This line has a slope of  $-2.6 (\pm 0.4 \text{ p.e.})$  for  $\log P$ . We note that the slope of  $\log P$  is remarkably similar to that predicted by equation (6).

The effect of temperature on the period is much smaller and more problematical than the effect of luminosity. Most of the observed correlation shown in Figure 5 is not due to a real dependence of  $\log P$  on temperature. It would be real only if our data were distributed randomly in  $M_v$  and  $b - y$  so that  $M_v$  and  $b - y$  are two independent parameters. This is not the case, since a small dependence of  $M_v$  on  $b - y$  exists. The thirty-five variable stars that could be used for a three-parameter least-squares solution to solve for the dependence of both the  $M_v$  and the  $b - y$  on the period are too few, and the periods and absolute magnitudes are not accurate enough, to warrant such an analysis. We have considered (i) the period-color dependence corrected statistically for the correlation between  $M_v$  and  $b - y$  and (ii) the color dependence of the deviations

from the P-L relation shown in Figure 4. A unique relation between  $\log (T_e/T_{e\odot})$  and  $b - y$  has then been assumed for the  $\delta$  Sct region and has been calibrated through effective-temperature values determined by present comparisons of model atmospheres with spectrophotometric scans, as well as determinations by Wolff, Kuhi, and Hayes (1968), Powell (1968), and Harris (1963). We find that a real temperature effect on the period exists and is of the same order as that predicted by equation (6). Since that temperature effect is much smaller than the dependence of the period on  $M_v$ , and is also affected by uncertainties in the determination of  $\log (T_e/T_{e\odot})$  and by mass and bolometric corrections, as well as by the effect of rotation and aspect on the observed colors, the limited data do not permit a more detailed analysis.

We conclude that within the errors of the available data the coefficients of  $\log P$  and  $\log (T_e/T_{e\odot})$  are as given by equation (6), and the P-L-C relation becomes

$$M_v = -2.5 \log P + 5.5(b - y) - 2.0. \quad (7)$$

Expected absolute magnitudes were then calculated from this P-L-C relation. The average difference between the predicted and the measured absolute magnitudes was less

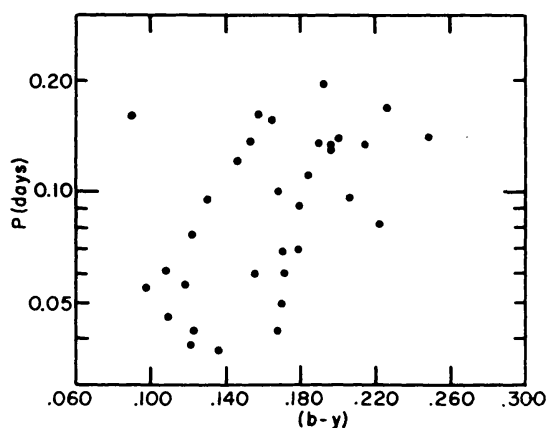


FIG. 5.—Observed period-color relation for  $\delta$  Sct stars

than 0.3 mag, which is also the expected mean error in determining  $M_v$  by photometry. We also note that the average deviation from the P-L-C relation is slightly less than that obtained by using only a P-L relation as shown in Figure 4.

#### b) Metal Abundance

Metal abundance as measured by the  $\Delta m_1$  index has no effect on the deviations from the P-L-C relation, as can be seen from the complete lack of correlation in Figure 6.

#### c) Rotational Velocity $v \sin i$

The influence of rotation on the P-L-C relation has been examined in Figure 7. For the Hyades and Praesepe clusters, it has been shown by Kraft and Wrubel (1965), Strittmatter (1966), and Dickens, Kraft, and Krzeminski (1968) that the surface gravity (i.e., the  $c_1$  index and hence the luminosity) is weakly affected by the measured rotational velocity. The models of Roxburgh and Strittmatter (1966) suggest large rotational effects on the colors and luminosity. However, it has not been established that, in the determination of a star's pulsational properties (such as period and amplitude), the important parameters are really the luminosity and effective temperature that the star would have if it were not rotating, rather than its actual measured luminosity and effective temperature. The fast-rotating variables (approaching the upper limit of rota-

tion) especially can be expected to have an internal structure different from that of a nonrotator. The aspect,  $\sin i$ , also complicates such a discussion even more. Figure 7 shows that the deviations from the P-L-C relation appear independent of the measured rotational velocity.

The star that deviates most from the P-L and P-L-C relations is HD 24550. (This star was not included in the derivation of the relations.) To come into line with the other less luminous variables of similar period (0<sup>d</sup>076), it would need to be a double-lined spectroscopic binary such as  $\delta$  Del. If, on the other hand, we assume a single star with a mass of about  $2.2 M_{\odot}$ , the  $Q$ -value for HD 24550 becomes 0.017, indicating pulsation in a higher mode than the first harmonic.

The question arises of whether the  $\delta$  Sct P-L-C relation could be an extension of the classical Cepheid relation, because of the similarity of the coefficients. A unique relation among  $M_{\text{bol}}$ , period, and  $T_e$  does not imply a unique relation among  $M_v$ , period, and color, since parameters such as bolometric corrections, mass and mass loss, composition

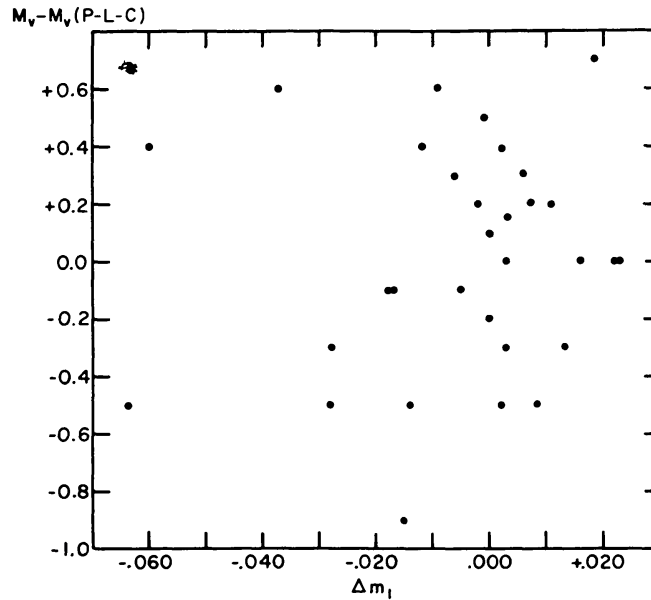


FIG. 6.—Dependence on metal abundance of the deviations from the P-L-C relation

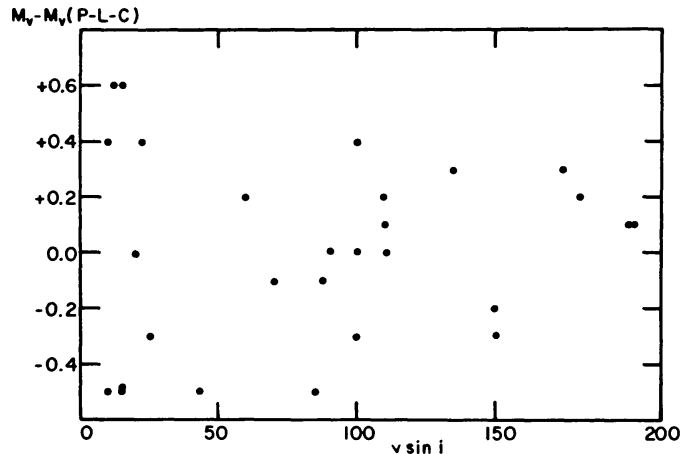


FIG. 7.—Dependence on the rotational velocity,  $v \sin i$ , of the deviations from the P-L-C relation

differences, and the  $(B - V, T_e)$ -relation cannot be the same for Cepheids and  $\delta$  Sct stars. There is some disagreement among computed P-L or P-L-C relations for classical Cepheids. The fact that some of these relations correctly extend into the  $\delta$  Sct region (e.g., Sandage and Tammann 1968) might be coincidental. We conclude that the classical Cepheid P-L and P-L-C relations should and actually do fit the  $\delta$  Sct stars only approximately.

#### VII. DISCUSSION OF PULSATION IN THE LOWER HERTZSPRUNG GAP

The instability strip of Population I Cepheids extends into the lower Hertzsprung gap down to an including the main sequence. Some of the variables are brighter than the RR Lyrae stars but have shorter periods. The borders of the instability strip intersect the main sequence at spectral types A4 and F2. At least 20 percent of the stars in the lower instability strip are variables. The large proportion of variables with small amplitudes close to the limit of detection suggests a much larger incidence of variability inside the instability strip than is observed. These  $\delta$  Sct variables have masses normal for their position in the H-R diagram and show a wide range in metal abundances and rotational velocities (up to  $200 \text{ km sec}^{-1}$  for  $v \sin i$ ). Metal abundance and binary characteristics do not appear to have any influence on the incidence of pulsation, which is largest near the center and the top of the instability strip. The only physical difference detected between the variables and nonvariables inside the instability strip is rotational velocity. While the small-amplitude variables ( $A_v < 0.02 \text{ mag}$ ) share the high rotation of the nonvariables, the development of large amplitudes depends critically on low  $v \sin i$ . The mean rotational velocity of the variables 2 mag above the main sequence is less than 0.3 of that of similar nonvariables. This effect decreases as the main sequence is approached, and on the main sequence the average variable and nonvariable have similar rotational velocities. It is therefore possible that the effect of rotation on pulsation also depends on the luminosity of the star.

Many variables show beat phenomena, indicating more than one period, as well as a possibly intrinsically varying amplitude. The best example is  $\iota$  Mon (HR 2107), which according to Danziger and Dickens (1967) showed amplitudes as different as 0.3 and 0.03 mag on two occasions. The star  $\tau$  Cyg (HR 8130) has been reported by Paraskevopoulos (1921) and Henroteau (1922) to be variable with a  $\delta$  Sct-like period of 0.143 days. Light variations of 0.02 mag in the yellow were also observed by Pande (1960). Abt (1961b) could find no velocity variations, while Breger (Paper III) as well as Millis (1967) could find no light variations. These data suggest that  $\tau$  Cyg has a very variable amplitude of pulsation or that pulsation could also stop altogether. Another star inside the instability strip,  $\gamma$  Boo, once showed periodic variations with a regular period of 0<sup>d</sup>.3 and sometimes shows erratic light or spectral variations or even constancy (Baglin, Praderie, and Perrin 1968). This star, however, could be a spectrum variable. The observed variations in the size of the amplitudes of most variables, however, are generally within a factor of 2 within a time period of several years.

The calculated  $Q$ -values for the  $\delta$  Sct variables are extremely sensitive to the assumed mass, temperature, and period. At this stage it can only be concluded that the  $\delta$  Sct stars pulsate in the first harmonic and/or the fundamental. We cannot state definitely whether *any*  $\delta$  Sct variable pulsates in the fundamental, although  $\delta$  Sct itself might do so. However, Christy (1966b) has proposed a transition period,  $P_{\text{tr}}$ , *independent of mass and composition*, which is the shortest period for a pulsator in the fundamental mode:

$$P_{\text{tr}} = 0.057(L/L_{\odot})^{0.6} \text{ days} . \quad (8)$$

According to the luminosity limits given by Christy, this expression should be valid for  $\delta$  Sct variables, and no  $\delta$  Sct variables should pulsate in the fundamental mode. In the RR Lyrae stars, the two types of pulsators can be separated by the asymmetry of the

light curve and somewhat by color (Bailey types *ab* and *c*). No such separation can be made for the  $\delta$  Sct stars.

The lower Hertzsprung gap contains another type of short-period pulsating variable, the dwarf Cepheid. By previous convention, all short-period variables with amplitudes larger than 0.30 mag are called dwarf Cepheids and are believed to be a separate type of pulsating variable belonging to the intermediate disk population. However, until it has been established that no  $\delta$  Sct variable can have a larger amplitude, and even more important, *until it has been established that the dwarf Cepheids cannot have a visual amplitude smaller than 0.30 mag*, the separation between the two types of variables on the basis of their amplitude alone is dangerous at best.

We wish to propose a difference between the  $\delta$  Sct stars and the dwarf Cepheids without using the size of their amplitudes: The  $\delta$  Sct variables are the short-period Population I variables similar to the A and F stars close to the Sun and have "normal" masses and luminosities. Their periods are less than 1 day, while the Population I stars of longer periods are classical Cepheids.

The dwarf Cepheids, on the other hand, are the short-period variables pulsating in the lower Hertzsprung gap, not obeying the above criterion. At least two dwarf Cepheids, SX Phe and AI Vel, have well-determined trigonometric parallaxes which place these stars 1 mag below the main sequence. This luminosity difference might be explained if the dwarf Cepheids have low masses (see below). Otherwise, very little is known about the dwarf Cepheids, except that it is difficult to determine at what period a dwarf Cepheid should be called an RR Lyrae star or a dwarf Cepheid. Solar-motion solutions made by Smith (1955) and by Woltjer (1956) suggest that possibly the dwarf Cepheids are members of an intermediate population.

We cannot expect the same P-L-C- $\mathcal{M}$  relation to be applicable to all types of pulsating stars (Ferne 1965), but we may inquire whether the assumption of very similar relations can explain the observed differences between the various types of pulsating variables.

#### a) Classical Cepheids and $\delta$ Scuti Stars

In § VI it has been shown that the periods and luminosities of the  $\delta$  Sct variables form approximately an extension of the classical Cepheid relation.

#### b) RR Lyrae and $\delta$ Scuti Stars

Several  $\delta$  Sct stars are as bright as the RR Lyrae stars (e.g., HR 432 and 515), but their periods are only about 0.16 days. The  $\delta$  Sct stars still have their original main-sequence mass, while the RR Lyrae stars are believed to have lost mass and to have a mass of about  $0.5 \mathcal{M}_{\odot}$  (Christy 1966a). Because of their identical positions in the H-R diagram, the mass term of the P-L-C- $\mathcal{M}$  relation points toward the difference between the two types. If we apply the  $\delta$  Sct relation to the RR Lyrae stars, then from equation (6) we obtain

$$\Delta \log P \sim -0.5 \Delta \log (\mathcal{M}/\mathcal{M}_{\odot}) . \quad (9)$$

This predicts a period of about 0.43 for the RR Lyrae stars (pulsating in the same harmonic as do the  $\delta$  Sct stars). This is a typical period for the first overtone pulsator RR Lyr stars (Bailey type *c*).

#### c) Dwarf Cepheids and $\delta$ Scuti Stars

Bessel (1967b) has derived masses of about  $0.5 \mathcal{M}_{\odot}$  for the two dwarf Cepheids SX Phe and AI Vel on the basis of spectrophotometric scans (by using a revised calibration of the absolute energy distribution of Vega) and observed period ratios in these stars. The latter assumes that the two measured periods in each of these stars can be identified with the fundamental and first-overtone periods. On the basis of different masses, equa-

tion (6) correctly predicts the observed difference in absolute magnitude between the two dwarf Cepheids SX Phe and AI Vel and  $\delta$  Sct variables of similar periods.

*d) RR Lyrae Stars and Dwarf Cepheids*

If the above ideas were correct, then these two types of variables should have similar low masses but different luminosities. The P-L-C-M relation then predicts the observed period differences.

The definition of RR Lyr stars should then be extended to include dwarf Cepheids that are simply similar evolved low-mass stars lower in the instability strip. Furthermore,  $\delta$  Sct variables of periods of about half a day should occur near  $M_v = -1$ . This region has not been studied for variability in the past. A search for such variables would be complicated because (i) it is difficult, if not presently impossible, to determine the absolute magnitude and mass of such stars observationally, (ii) detection of small amplitudes of a hundredth of a magnitude becomes more difficult with longer periods, and (iii) such stars may show small, irregular variations that mask any regular pulsation. Several supergiants have been tested for short-period regular variability, so far with negative results (i.e., no periods could be derived).

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