

THEORETICALLY PREDICTED COLOR-MAGNITUDE DIAGRAMS FOR CLUSTERS AND THE OBSERVATIONS

BARRY M. SCHLESINGER

Department of Astronomy and Astrophysics, University of Chicago

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ABSTRACT

Stellar-evolution calculations by Iben have been used to predict the appearance of the color-magnitude diagram for clusters of six different ages. The predicted C-M diagrams are compared with observations. The relative number of stars in different parts of the C-M diagram receives special attention.

Comparisons at particular ages help in the interpretation of the available observations. An examination of the relation between the luminosity of the stars at the top of the main sequence and that of the stars on the giant branch of young clusters reveals that some of the differences between the C-M diagrams of Magellanic Cloud clusters and of Galaxy clusters that are usually attributed to composition may be age effects. The possibility of a composition difference between M41 and M11 is discussed. If the distance moduli derived at the time of the original observations were correct, the differences between the C-M diagrams of M41 and M11 indicate a difference in composition. It is suggested that most yellow giants in clusters aged 5×10^8 to 10^9 years are burning core helium.

I. INTRODUCTION

Since 1964, theoretical evolutionary tracks which extend into advanced stages of evolution have become available. For stars of less than $3 M_{\odot}$ several tracks have been computed up to the helium flash, and a few have been computed beyond it; for stars of $3 M_{\odot}$ or more, computations extend through core-helium burning. Iben (1965*a,b*, 1966*a,b,c*, 1967*a,b*, referred to hereinafter as Iben I–VII) has computed the most extensive series for a given chemical composition: $X = 0.71$, $Z = 0.02$. Tracks for stars of eight different masses in the range 1 – $15 M_{\odot}$ were calculated. With such a set of evolutionary tracks, it is possible to obtain the dependence upon stellar mass of the time required for a star to pass through a particular evolutionary stage.

To predict the color-magnitude (C-M) diagram of a cluster of a given age first requires the locus in the $(M_{\text{bol}}, \log T_e)$ diagram for stars of a range of masses at that age. Interpolation between a series of evolutionary tracks yields more accurate results than does use of the homology arguments which the scarcity of evolutionary tracks then available forced on Sandage (1957) in his pioneering work in the prediction of cluster C-M diagrams from theory.

Section II describes the method used to construct the C-M diagrams from the theoretical tracks. In § III there is a discussion of the figures and tables. Sections IV–V compare the theoretical predictions for clusters younger than 5×10^8 years with observations of h and χ Per, NGC 1844, M41, and M11. Section VI contains a discussion of the relative luminosities of the giant branch and the top of the main sequence in young clusters. Sections VII and VIII discuss clusters older than 5×10^8 years, with particular emphasis on the distribution of stars in the C-M diagrams. Finally, § IX suggests possible future observations which would help solve the problems incompletely settled in the present study.

II. CONSTRUCTION OF THE COLOR-MAGNITUDE DIAGRAMS

Each cluster is a collection of stars of different masses at the same age. The masses were distributed according to the Salpeter (1955) mass function:

$$N(M) dM = \text{const.} \times M^{-1.35} dM. \quad (1)$$

Evaporation of stars from the cluster was neglected. The evolutionary state of each star at the age of the cluster was then computed. The resultant C-M diagrams are appropriate to Iben's composition: $X = 0.708$, $Z = 0.02$.

The evolutionary history of a star can be divided into several stages; within each stage, the time variation of luminosity and of effective temperature is approximately linear. Well-defined events or conditions in the star, described in Table 1, separate these stages. The event numbers correspond, as far as is practical, to those of Iben. Because the evolution of a star can be described as a series of smooth stages, the following procedure could be used to obtain the evolutionary stage and position in the C-M diagram for the given star at the desired age.

Starting at the main sequence, the time to pass from one stage to the next was computed until the total time elapsed at the end of a given stage exceeded the age of the cluster. This procedure located the stage through which the star was evolving. For

TABLE 1
PRINCIPAL EVENTS OF EVOLUTIONARY TRACK

Event Number	Description
1 . . .	Zero-age main sequence
2 . . .	Half of initial central hydrogen is depleted
3 . . .	Overall contraction starts, effective temperature increases
4 . . .	T_{eff} at maximum after overall contraction
6	Luminosity peak during thick-shell burning
10	Giant base, rapid gravitational contraction
13	Giant tip
15	Luminosity minimum at return from giant tip, core-helium burning
17	Rapid envelope contraction
18	Blue point of core-helium burning
20	Core-helium exhaustion

calculation of the time elapsed during pre-main-sequence contraction and during core-hydrogen burning, the following formulae were fit to the data from Iben I–VII:

$$(\text{pre-main sequence}) \log \tau = 7.7368 - 2.8282 \log \mathfrak{M} + 0.169 (\log \mathfrak{M})^5, \quad (2)$$

$$(\text{core-hydrogen burning: Iben points 1–2}) \\ \log \tau = 9.5650 - 3.8075 \log \mathfrak{M} + 1.31 (\log \mathfrak{M})^7, \quad (3)$$

$$(\text{core-hydrogen burning: Iben points 2–3}) \\ \log \tau = 9.4817 - 4.293 \log \mathfrak{M} + 1.6312 (\log \mathfrak{M})^7, \quad (4)$$

where $\mathfrak{M}$ is in solar masses and τ is in years. These formulae yielded values of $\log \tau$ to an accuracy of ± 0.02 . For later stages, a table was constructed listing the logarithm of the time between events for each stage at each mass studied by Iben; the logarithm of the time between events for intermediate masses was calculated by linear interpolation as a function of the logarithm of the mass.

Main-sequence values of $\log L$ and $\log T_e$ came from the following formulae derived by fitting curves to Iben's results:

$$\log L = -0.1514 + 5.293 \log \mathfrak{M} - 1.31 (\log \mathfrak{M})^7, \quad (5)$$

$$\log T_e = 3.7486 + 0.678 (\log \mathfrak{M})^8, \quad (6)$$

with $\log \mathcal{M}$ again in solar masses. The error in $\log L$ was ± 0.03 ; that in $\log T_e$, ± 0.01 . For later stages, a table of values for $\log L$ and $\log T_e$ was constructed, including all of Iben's masses. The table consisted primarily of differences from the main-sequence values of the values of $\log L$ and $\log T_e$ at each stage of evolution. There then resulted a smoother variation with mass than would have resulted with the actual values of $\log L$ and $\log T_e$. On the red-giant branch (events 10, 13, and 15), however, the actual values of $\log T_e$ were used. Interpolation in this table, considering $\log L$ and $\log T_e$ functions of the logarithm of the mass and the logarithm of the time, yielded the position of the star in the theoretical H-R diagram.

In order to obtain $B - V$ colors, bolometric corrections, and spectral types from the effective temperatures, a table based on the work of several authors was compiled. The review article by Harris (1963) supplied an evaluation and summary of earlier work for luminosity classes III and V, while the best early summary for supergiants was that of Kraft (1961). The work of Kopylov (1963) provided an independent set of values. The results of Johnson (1966), including data based on new observations in the infrared, were the most recent complete work; consequently, they were given strong weight. Because there have been few observations in the ultraviolet, the theory of Mihalas (1965) was considered in studying early spectral types. In constructing the table, all data available from the above sources were considered. Appropriate weights were assigned to each available value, and a weighted mean was computed. Table 2 presents the values adopted. Using this table, $(\log L, \log T_e)$ could be converted to $(M_V, B - V)$ for comparison with observations. A value of 4.72 mag was used for the bolometric magnitude of the Sun.

III. THE COLOR-MAGNITUDE DIAGRAMS

Figure 1 shows the resultant "color-magnitude diagrams" derived from theory. The solid lines represent regions of high star density, but the individual stars are shown wherever possible, as on the giant branch. The actual relative number of stars in the different parts of the C-M diagrams appears in Tables 3 and 4. In order to afford convenient comparison with the observations and to illustrate the variation with cluster age of the ratio of giants and supergiants to main-sequence stars, the figures in Table 3 have been normalized to 100 stars in the upper two magnitudes of the main sequence of each cluster. Table 4 presents the distribution in the C-M diagram of the stars in a single cluster at different ages; the density corresponds to $\Delta \log \mathcal{M} = 0.00113$ at $\mathcal{M} = \mathcal{M}_\odot$. This table illustrates the change with time of the population of the different sections of the C-M diagram.

IV. NGC 1844 AND OTHER YOUNG CLUSTERS

Hodge (1961) has called attention to the vertical giant branch of the Large Magellanic Cloud cluster NGC 1844, an observation apparently at variance with the theoretical result that massive stars evolve horizontally in the H-R diagram after the exhaustion of core hydrogen. Comparison with the C-M diagrams of clusters A and B helps explain the observations. Two of the blue stars in NGC 1844 stand apart, one 0.5 mag and the other 1.5 mag above the top of the main sequence, curving toward the red. A similar situation has been observed in the clusters h and χ Per (Wildevy 1964) and in NGC 4755 (Arp and Van Sant 1958). It has been suggested (Hayashi and Cameron 1962; Schild 1967) that the brightest blue stars in h and χ Per are burning core helium. Comparison with the theoretical clusters of the present study reinforces this point of view. In cluster A, of the eleven blue stars above the tight main sequence, ten are burning helium in the core after returning from the giant tip, and one is in the phase of evolution in which hydrogen is burned in the shell. Therefore, of the two brightest blue stars in NGC 1844, at least one, and possibly both, are burning core helium.

Now, consider a cluster containing, as does NGC 1844, nineteen stars within 2.0 mag of the top of the main sequence. At the age of 22.5 million years, it would contain, according to Table 3, one blue supergiant and, at the age of 70 million years, two blue supergiants and two red supergiants. Hodge suggested that the faintest star among the red giants of NGC 1844 might be a nonmember; the observed cluster thus contains one or two blue supergiants and three or four red supergiants. It is not inconsistent with theory (Hofmeister, Kippenhahn, and Weigert 1964) that the brightest red supergiant

TABLE 2

RELATION BETWEEN SPECTRAL TYPE, COLOR, TEMPERATURE, AND BOLOMETRIC CORRECTION

Spectral Type	$B-V$		Effective Temperature ($^{\circ}$ K)			Bolometric Correction			
O7	-0 32		39000			-3 48			
O8	-0 31		37600			-3 36			
O9	-0 31		33900			-3 12			
B0	-0 30		30400			-2 93			
B1	-0 26		24500			-2 45			
B2	-0 24		21000			-2 06			
B3	-0 20		18100			-1 69			
B5	-0 16		15800			-1 35			
B7	-0 12		13800			-1 01			
B8	-0 09		12400			-0 73			
B9	-0 06		11400			-0 58			
A0	0 00		10100			-0 30			
A1	+0 03		9750			-0 25			
A2	+0 06		9290			-0 20			
A3	+0 09		9000			-0 15			
A5	+0 15		8400			-0 11			
A7	+0 20		8010			-0 08			
F0	+0 33		7280			-0 07			
F2	+0 38		6910			-0 06			
LUMINOSITY CLASS									
	Ib	III	V	Ib	III	V	Ib	III	V
F5	0 37	0 43	0 45	6630	6510	6550	-0 03	-0 04	-0 05
F6	0 44	0 48	0 47	6400	6240	6420	-0 04	-0 04	-0 06
F8	0 55	0 54	0 53	6020	6010	6120	-0 07	-0 05	-0 07
G0	0 70	0 66	0 60	5670	5760	5880	-0 17	-0 10	-0 09
G2	0 85	0 73	0 64	5540	5430	5720	-0 28	-0 15	-0 11
G5	1 01	0 83	0 68	5000	5180	5590	-0 37	-0 23	-0 13
G8	1 03	0 86	0 72	4830	4940	5470	-0 48	-0 28	-0 17
K0	1 12	1 01	0 81	4580	4720	5240	-0 67	-0 39	-0 22
K1	1 18	1 09	. . .	4400	4530	. . .	-0 71	-0 47	. . .
K2	1 25	1 16	0 92	4260	4350	4850	-0 79	-0 54	-0 30
K3	1 38	1 29	0 98	4140	4050	4660	-0 86	-0 74	-0 41
K4	1 49	1 40	. . .	4010	3840	. . .	-0 95	-0 96	. . .
K5	1 62	1 52	1 18	3690	3610	4140	-1 27	-1 20	-0 76
K7	. . .	. . .	1 38	. . .	. . .	3710	. . .	. . .	-1 17
M0	1 63	1 53	1 42	3580	3480	3570	-1 61	-1 40	-1 39
M1	1 63	1 54	1 49	3530	3360	3450	-1 69	-1 48	-1 65
M2	1 64	1 57	1 51	3470	3310	3310	-1 73	-1 60	-1 98
M3	1 64	1 58	1 53	3310	3300	3240	-1 04	-1 93	-2 20
M4	1 64	1 58	1 56	3100	2980	3140	-2 60	-2 57	-2 33
M5		1 58	1 60		2870	3040		-3 36	-2 48
M6			1 63			2920	. . .		-2 82

may be returning to the red after core-helium burning. When the blue supergiants are included, the giant branch is then nearly horizontal. The stars are in two segments widely separated in color, with the intermediate region devoid of stars because the evolving stars pass through quickly during stages of rapid envelope contraction or expansion. Such a giant branch appears in theoretical cluster B. The observations, then, can be explained by theory.

Note that there is no giant-tip brightness peak in cluster B, in contrast with the sharp giant-tip luminosity peak in the theoretical tracks. Two processes in the use of an

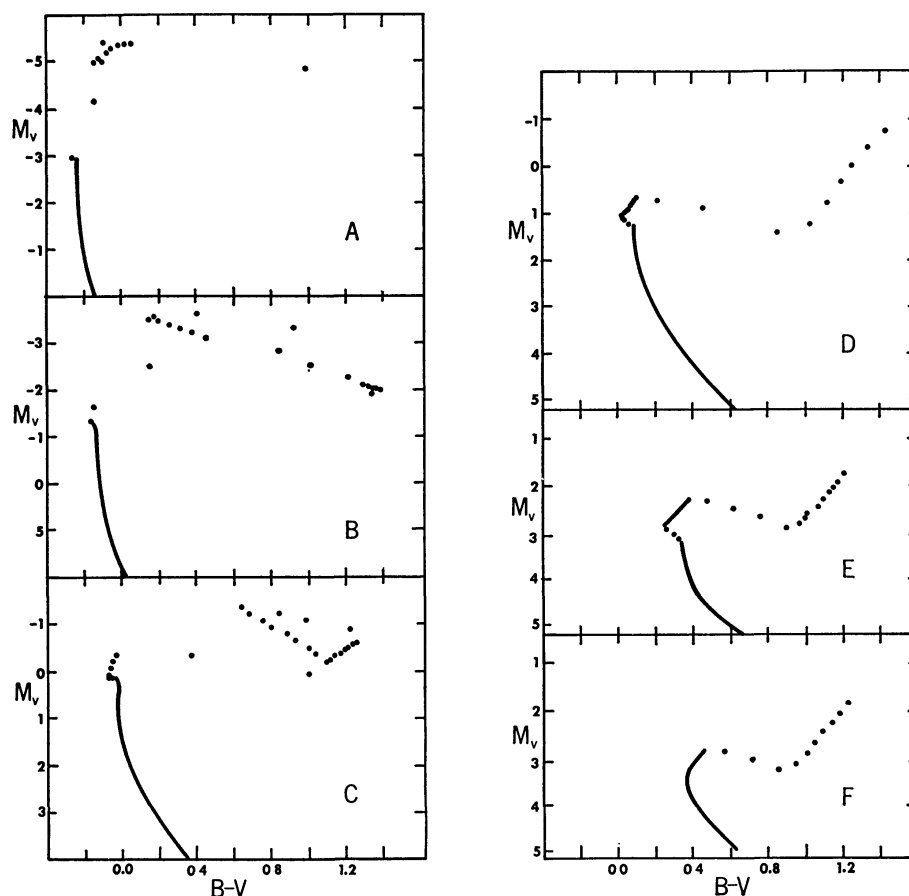


FIG. 1.—Theoretically predicted C-M diagrams. Heavy lines, regions of high star density. Cluster ages in millions of years are as follows: A, 22½; B, 70; C, 250; D, 600; E, 2250; F, 4000.

evolutionary track in the theoretical H-R diagram to obtain a cluster C-M diagram increase the brightness of the bluer stars relative to that of the reddest stars:

1. Although the redder stars of the red-giant branch are more luminous than the bluer stars, a larger fraction of their energy is emitted in the invisible infrared.

2. Nearly all of the red giants in the cluster are returning from the giant tip. Since evolution on the giant return is from red to blue, the bluer stars are further evolved. The further-evolved stars are more massive, hence, intrinsically more luminous at any evolutionary stage. The mass range on the giant branch is sufficiently large that its effect will be significant; stars which have exhausted core helium are 10 percent more massive than those about to cross the Hertzsprung gap.

As a result of these two factors, the descending giant branch of theory becomes nearly flat in practice.

V. M41 AND M11

Cox (1954) published a C-M diagram for M41 and derived a distance modulus. Johnson, Sandage, and Wahlquist (1956) have similarly studied M11. Since the top of the main sequence of the published C-M diagram of M41 is at $M_V = -1.2$ mag, and that of M11 at $M_V = -0.9$ mag, both clusters are best compared with the theoretical constructs B and C. There results an age for the clusters of $1-2 \times 10^8$ years. The M41 giant branch is somewhat redder than the theory predicts, but since the uncertainties in the theory of convection make predictions of the effective temperature unreliable to the red of $\log T_e = 3.700$ or $B - V = 0.85$ mag (Henyey, Vardya, and Bodenheimer 1965), this difference need not be of concern. A quantitative study is possible of the denser M11. Since 102 stars are within 2 mag of the top of the main sequence, and since the published C-M diagram includes a statistical bias of 2 in favor of the giants, Table 4 predicts from thirty to fifty giants. Only twenty-four are present.

Significant differences exist between the C-M diagrams of M41 and M11 which are difficult to explain if both clusters are assumed to have the same composition. Although the top of the main sequence of M41 is 0.3 mag brighter than that of M11, the giant

TABLE 3
STAR DISTRIBUTION AMONG EVOLUTIONARY STAGES IF 100 STARS
ARE WITHIN 2.0 MAG OF TOP OF MAIN SEQUENCE

CORRESPONDING CLUSTER OF FIGURE 1	AGE (million years)	NUMBER				
		Top Magnitude of Main Sequence	Hertzsprung Gap; Subgiants	Rising Giants (Lowest Magnitude)	Red Supergiants	Blue Supergiants
A	22 5	38 5	.	.	1 3	6 5
B	70	39 3	0 9	.	8 9	8 0
C	250	37 0	1 4	.	27 4	0 0
D	600	36 6	2 2	7 5	.	.
E	2250	33 0	3 3	11 0	.	.
F	4000	35 3	2 4	9 4	.	.

TABLE 4
STAR DISTRIBUTION AMONG EVOLUTIONARY STAGES
FOR ONE CLUSTER AT DIFFERENT AGES

AGE (million years)	NUMBER					
	Top 2 0 Mag of Main Sequence	Top 1 0 Mag of Main Sequence	Hertzsprung Gap Subgiants	Rising Giants (Lowest Magnitude)	Red Supergiants	Blue Supergiants
22 5	106	40 4	0	.	3 2	16 2
70	130	51 0	1 2	.	11 6	10 4
250	101	37 6	1 4	0	27 8	0
600	93	34	2	7	.	.
2250	91	30	3	10	.	.
4000	85	32	2	8	..	.

branch is a full magnitude brighter. The discrepancy is not due to differences in the appropriate bolometric corrections; the giants of M11 are, if anything, bluer than those of M41, and the differences in bolometric magnitude are therefore at least as large as those in the visual. It might also be suggested that there is a false shortening at the top of the main sequence of M41 because it is only one-fourth as dense as M11, but even if only every fourth star in M11 were considered, the top of the main sequence would drop at most 0.2 mag. An examination of clusters B and C shows a trend for the red end of the giant branch to decrease in magnitude more rapidly than the top of the main sequence as the cluster grows older, but the trend is not nearly as dramatic as the differences between M41 and M11. It is possible that a complete reexamination of the cluster fit to the zero-age main sequence (ZAMS), using a ZAMS derived since the original studies and spectral types of cluster members, would change the derived luminosity of one or both of the cluster main sequences. However, if the present distance moduli are correct, the differences between the C-M diagrams of M41 and M11 suggest a difference in chemical composition.

Other problems arise when M11 itself is compared with the theory. In the theoretical cluster C, aged 250 million years, the red giants lie 0.0–0.5 mag above the top of the main sequence, but those of M11 are 0.1–0.9 mag below it. Since the M11 giants are redder than those of the theoretical cluster, 0.2 mag can be explained by the larger bolometric correction of the M11 giants, but a significant difference (about 0.7 mag) remains. The pronounced curve to the right at the top of the main sequence of M11 is not found in the theoretical clusters. If these stars are burning hydrogen in a thick shell, more are present than theory predicts. Consideration of rotation and mass loss may improve our understanding of the top of the main sequence and explain the rightward curve. Some modification of Iben's results is necessary to explain adequately the distribution of the stars in the C-M diagram of M11. One such modification would be the inclusion of line effects in the opacities, which would reduce the magnitude of the giant branch of the evolutionary tracks relative to the main sequence. This question will be discussed in more detail with reference to stellar-evolution calculations by the present author and other authors in a subsequent paper.

VI. MAIN SEQUENCE AND GIANT BRANCH

Hodge (1963) has noted that the difference in magnitude between the red giants and the top of the main sequence is not the same for Magellanic Cloud clusters as for those of the Galaxy. In clusters of the Galaxy such as η Per and M41, the giant branch is fainter than the top of the main sequence, but in the LMC cluster NGC 1866, whose age falls between those of the Galaxy clusters mentioned above, the giants are brighter than the top of the main sequence. The age difference between the Galaxy and Cloud clusters hampers actual comparison, and care must be taken in interpreting differences in the C-M diagram as due to composition, because the effect discussed by Hodge may be expected even at constant composition, as will now be shown.

Consider the theoretical clusters A, B, and C. In the youngest clusters, like cluster A, the stars burning helium in the core return so far to the blue that they can be—and, in fact, have been—confused with main-sequence stars, although they are 1 or 2 mag brighter than the top of the true main sequence where stars are burning hydrogen in the core. This problem has already been noted in the discussion of NGC 1844 in § IV. As the main sequence carries a spurious extension of 1–2 mag, the red-giant branch appears fainter than the top of the main sequence. In the somewhat older cluster B, stars burning helium in the core are far enough to the red to be distinguished from the main-sequence stars and appear as a giant branch perhaps a magnitude above the main sequence. A cluster of the age of the Pleiades but denser would show this kind of C-M diagram. In the still older cluster C, the effects of changing bolometric corrections at both the blue and the red ends combine to lower the giant branch with respect to the

main sequence. As a cluster ages, both the top of the main sequence and the giant branch become redder. At the top of the main sequence, as the spectral type of the brightest stars moves from early B to late B, the bolometric correction decreases. The bluest giants, meanwhile, in changing from F to K, are radiating an increasing proportion of their light in the invisible infrared.

To summarize, clusters of an age between that of η Per and that of M41–M11 will have giants brighter with respect to the main sequence than will the younger or older clusters. In the present study, the difference, giant branch minus top of main sequence, is -0.3 mag at 22.5 million years, -1.7 mag at 70 million years, and $+0.3$ mag at 250 million years. Real differences in composition may exist between the Clouds and the Galaxy, and real differences may exist in the H-R diagram, but the phenomenon discussed in this section must be considered before such differences are interpreted.

VII. THE HYADES GENERATION

A main sequence extending to between 0.0 and 1.0 mag in absolute visual magnitude and a few G giants at the same magnitude as the top of the main sequence characterize the C-M diagrams of the Hyades (Johnson 1960), Praesepe (Johnson 1952), and NGC 6633 (Hiltner, Iriarte, and Johnson 1958). Comparison with theory yields an age of 500–750 million years; the 600-million-year cluster will serve to analyze the observations.

The term *main sequence* has so far been used somewhat loosely. In clusters of the age of the Hyades or older, the continuous band of stars extending up from the top of the main sequence includes not only stars burning hydrogen in the core but also stars burning hydrogen in a thick shell. In the younger clusters, the shell-burning stars are not present in numbers sufficient to create a problem. In the interest of precision, the top of the band of stars in older clusters which extends continuously from the main sequence will be referred to hereinafter as the *extended main sequence* (EMS).

The nature of the stars on the giant branch of these clusters is particularly interesting. Giants approaching the helium flash should, according to theory, be spread out from the base to the tip of the rising-giant branch. Because of the method used in constructing them, theoretical clusters E and F show a somewhat more even distribution than detailed evolutionary tracks predict. Star densities should be about 20 percent higher near the base of the rising giant-branch, and 20 percent lower near the tip of the theoretical clusters. In the $2.25 M_{\odot}$ model, on which cluster D is based, the rate of luminosity increase is more nearly constant with time than for the less massive stars. Although the luminosity decreases on the subgiant branch at a rate comparable with that of the increase on the rising-giant branch, the subgiant branch is less densely populated because stars change color and temperature more rapidly on the subgiant branch than on the rising-giant branch.

In the clusters of the age of the Hyades, however, the giants show no spread along the rising-giant branch; all fall in a tiny region of the C-M diagrams. Observe the number of giants relative to the number of main-sequence stars. Each cluster has approximately twenty stars in the upper two magnitudes of the extended main sequence. From Table 3, only one or two stars are to be expected in the lowest magnitude of the rising-giant branch; yet each cluster contains at least three giants, and NGC 6633 contains five. If these stars are assumed to be on the rising-giant branch, the observations contradict theory. To suggest that the giants are burning core helium after undergoing the helium flash, however, is consistent with the observations. Calculations at higher masses show more stars burning core helium than are rising to the giant tip; the problem of the excessive number of giants is solved. Furthermore, giants will cluster closer in color to the giant tip at lower masses. Extrapolation to lower masses from the $3 M_{\odot}$ data predicts that all giants will have nearly the same color—exactly what is observed. Since the giants of the age of the Hyades are at the upper limit of the helium flash, this interpretation of the observations implies that, for the case of a mild flash, there is no catastrophe.

VIII. THE OLDEST GALACTIC CLUSTERS

It is difficult to obtain an accurate count of the number of stars on the giant branch of clusters older than the Hyades that can be compared with theory. Iben's evolutionary tracks were not carried to the helium flash, and only recently have the helium flash and subsequent events been treated in detail. No series of models of differing masses evolving beyond the helium flash comparable with Iben's pre-flash series yet exists. Because stars may return to the upper part of the giant branch after helium burning, it is wrong to compare the star density along the entire giant branch with theory as if all stars there were burning shell hydrogen. Such error can be avoided by considering only the lowest magnitude of the rising giant-branch. A further problem, however, is that detailed membership studies have not been made for most older clusters.

In the case of NGC 2158, the membership difficulty is less severe. Arp and Cuffey (1962), by comparing the number of stars from selected sections of the C-M diagram in the inner area of the cluster with the number in an outer ring, were able to estimate the percentage of field stars in these areas of the C-M diagram. They thus provided an accurate value for the number of cluster members in these areas. The luminosity of the top of the EMS suggests an age near 2×10^9 years. Cluster E is thus appropriate for comparison. About 475 stars occupy the upper 2.5 mag of the EMS, or about 400 the upper 2.0 mag. From Table 3 comes the prediction of eleven subgiants and thirty-six stars in the lowest 0.8 mag of the rising-giant branch (note that the theoretical cluster does not cover the entire distance to the giant tip). The numbers observed are twelve and twenty-seven, in excellent agreement with the predictions.

Burbidge and Sandage (1958) did not analyze NGC 7789, a cluster of approximately the same age as NGC 2158, in such detail. The prediction from Table 3 is eight subgiants and thirty rising giants; the actual numbers are three and fifty-six.

The most discussed cluster of this age, NGC 752, is a poor subject for analysis by star count. Because the base of the main sequence halts well above the plate limit (Johnson 1953), it is difficult to judge the density of the cluster. The abnormal main sequence further suggests that NGC 752 may have additional anomalies in the distribution of stars in the C-M diagram.

The cluster NGC 2360, studied by Eggen (1968), is intermediate in age between the clusters studied so far in this section and the Hyades. Most striking is the nearly horizontal giant branch which contrasts with the sloped rising-giant branches of NGC 2158 and NGC 7789. Too many stars are present for a rising-giant branch; from the main-sequence density, one subgiant and four rising giants in the lowest magnitude are expected, but there are, in fact, twelve stars in the first magnitude of the giant branch. Some of the giants may be burning core helium, as has been suggested in § VII for the Hyades. In this connection, it is interesting to note that a curve similar to the $3 M_{\odot}$ evolutionary track can be drawn through the stars of the giant branch of Eggen's C-M diagram (see Fig. 2).

In cluster F, whose C-M diagram resembles that of M67, the increase in density of the subgiant branch relative to the main sequence and the rising giants makes it difficult to decide where the break between the EMS and the subgiant branch lies. The most logical location is where the density of stars drops moving redward. In M67 (Eggen and Sandage 1964), forty-one stars can then be assigned to the first magnitude of the EMS, and Table 3 predicts three subgiants and ten rising giants in the first magnitude. The actual numbers are five subgiants and six rising giants.

When using the theoretical clusters as a basis for a study of the distribution of stars over the C-M diagrams of clusters, it is important to bear in mind that the Salpeter mass function used in this paper does not represent exactly the true mass function for stars throughout the Galaxy. Some differences between the "clusters" of the paper and observed clusters may result from this error in the mass function.

Qualitative trends show agreement between theory and observation. With increasing

age, stars fill in the Hertzsprung gap and cause it to shrink. The size of the Hertzsprung gap and subgiant region predicted by the theory is larger than that observed. For example, the length of the subgiant branch of M67 that cluster F predicts is 0.4 mag in $B - V$; the observed value is 0.25 mag. This discrepancy is easily explained. Iben's choice of chemical composition may not correspond to the actual cluster composition, and a change in chemical composition changes the location of both the main sequence and the giant branch. Further errors in the placement of the giant branch result from the unavailability of a rigorous treatment of nonadiabatic convection.

IX. ON FUTURE OBSERVATIONS

To compare theory and observation convincingly, spectral types must be available for stars of the cluster under study. The standard procedure for determination of colors

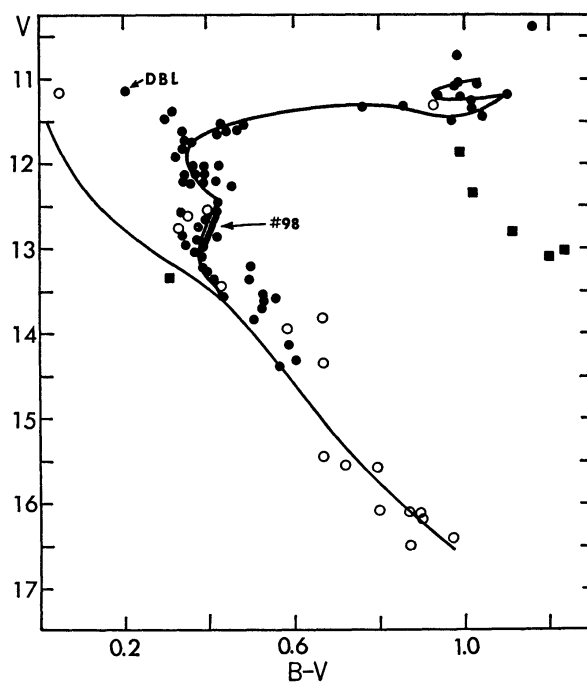


FIG. 2.—Possible evolutionary path of stars in NGC 2360. C-M diagram is from Eggen (1968); the straight line has been added.

and magnitudes of cluster stars involves a fit to a zero-age main sequence. Yet, because of the scatter in the observations, individual judgment is unavoidable in the fitting. The difficulty in determining reddening creates errors in $B - V$. For early B stars, a change in $B - V$ of 0.02 mag may mean a difference of 0.050 in the value of $\log T_e$ assigned to the star. If the color of the bluest stars in the cluster is used to set the luminosity at the main-sequence turnoff, errors in color will lead to errors in luminosity and in the derived distance modulus. Spectral types independently determine the true temperature of the stars at the top of the main sequence, and they do not have to be corrected for reddening. In the youngest clusters, the luminosity class can be used to distinguish stars which have only recently exhausted core hydrogen from stars which are burning core helium. The helium-burning stars will show a supergiant spectrum. It would therefore be most useful if spectrograms were to be taken, where possible, of the brightest stars in those clusters for which C-M diagrams are already available.

Because C-M diagrams of clusters often include both member and nonmember stars,

care is required in basing any conclusions on the number of stars in different regions of the H-R diagram. Problems may arise in several areas. In the youngest clusters, the distribution of stars along the giant branch can be compared with the detailed theory of helium-burning evolution. Not only does the inclusion of nonmembers spoil counts in such cases, but attempts to use the theory to explain the irrelevant observation may lead to unfounded criticism of the theory. In the clusters of the age of the Hyades and NGC 2360, careful star counts can help decide whether or not helium ignition has taken place in the red giants. In addition, the changes in the shape of the observed giant branches between clusters of the age of the Hyades and those of the age of NGC 7789 would be clearer if the shape of the giant branches were not confused by the inclusion of nonmember stars. Finally, if C-M diagrams of old clusters consisting of only member stars were available, it would be possible to compare the theory of giant evolution with observations. Many of these clusters are sufficiently rich for statistically significant comparisons to be made between the predicted evolutionary time scales and the distribution of stars on the giant and subgiant branches. In order for star counts to become a useful tool for testing the theory of stellar evolution, it is imperative that careful membership studies be made, where possible, of those clusters for which C-M diagrams have been published.

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Note added March 28, 1969.—After this paper had been passed for publication, it was noticed that the main-sequence luminosity assigned to the $5 M_{\odot}$ model was, through an error, 0.2 mag higher than Iben's value. Since the mass-luminosity relation used to locate the main sequence was a curve fit to the Iben models, the error in the location of the main sequence contributed only to the estimated error in the derived $\log L$, 0.1 mag. The difference between the fit value and the correct value for L at $5 M_{\odot}$ is within this error. The discussion of the relation between the luminosity at the top of the main sequence and that of the giant branch is not affected. The top of the main sequence is at the end of core-hydrogen burning, stage 2. Since the luminosities of stage 2 and of the giant branch are both measured relative to the main sequence, their relative positions are unchanged. However, because the rise in magnitude during core-hydrogen burning used in the paper was too low at $5 M_{\odot}$, the number of stars in the upper magnitudes of Cluster B should be reduced 15 percent.

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