Parallax, Proper Motion, Acceleration, and Orbital Motion of Barnard's Star

PETER VAN DE KAMP
Sproul Observatory, Swarthmore College, Swarthmore, Pennsylvania

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Twenty-four early plates (1916–1919) and thirty consecutive years of photographic observations of Barnard's star covering the interval 1938–1967 confirm orbital motion with a period of 25 years and semi-axis major of 0".0275. The total number of plates is 3036, with 10 452 exposures, taken on 766 nights, with a total weight of 2056. Assuming a value of 0.15 ⊙ for the mass of Barnard's star, the mass of the companion is found to be 0.0016 ⊙, or 1.7 times the mass of Jupiter.

I. PARALLAX

THIS presentation augments the analysis made in 1963, and includes five more years of photographic observations. A number of plates, in years with a below-average number of plates, have been remeasured, as well as plates in the critical years 1949–1952 within which periastron passage occurs. All of these plates were measured on the Gaertner machine by Mary Jackson.

The plates were reduced to the standard frame of three reference stars used in the earlier investigation (van de Kamp 1963; erratum: the position of Barnard's star in Table I of that investigation holds for 1930.000, not for 1940.000). The dependence method was used and the same color equation applied as before. A correction of $+0^{mm}.0005$ was applied to declination measures from plates taken on Eastman C emulsion over the interval 1939–1949, to reduce to the standard system of G emulsion (Moore 1967). The plate scale is 1 mm $=18''.87$.

A combined solution (1), including 56 McCormick plates, was made for proper motion, quadratic terms, and parallax covering the interval 1916–1967. No allowance was made at this stage for personal equation between the different measurers. The results for parallax are listed below.

$$
\begin{array}{lll}
\text{Separate solutions:} & \text{weight} & \\
\quad \pi_x = +0''.5471 \pm 0''.0009, & 963.81, & \\
\text{p.e.}1_x = \pm 0''.0290; & & \\
\quad \pi_y = +0''.5812 \pm 0''.0048, & 39.16, & (1) \\
\text{p.e.}1_y = \pm 0''.0303; & & \\
\text{combined solution:} & & \\
\quad \pi = +0''.5484 \pm 0''.0009, & 1002.98, & \\
\text{p.e.}1 = \pm 0''.0299. & &
\end{array}
$$

The range in parallactic displacement in declination is very small, hence the discrepancy of the weights in the separate solutions. The systematic difference $\pi_y - \pi_x = +0''.034$ may be partly attributed to the seasonal effect in declination, referred to in the earlier investigation. The reduction to absolute parallax is $+0''.0034$ (van de Kamp 1963), thus yielding an absolute parallax of Barnard's star of $+0''.552 \pm 0''.001$.

II. PROPER MOTION AND ACCELERATION

The remainders from the combined solution were grouped into yearly means, which closely confirm the orbital pattern of the earlier solution. The McCormick material, while in close agreement with the Sproul measures, was not included in this analysis. The yearly mean remainders were next corrected for:

(a) the perturbing effects of Jupiter and Saturn on the sun, which have a combined range of 0".008 in x and 0".004 in y;

(b) allowance for personal equation between the two principal measurers in the sense that corrections of $-0^{mm}.0007$ and $+0^{mm}.0007$ in x and $-0^{mm}.0005$ and and $+0^{mm}.0005$ in y, were applied to the measurements by Lippincott and Jackson, respectively; no other personal equations were allowed for since they are considered not to be well established;

(c) the seasonal diameter effect (van de Kamp 1963).

Several provisional orbital solutions were made; all of which indicated corrections to proper motion and quadratic terms, which were added to the values obtained from the combined solution (1) for parallax, proper motion, and quadratic terms. The following final results for proper motion and quadratic terms were adopted:

$$
\begin{aligned}
\mu_x &= -\ 0''.8038 \quad \pm 0''.0001, \\
\mu_y &= +10''.2745 \quad \pm 0''.0001, \\
q_x &= +\ 0''.000082 \pm 0''.000007, \\
q_y &= +\ 0''.000239 \pm 0''.000007, \\
\text{p.e.}1 &= \pm\ 0''.0376.
\end{aligned} \tag{2}
$$

The increased value for p.e. 1 is easily explained by the effect of various, unallowed for, systematic effects in the high weight yearly means, from which this p.e.1 is derived.

The corresponding values for the observed annual acceleration are

$$+0''.000164 \pm 0''.000014 \text{ in } x,$$
$$+0''.000478 \pm 0''.000014 \text{ in } y.$$

Correcting for the spurious acceleration due to the proper motions of the reference stars (van de Kamp

TABLE I. Yearly mean remainders, based on relative parallax of 0″.5484, (1) proper motion and quadratic terms and (2), the corrections (a), (b), (c) given in Sec. II. The residuals in the last two columns hold for the orbital solution given in Sec. III.

Epoch	Nights	Number of plates	Exposures	Total weight	Remainders R_x (unit 0^{mm}.0001)	Remainders R_y (unit 0^{mm}.0001)	Residuals v_x (unit 0^{mm}.0001)	Residuals v_y (unit 0^{mm}.0001)
1918.11	21	24	50	24	−8	−2	0	−2
38.49	23	77	171	48	−4	+4	+1	+7
39.51	21	62	199	48	−4	−10	+2	−7
40.48	28	108	381	79	−6	0	+1	+2
41.51	46	192	656	137	−8	+2	−1	+2
42.54	29	142	526	94	−12	−4	−5	−3
43.52	21	130	506	73	−5	−3	+2	−3
44.52	29	114	427	83	−8	0	−2	−1
45.52	25	93	333	68	−3	+7	+2	+4
46.54	41	230	820	124	−1	+1	+3	−2
47.51	40	183	538	105	−6	+2	−5	−1
48.55	25	95	350	67	+3	+7	0	+3
49.43	32	152	528	88	+17	+5	+8	+2
50.46	19	65	230	44	+8	−3	−8	−2
51.55	24	92	315	64	+16	−8	−1	−4
52.53	29	108	391	80	+16	−4	+1	0
53.51	19	64	194	46	+14	+7	+2	+8
54.54	19	65	183	43	+10	−5	+1	+1
55.51	15	57	176	41	0	−8	−7	−2
56.53	13	46	174	36	−3	−9	−8	−3
57.53	15	54	206	41	+6	−12	+2	−6
58.50	17	63	231	48	+4	−5	+2	+1
59.48	14	55	184	37	+3	−6	+3	−1
60.51	6	21	80	16	−4	−2	−2	+2
61.44	12	48	183	34	+2	+2	+5	+6
62.47	19	73	278	55	−8	0	−4	+3
63.46	47	186	626	122	−2	−6	+3	−3
64.45	36	136	449	90	−4	+3	+2	+5
65.44	43	157	541	112	−7	−2	0	−2
66.27	17	62	224	48	−9	−5	−2	−4
67.44	21	82	302	61	−11	+1	−4	+1
Total	766	3036	10452	2056				

1963), the corresponding values for the annual secular acceleration are found to be

$$-0''.00010 \pm 0''.00004 \text{ in } x,$$

$$+0''.00115 \pm 0''.00004 \text{ in } y,$$

agreeing closely with the predicted perspective acceleration:

$$-0''.00010 \pm 0''.00001 \text{ in } x,$$

$$+0''.00125 \pm 0''.00003 \text{ in } y.$$

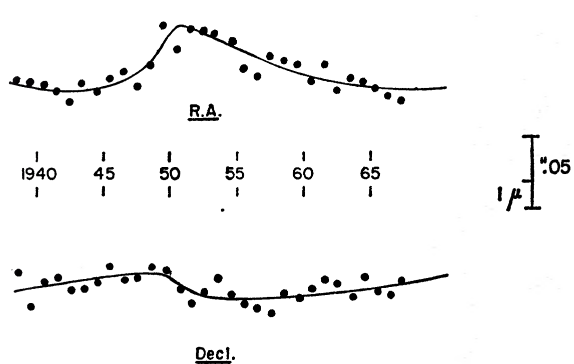

FIG. 1. Barnard's star: Yearly means, averaging 100 plates and weight 68; time–displacement curves for $P=25$ yr, $e=0.75$, $T=1950$.

III. ORBITAL MOTION

An orbital analysis covering the interval 1938–1967 was made from the yearly mean remainders based on the parallax value from solution (1), and the proper motion and quadratic term values from solution (2). The yearly remainders are given in Table I. The yearly number of plates ranges from 21 to 230 and averages 100; the yearly weights range from 16 to 137 and average 68.

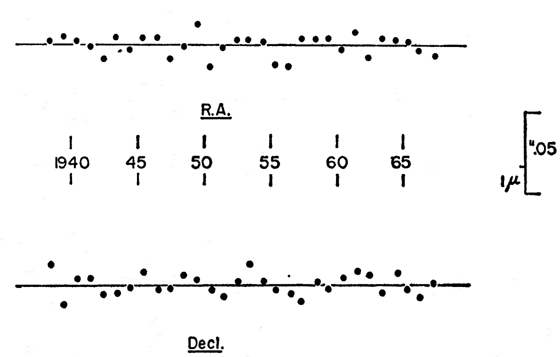

FIG. 2. Barnard's star: Yearly mean residuals from $P=25$ yr, $e=0.75$, $T=1950$.

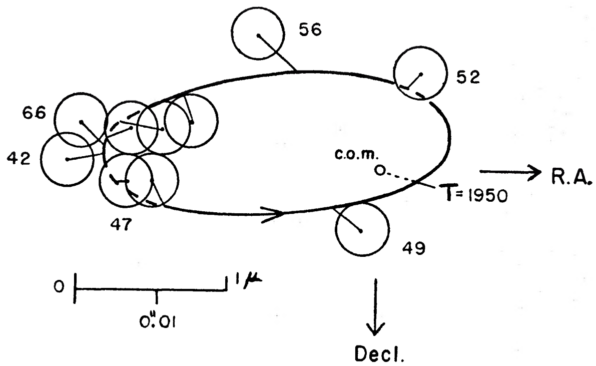

FIG. 3. Barnard's star: Apparent orbit; ten normal points, averaging 301 plates and weight 203; radii of circles indicate probable errors; $\alpha=1.46$, $\mu=0''.0275$, $i=69°$.

To derive the orbital elements, ten normal points were formed with weights ranging from 161 to 231. Several orbital solutions were made with P ranging from 22 to 29 years, e from 0.60 to 0.90, and T from 1948.0 to 1952.0. Periods as low as 22 or as high as 27 appear to be excluded. The present extended material suggests no substantial change in the earlier dynamical elements, except for higher eccentricity. The following elements were adopted, as satisfying best both x and y coordinates:

$$P=25 \text{ years},$$
$$e=0.75,$$
$$T=1950.$$

With these values, the yearly means yield the following geometric elements:

$$(A)=+\ 2.3,\quad (B)=+\ 8.9,\quad (F)=-\ 6.1,\quad (G)=+11.4. \qquad \text{unit}\ 0^{\mathrm{mm}}.0001$$

The corresponding Keplerian motion is illustrated in Fig. 1.

The residuals for the yearly means are given in Table I and Fig. 2. Compared with the earlier solution, the residuals have been improved, the extreme residuals of $-0^{\mathrm{mm}}.0012$ in x (1956) and $+0^{\mathrm{mm}}.0010$ in y (1953) from the 1963 solution now having been replaced by extreme residuals of $\pm0^{\mathrm{mm}}.0008$; in particular, the large residuals in x for 1955 and 1956 were reduced from $-0^{\mathrm{mm}}.0008$ and $-0^{\mathrm{mm}}.0012$ to $-0^{\mathrm{mm}}.0007$ and $-0^{\mathrm{mm}}.0008$, respectively.

The probable errors of unit weight are

$$\text{p.e.}1_x=\pm0^{\mathrm{mm}}.00210=\pm0''.040,$$
$$\text{p.e.}1_y=\pm0^{\mathrm{mm}}.00200=\pm0''.038.$$

The probable errors of unit weight are appreciably smaller than the earlier values $\pm0''.050$ (x) and $\pm0''.044$ (y) from the earlier analysis (van de Kamp 1963). Contributing causes are the second measurements of a large number of plates and the corrections (a) and (b) given in Sec. 2.

The recently measured yearly mean for 1968, based on 144 plates taken on 36 nights agrees very closely with the ephemeris based on the present 1916–1967 solution, the residuals being $0^{\mathrm{mm}}.0000$ in x and $+0^{\mathrm{mm}}.0003$ in y.

IV. DYNAMICAL INTERPRETATION

The geometric elements yield values of $\alpha=0^{\mathrm{mm}}.00146=0''.0275$ for the semi-axis major of the photocentric orbit, and $i=69°$ for the inclination. The probable errors of the geometric elements as well as of the semi-axis major are below $\pm0^{\mathrm{mm}}.0001$ or $\pm0''.002$. The apparent orbit is illustrated in Fig. 3; the yearly means have been combined into ten normal points of average weight 203.

An adopted mass of $0.15\odot$ for Barnard's star leads to a value of

$$(0.15\times25^2)^{\frac{1}{3}}=4.54 \text{ a.u. or } 2''.51$$

for the semi-axis major of the relative orbit of the companion and primary. This in turn leads to a mass ratio

$$\frac{\mathrm{m}}{\mathrm{M}}=\frac{\alpha}{a}=0.011.$$

Hence for $\mathrm{M}=0.15\odot$ we find $\mathrm{m}=0.0016\odot$, or 1.7 times the mass of Jupiter for the mass of Barnard's star B. A comparison of the present and the earlier results is given in Table II.

TABLE II. Comparison of 1968 with 1963 orbital analysis of Barnard's star.

	1968 solution	1963 solution
Interval:	1916–1967	1916–1962
Number of nights:	766	609
Number of plates:	3036	2413
Total weight:	2056	1621
P	25 yr	24 yr
e	0.75	0.6
T	1950	1950
α	$1.46\ \mu=0''.0275$	$1.3\ \mu=0''.0245$
i	69°	77°
Mass of companion:	$0.0016\odot$	$0.0015\odot$

The present stronger material essentially confirms the results derived five years ago. The slightly increased value for α and m and the smaller value for i are principally due to the reduction in declination from C to G emulsion, which had been neglected in the earlier analysis.

ACKNOWLEDGMENT

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REFERENCES

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