

THE RATE OF THE PROTON-PROTON REACTION AND SOME RELATED REACTIONS

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ABSTRACT

The results of detailed calculations of the rate of the proton-proton reaction, $p + p \rightarrow {}^2\text{D} + e^+ + \nu$, and the related electron-capture reaction, $p + e^- + p \rightarrow {}^2\text{D} + \nu$, are presented. Estimates of the likely errors are given. The rates of some other electron-capture reactions that are of interest in connection with solar-neutrino experiments are also calculated.

I. INTRODUCTION

We present the results of detailed calculations of the rate of the proton-proton reaction,



and the related electron-capture reaction (henceforth designated *pep*),



under conditions that are believed to exist in stellar interiors. In addition, we provide more accurate expressions for the rates of some other electron-capture reactions that can also occur in the proton-proton chain. Our results for reaction (1), the rate of which determines the conditions under which nuclear energy generation takes place in main-sequence stars of one solar mass or less, include various refinements that have become possible since the original calculation by Bethe and Critchfield (1938) and the authoritative rediscussion by Salpeter (1952). Our numerical results for the rate of reaction (1) have been presented earlier (Bahcall and May 1968) without detailed justification. Schatzman (1958), among others, has pointed out the possible role of reaction (2) in white dwarfs.

Recently Bahcall and Shaviv (1968) have shown that the values derived from solar models of certain quantities, such as the solar central temperature and the predicted neutrino fluxes, are sensitive to the precise rate of reaction (1). On the other hand, the predicted solar-neutrino flux from the *pep* reaction is almost independent of the solar model (Bahcall, Bahcall, and Shaviv 1968). Hence a measurement of the neutrino flux from reaction (2) could be used as a test of the basic hypothesis that nuclear fusion reactions, initiated by the proton-proton reaction, are responsible for the luminosity of main-sequence stars like the Sun.

The present work was undertaken to provide more accurate expressions for the rates of the nuclear reactions mentioned above, to estimate the likely errors, and to indicate how changes in the nuclear data used in the calculations would affect the results presented. In § II we give a general formula for the rate of the proton-proton reaction and define the quantities which must be evaluated. We present in § III the results of our calculations of the nuclear, radial matrix element which were carried out using contemporary proton-proton scattering data and deuteron wave functions. Estimates for the likely errors were made by carrying out the calculations with a variety of wave functions

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and experimental parameters. Section IV contains a brief discussion of the phase-space integrals and the Gamow-Teller coupling constant used in our work. Our final results for reaction (1) are summarized in § V, and our results for the $p\bar{p}$ reaction are presented in § VI. In § VII we give the rates of some other electron-capture reactions that are of interest in connection with solar-neutrino experiments.

II. GENERAL FORMULAE

In this section we present the general formulae, and introduce the notation, by which the rate for reaction (1) can be expressed in terms of quantities that can be calculated more or less directly from experimentally measured quantities. The number of reactions per unit of time per volume V is

$$R_{p-p} = \frac{2\pi}{\hbar V} \sum_{\text{spins}} \int d^3n_{p_1} d^3n_{p_2} \phi(E_1) \phi(E_2) d^3n_e d^3n_\nu d^3n_d \delta(E_i - E_f) |\langle f | H_\beta | i \rangle|^2, \quad (3)$$

where $d^3n_i = [V/(2\pi\hbar)^3] d^3p_i$, $\langle f | H_\beta | i \rangle$ is the matrix element of the usual beta-decay Hamiltonian (Konopinski 1967) between initial state $|i\rangle$ and final state $|f\rangle$, and $\phi(E_i)$ are the standard Maxwell-Boltzmann distribution functions for the two protons. Many of the quantities appearing in equation (3) also appear in the expression for the theoretical beta-decay rate of the neutron. Eliminating most of the constants by making use of the beta decay of the neutron and also taking account of the fact (Salpeter 1952) that only the singlet proton-proton state contributes to the stellar reaction rate, one can re-write equation (3) as:

$$R_{p-p} = V^{-1} \int d^3n_{p_1} d^3n_{p_2} \phi(E_1) \phi(E_2) [\lambda_n (f_{p-p}/f_n) (\xi_{p-p}/\xi_n)] . \quad (4)$$

Here $\lambda_n = \ln 2/(\tau_{1/2})_n$ is the transition probability for the beta decay of the neutron, f_{p-p} and f_n are the respective phase-space integrals (Bahcall 1966a), and the quantities ξ are the usual (Konopinski 1967) reduced matrix elements

$$\xi = \langle 1 \rangle^2 G_V^2 + \langle \sigma \rangle^2 G_A^2 . \quad (5)$$

Here G_V and G_A are, respectively, the vector and axial vector coupling constants. For $O^+ \rightarrow O^+$ transitions like ^{14}O , $\xi = 2G_V^2$; for the neutron decay, $\xi = G_V^2 + 3G_A^2$. For the proton-proton reaction, in Salpeter's notation,

$$\xi_{p-p} = 6G_A^2 |\int \psi_d^* \psi_{pp} d^3r|^2, \quad (6)$$

or, equivalently,

$$\xi_{p-p} = 96\pi^2 \alpha (c/v) e^{-2\pi\eta} (\Lambda^2/\gamma^3) V^{-1} G_A^2 . \quad (7)$$

In equation (7), η is the Coulomb parameter, $\eta = e^2/\hbar v$, v is the proton center-of-mass velocity, γ^{-1} is the radius of the deuteron, and Λ is the orbital matrix element defined by equation (15) of Salpeter,

$$\Lambda = (\gamma^3/8\pi C_0^2)^{1/2} \int \psi_d^* \psi_{p-p} d^3r, \quad (8)$$

with $C_0 = [2\pi\eta/(e^{+2\pi\eta} - 1)]^{1/2}$.

It is both conventional and convenient to introduce the quantity $S(E)$ defined in terms of the cross-section σ in the center of mass by the relation

$$S(E) = (\sigma E) e^{2\pi\eta} . \quad (9)$$

The reaction rate is usually calculated from the equation

$$R_{p-p} = (N_p^2/2) \langle \sigma v \rangle, \quad (10)$$

where N_p is the proton number density and the indicated average is over a Maxwell-Boltzmann distribution for the center-of-mass quantities. It is convenient to use the relation

$$(G_A/G_V)^2 = \frac{1}{3} \left\{ \left[\frac{2(f\tau_{1/2})_{^{14}\text{O}}}{(f\tau_{1/2})_n} \right] - 1 \right\}. \quad (11)$$

For comparison purposes, we note that Salpeter's $g = 2 \ln 2 (G_A/G_V)^2 / (f\tau_{1/2})_{^{14}\text{O}}$. Combining equations (4), (9), (10), and (11), we obtain the basic formula

$$S(E) = 4\pi^2 m_p c a \ln 2 [\Lambda^2(E)/\gamma^3] [f_{p-p}/(f\tau_{1/2})_n] \left[1 - \frac{(f\tau_{1/2})_n}{2(f\tau_{1/2})_{^{14}\text{O}}} \right], \quad (12)$$

where m_p is the proton mass. Electromagnetic radiative corrections can be approximately taken account of (Bahcall and May 1968) by using in equation (11) the $f\tau$ -values, taken from experiment, which have not been altered by theoretical corrections for radiative processes (cf. § IV). We have written equation (12) in the above form in order to make explicit the effect of uncertainties in various quantities on the computed values of $S(E)$ and hence on the total reaction rate. We observe that in order to evaluate $S(E)$ we need to: (1) calculate $\Lambda(E)$; (2) calculate phase-space integrals such as f_{p-p} ; and (3) determine the most accurate values for the neutron and ^{14}O ($f\tau$)-values.

III. THE MATRIX ELEMENT $\Lambda(E)$

In this section we evaluate the (dimensionless) matrix element $\Lambda(E)$ defined in equation (8). Since we only seek to calculate $S(0)$ and $S'(0)$, it is sufficient to evaluate the low-energy behavior of $\Lambda(E)$, characterized by $\Lambda(0)$ and $\Lambda'(0)$:

$$\Lambda(E) = \Lambda(0) + \Lambda'(0)E + O(E^2). \quad (13)$$

For energies corresponding to temperatures in the center of the Sun, the second term in equation (13) contributes only about 0.5 per cent to the magnitude of the matrix element. We calculate the total matrix element using various accurate numerical wave functions for ψ_d and ψ_p , and also, for comparison, in an effective range approximation.

For ψ_d in equation (8), one only needs the S -state deuteron wave function since, as Fieman and Motz (1953) have shown, the deuteron D -state makes a negligible contribution to the matrix element. (The normalization of ψ_d , of course, takes into account the D -state admixture.) The first excited state of the deuteron does not make a significant contribution to the reaction rate because, for temperatures less than 10^9 °K, there are negligibly few protons with enough energy to reach this state. *Asymptotically*, the S -state wave function has the form $\tilde{\psi}_d$,

$$\tilde{\psi}_d = \left(\frac{2\gamma}{4\pi} \right)^{1/2} N' \frac{e^{-\gamma r}}{r}. \quad (14)$$

Here

$$N' = [(1 + \delta^2)(1 - \gamma\rho_d)]^{-1/2}, \quad (15)$$

where δ is the *asymptotic* ratio of D - to S -state wave functions (which is to be distinguished from, and is much smaller than, the D -state probability; in fact, $\delta^2 \sim 0.0007$). Equation (14), of course, follows from the definition of the deuteron (triplet) effective range $\rho_d(-\epsilon, -\epsilon)$.

The deuteron wave function ψ_d is chosen to fit experimental data on the static properties of the deuteron, such as the binding energy, photodisintegration, and percentage D -state. One such wave function which is tailored to fit not only the static properties of the deuteron but also the low-energy n - p scattering data up to 300 MeV, has recently been given by McGee (1966). This wave function is constructed with six exponentials

McGee shows it to be essentially indistinguishable from purely numerical wave functions such as that of Partovi (1964). We shall henceforth refer to McGee's wave function as the "accurate ψ_d ." The most significant parameters which enter the calculation of $\Lambda(E)$ are the deuteron binding wave number $\gamma^{-1} = 4.316f$, the effective range $\rho_d = 1.75 (\pm 0.03)f$, and the D-state probability $7 (\pm 1)$ per cent. (For a more detailed discussion see, e.g., Wilson 1963 or Moravcsik 1963.) To test the sensitivity of our results to the choice of ψ_d , we also calculate Λ^2 using two Hulthén wave functions taken from the literature: we call "Hulthén A" that given by Gourdin *et al.* (1965), and "Hulthén B" the earlier one of Yamaguchi and Yamaguchi (1954) (which is fitted to older values of the static deuteron parameters).

The wave function for the initial two-proton system ψ_p is normalized to a plane wave of unit amplitude. We need only the *S*-state part of this wave function, which thus has the asymptotic form $\tilde{\psi}_p$,

$$\tilde{\psi}_p = \frac{C_0 R}{K} \frac{1}{r} \left[\theta(r) + \left(\frac{r}{R} \right) K \phi(r) \right]. \quad (16)$$

Here ϕ and θ are the usual regular and irregular Coulomb functions, defined so that at the origin $\theta(0) = \phi(0) = 1$; and R is the proton "Bohr radius," $R = \hbar^2/(m_p e^2) = 28.81f$. The quantity K is related to the *S*-wave phase shift $\delta(k)$ by the familiar relation

$$K(k) = C_0^2 k R \cot \delta(k) = -\frac{R}{a} + \frac{1}{2} R \rho_p k^2 - P \rho_p^3 R k^4 - h(kR), \quad (17)$$

where a is the scattering length, ρ_p the effective range, and P the shape parameter for *p-p* *S*-wave scattering. For small x , the function $h(x) = (x^2/3)[1 + (x^2/10) + \dots]$. It is convenient to note that the wave number k is related to the initial center-of-mass proton energy E by $k^2 R^2 = 20.0E$ (MeV). Following Salpeter, we also define the zero energy limit of K as $K_\infty = -(R/a)$.

To get an accurate numerical proton wave function, one first follows the usual procedure of employing the scattering length and effective range to get a *p-p* nuclear potential (of an assumed square, exponential, or Yukawa shape), and then using this potential plus the Coulomb potential to calculate ψ_p . Following Noyes (1964), we take $a = -7.824 (\pm 0.04)f$ and $\rho_p = 2.79 (\pm 0.06)f$. The sensitivity of the results to the choice of these parameters and to the choice of potential shape is discussed below.

As a preliminary, it is interesting (and also convenient in subsequent numerical computations) to first calculate the matrix element $\tilde{\Lambda}$ obtained by using the *asymptotic* wave functions $\tilde{\psi}_d$ and $\tilde{\psi}_p$ in equation (8). One thus gets immediately

$$\tilde{\Lambda}(E) = \frac{\gamma^2 R N'}{K} \int_0^\infty e^{-r} \left(\theta + \frac{r}{R} K \phi \right) dr. \quad (18)$$

Our expression for $\tilde{\Lambda}$ differs from Salpeter's Λ_0 only by the normalization factor N' . It is simple to show that

$$\tilde{\Lambda}(0) = N' \left\{ 1 + \frac{a}{R} \left[E_1(\chi) - \frac{e^{-\chi}}{\chi} \right] \right\} e^\chi, \quad (19)$$

where $E_1(\chi)$ is the usual exponential integral, and we have for convenience defined $\chi = (\gamma R)^{-1}$. Equation (19) is obtained with the help of the low-energy expansions for θ and ϕ (cf. Abramovitz and Stegun 1965). Higher-order terms in the expansion (13) are similarly simple to calculate.

An approximate evaluation of Λ based on $\tilde{\Lambda}$ can be effected by the so-called effective range approximation (Salpeter 1952; Ellis and Bahcall 1968). Following the definitions

of the effective ranges, one can write that approximately

$$\Lambda = \tilde{\Lambda} - \left(\frac{\gamma^2 R N'}{4K} \right) (\rho_d + \rho_p). \quad (20)$$

With the values of a , ρ_d , and ρ_p listed above, this approximate result gives $\Lambda^2(0) = 7.05$.

For an accurate calculation, we use the “accurate ψ_d ” and numerical ψ_p described above, to get the following results. When McGee’s “accurate ψ_d ” is used in conjunction with a square-well ψ_p ,

$$\Lambda^2(0, \text{SW}) = 7.01; \quad (21a)$$

when an exponential well is used for ψ_p ,

$$\Lambda^2(0, \text{exp}) = 7.08; \quad (21b)$$

and for a Yukawa well,

$$\Lambda^2(0, \text{Y}) = 7.21. \quad (21c)$$

In all cases the energy dependence is (to two significant figures, which is more than adequate for our purposes)

$$\Lambda^2(E) = \Lambda^2(0)[1 + 2.2E \text{ (MeV)} + \dots]. \quad (21d)$$

In order to examine the sensitivity to the choice of ρ_p , we use a square-well ψ_p and “accurate ψ_d ,” but allow ρ_p to take the values at the extreme of the allowed range; we get $\Lambda^2(0) = 6.97$ for $\rho_p = 2.85f$, and $\Lambda^2(0) = 7.05$ for $\rho_p = 2.73f$. Conversely, to examine the sensitivity to the choice of ψ_d , we take the exponential-well ψ_p in conjunction with the less accurate “Hulthén A” wave function ψ_d defined above and get $\Lambda^2(0) = 7.01$; if we use in the same way the “Hulthén B” ψ_d , we get $\Lambda^2(0) = 7.00$. The effects of varying a and ρ_d can be adequately estimated from the approximate form (eq. [20]).

In principle, a unique choice of the proton-proton potential shape can be made by knowledge of the shape parameter P in equation (17): Thus P is -0.033 for a square well, -0.019 for a Gaussian, $+0.009$ for an exponential, and $+0.055$ for a Yukawa potential. However, it is difficult to determine P to such an accuracy; typical experimental values are $P = 0.026 \pm 0.014$ (Noyes 1964) and $P = 0.008 \pm 0.035$ (Wilson 1963), which favor the exponential shape but do not dictate it as the unique choice. This question was discussed fully by Salpeter (1952), who showed that different potential shapes produce in Λ^2 differences of about 1 per cent from the value calculated by using the exponential. *Consequently, we choose equation (21b) as our best estimate of Λ^2 .*

Salpeter’s (1952) evaluation of Λ^2 was carried out using essentially an effective range approximation, with the p - p scattering being characterized by the scattering length and effective range given by Blatt and Jackson (1950), which differ from current values by as much as 2 per cent. It is interesting to note that our present value is equal to Salpeter’s 1952 estimate to within 1 per cent, with the various small changes and refinements happening to cancel out. However, the value of Λ^2 actually used by Salpeter to compute the reaction rate was not his own estimate, but rather a contemporary numerical calculation¹ by Frieman and Motz (1953), which is about 4 per cent smaller than our value; these latter authors were using pre-Blatt-Jackson proton-scattering parameters.

Surveying the above discussion to judge the accuracy of our estimate of Λ^2 , we see that the significant sources of error are ~ 1.5 per cent from the shape of the p - p potential,

¹ Frieman and Motz used the numerical deuteron wave function obtained by Feshbach and Schwinger (1951). For ψ_p they used a square well which corresponded to a scattering length of $-7.4f$. It is easy to show that, had they used the Blatt-Jackson parameters, they would have been within 1 per cent of Salpeter’s effective range calculation (they would have obtained $\Lambda^2 \cong 7.12$).

~ 1.2 per cent from the deuteron effective range ($\rho_d = 1.75 \pm 0.03$), ~ 0.6 per cent from the proton scattering length ($a = -7.824 \pm 0.04$), and ~ 0.6 per cent from the proton effective range ($\rho_p = 2.79 \pm 0.06$). We have been conservative in assigning to ρ_d , ρ_p , and a errors larger than those usually quoted in the literature: Our errors are based on the changes in these quantities which arise from different methods of analysis of the raw experimental data (Wilson 1963; Moravcsik 1963; Noyes 1964; Slobodrian *et al.* 1966). Since the above errors are independent, we assign a conservative error of 2.5 per cent to our value of the matrix element:

$$\Lambda^2(E) = (7.08 \pm 0.18)[1 + 2.2E \text{ (MeV)} \dots] . \quad (22)$$

The effective range approximation, equation (20), which is given in analytic form via equation (19), should be accurate enough to describe the effects of changes in equation (22) due to any future revisions in the values of the various basic experimental quantities.

IV. PHASE-SPACE INTEGRALS

The numerical calculations of the various phase-space integrals that appear in equation (12) can be carried out to a sufficiently high accuracy (much better than 1 per cent) so that the remaining uncertainty does not make a significant contribution to the overall error in $S(E)$. Our results for the phase-space factors have been reported earlier (Bahcall and May 1968) and will only be summarized here. Using the computer program of Bahcall (1966*a*), we obtained

$$f_{p-p}(E) = 0.142[1 + 9.04E \text{ (MeV)}] , \quad (23)$$

where E (MeV) is the center-of-mass energy, in MeV, of the two colliding protons. Our value for f_{p-p} is 2 per cent smaller than the one used by Salpeter (1952). The value of $(f\tau)_{O^+ \rightarrow O^+}$ can be obtained from the accurate experiment (Bardin *et al.* 1962) on the decay of ^{14}O . The $f\tau$ -value without radiative corrections is

$$(f\tau)_{^{14}\text{O}} = 3074(1 \pm 0.005) \text{ sec.} \quad (24)$$

Using the most recent and accurate measurement of the neutron half-life (Christensen *et al.* 1967) and the accurately calculated f -value for the neutron (Bhalla 1966; Bahcall 1966*a*), we found

$$(f\tau_{1/2})_n = 1098(1 \pm 0.015) \text{ sec.} \quad (25)$$

The new neutron lifetime of Christensen *et al.* (1967) increases the calculated $S(0)$ by about 10 per cent compared with the value given by Fowler, Caughlan, and Zimmerman (1967).

We have based our estimate of $(G_A/G_V)^2$, as it enters equation (12), on the beta decay of the neutron. Other estimates can be made (e.g., Salpeter 1952), using more complicated nuclei such as ^6He . However, for a nucleus as complicated as ^6He it is difficult to be certain of the precise theoretical value for $\langle \sigma \rangle^2$. More recently Blin-Stoyle and Papageorgiou (1964) have used the difference between $(f\tau_{1/2})_n$ and that for the decay of ^3H to calculate, for reaction (1), a mesonic exchange correction of 3 per cent to Λ^2 . This result includes mesonic effects on Λ , but not on G_A , and hence cannot be consistently applied (cf. Frank-Kamenetskii 1962). Moreover, the endpoint energy for the decay of ^3H is difficult to measure, and hence the calculated f -value for the decay of ^3H is uncertain by about 10 per cent (Bahcall 1966*a*). We do not expect mesonic effects to alter significantly the rate of reactions (1) and (2), but a consistent calculation of their magnitude would be interesting and useful.

V. FINAL RESULTS FOR $p + p \rightarrow {}^2\text{D} + e^+ + \nu$

Combining equations (12), (22), (23), (24), and (25), we obtain for reaction (1)

$$S(E = 0) = (3.78 \pm 0.15) \times 10^{-25} \text{ MeV-barns} \quad (26a)$$

and

$$\left(\frac{1}{S} \frac{dS}{dE}\right)_{E=0} = (11.2 \pm 0.1) \text{ MeV}^{-1}. \quad (26b)$$

Our results for the effective low-energy cross-section factor (cf. Salpeter 1952; Bahcall 1966*b*) are:

$$S_{\text{eff}} \cong (3.78 \pm 0.15)(1 + 0.417\tau^{-1} + 12.6\tau^{-2} + 36.6\tau^{-3}) \times 10^{-25} \text{ MeV-barns}, \quad (27)$$

where $\tau = 33.80T_6^{-1/3}$ and T_6 is the temperature in units of 10^6 °K.

VI. THE RATE OF $p + e^- + p \rightarrow {}^2\text{D} + \nu$

The rate of the pep reaction, $p + e^- + p \rightarrow {}^2\text{D} + \nu$, can be expressed in terms of the rate of reaction (1), ${}^1\text{H} + {}^1\text{H} \rightarrow {}^2\text{D} + e^+ + \nu$, as follows (Bahcall 1964*b*):

$$R_{p+e+p} = (K_{p-p}/\bar{f}_{p-p})R_{p-p}, \quad (28)$$

where R_{p-p} is the rate of reaction (1), $\bar{f}_{p-p} = f(E_0)$, and E_0 is the most probable center-of-mass energy ($\approx 3.9T_6$ keV) for the collisions that lead to reaction (1). In equation (28), K_{p-p} is the generalized phase-space factor (Bahcall 1964*b*) appropriate for electron captures. We find

$$K_{p-p} = 2^{3/2}\pi^{5/2}an_e z(\hbar/m_e c)^3 \left(\frac{W_0 + m_e c^2}{m_e c^2}\right)^2 \left(\frac{m_e c^2}{kT}\right)^{-1/2} \times I(\beta) \quad (29a)$$

with

$$I(\beta) \equiv \int_0^\infty dx e^{-x} [1 - \exp - (2.50\beta x^{-1/2})]^{-1} \quad (29b)$$

and

$$\beta \equiv zT_6^{-1/2}. \quad (29c)$$

In equations (29), $W_0 + m_e c^2$ is the energy, 1.442 MeV, of the neutrino that is produced by reaction (2); n_e is the electron number density; and z is the effective charge (2) on the initial nuclear system. Assuming that the mass density is due mainly to ionized hydrogen and helium [i.e., $\mu_e^{-1} = 0.5(1 + X)$]:

$$R_{p+e+p} = 5.35 \times 10^{-5} \rho (1 + X) T_6^{-1/2} I(2T_6^{-1/2}) R_{p-p}, \quad (30)$$

where ρ is the density in grams per cubic centimeter. Table 1 contains the values of $I(\beta)$ computed by numerical integration for some values of β that are relevant for calculating the rate of neutrino production by electron capture in the Sun. We note that

$$I(\beta) \approx (\pi^{1/2}/5\beta) + \frac{1}{2}, \quad \beta \ll 1, \quad (31a)$$

and

$$I(\beta) \approx 1, \quad \beta \gg 1. \quad (31b)$$

For the temperatures of main interest in the considerations of neutrino production in the solar interior, i.e., $10 \leq T_6 \leq 16$, we obtain the following approximate formula:

$$R_{p+e+p} \cong 5.51 \times 10^{-5} \rho (1 + X) T_6^{-1/2} (1 + 0.02T_6) R_{p-p}. \quad (32)$$

Note that the pep reaction is competitive with the proton-proton reaction, in non-degenerate situations, only for densities greater than or of the order of 10^4 g cm^{-3} .

Equation (32) has been used by Bahcall, Bahcall, and Shaviv (1968) to calculate the rate of neutrino production in the Sun by reaction (2). According to the solar models of these authors, the pep reaction occurs only once for every four hundred ordinary proton-proton reactions in the Sun.

Equation (28) also applies if the electrons are degenerate. The expression for K_{p-p} in the degenerate limit has been given by Bahcall (1962, 1964*b*). Some numerical values of K_{p-p} (degenerate) are listed in Table 2. The following interpolation formula reproduces

TABLE 1
NUMERICAL VALUES FOR $I(\beta)^*$

β	$I(\beta)$	β	$I(\beta)$
5.0	1 000	0 9 . . .	1 11
4 0	1 001	0 8 . . .	1 15
3.0	1 002	0 7 . . .	1 19
2 0	1 01	0 6 . . .	1 26
1 5	1 03	0 5 . . .	1 35
1.3	1.05	0 4 . . .	1.50
1.1	1 07	0 3 . . .	1 77
1.0	1 09	0 2	2 33

* The quantity $\beta = zT_e^{-1/2}$.

TABLE 2
SOME NUMERICAL VALUES OF K_{p-p} (DEGENERATE)

$2\rho/\mu_e$ (g cm^{-3})	K_{p-p} (degenerate)	$2\rho/\mu_e$ (g cm^{-3})	K_{p-p} (degenerate)
1	3.6×10^{-5}	3×10^5	8.5×10^{-1}
10^1	1.7×10^{-4}	10^6	3 1
10^2	7.8×10^{-4}	3×10^6	1 1×10^1
10^3	3.6×10^{-3}	10^7	4 6×10^1
10^4	2.6×10^{-2}	10^8	1 0×10^3
3×10^4	8.0×10^{-2}	10^9	3 0×10^4
10^5	2.7×10^{-1}	10^{10}	1.1×10^6

the results of detailed calculations to an accuracy of better than 15 per cent for densities in the range of $1\text{--}10^{11} \text{ g cm}^{-3}$:

$$\log_{10} K_{p-p}(\text{degenerate}) = -4.3179 + 4.7323 \times 10^{-1} \nu$$
$$+ 5.3884 \times 10^{-2} \nu^2 + 1.9559 \times 10^{-4} \nu^3,$$

(33a)

where

$$\nu \equiv \log_{10} (2\rho/\mu_e),$$

(33b)

and μ_e is the molecular weight per electron [$\approx 2/(1 + X)$].

VII. SOME OTHER ELECTRON-CAPTURE RATES

Several other electron-capture reactions are also of interest in connection with solar-neutrino experiments (Bahcall 1964*a*). The most important of these electron-capture

reactions is ${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$, which is discussed in detail in the following paper (Bahcall and Moeller 1969). In addition, the capture in the Sun of electrons by ${}^8\text{B}$, ${}^{13}\text{N}$, and ${}^{15}\text{O}$, all of which are positron emitters in the laboratory, gives rise to some weak neutrino lines from the solar interior. The ratio of electron capture from the continuum to positron emission is (Bahcall 1964b)

$$(\lambda_{\text{cont. electron capture}}/\lambda_{\text{positron emission}}) = (K/f), \quad (34)$$

where K is given by formula (29) of the preceding section and f is the phase-space factor for the positron decays. The capture of bound electrons will contribute significantly to the total decay rate for the cases considered here and must be computed separately. Making use of the numerical results of Table 1, f -values calculated with the program of Bahcall (1966a), and the decay energies given by Ajzenberg-Selove and Lauritsen (1959), we find

$$\left(\frac{\lambda_{\text{cont. electron capture}}}{\lambda_{\text{positron emission}}}\right)_{8\text{B}} = 3.7\rho(1+X)T_6^{-1/2} \times 10^{-9}, \quad (35a)$$

$$\left(\frac{\lambda_{\text{cont. electron capture}}}{\lambda_{\text{positron emission}}}\right)_{13\text{N}} = 8.3\rho(1+X)T_6^{-1/2} \times 10^{-6}, \quad (35b)$$

and

$$\left(\frac{\lambda_{\text{cont. electron capture}}}{\lambda_{\text{positron emission}}}\right)_{15\text{O}} = 3.1\rho(1+X)T_6^{-1/2} \times 10^{-9}, \quad (35c)$$

for $10 \leq T_6 \leq 16$.

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