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Effects of Velocity Dispersion on the Evolution of a Disk of Stars

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A computer model of a thin disk galaxy has been used to study the stabilizing effect of velocity dispersion on the evolution of a disk galaxy. The motion of 50 000 model stars are computed stepwise in time as they move in their mutual gravitational fields. A cold balanced disk is found to be violently unstable. A velocity dispersion of 27% of the velocity at the edge of the disk stabilizes the system. These results agree with the theoretical predictions of Toomre.

I. INTRODUCTION

THE development of computer models of star systems makes possible the experimental verification, on the computer, of theoretical predictions of the behavior of such systems. In this paper we are concerned with the effect of velocity dispersion on the stability of a thin disk of stars and we obtain good agreement with the theoretical predictions of Toomre (1964). A full description of the model and the demonstration of the violent instability of the cold thin disk is given elsewhere (Hohl and Hockney 1969). We include below only a summary of the model and follow it with a description of the results obtained with varying amounts of velocity dispersion.

II. THE MODEL

The star system is represented by calculating the individual motion of a large number of representative stars (here 50 000) as they move according to Newton's laws of motion in the mutual gravitational field of the whole star system. The motion of the stars is confined to a plane and the gravitational potential of each star approximates to the r^{-1} potential variation of a point star. Hence one has a model of point stars moving in a plane or of a thin disk of stars. The potential calculation is performed on a fixed mesh of points (here 64×64) by Fourier transform methods which are described in detail by Hockney (1969). The coordinates and velocities of each star are stored in the computer and the calculation is performed stepwise in time as follows:

(a) The star coordinates are examined and a mass distribution is built up on the fixed mesh.

(b) The gravitational potential corresponding to this mass distribution is determined.

(c) The gravitational potential is assumed to remain constant for a short time interval and is used to accelerate all the stars to new positions and velocities. The cycle then repeats at step (a).

The time for the cycle on the CDC 6600 with 50 000 stars and a 64×64 mesh is 5.56 sec. The effect of variations in the time interval, the number of mesh points, and the number of stars is given by Hohl and Hockney (1969). In particular, it is shown that the results obtained during the first few rotations taking 200 time steps per rotation of the system (as used here) are qualitatively indistinguishable from those obtained taking 400 time steps per rotation, and that the results with 50 000 stars are indistinguishable from results with 200 000 stars. Except where expected, results obtained on a 128×128 fixed mesh are the same as those on a 64×64 mesh. Hence we believe that the parameters used in these experiments give a good model of a star system. The independence of the results on the numbers of stars shows that at least for the first few rotations one has a good representation of a collisionless star system.

We now describe the initial conditions used in the experiments and the results obtained.

III. INITIAL CONDITIONS

Various authors (Toomre 1963, 1964; Wyse and Mayall 1942; Hunter 1963; Brandt 1960; Ng 1967) have constructed time-independent models for disk-shaped galaxies. An initial condition used extensively in the present calculations is the familiar simple model of a uniformly rotating disk with a surface density given by

$$\sigma(r)=\sigma(0)(1-r^2/R_0^2)^{\frac{1}{2}}$$
$$=\frac{3M}{2\pi R_0^2}(1-r^2/R_0^2)^{\frac{1}{2}}, \quad (1)$$

where r is the radial coordinate, R_0 is the radius of the disk, and M is the total mass of the disk. The uniform angular velocity required to balance the gravitational attraction is then given by

$$\omega=(3G\pi Nm/4R_0^3)^{\frac{1}{2}}, \quad (2)$$

and the square of the rotation speed is

$$V_\theta(r)^2=r^2\omega^2=\frac{3\pi GNm}{4R_0}\left(\frac{r}{R_0}\right)^2, \quad (3)$$

where N is the total number of stars, and m is the mass

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of each star. The initial positions of the stars are obtained by using a pseudo-random number generator which gives a uniform distribution between zero and one to generate three numbers, x_r, y_r, and z_r. If

$$x_r^2+y_r^2+z_r^2\leq 1, \tag{4}$$

the initial position of a star is then given by the $x-y$ coordinates,

$$x(t=0)=2R_0(x_r-0.5) \tag{5}$$

and

$$y(t=0)=2R_0(y_r-0.5). \tag{6}$$

The initial circular velocity of the stars is given by Eq. (3). For the cases where the stars have an initial velocity dispersion, a uniform velocity dispersion $\sigma_v=\langle v_r^2\rangle^{\frac{1}{2}}=\langle v_\theta^2\rangle^{\frac{1}{2}}$ is superimposed on the circular velocity given by Eq. (3).

IV. RESULTS

All the calculations for uniformly rotating disks presented in this section were performed with disks containing 50 000 stars. There are 200 time steps per rotation and the gravitational field was calculated by the Fourier method (Hohl and Hockney 1969). The active mesh used for the potential calculation is 64×64.

Using a dimensional order-of-magnitude argument, Toomre (1964) predicted that a cold (zero velocity dispersion), infinitesimally thin disk of stars is violently unstable. The growth time of the unstable modes is approximately proportional to the square root of their wavelength (Hunter 1963, 1965). For this reason, the small wavelength disturbances will be the most unstable. Toomre (1964) finds that the root-mean-square radial-velocity dispersion required to stabilize all axisymmetric disturbances anywhere in the disk is

$$(\sigma_v)_r=\langle v_r^2\rangle^{\frac{1}{2}}=3.36G\sigma/K, \tag{7}$$

where σ and K are the local values of the density and epicyclic frequency, respectively. The epicyclic frequency is given by

$$K^2=\left(\frac{\partial^2\phi}{\partial r^2}+\frac{3}{r}\frac{\partial\phi}{\partial r}\right), \tag{8}$$

where ϕ is the gravitational potential. Initially, the cold uniformly rotating disk is balanced and

$$\frac{\partial\phi}{\partial r}=\frac{V_\theta^2(r)}{r}=\frac{3\pi GNm}{4R_0^2}r, \tag{9}$$

where $V_\theta^2(r)$ is given by Eq. (3). Using Eq. (9) in (8), the value of K^2 becomes

$$K^2=\frac{3\pi GNm}{R_0^3}=\left(\frac{2V_\theta(R_0)}{R_0}\right)^2. \tag{10}$$

The expressions for the density and K^2, as given by Eqs. (1) and (10), can be used in Eq. (7) to obtain the radial-velocity dispersion needed to stabilize the disk:

$$(\sigma_v)_r=0.342[1-(r/R_0)^2]^{\frac{1}{2}}V_\theta(R_0). \tag{11}$$

The wavelength of the longest axisymmetric instability of a thin pressure-free disk was given by Toomre as

$$\lambda_{\text{crit}}=4\pi^2G\sigma/K^2. \tag{12}$$

For the uniformly rotating disk, this reduces to

$$\lambda_{\text{crit}}=R_0[1-(r/R_0)^2]^{\frac{1}{2}}. \tag{13}$$

Plate IV shows the evolution of an initially balanced uniformly rotating disk with zero velocity dispersion. The time is in rotational periods,

$$\tau_r=2\pi(3\pi GNm/4R_0^3)^{-\frac{1}{2}}. \tag{14}$$

As predicted by Toomre (1964), the disk is violently unstable and the growth time for the instabilities is a fraction of the rotational period. Since the growth time of a disturbance is approximately proportional to its wavelength, the fast appearance of the small-scale condensations is to be expected. After 1.1 rotations there remain five main condensations. The five smaller clusters which remain after 1.1 rotations exist mainly because the pressure caused by the velocity dispersion of the stars counteracts the gravitational attraction towards the center of each cluster. If the trajectory of individual stars is plotted it is found that they oscillate in the potential wells set up by the five condensations of stars. This fast randomization must be caused by collective effects or violent relaxation as discussed by Lynden-Bell (1967) and Hohl and Campbell (1968).

Toomre (1964) predicted that the short-wavelength modes are stabilized by the effects of velocity dispersion. To check his predictions, all the stars in the initially uniformly rotating disk were given a uniform velocity dispersion. Plate V shows the evolution of such a disk of stars where the rms value of the random velocity is $0.068V_\theta(R_0)$ or 6.8% of the circular velocity at the edge of the cold balanced disk. The initial positions of the stars are the same as for the disk shown in Plate IV. The disk is still violently unstable, but when compared with the evolution of the cold disk shown in Plate IV, it can be seen that the initial condensations take longer to form and are no longer as concentrated as for the cold disk. The smallest condensations formed are now of the order of three cell sizes.

The evolution of the disk with a velocity dispersion equal to 13.6% of the circular velocity at the edge of the cold disk is shown in Plate VI. The smallest condensations formed are now of the order of six cell dimensions. The evolution of the disk clearly shows that the small-wavelength modes have been stabilized. However, the disk is still violently unstable in respect to large-wavelength modes.

An initial velocity dispersion of 20.4% of the circular velocity at the edge of the cold balanced disk results in the evolution shown in Plate VII. The disk is now nearly stabilized, but it still forms a bared spiral-like condensation about ten cell sizes across.

Finally, the velocity dispersion was increased to 27.2% of the circular velocity at the edge of the cold, balanced disk. The resulting evolution shown in Plate VIII shows that the over-all system is now relatively stable. Equation (11) gives the maximum velocity dispersion to stabilize the disk at $r=0$ as 34.2% of the circular velocity of the cold, balanced disk. The average value of the velocity dispersion required to stabilize the disk should be less than the maximum value. Therefore, the value of 27.2% found from the computer simulation is in good agreement with the predictions of Toomre (1964). Plate V indicates that there is an increase in the central density as the system evolves.

Plate IX shows the results obtained using exactly Toomre's expression for the dispersion necessary to stabilize the disk as given by Eq. (11). In this case, the dispersion is a function of radius according to

$$\langle v_r^2\rangle^{\frac{1}{2}}=0.342[1-(r/R_0)^2]^{\frac{1}{2}}V_\theta(R_0). \qquad (15)$$

In order to balance the pressure due to this dispersion, the angular velocity for balance is given by

$$\omega^2=\omega_0^2-\frac{1}{r\sigma}\frac{dP}{dr}, \qquad (16)$$

where the pressure

$$P=\tfrac{1}{3}\sigma\langle v_r^2\rangle^{\frac{1}{2}}. \qquad (17)$$

As before, we have taken the azimuthal and radial dispersion to be the same, $\langle v_\theta^2\rangle^{\frac{1}{2}}=\langle v_r^2\rangle^{\frac{1}{2}}$. The results show that for this dispersion the disk is just barely stable. The long-wave disturbances that are seen grow slowly. They have an azimuthal dependence and are not covered by the theory of Toomre.

V. CONCLUSION

Computer experiments have been described on the effect of velocity dispersion on the stability of a thin disk system of stars. The results are in agreement with the theoretical predictions of Toomre.

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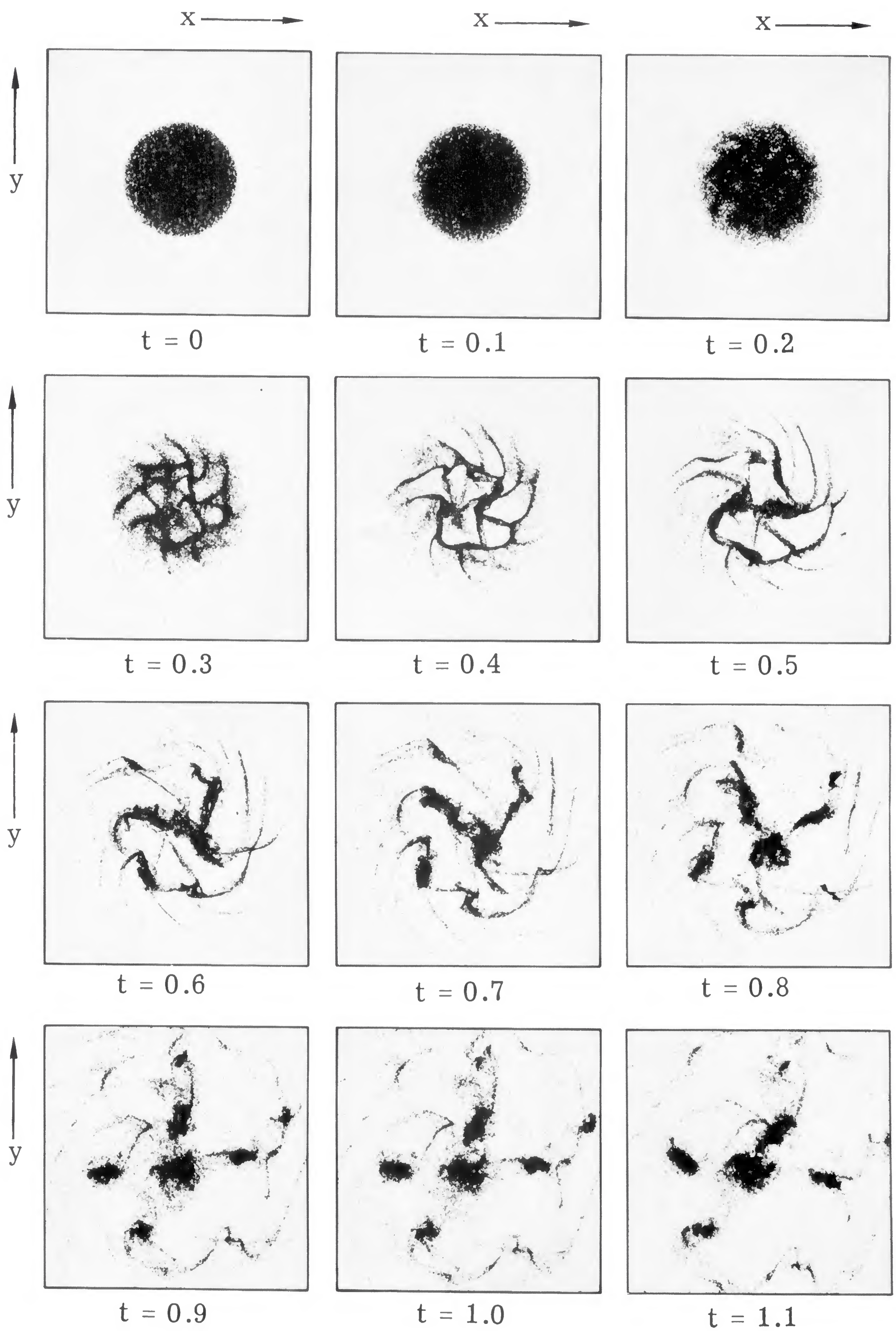

PLATE IV (No. 1, Hockney and Hohl). The evolution of an initially cold thin disk of stars, given an initial solid-body rotation, just sufficient to balance gravitational attraction (time in rotations).

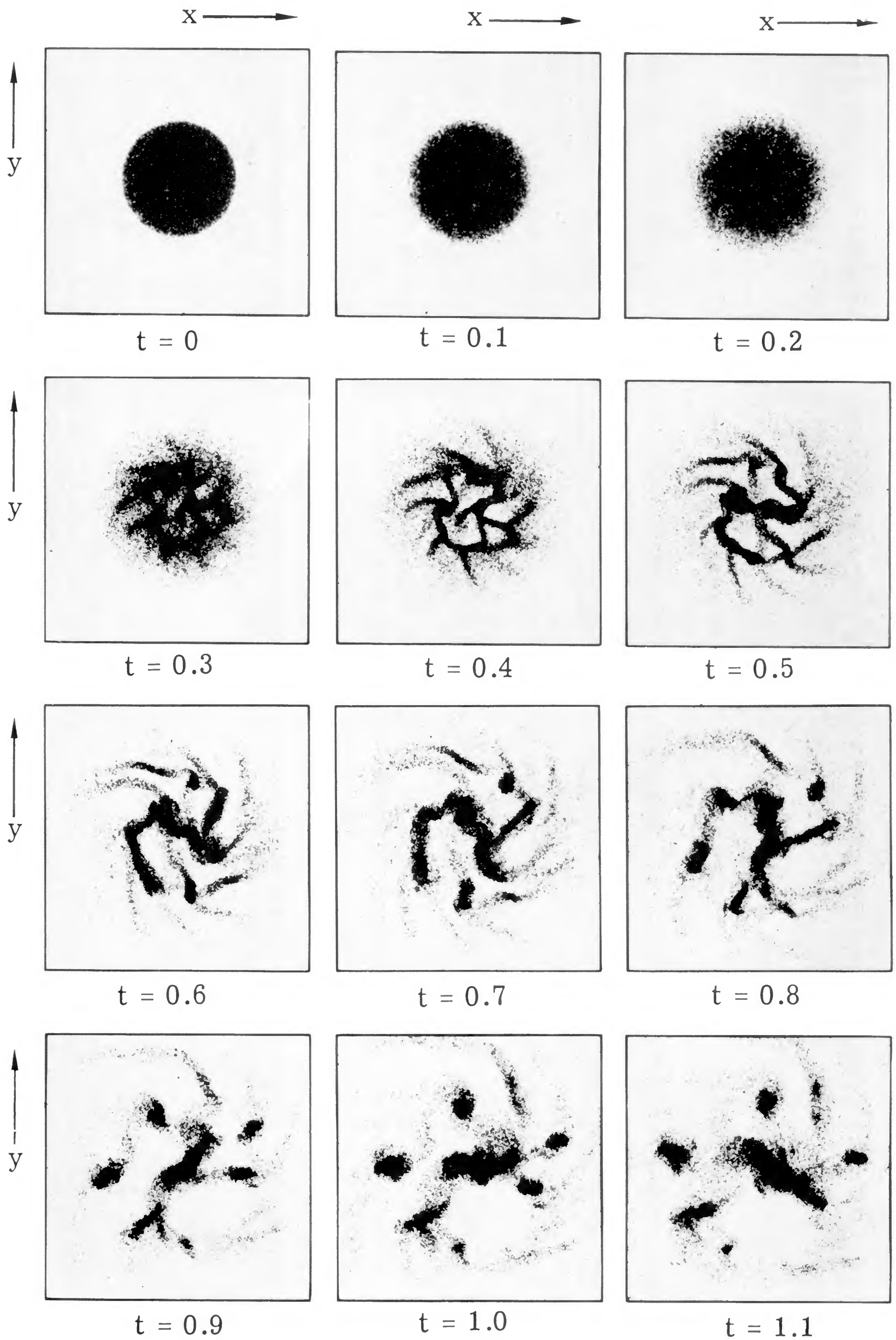

PLATE V (No. 2, Hockney and Hohl). The evolution of a thin disk of stars with a constant velocity dispersion equal to 6.8% of the circular velocity at the edge of the cold balanced disk (time in rotations).

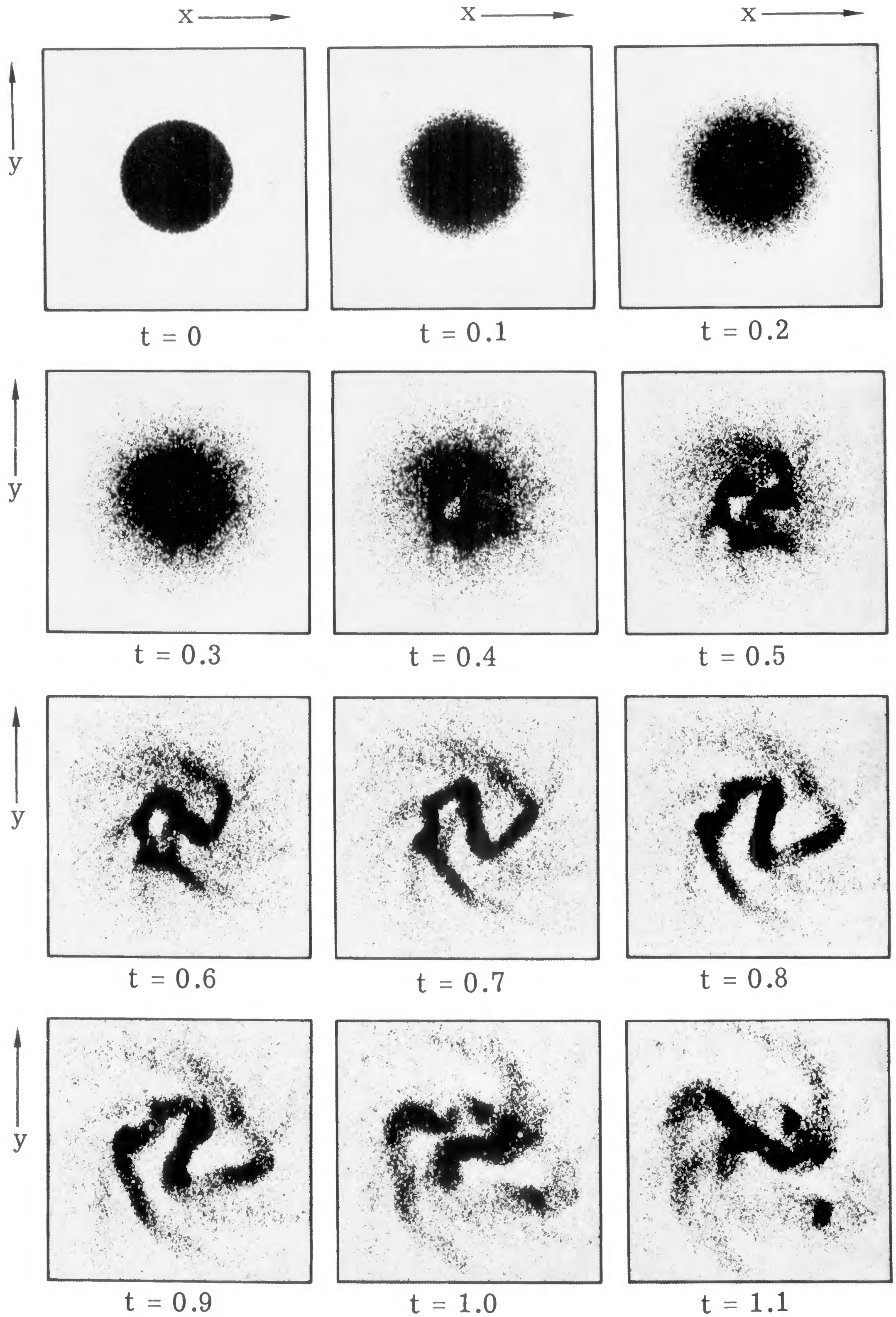

Plate VI (No. 3, Hockney and Hohl). The evolution of a thin disk of stars with a constant velocity dispersion equal to 13.6% of the circular velocity of the edge of the cold balanced disk (time in rotations).

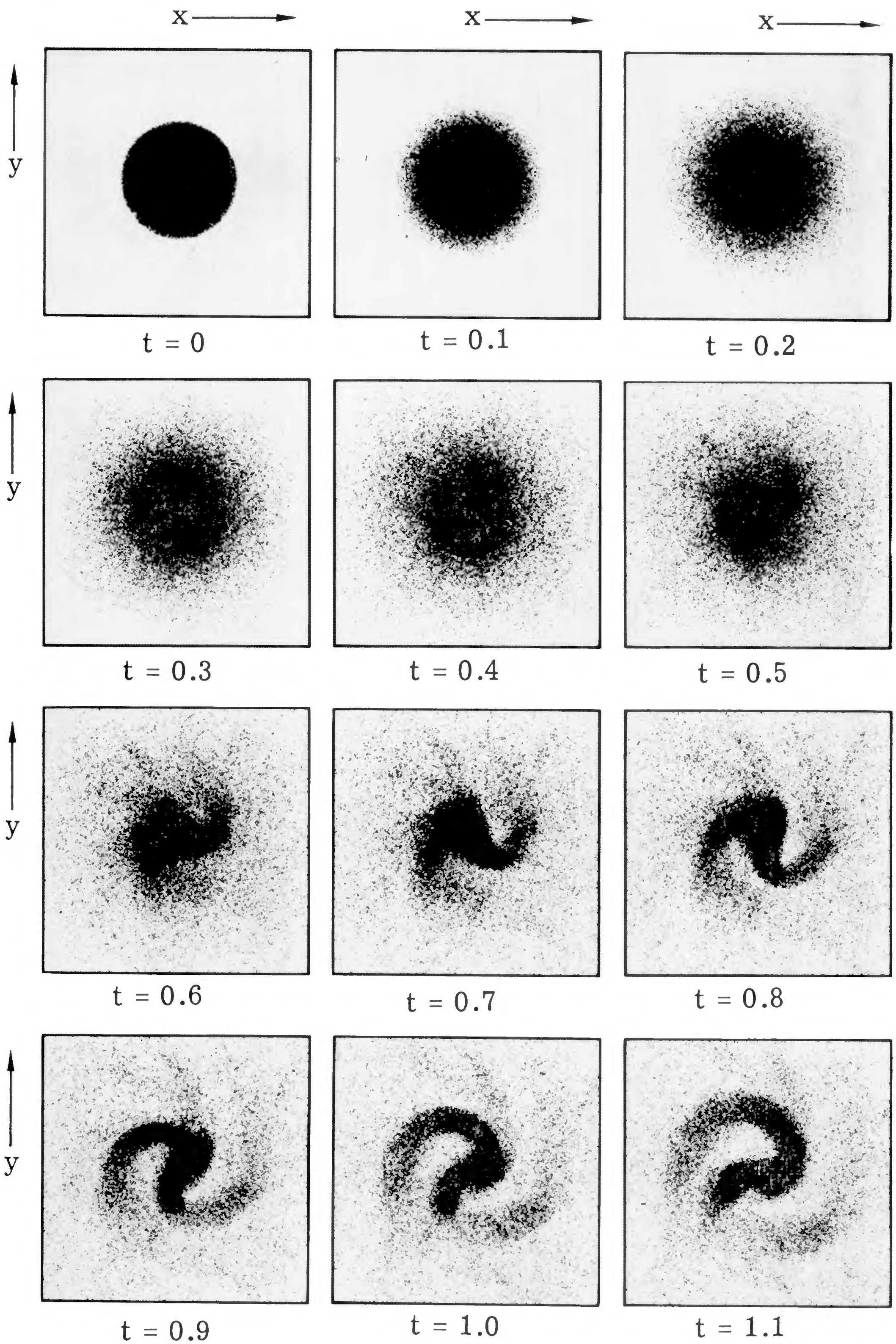

PLATE VII (No. 4, Hockney and Hohl). The evolution of a thin disk of stars with a constant velocity dispersion equal to 20.4% of the circular velocity at the edge of the cold balanced disk (time in rotations).

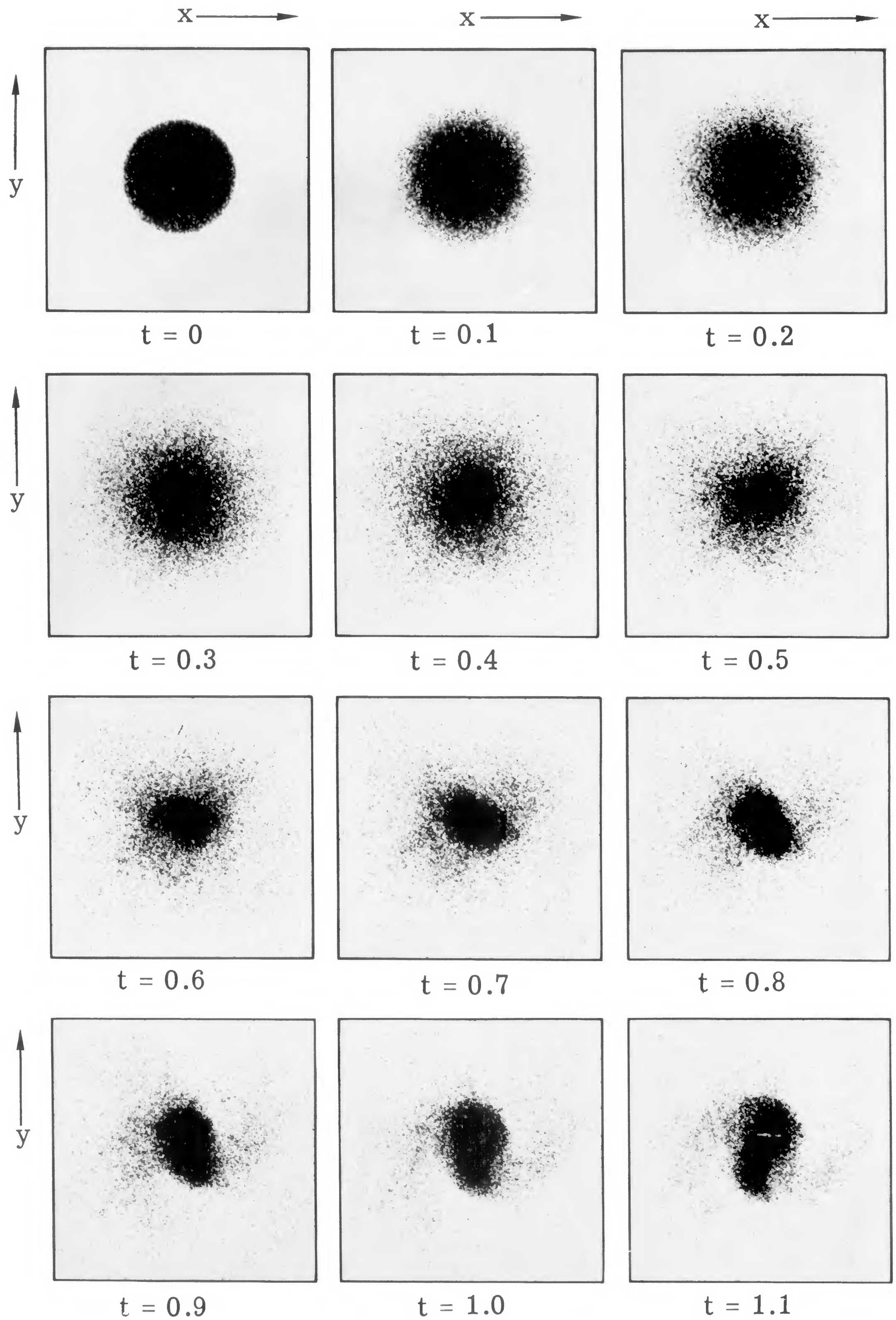

PLATE VIII (No. 5, Hockney and Hohl). The evolution of a thin disk of stars with a constant velocity dispersion equal to 27.2% of the circular velocity of the edge of the cold balanced disk (time in rotations).

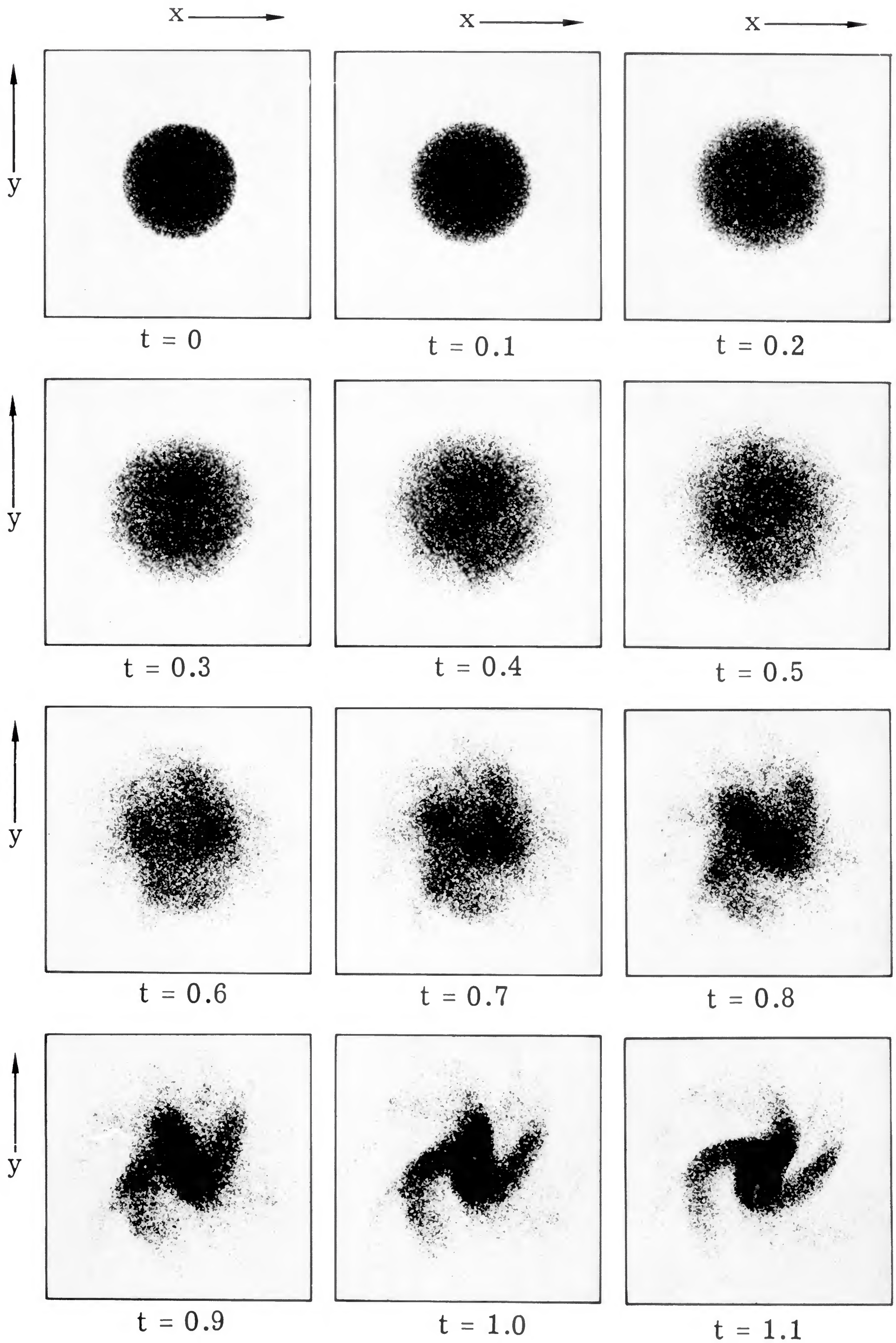

PLATE IX (No. 6, Hockney and Hohl). The evolution of a thin disk of stars given the variable velocity dispersion of Eq. (15).