THE ASTROPHYSICAL JOURNAL, Vol. 158, October 1969

THE VARIABILITY OF 21 PERSEI

GEORGE W. PRESTON
Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology
Received 1969 March 7

ABSTRACT

Radial velocities and line intensities in the spectrum of 21 Per vary periodically in the 2ᵈ.88 photometric cycle derived by Stępień. The velocity curve for the singly ionized rare earths consists of two branches that overlap near primary light maximum and overlap again one half-cycle later, when a weak secondary light maximum may occur. The range of the velocity variation is about 30 km sec^{-1}. The velocity variation for Ti II and Mn II resembles those for the rare earths, while lines of Si II, Sr II, and Fe II yield velocity curves of small amplitude with double waves. The velocity curve for Cr II lines varies in antiphase with the other elements. The line components of the rare earths appear as sharp, weak, shortward-displaced features that first wax and then wane in strength as they move longward. They reach maximum strength when their velocity equals the systemic velocity. The magnetic field of 21 Per is weak and cannot be measured with precision because of the complicated kinematic effects. The interpretation of these results in terms of a rigid-rotator model is discussed briefly.

I. INTRODUCTION

Long ago Morgan (1935) called attention to the unusual spectrum of 21 Per which contains abnormally lines of Si II, Cr II, Mn II, Sr II, and Eu II. However, apart from a brief report on its magnetic field by Babcock (1958) and its inclusion in occasional survey discussions of the Ap stars (see, for example, Osawa 1965; Searle and Sargent 1964), this star has received little attention, and it was not until 1962 that Rakos (1962) found it to be variable in light with a range of about 0.03 mag and a period of 1ᵈ.729. In order that the possible periodicity of its magnetic field could be investigated, 21 Per was included in a routine program of magnetic observations at the coudé spectrograph of the Lick Observatory 120-inch reflector in 1966 and 1967. The spectroscopic observations, listed in Table 1, were accompanied by *UBV* photometric observations that are described by Stępień (1968).

Examination of the 1966 Zeeman spectrograms (dispersion 4 Å mm^{-1}; total widening 0.65 mm) verified the diversity of line widths reported by Babcock and indicated the occasional occurrence of many weak, sharp features at the limit of detectability. Therefore, in 1967 a series of 4 Å mm^{-1} Zeeman spectrograms with greater widening (1.3 mm) was obtained. It was quickly established that at intervals of about 3 days a host of extremely weak, sharp lines appear in the spectrum, and on closer examination it was found that most or all of them could be interpreted as components of doublets with a separation in velocity units of $\sim$25 km sec^{-1}. There are so many line components in some spectral regions at these times that the above interpretation is not so obvious as one might expect it to be; the clue to the interpretation is provided by stronger identifiable lines of Ti II, Mn II, and Fe II, many of which are more or less clearly double on these well-widened spectrograms.

The question of the period was settled by Stępień (1968), who found that his photometric observations are not represented by Rakos's period, but do appear to be periodic with the provisional elements

$$\mathrm{JD}_{\odot}\,(\text{max light}) = 2439837.7 + 2^{\mathrm{d}}.883\,E\,. \qquad (1)$$

TABLE 1

SPECTROSCOPIC DATA FOR 21 PERSEI

Plate No. ECZ-	JD$_\odot$ 2439000+	Phase Eq.(2)	$v_{\rm rad}$ (km sec^{-1})*					H_e ± p.e. (gauss)
			R.E.	Ti II, Mn II	Fe II	Cr II	Si II, Sr II	
5096.....	334.02	0.30	−2.5 +24.0	−4.5 +19.5	−1.5 +17.5	+7.5	+9.5	....
5117.....	339.02	.04	+10.5	+7.0	+11.0	+5.5	+9.0	−370 ± 360
5145.....	342.99	.42	+6.0	+4.5	+8.5	+7.0	+7.5	....
5158.....	344.03	.78	−3.5 +25.0	−5.0 +18.5	+5.0	+12.0	+9.0	....
5167.....	363.05	.37	−2.0	+4.0	+10.5	+11.0	+9.5	....
5255.....	400.85	.48	+6.0	+5.5	+9.5	+7.5	+7.5	....
5263.....	401.88	.83	−1.5 +21.0	−2.5 +16.0	+7.0	+9.0	+5.0	....
5270.....	402.80	.15	−12.0 +21.0	−7.5 +16.0	−3.5 +20.0	+6.0	+12.0	+30 ± 220
5375.....	453.83	.85	+0.5 +20.0	0.0 +22.5	+6.5	+11.5	+7.5	....
5390.....	454.86	.20	−5.5 +23.0	−6.0 +18.5	−2.0 +17.5	+6.0	+10.0	....
5399.....	455.80	.53	+12.0	+6.5	+9.0	+6.0	+7.5	....
5407.....	456.68	.83	−1.0 +26.5	−3.5 +17.5	+5.0	+10.0	+10.5	....
5416.....	459.70	.88	+2.5	+2.0	+7.0	+10.5	+6.0	....
5464.....	491.71	.98	+7.5	+4.5	+10.5	+13.0	+9.0	−280 ± 230
5474.....	492.71	.33	+1.5	+5.0	+8.5	+5.0	+9.5	....
5854.....	722.02	.83	−2.0	−3.5 +19.0	+6.5	+9.5	+7.0	−260 ± 160
5866.....	723.01	.17	−4.5 +22.5	−8.0 +18.0	(+3.0) (+22.0)	+8.0	+12.5	+790 ± 190
5878.....	724.03	.53	+11.0	+6.5	+10.5	+5.0	+9.0	−150 ± 110
5911.....	727.98	.90	+2.0	+2.0	+8.0	+10.5	+6.5	−290 ± 210
5925.....	729.02	.26	−4.0 +29.0	−7.0 +18.5	+10.5	+7.0	+8.0	−240 ± 140
6086.....	775.95	.53	+9.0	+7.0	+10.0	+6.5	+10.0	−240 ± 170
6089.....	776.96	.88	+1.5	+1.5	+7.0	+12.0	+7.5	+280 ± 260
6100.....	779.04	.60	(+3.0) +13.5	+7.5	+11.5	+5.0	+9.5	−100 ± 150
6102.....	779.89	0.90	+2.5	+2.0	+6.5	+10.5	+5.0	+370 ± 200

*Parentheses () denote uncertain values based on a single line.

By means of these elements, phases were calculated for all the spectrograms. It could then be established that a complex sequence of spectroscopic variations takes place, as described below.

II. MEASUREMENTS AND REDUCTIONS

a) *Radial Velocities*

As a preliminary to measurement of all the plates, two spectrograms, ECZ 5911 and 6102, on which the lines are (generally) sharp and single, were measured for the purpose of line identification. Of some 600 features measured on λλ3740–4640 on both spectrograms, about half can be identified as lines of neutral or singly ionized Si, Ca, Ti, Cr, Mn, Fe, and Sr, roughly one-quarter can be identified with the rare-earth elements Ce II, Nd II, Eu II, Gd II, and one-quarter of the lines cannot be identified by reference to the revised multiplet table (Moore 1945) or the *Tables of Spectral Line Intensities* prepared by Meggers, Corliss, and Scribner (1961). Needless to say, many of the above-mentioned identifications refer to blends, and literally hundreds of very weak and/or diffuse features were not even measured. The unidentified lines with few exceptions are weak features that become double at certain phases. For reasons that will be made clear below, we believe that most of these unidentified lines are due to the rare earths.

TABLE 2

LINES MEASURED IN THE SPECTRUM OF 21 PERSEI

λ(Å)	El (RMT)	λ(Å)	El (RMT)	λ(Å)	El (RMT)	λ(Å)	El (RMT)
3819.67....	Eu II(1)	4020.87...	Nd II	4261.92....	Cr II (31)	4416.82...	Fe II (27)
3853.66....	Si II (1)	4038.03...	Cr II (194)	4282.48....	Mn II	4435.58...	Eu II (4)
3862.59....	Si II (1)	4076.87...	Cr II (19)	4292.25....	Mn II (6)	4483.33...	Gd II (62)
3863.37....	Nd II	4077.71...	Sr II (1)	4294.10....	Ti II (20)	4493.53...	Ti II (18)
3865.59....	Cr II (167)	4082.30...	Cr II (165)	4312.86....	Ti II (41)	4501.27...	Ti II (31)
3900.55....	Ti II (34)	4128.05...	Si II (3)	4326.76....	Mn II (6)	4508.28...	Fe II (38)
3930.50....	Eu II (5)	4129.73...	Eu II (1)	4385.38....	Fe II (27)	4515.34...	Fe II (37)
3935.94....	Fe II (173)	4215.52...	Sr II (1)	4395.03....	Ti II (19)	4522.63...	Fe II (38)
4012.50*...	Cr II (183)	4251.73...	Gd II (15)	4395.85....	Ti II (61)		

* Blended at some phases: see text in § II*b*.

The thirty-five lines listed in Table 2 were selected for measurement on all twenty-four spectrograms. Except for the strong lines of Si II and Sr II, they are sufficiently weak to exhibit line doubling when it occurs, although *all* of the lines in the list are not double on any spectrogram. The Eu II identifications seem secure, except that λ3930.50 is probably contaminated by Fe II (3) λ3930.31. We have less confidence in the identifications for the two lines of Nd II and one of Gd II at λ4483.33. However, the profile and velocity variations of these lines are similar to those of Eu II, and they show no systematic velocity residuals relative to Eu II.

Radial velocities based on these thirty-five lines are listed separately for the rare earths (R.E.), Ti II + Mn II, Fe II, Si II + Sr II, and Cr II in Table 1; they are plotted versus phase (also given in Table 1) in Figure 1 by means of equation (2) derived in § II *b* below. The velocity variations are unique among known Ap stars. The Eu II velocity curve consists of two branches. The line components of these branches appear as weak shortward-displaced features that first wax and then wane as they move longward until they are replaced by the components of the next branch. On the phase interval $0.75 < \phi < 0.30$ the variation in velocity of Ti II + Mn II resembles that of the rare earths except for a slightly smaller velocity amplitude. However, on the interval $0.3 < \phi < 0.7$ the second branch of the velocity curve is poorly defined. Examination of the profiles

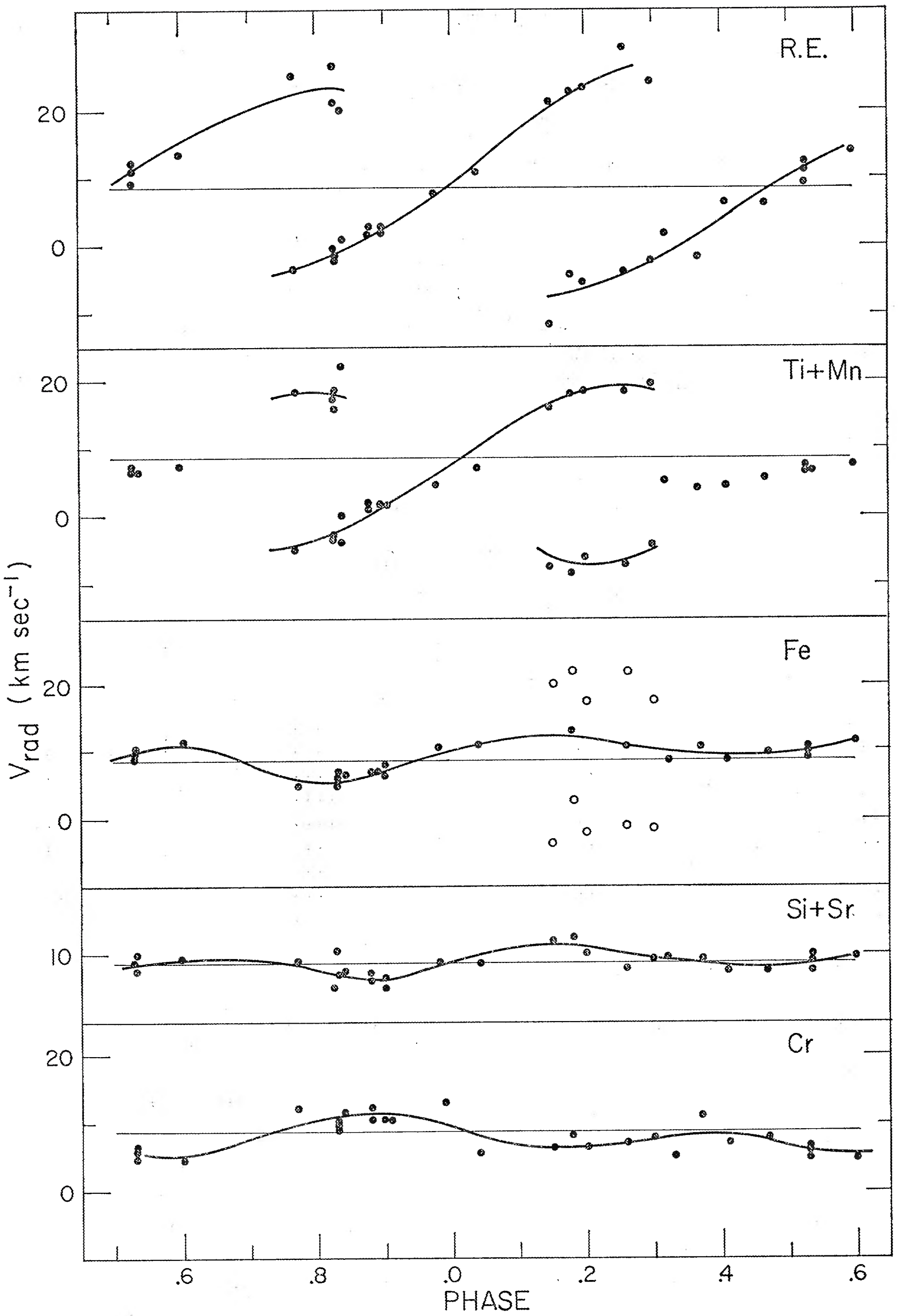

FIG. 1.—Radial-velocity variations of 21 Per for elements grouped as indicated in the upper right corner of each panel. The abbreviation R.E. refers to the rare-earth lines listed in § II*a*. Open circles in the third panel refer to a few lines of Fe II that are double near phase 0.2. The horizontal straight lines at +8.8 km sec^{-1} denote the systemic velocity as inferred in § II*a*.

suggests the reason: whereas the rare-earth lines are relatively sharp after phase 0.25, those of Ti II and Mn II are broad with incipient double structure. This suggests that two unresolved components of Ti II and Mn II lines are present at these phases and are responsible for the approximately constant velocities on $0.3 < \phi < 0.7$. The behavior of the Fe II, Si II, and Sr II lines indicates an enhancement of the effect described for Ti II and Mn II. Line doubling is seen only near $\phi = 0.25$ for Fe II, and then only in some of the lines (as indicated by the open circles in panel 3 of Figure 1. Occasional double lines of Si II, Cr II, and Sr II also occur. The forms of the smoothed low-amplitude variations of Fe II and Si II + Sr II are consistent with the notion that they are produced by changes in displacement and intensity of line components similar to those seen for Ti II and, more particularly, the rare earths. Our inability clearly to resolve the branches is due in part to the fact that the Fe II, Si II, and Sr II lines are systematically stronger than those of the rare earths, perhaps to the fact that the doublet separations are systematically smaller than those of the rare earths, and perhaps to other reasons that are discussed in § III.

The Cr II lines behave rather like the Fe II and Si II + Sr II lines except that their variation appears to be approximately 180° out of phase. Thus, while 21 Per is unique among known spectrum variables in that its rare earths show a pronounced double wave in the photometric period, it conforms to a very general rule, namely, that the chromium variations are in antiphase with the rare earths. This follows from the fact that the descending branches of the Cr II variations, which are the analogues of the line-doubling phases for the rare earths, occur at phases near $\phi = 0.0$ and $\phi = 0.5$.

The center-of-mass velocity can be estimated from the time-average velocities of the various curves in Figure 1 with the following results:

	$\langle v_{\rm rad} \rangle$ (km sec^{-1})
Rare earths (branch I) $0.725 \leq \phi \leq 0.275$	+9.68
Rare earths (branch II) $0.15 \leq \phi \leq 0.85$	+8.57
Fe II (continuous curve in Fig. 1)	+9.55
Si II + Sr II (continuous curve in Fig. 1)	+8.85
Cr II (continuous curve in Fig. 1)	+7.95

No attempt was made to obtain an average velocity for Ti II because the branch variations are resolved in one half of the cycle, but not in the other half. Because the individual branch velocities for the rare earths depend on the phase interval covered by the observations, we exclude the rare earths from the adopted mean value of 8.8 km sec^{-1}. This value is indicated by the horizontal straight lines in Figure 1, where it can be seen that the rare-earth velocity equals the time-average (systemic) velocity very nearly at phases 0.00 and 0.50.

b) The Period

Stępień's elements, derived from photometric observations that were concurrent with our spectroscopic observations, are adequate to define the velocity variations described above. However, it has proven possible to improve his period by means of additional spectroscopic data. Advantage is taken of the peculiar behavior of the absorption feature at λ4012. This strong feature becomes clearly double near phase 0.8 and remains double for nearly 0.1 cycles after the rare-earth line doubling ceases. The displacements of this line at this and other phases in the $2\overset{\rm d}{.}88$ cycle can be understood if the longward component of λ4012 is attributed primarily to Cr II (183) λ4012.50, while the shortward component is due to Ti II (11) λ4012.37. The details of this argument will be given elsewhere in a discussion of the λ4012 feature in the Ap stars. For our purposes it is necessary only to note empirically that λ4012 is double only in one restricted phase interval in the $2\overset{\rm d}{.}88$ cycle. Within this phase interval the relative intensities of the shortward- and longward-displaced components give even finer phase resolution. We estimate that the phase of any spectrogram of 21 Per on which λ4012 is double can be determined with an error no greater than ±0.05 period, or $\pm 0^{\mathrm{d}}.15$. Of twenty-eight coudé spectrograms of 21 Per, mostly due to Babcock, on file at the Mount Wilson and Palomar Observatories, λ4012 is double on the seven listed in Table 3. For each spectrogram a phase was estimated and the Julian date was corrected to the nearest epoch of zero phase. If we adopt JD☉ 2439491.77 as the mean epoch of zero phase for the 1966–1967 observations, then division of the elapsed intervals ΔJD by the integral numbers of elapsed cycles ΔE in columns (5) and (6) of Table 3 leads to the periods in column (7). If the values of ΔE are employed as weights, the weighted mean period leads to the elements

$$\begin{aligned} \text{JD (primary maximum)} &= 2439491.77 + 2^{\mathrm{d}}.88422\,E \\ &\quad\text{(p.e.)} \pm 0.03 \pm 0^{\mathrm{d}}.00003\,. \end{aligned} \tag{2}$$

The meaning of "primary maximum" will be made clear in the sections that follow. These elements satisfy not only the seven observations in Table 3 but also the appearance of λ4012 and other features on the remaining twenty-one spectrograms obtained between 1948 and 1954, and, of course, the observations of 1966 and 1967.

TABLE 3

DATA FOR IMPROVEMENT OF THE PERIOD OF 21 PERSEI

Plate No. (1)	J.D. 2430000+ (2)	Estimated Phase (3)	Nearest J.D. for $\phi=0.00$ (4)	ΔJ.D. (5)	ΔE (6)	P (7)	Phase (eq. [2]) (8)
Ce 5302	2785.97	0.00	2785.97	6705.80	2325	2.88422	0.00
Ce 5543	2958.67	≤ .9	2958.95	6532.82	2265	2.88425	.88
Ce 6544	3549.94	.85	3550.37	5941.40	2060	2.88417	.88
Pb 735	4380.69	.95	4380.83	5110.94	1772	2.88428	.92
Pb 1244	4706.83	.00	4706.83	4784.94	1659	2.88423	.99
Ce 9050	4732.73	.95	4732.87	4758.90	1650	2.88418	.97
Pb 1868	5084.74	≥0.0	5084.74	4407.03	1528	2.88418	0.02

c) The Photometric Variation

The B and V photometric observations of Rakos (1962) and Stępień (1968), phased together by means of equation (2), are plotted in the upper panels of Figure 2. Rakos's data are given as magnitude differences on an instrumental system. They have been converted to B and V approximately by adding 5.44 and 5.36 magnitudes, respectively, to Rakos's blue and yellow magnitude differences. Mean points for observations in each tenth of a cycle, 0.00–0.09, 0.10–0.19, and so forth, are plotted in the lower panels of Figure 2. Smooth curves drawn through these mean points indicate the presence of a weak secondary maximum in both the B and V light variations. Two high points of Rakos produce most of the secondary maximum in B, but the secondary maximum in V is present with or without the Rakos data. Primary maximum coincides with phase zero calculated by equation (2), while the secondary maximum occurs very nearly at phase 0.5. An effort to verify the reality of this small secondary effect in the curve of 21 Per would be desirable.

d) Spectrum Variability

As mentioned earlier, the intensities of the rare-earth line components vary regularly in the $2^{\mathrm{d}}.88$ period. This is illustrated in Figure 3, where means of eye-estimated intensities on an arbitrary scale for Eu II λ3819 and λ4129 are plotted versus phase, together with the radial-velocity data and the mean V light curve. The line intensities were estimated in two ways: (1) as ratios relative to features at λ3820 (Fe I?) and λ4122 (Fe II) on each

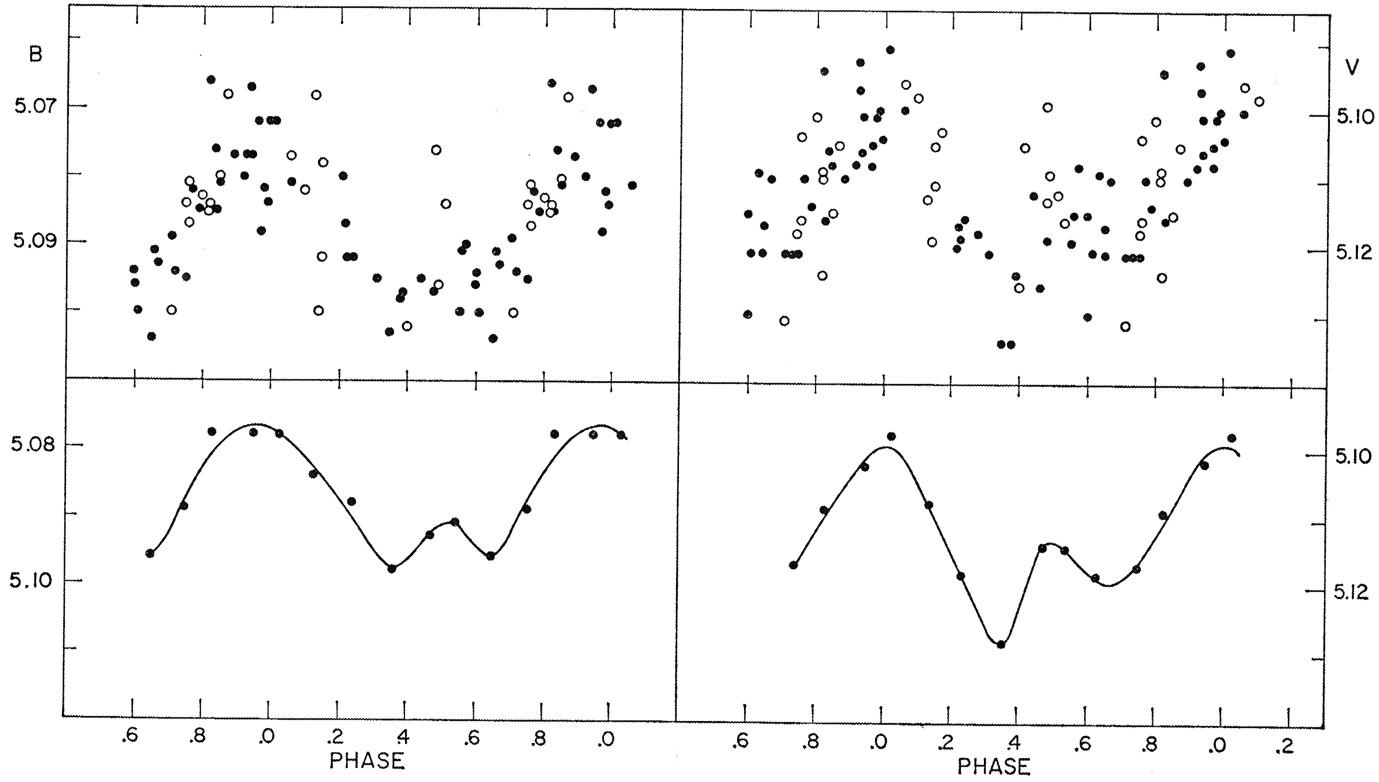

FIG. 2.—*Upper panels:* Photometric B and V variations of 21 Per by Stępień (*filled circles*) and by Rakos (*open circles*). The instrumental magnitudes of Rakos were converted to the UBV system by an approximate procedure described in the text. *Lower panels:* The mean variations in B and V obtained by the procedure described in the text.

spectrogram, and (2) by comparison with the Eu II lines on spectrogram ECZ 5911, which was used as a standard plate. Both sets of estimates gave qualitatively similar results and therefore were averaged together. At phase zero (*a*) the Eu II lines go through maximum strength, (*b*) the Eu II radial velocity equals the systemic or, more properly, the time-average velocity, and (*c*) primary light maximum occurs. At phase 0.5 the procedure repeats; the strength of the Eu II line appears to be slightly weaker, and a small secondary light maximum appears to be present. If the spectrum is observed at lower dispersion, it seems possible that a small-amplitude spectrum variation of Eu II with a double wave might be detectable, but it is now clear that a strong Eu II spectrum variation such as is found in α^2 CVn or HD 125248 can be detected for 21 Per only if high-resolution spectrograms are available. Variations of Ti II and Mn II lines qualitatively similar to those of Eu II occur, while no conclusive variations of Cr II, Fe II, or Sr II could be detected in this study.

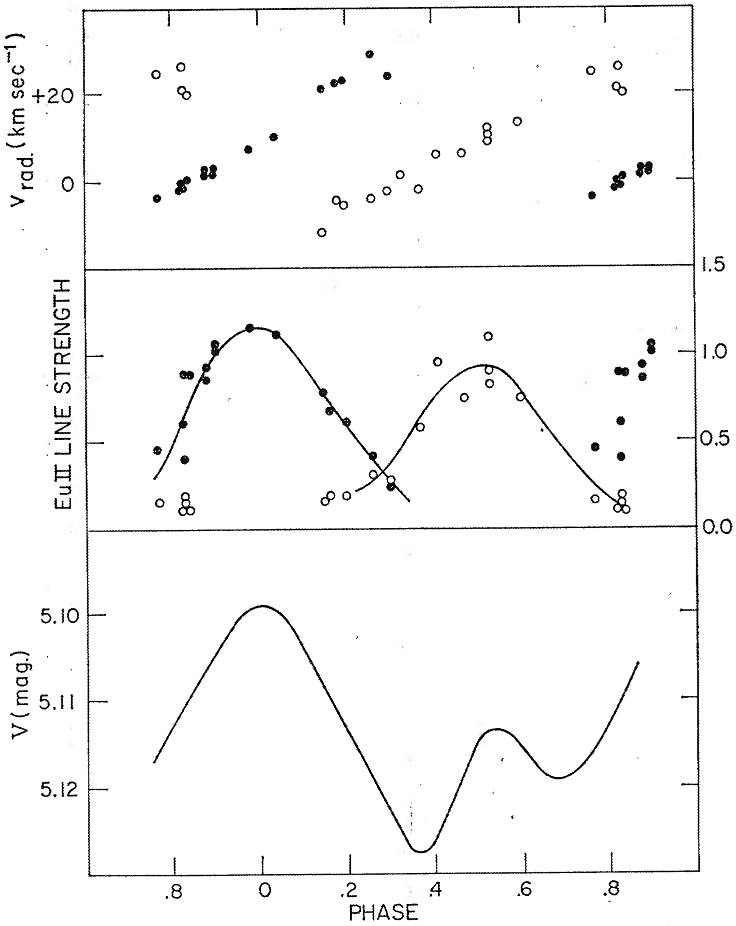

FIG. 3.—The rare-earth velocity variation, Eu II line-intensity variation, and the mean V light variation of 21 Per. Filled and open circles refer to the primary and secondary branches, respectively, of the rare-earth variations.

e) The Magnetic Field

Eyepiece examination of our spectrograms indicates that the Zeeman effect is not pronounced in 21 Per. Furthermore, precise measurements of field strength are difficult to obtain because of the broadened, often asymmetrical, and sometimes double, absorption lines. For these reasons only eleven of the twenty-four spectrograms, primarily the well-widened ones of 1967, were measured for the Zeeman effect, as indicated in Table 1. For practical reasons, no doubled lines were measured for the Zeeman effect. It should be mentioned that no pronounced Zeeman effect can be seen in any of the components of double lines. The data in Table 1, plotted with their error bars versus phase in Figure 4, are inconclusive as regards the periodicity of the magnetic field. Only one or two of the field determinations differ from zero by as much as 2 (formal) probable errors, and it may be expected that systematic errors comparable to the probable errors derived from internal scatter may be present for a variety of reasons (an error of 1 μ corresponds to an error of 200 gauss for a normal Zeeman triplet observed with a dispersion of 4 Å mm^{-1}). Therefore, it is the opinion of the writer that the present data cannot delineate any small magnetic variation that may be present, and they do not even clearly demonstrate the presence of a magnetic field in 21 Per in 1966 and 1967. The only conclusion to be drawn from the present magnetic data is that the complex spectroscopic behavior of 21 Per is not due to the prevalence of a very strong magnetic field. To the contrary, the magnetic field of 21 Per is at most prosaic compared to other well-known periodic magnetic stars.

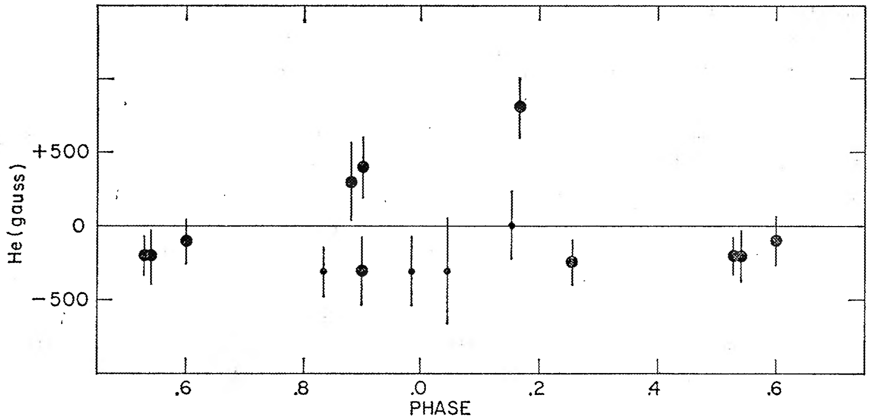

FIG. 4.—Effective magnetic fields for 21 Per phased together by equation (2). The formal probable errors are indicated by vertical bars. Small and large filled circles denote the 1966 and 1967 observations, respectively.

III. DISCUSSION

a) General Remarks

Strong spectrum variations have beeen found in only a modest fraction, about 10 percent (see Table 4 of Ledoux and Renson 1966), of some 220 known Ap stars (Osawa 1965), and it is also known that Ap stars with periodic light and magnetic variations, e.g., HD 32633 (Preston and Stępień 1968*a*) and HD 10783 (Preston and Stępień 1968*b*) have no readily apparent spectrum variations. However, the present results for 21 Per indicate that the statistics of spectrum variability probably have been biased in favor of stars with single waves. If one of the rare-earth branches in 21 Per were suppressed, it would be a pronounced spectrum variable. This conclusion is supported by the fact that weak secondary waves in the line-intensity variations have been discovered in four of the twenty-four known spectrum variables: for He and Si in 56 Ari (Deutsch 1956; Peterson 1966) and HD 124244 (Peterson 1966), for Ti and Ca in HD 125248 (Deutsch 1958), and for several elements in 73 Dra (Preston 1967). In the case of HD 125248, a double wave also occurs in the velocity curve (Babcock 1951). We suggest that these secondary waves were discovered in part *because* they are weak. If they were too strong, the overlapping branches would reduce the ranges of the variations and hence reduce the probability of discovery. Further complications are suggested by recent work of Pyper (1968), who finds evidence for *four* branches in the velocity curve and line intensities of the Fe-peak elements in α^2 CVn, a phenomenon that is detected only when the spectrum is observed with very high dispersion ($\sim$2 Å mm^{-1}). For these reasons it is necessary to consider the possibility that a major fraction or perhaps all of the Ap stars possess periodic spectrum variations of small amplitude and perhaps complicated form that have gone undetected by the methods of observation employed heretofore. Similar remarks may apply to the light variations, as suggested by the weak secondary light maximum in 21 Per.

b) The Rigid-Rotator Model

As is the case for other periodic spectrum variables, the spectroscopic behavior of 21 Per can be understood qualitatively in terms of the modulations in line intensity and radial velocity produced by a spotted, rotating star (Deutsch 1956). The velocity ranges for the rare earths and Fe-peak elements conform to the relation between period and line width (Deutsch 1953, 1956), and the line intensities and radial velocities vary in quadrature as required by the rotator model. Furthermore, the line components of the rare earths are relatively sharp when they are double and broadest when, on the rotator model, they transit the center of the disk and thus occupy a major fraction of it. The rotation is rigid in the sense that the spectroscopic phenomena appear to have repeated in a simply periodic manner for the past twenty years. The case of 21 Per is of particular interest because it provides the first evidence that the rare earths may be concentrated in two spots rather than one and, by inference, perhaps in several spots on other stars.

From this point of view the known periodic spectrum variables possess the least amount of fine structure on their surfaces, present the largest variations in integrated light, and thus were discovered first. These ideas certainly are not new. We simply call attention to the fact that our results for 21 Per tend to support them. Finally, this study reduces by one the number of reportedly irregular spectrum variables that stand as evidence against the rigid-rotator model.

The writer wishes to acknowledge helpful discussions with Dr. Armin J. Deutsch. A portion of this research was supported by a grant to the writer from the National Science Foundation.

REFERENCES

Babcock, H. W. 1951, *Ap. J.*, **114**, 1.
———. 1958, *Ap. J. Suppl.*, **3**, 141 (No. 30).
Deutsch, A. J. 1953, *A.J.*, **58**, 37.
———. 1956, *Pub. A.S.P.*, **68**, 92.
———. 1958, *I.A.U. Symp. No. 6*, p. 209.
Ledoux, P., and Renson, P. 1966, *Ann. Rev. Astr. and Ap.*, **4**, 293.
Meggers, W. F., Corliss, C. H., and Scribner, B. F. 1961, *N.B.S. Monog.*, No. 32, Part 2.
Moore, C. E. 1945, *Contr. Princeton Univ. Obs.*, No. 20.
Morgan, W. W. 1935, *Pub. Yerkes Obs.*, Vol. **7**, Part 3.
Osawa, K. 1965, *Ann. Tokyo Astr. Obs.*, 2d ser., **9**, 123.
Peterson, B. A. 1966, *Ap. J.*, **145**, 735.
Preston, G. W. 1967, *Ap. J.*, **150**, 871.
Preston, G. W., and Stępień, K. 1968*a*, *Ap. J.*, **151**, 577.
———. 1968*b*, *ibid.*, **154**, 971.
Pyper, D. M. 1968, unpublished thesis, University of California at Santa Cruz.
Rakos, K. D. 1962, *Lowell Obs. Bull.*, **5**, 227.
Searle, L., and Sargent, W. L. W. 1964, *Ap. J.*, **139**, 793.
Stępień, K. 1968, *Ap. J.*, **154**, 945.