The Mass of Neptune and the Orbit of Uranus

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(Received 28 April 1969)

Analysis of the observations of Uranus from 1781–1968 indicates a reciprocal mass of Neptune of 19349 ±28 (m.e.).

I. INTRODUCTION

THE mass of Neptune has not been determined from its perturbation of the motion of Uranus since 1910 when Gaillot derived a reciprocal mass of 19 094 based on the observations of Uranus from 1690 to 1903. This value differs markedly from Newcomb's (1898) result of 19 314 based on observations of Uranus from 1781 to 1896. Analysis of the observations of Triton by Eichelberger and Newton (1926) yielded a reciprocal mass of 19 331 in good agreement with Newcomb. A recent determination by Gill and Gault (1968) based on observations of Triton from 1887 to 1958 gave a reciprocal mass of 19 296. On the other hand, Van Biesbroeck's determination (1957) based on observations of Nereid from 1949 to 1955 gave a reciprocal mass of 18 889. In view of the discordance of the previous analyses and the extensive series of observations of Uranus available since the last determination, it was felt advisable to attempt a new determination of the mass of Neptune.

II. OBSERVATIONAL MATERIAL

The observations of Uranus from 1781 to 1938 as discussed by Wylie (1947), and given in Table 3 of Eckert, Brouwer, and Clemence (1951), were incorporated in the solution with the weights given by Wylie. These observations were augmented by the Six-Inch Transit Circle observations of Uranus from 1960 to 1968, as well as the prediscovery observations from 1690 to 1771 discussed by Woolard (1952). In this analysis these additional observations were assigned weights consistent with the weighting system of Wylie (1947).

TABLE I. Residuals ($O-$Int.) in orbital longitude and latitude for Uranus observations 1960–1968.

Date	Longitude ($O-$Int.)	Latitude ($O-$Int.)
1960.18	−0″.31	−0″.40
1961.17	−0.18	−0.33
1962.15	−0.32	−0.45
1963.25	−0.38	−0.54
1964.12	−0.42	−0.50
1965.22	−0.58	−0.52
1966.20	−0.48	−0.27
1967.19	−0.64	−0.36
1968.17	−0.69	−0.55

The observations of Neptune from 1795–1968 utilized by Duncombe, Klepczynski, and Seidelmann (1968), and the observations of Pluto from 1914–1965 discussed by Cohen, Hubbard, and Oesterwinter (1967) were also employed.

Table I gives a continuation of Table 3 of Eckert, Brouwer, and Clemence (1951).

III. PROCEDURE

Assuming a range of values for the mass of Neptune, a simultaneous numerical integration of the orbits of the five outer planets was generated for each discrete value of the mass. The reciprocal masses of Jupiter, Saturn, and Uranus were those used by Eckert, Brouwer, and Clemence (1951), while the value 1 812 000 was used for Pluto (Duncombe, Klepczynski, and Seidelmann 1968). An elliptic adjustment was then made to the observations of Uranus (1781–1968), Neptune (1846–1968), and Pluto (1914–1965). The integration was then repeated to obtain final orbits of these three planets consistent with the particular mass of Neptune. Two classes of solutions were made separately: one, a six-unknown solution for corrections to the orbital elements of Uranus; the other, a seven-unknown solution for corrections to the orbital elements of Uranus and, simultaneously, for a correction to the mass of Neptune.

Table II gives the sums of the squares of the residuals (Σv^2) in orbital longitude and latitude resulting from the six-unknown solutions for the observations from 1781–1968 with the various values of the reciprocal mass used. The values of the reciprocal mass determined from the seven-unknown solutions of the same observational material are also given in Table II.

TABLE II. Uranus longitude and latitude Σv^2 for test values of the reciprocal mass of Neptune.

Reciprocal mass test values	Σv^2 (1781–1968) $\Delta\lambda$	$\Delta\beta$	Reciprocal mass (seven-unknown solution)
18 800	887.55	85.12	
19 234	95.38	87.24	19 353
19 314	65.67	87.45	19 350
19 828	641.02	90.12	
19 349	63.17	87.85	19 349

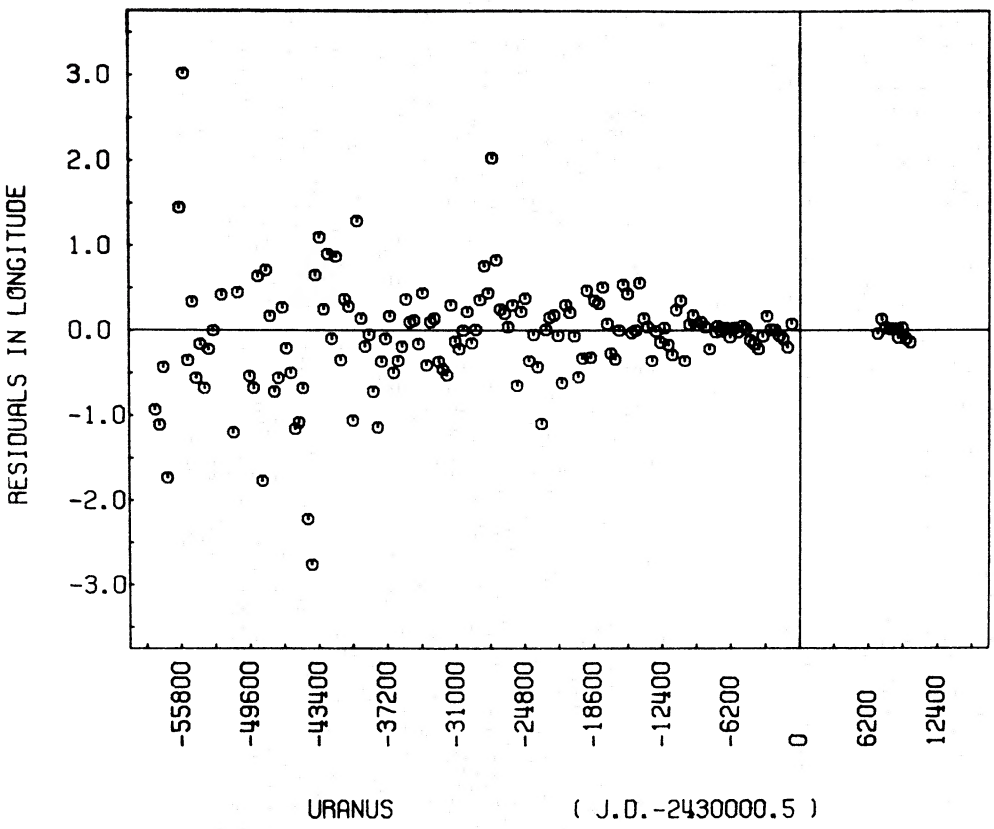

FIG. 1. $(O-C)$'s in orbital longitude (seconds of arc) of Uranus with respect to an integration utilizing a reciprocal mass of Neptune of 19 349.

IV. RESULTS

Differentiating the equation of the parabola fitted through the sums of the squares of the longitude residuals for the first four values given in Table II indicates a reciprocal mass of 19 346. This agrees well with the values determined from the seven-unknown solutions, using longitude observations only, or both longitude and latitude observations; their mean value is 19 352. The average of these two values, 19 349, was used in a new integration. The seven-unknown solutions based on this integration confirmed the value of the reciprocal mass as 19 349 with a formal mean error of ± 28, as shown in the last line of Table II.

The residuals from the new integration in orbital longitude are given in Fig. 1. The residuals in latitude were not improved by the new solution, nor, as noted by Wylie (1947) and Newcomb (1898), was the secular effect removed. The resulting osculating position and velocity vectors of Uranus, Neptune, and Pluto at epoch JD 2 430 000.5 using the new mass of Neptune are given in Table III. The associated osculating elements of Uranus, Neptune, and Pluto at the same epoch are given in Table IV.

TABLE III. Osculating position and velocity vectors—epoch JD 243 0000.5.

	Coordinates	Velocities (40 day)
	Uranus	
x	11.263 033 446 360 2	−0.130 130 303 525 981
y	14.695 261 962 151 6	0.075 882 475 306 815 9
z	6.279 611 704 260 49	0.035 090 573 884 929 7
	Neptune	
x	−30.155 409 825 692 3	−0.009 620 288 675 089 81
y	1.657 040 061 559 37	−0.115 063 702 348 076
z	1.437 915 134 924 28	−0.046 887 662 432 113 1
	Pluto	
x	−21.123 744 008 387 7	−0.070 743 932 637 024 6
y	28.446 371 831 496 7	−0.086 556 601 373 562 5
z	15.388 196 441 391 4	−0.005 945 729 301 127 34

TABLE IV. Osculating elliptic elements representing the new numerical integrations for epoch JD 243 0000.5. M, mean anomaly; ω, argument of perihelion; Ω, longitude of the ascending node; i, inclination; e, eccentricity; and a, semimajor axis (a.u.).

	Uranus	Neptune	Pluto
M	256°41′36″.829	133°44′10″.597	279°16′31″.012
ω	89°14′35″.956	270° 3′29″.996	113°20′46″.930
Ω	73°48′16″.500	131°16′41″.901	109°36′24″.721
i	0°46′23″.615	1°46′33″.653	17° 7′21″.448
e	0.044 735 041 13 14	0.011 857 134 77 67	0.245 934 737 3 40
a	19.316 075 623 8	29.987 126 204 0	39.517 199 215 6

The low accuracy of the Uranus observations prior to 1780, as evidenced by their large observation residuals, prevents application of the parabolic solution to these early observations. A seven-unknown solution, using all observations of Uranus 1700–1968, however, yields a reciprocal mass of Neptune of 19 341, in good agreement with the value adopted above.

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