SOLAR MODULATION OF GALACTIC COSMIC RAYS

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ABSTRACT

Beginning with the equations of the convection-diffusion model including energy losses, it is shown that the streaming of galactic cosmic rays in the solar-wind cavity can be neglected above a few hundred MeV per nucleon. For the case in which the diffusion coefficient is separable into radially and rigidity dependent parts, an integral is derived which relates the intensity at points within the cavity to that at infinity through an energy-loss parameter. This parameter is a function of energy; it is defined in terms of observable quantities and is identified, tentatively, as the mean energy lost by the cosmic-ray particles in moving into the interplanetary region. The integral is formally equivalent to that obtained by using Liouville's theorem. Experimental data for the modulation of protons and helium ions in 1963–1965 and electrons and protons in 1965–1966 are shown to behave as predicted, down to kinetic energies of a few hundred MeV per nucleon. The diffusion coefficient was proportional to the magnitude of the charge of the cosmic-ray species during these periods, and the analytical results are formally equivalent to those obtained for a heliocentric force field proportional to the magnitude of the charge of the cosmic-ray species. Making use of the observed lower limits of validity, it is shown that the *e*-folding distance for the diffusion coefficient lies between 0.8 and 1.6 a.u., thus setting limits on the radial dependence of the scattering process. Estimates are given of the changes in energy loss in the periods 1963–1965 and 1965–1966 and of the energy loss at solar minimum.

I. INTRODUCTION

In this paper we discuss the solar-cycle modulation of galactic cosmic rays on the basis of equations which we have derived elsewhere (see Gleeson and Axford 1967; hereinafter referred to as "Paper I"). We derive an integral which is effectively an asymptotic solution of the equations valid for sufficiently large energies. This integral, which is formally similar to the Liouville theorem, appropriate to the case of modulation by a conservative field, is shown to describe the observed modulation of galactic cosmic rays rather well at energies greater than a few hundred MeV per nucleon. Since this result makes allowance for energy changes of the cosmic rays, it represents an improvement over that obtained from the simple convection-diffusion theory, in its range of validity.

The equations given in Paper I describe the effects of convection and scattering of cosmic-ray particles by "magnetic scattering centers" carried along by a radially moving solar wind, with the assumption of spherical symmetry:

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 S) = -\frac{V}{3} \frac{\partial^2}{\partial r \partial T} (\alpha T U) , \tag{1}$$

$$S = VU - \kappa \frac{\partial U}{\partial r} - \frac{V}{3} \frac{\partial}{\partial T} (\alpha T U) . \tag{2}$$

Here $U(r,T)$ is the differential density and $S(r,T)$ the radial current density (or streaming) of the cosmic-ray particles in the kinetic energy range $(T, T + dT)$; r is the radial

distance from the Sun; $V(r,t)$ is the solar-wind speed; κ is the diffusion coefficient; and $\alpha = (T + 2\mathfrak{E}_0)/(T + \mathfrak{E}_0) = (\mathfrak{E} + \mathfrak{E}_0)/\mathfrak{E}$, with $\mathfrak{E}_0$ the rest energy and $\mathfrak{E} = T + \mathfrak{E}_0$ the total energy of the cosmic-ray particles. Steady-state equations are appropriate, since the relaxation time of the distribution is short compared with the 11-year period of the solar cycle; thus the time, t, enters only as a parameter in the solutions.

Equations (1) and (2) reduce to the Fokker-Planck equation developed by Parker (1965, 1966) when S is eliminated between them. Parker (1965) has obtained a formal solution of this Fokker-Planck equation, using the method of separation of variables, assuming that α and κ are constant and that the cosmic rays move without scattering beyond a certain distance from the Sun. Jokipii (1967) has generalized this discussion by allowing $\kappa \propto T^{\alpha} r^{b}$ $(b < 1)$. These results are helpful as illustrations of certain aspects of the phenomena involved but are not immediately useful in the interpretation of observations.

Here we obtain an approximate but more general solution that can be applied directly to the observations with satisfactory results. The procedure adopted is to establish the conditions under which $S(r,T)$ can be considered negligible in equation (2) (by using analytic arguments supported by observational evidence), to set $S = 0$, and to examine some of the consequences.

It should be noted that the usual terms of the steady-state convection-diffusion theory are obtained from equations (1) and (2) by setting $\alpha = 0$. However, since $1 \leq \alpha \leq 2$, it is clear that there is no sound physical basis for the convection-diffusion theory of the solar-cycle modulation of galactic cosmic rays; but it is possible in some energy ranges for solutions of equations (1) and (2) to have a similar form to solutions of the usual convection-diffusion equations with modified V/κ.

Clearly, then, there is no sound theoretical basis for the many attempts which have been made to force the observations to fit the convection-diffusion theory by suitably adjusting the form of $\kappa(r,T,t)$. The convection-diffusion theory should not be regarded as a valid alternative to the present approximate theory or to the more exact theory based on the Fokker-Planck equation.

II. THE MODULATION INTEGRAL

A solution of equations (1) and (2) in series form has been obtained by Gleeson and Axford (1968; hereinafter referred to as "Paper II"). In that solution S and U are given as functions of (r,T) for the case $V = $ constant, $\kappa = P\beta \exp{(r/R)}$, where P is the particle rigidity, $\beta = v/c = $ (particle speed/speed of light), and R is a length characteristic of the radial variation of the diffusion coefficient. It was assumed that $U(r,T) \sim \mathfrak{E}^{-\mu}$, $S \to 0$ as $r \to \infty$, and $r^2 S \to 0$ as $r \to 0$. With values of κ, R, V, and μ appropriate to the solar-modulation problem, it was found that S is negligible when $T \geq 400$ MeV per nucleon for protons, and $T \geq 200$ MeV per nucleon for alpha particles. At lower energies S becomes significant, and at very low energies the series solutions are divergent.

In general, it can be shown by order-of-magnitude arguments that, when there are no sources or sinks at $r = 0$,

$$\eta = S \Big/ \left[VU - \tfrac{1}{3}V\frac{\partial}{\partial T}(\alpha T U) \right] \sim VR/\kappa , \qquad (3)$$

if $VR/\kappa \ll 1$; this result is independent of the specific details of the model. Thus, if $VR/\kappa \ll 1$, then S is negligible in equation (2). For the bulk of the particles detected at ground level by neutron monitors (i.e., with kinetic energies of the order of a few BeV per nucleon), $\kappa \approx 10^{22}$ cm^2 sec^{-1}, and hence with $V = 4 \times 10^7$ cm sec^{-1} we would expect $VR/\kappa \ll 1$, provided that $R \leq 1$ a.u.

These considerations are verified by observations of the diurnal variation of the

cosmic-ray intensity at ground level (McCracken and Rao 1966). These show an anisotropy of about 0.4 per cent directed at $89°5 \pm 1°6$ to the radial direction, hence $|\eta| \lesssim 0.04$. Note that S refers to the *radial* component of the differential current density $\vec{S}$; the tangential component is attributed to corotation with the interplanetary magnetic field and is not considered here.

These arguments show that S is negligible in equation (2) at sufficiently high energies, where $VR/\kappa \ll 1$. Dropping S, regrouping terms, and using the total energy $\mathfrak{E}$ as an independent variable, we find that equation (2) becomes

$$\kappa \frac{\partial U}{\partial r} + \tfrac{1}{3} V(\mathfrak{E}^2 - \mathfrak{E}_0^2)^{3/2} \frac{\partial}{\partial \mathfrak{E}} \left[\frac{U}{\mathfrak{E}(\mathfrak{E}^2 - \mathfrak{E}_0^2)^{1/2}} \right] = 0 . \tag{4}$$

The solution of this equation together with equation (1) (solved by quadrature) can be regarded as an asymptotic solution of the correct equations (1) and (2) valid at high energies. In § III we discuss how the solution can be used to give a more precise estimate of the range in which S can be considered sufficiently small for this procedure to be valid.

The diffusion coefficient has the form $\kappa = \tfrac{1}{3} v \lambda$, where $\lambda = \lambda(r,P,t)$ is the scattering mean free path. The rigidity dependence of λ follows, because the path of a charged particle in a steady magnetic field is completely determined by its initial direction and its rigidity. We assume that λ is a separable function of r and P such that

$$\kappa = \beta \kappa_1(r,t) \kappa_2(P,t) . \tag{5}$$

There is no clear evidence that this step is justified, although O'Gallagher and Simpson (1967) have concluded that κ is separable in this manner on the basis of observations made in the range $1.0 \le r \le 1.6$ a.u. during the period December 1963—June 1965.

With κ given by equation (5), equation (4) can be integrated to give

$$\frac{4\pi}{c} \frac{J(r,\mathfrak{E},t)}{(ZeP)^2} = \frac{U(r,\mathfrak{E},t)}{\mathfrak{E}(\mathfrak{E}^2 - \mathfrak{E}_0^2)^{1/2}} = H\left\{ \left[\int^{\mathfrak{E}} \frac{\kappa_2(P',t)}{(\mathfrak{E}'^2 - \mathfrak{E}_0^2)^{1/2}} d\mathfrak{E}' - \int^r \frac{V(s,t)}{3\kappa_1(s,t)} ds \right], t \right\} . \tag{6}$$

Here Z is the particle charge in units of electronic charge e, J is the differential intensity, with

$$J(r,\mathfrak{E},t) = vU(r,\mathfrak{E},t)/4\pi = (c/4\pi\mathfrak{E})(\mathfrak{E}^2 - \mathfrak{E}_0^2)^{1/2} U(r,\mathfrak{E},t) , \tag{7}$$

and $H(x,t)$ is an arbitrary function to be determined from the boundary conditions or by matching observations at, say, the orbit of Earth. The determination of H requires the form of $\kappa_2(P,t)$ to be known, and in general $H(x,t)$ will be different for each species. In this application $J(r,\mathfrak{E},t)$ is to be determined in terms of $J(\infty,\mathfrak{E})$, the mean differential intensity beyond the termination of the solar wind at $r = r_b(t)$.

We introduce the quantities

$$\zeta(\mathfrak{E},Z,t) = \int_{\mathfrak{E}_0}^{\mathfrak{E}} \frac{\kappa_2(P',t)}{(\mathfrak{E}'^2 - \mathfrak{E}_0^2)^{1/2}} d\mathfrak{E}' , \qquad \phi(r,t) = \int_r^{r_b(t)} \frac{V(x,t)}{3\kappa_1(x,t)} dx , \tag{8}$$

and the inverse function $\psi(\zeta,Z,t)$ such that, given (ζ,Z,t), then $\mathfrak{E} = \psi(\zeta,Z,t)$. Then we define

$$\Phi(r,\mathfrak{E},Z,t) = \psi(\zeta + \phi,Z,t) - \mathfrak{E} = \psi(\zeta + \phi,Z,t) - \psi(\zeta,Z,t) . \tag{9}$$

The boundary conditions applied to equation (6) now give

$$H(\zeta,t) = (4\pi/c)J(\infty,\mathfrak{E})/(\mathfrak{E}^2 - \mathfrak{E}_0^2) , \tag{10}$$

and it follows from equation (6) and the above that

$$\frac{J(r,\mathfrak{E},t)}{\mathfrak{E}^2 - \mathfrak{E}_0^2} = \frac{J(\infty,\mathfrak{E} + \Phi)}{(\mathfrak{E} + \Phi)^2 - \mathfrak{E}_0^2} . \tag{11}$$

This equation specifies the differential intensity at $(r,\mathfrak{E},t)$ in terms of the undisturbed intensity at infinity. It is identical in form with the well-known Liouville theorem for conservative force fields: $J/p^2 =$ constant following the motion, with $p = (\mathfrak{E}^2 - \mathfrak{E}_0^2)^{1/2}/c$ the particle momentum. The quantity Φ is the potential energy or energy loss experienced in coming from infinity; in the classical Liouville result, Φ is independent of energy, but here it is a function of both energy and species.

The modulation function Φ is completely determined by $\phi(r,t)$ and the form of $\kappa_2(P,t)$; ϕ and κ_2 are independent of the species of cosmic-ray particles, but Φ is a function of $\mathfrak{E}$, $\mathfrak{E}_0$, Z, $\phi(r,t)$, and t and is, in general, different for each species. The functional form of $\kappa_2(P,t)$ can be determined directly from the interplanetary magnetic-field power spectrum observed locally (Jokipii 1966). However, $\phi(r,t)$ involves an integral over the region beyond Earth's orbit and thus cannot be evaluated easily.

Parker (1966) solved the Fokker-Planck equation under the simplifying assumptions of κ, V, and $a(T)$ constant. With $Vr_b/\kappa \ll 1$, he determined the distribution of particles at $r = 0$ resulting from the injection of monoenergetic particles at $r = r_b$. The mean loss of energy for particles reaching $r = 0$ was $\frac{1}{3}aT(Vr_b/\kappa)$, to a good approximation.

When $\Phi \ll \mathfrak{E}_0$, i.e., $\phi \ll \zeta$, it follows from equation (9) and the definitions of ζ and ψ that

$$\Phi \simeq \frac{\partial \psi}{\partial \zeta} \phi = \frac{ZeP}{\kappa_2(P,t)} \phi = \frac{aT}{3} \int_r^{r_b} \frac{V(x,t)}{\kappa(x,\mathfrak{E},t)} dx . \tag{12}$$

When κ and V are constant and $r = 0$, this equation yields $\Phi = \frac{1}{3}aT(Vr_b/\kappa)$, the mean energy loss given by Parker for this special case.

This correspondence and the formal correspondence between equation (11) and the Liouville theorem make it highly likely that Φ can be identified with the mean energy loss of the particles. The approximate expression (12) is a generalization of Parker's result.

It is possible in principle to determine Φ as a function of time for each species if we have available observational data on J at times t_i, $i = 1, \ldots, n$, say, and $\kappa_2(P,t)$ has a known but different form at two of these times. This problem is deferred until a more complete knowledge of $\kappa_2(P,t)$ is available.

In the remainder of this paper we restrict our attention to the case with κ_2 independent of t. This leads to a simple result which will be very useful if κ_2 remains approximately independent of time over a period of several years in the energy range above a few hundred MeV per nucleon, and there is evidence that suggests that this may be so.

When κ_2 is independent of t, ζ and ψ are also independent of t; then, from equations (11) and (9),

$$\frac{J(r,\mathfrak{E},t)}{(ZeP)^2} = \frac{J\{\infty,\psi[\zeta(\mathfrak{E},Z) + \phi(r,t)]\}}{\{\psi[\zeta(\mathfrak{E},Z) + \phi(r,t)]\}^2 - \mathfrak{E}_0^2} . \tag{13}$$

The quantity J/P^2 is a function of the single argument $\zeta(\mathfrak{E},Z) + \phi(r,t)$. The modulation now arises solely from the spatial term $\phi(r,t)$, which is a *single parameter describing the modulation* when the conditions on S, the separability of κ, and the time independence of κ_2 are satisfied.

When this part of the theory applies, we can utilize observations at t_i, $i = 1, 2, \ldots$, and fixed r to verify a given functional form of $\kappa_2(P)$ by plotting J/P^2 against $\zeta(\mathfrak{E})$,

given by equation (8), at each of these times. If the form of κ_2 is correct, the curves will be of identical shape but displaced along the ζ-axis as illustrated in Figure 1. The displacement along the ordinate gives $\Delta\phi(r,t_i,t_j) = \phi(r,t_j) - \phi(r,t_i)$, the change in the modulation function. A similarly displaced set of curves should be obtained for each species, and a crucial check on the applicability of this form of the theory is that $\Delta\phi(r,t_i,t_j)$ should be the same for all species. An example is given in the next section.

III. AN APPLICATION TO PROTONS AND ALPHA PARTICLES

In this section we use observational data to produce curves of the form given in Figure 1 and thus show that our result does describe the observations at higher energies during some periods at least. The data used are those appropriate to 1963 $(=t_1)$ and 1965
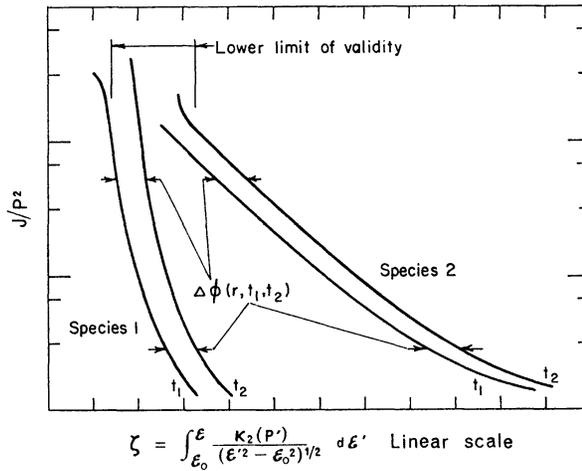

$$\zeta = \int_{\mathcal{E}_0}^{\mathcal{E}} \frac{\kappa_2(P')}{(\mathcal{E}'^2 - \mathcal{E}_0^2)^{1/2}} \, d\mathcal{E}' \quad \text{Linear scale}$$

Fig. 1.—Displacement of curves of J/P^2 for times t_1 and t_2, as predicted by the modulation integral. Different species have the same displacement in abscissa coordinate at fixed ordinate value.

$(=t_2)$ for protons and alpha particles at Earth $(r = r_e)$ given by Quenby (1967) (see Figs. 1 and 2 of that paper for details). We take $\kappa_2 = P$ from the result $\kappa \propto P$ deduced by Gloeckler and Jokipii (1966), using the power spectrum of the interplanetary magnetic field. This was shown to be consistent with observations by Gloeckler and Jokipii and by O'Gallagher and Simpson (1967) (see also Paper II).

Since $\mathcal{E}^2 = (ZeP)^2 + \mathcal{E}_0^2$, in this case

$$\zeta = \int_{\mathcal{E}_0}^{\mathcal{E}} \frac{\kappa_2(P')}{(\mathcal{E}'^2 - \mathcal{E}_0^2)^{1/2}} \, d\mathcal{E}' = \int_{\mathcal{E}_0}^{\mathcal{E}} \frac{d\mathcal{E}'}{|Z|e} = \frac{\mathcal{E} - \mathcal{E}_0}{|Z|e}, \tag{14}$$

and, accordingly, in Figure 2, J/P^2 is plotted against $(\mathcal{E} - \mathcal{E}_0)/Ze$ on a linear scale, as abscissa. It is seen that, except for a region at low energies, the curves for each species are of the form shown in Figure 1 and that, in accordance with the theory, the displacement of the curves is the same in each case: $\Delta\phi(r_e,t_1,t_2) = -80$ MV. This agreement with the integral (13) is better than might have been expected from the scatter of the data.

In this case, with $\kappa_2 = P$, the inversion of relation (14) gives $\psi(\zeta,Z)$, and Φ follows immediately from equation (9):

$$\psi(\zeta,Z) = \mathcal{E}_0 + |Z|e\zeta, \quad \Phi = |Z|e\phi(r,t). \tag{15}$$

Thus the intensity defined by equation (11) is given by

$$\frac{J(r,\mathfrak{E},t)}{\mathfrak{E}^2 - \mathfrak{E}_0{}^2} = \frac{J[\infty,\mathfrak{E} + |Z|e\phi(r,t)]}{[\mathfrak{E} + |Z|e\phi(r,t)]^2 - \mathfrak{E}_0{}^2} . \tag{16}$$

This result is precisely that obtained for positively charged particles, assuming a heliocentric electric field, $E(r,t) = \frac{1}{3}V(r,t)/\kappa_1(r,t)$ and with $\phi(r,t)$ the electric potential (Emhert 1960; Freier and Waddington 1965). Here it appears as a special case of a more general result giving a one-parameter description of the modulation. The formal correspondence between a scattering process and a heliocentric electric field when S is negligible and $\kappa \propto P\beta$ was established from the differential equations (1) and (2) in Paper I, but the modulation integral was not given.
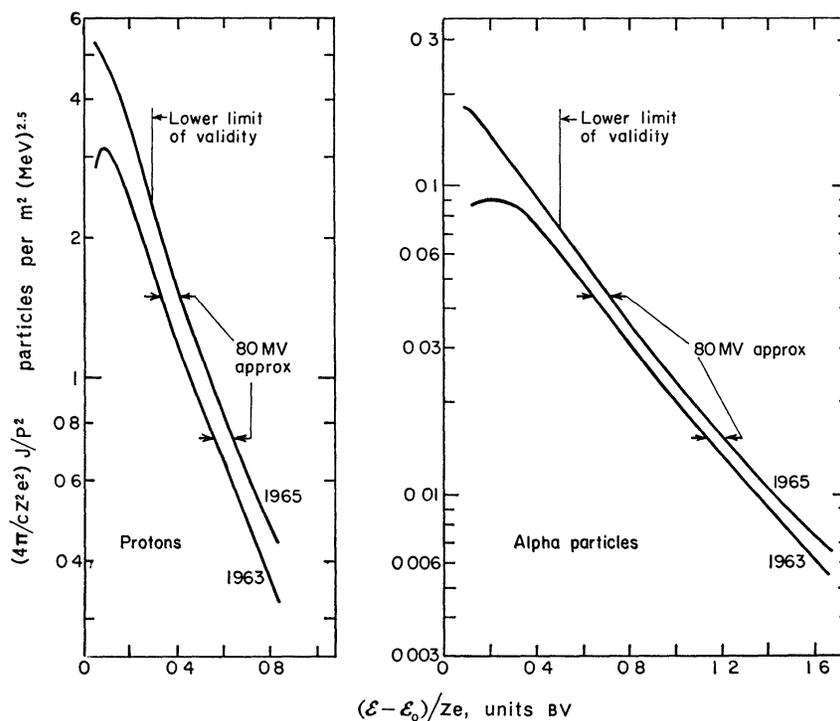

FIG. 2.—Curves of J/P^2 versus $\zeta = (\mathfrak{E} - \mathfrak{E}_0)/Ze$ for protons and alpha particles in 1963 and 1965. The constant displacement predicted by the modulation integral appears to hold, down to kinetic energies of 200–300 MeV per nucleon. Note that the ordinate is proportional to $H(\zeta + \phi)$.

The breakdown of parallel displacement which takes place in Figure 2 at approximately 300 MeV per nucleon for protons and 250 MeV per nucleon for alpha particles is interpreted to mean that S is no longer negligible below these energies, and the integral (16) is not valid there. This, of course, means that the heliocentric electric-field model will break down at these energies, which is consistent with the comments by Freier and Waddington (1965) and the deductions of Quenby (1967) and others.

IV. DETERMINATION OF S

The integral (6), obtained on the assumption that S is negligible in equation (2), can now be used in equation (1) to find an approximate expression for S. This, in turn, can

be used to determine the range of energy over which S is indeed negligible. We require $\kappa_1(r)$, $\kappa_2(P)$, and $V(r)$ to be known and $H(\zeta + \phi)$ to have been determined at the orbit of Earth, $r = r_e$, say, from observations of the intensity J.

With S assumed negligible, the integral (6) can be substituted in equation (4) to give

$$\kappa \frac{\partial U}{\partial r} = -\tfrac{1}{3} V(\mathfrak{E}^2 - \mathfrak{E}_0{}^2)^{3/2} \frac{\partial}{\partial \mathfrak{E}} [H(\zeta + \phi)] . \tag{17}$$

Using this result to eliminate $\partial U/\partial r$, we find that equation (1) becomes

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 S) = -\frac{V^2}{9\kappa_1} \frac{\partial}{\partial \mathfrak{E}} \left\{ \frac{\alpha T}{\beta \kappa_2} (\mathfrak{E}^2 - \mathfrak{E}_0{}^2)^{3/2} \frac{\partial}{\partial \mathfrak{E}} [H(\zeta + \phi)] \right\} . \tag{18}$$

Assuming no sources or sinks (i.e , $r^2 S \to 0$ as $r \to 0$), we find that integration of equation (18) gives

$$S(r,\mathfrak{E}) = -\frac{1}{r^2} \int_0^r \frac{s^2 V(s)^2}{9\kappa_1(s)} \frac{\partial}{\partial \mathfrak{E}} \left(\frac{\alpha T}{\beta \kappa_2(P)} (\mathfrak{E}^2 - \mathfrak{E}_0{}^2)^{3/2} \frac{\partial}{\partial \mathfrak{E}} \{ H[\zeta(\mathfrak{E}) + \phi(s)] \} \right) ds \tag{19}$$

as an approximate expression for S. Note particularly that S, and hence the modulation at a point $r = r_S$, is determined by the scattering throughout the whole range of r: the scattering in $r > r_S$ appears in ϕ and that in $r < r_S$ in the integral above. Thus accurate prediction requires κ and V to be known over the entire range $r > 0$.

The integration of expression (19) cannot be completed at present because we have no data on the form of $\kappa_1(r)$. So, instead of determining an energy below which S becomes significant, we shall accept the limits and spectra, given in § III, assume that $\kappa_1(r) \propto \exp (r/R)$, and use equation (19) to obtain estimates of the characteristic length R.

The streaming S is negligible in equation (2) if

$$\eta(r,\mathfrak{E}) = S/[VU - \tfrac{1}{3} V \partial(\alpha T U)/\partial \mathfrak{E}] \ll 1 . \tag{20}$$

In Appendix A we use the integral (19) to show that, for the data considered here and with $\kappa_1 \propto \exp (r/R)$,

$$\eta(r,\mathfrak{E}) \approx -0.15 Z \left[\frac{3\mathfrak{E}\mathfrak{E}_0}{\mathfrak{E}^2 - \mathfrak{E}_0{}^2} + \frac{\partial \log H}{\partial(\mathfrak{E}/\mathfrak{E}_0)} \right] \frac{e^{1/x}}{x^2}$$
$$\times [2X^3 - e^{-x/X}(x^2 X + 2xX^2 + 2X^3)] . \tag{21}$$

Here $x = r/r_e$ and $X = R/r_e$, and we have taken $\kappa_2 = P$, $\kappa = 10^{22}$ cm^2 sec^{-1} at $r = r_e$ for protons with $T = 6$ BeV and $V = 400$ km sec^{-1}. For convenience this is written in the form

$$\eta(r,\mathfrak{E}) \approx -0.15 Z F_1(\mathfrak{E}) F_2(x,X) , \tag{22}$$

with $F_1(\mathfrak{E})$ the terms of equation (21) contained in the first set of square brackets and $F_2(x,X)$ the terms containing x and X. Figure 3 shows the function $F_2(x,X)$, and Figure 4 shows $F_1(\mathfrak{E})$ derived from the upper curves of Figure 2. The function $F_1(\mathfrak{E})$ increases rapidly as $T \to 0$, peaks negatively near $T = 2$ BeV, and asymptotically approaches $(1 - \gamma)\mathfrak{E}_0/\mathfrak{E}$ as $\mathfrak{E} \to \infty$, $\gamma \simeq 2.65$ being the differential spectral index.

According to our interpretation of Figure 2, S becomes significant below about 300 MeV for protons, and Figure 4 shows that $|F_1(\mathfrak{E})| < 2.4$ for $T > 300$ MeV. Requiring $|\eta| < 0.15$, say, for S to be negligible, we find that equation (22) specifies $F_2(x,X) \le 0.4$, which, according to Figure 3, implies $0.8 \lesssim R/r_e \lesssim 1.6$. Repeating this analysis for
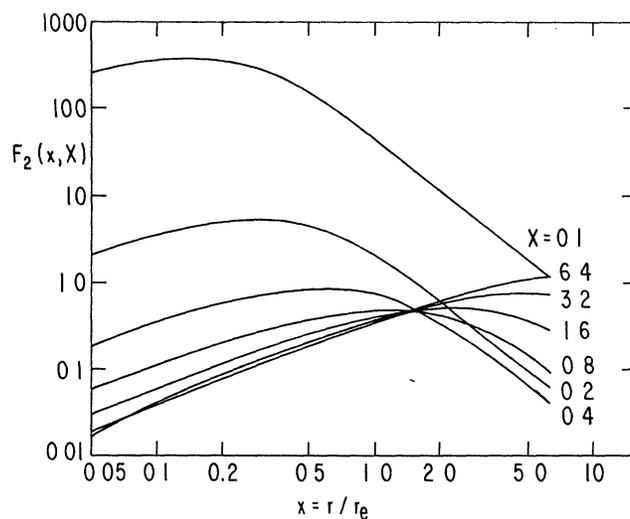
Fig. 3.—Variation of the function $F_2(x,X)$ with x for a range of the parameter $X = R/r_e$, when $\kappa \propto \exp (r/R)$. The variable x measures the radial distance from the Sun. Curves show approximately the manner in which the streaming S varies with radial distance for each value of X.
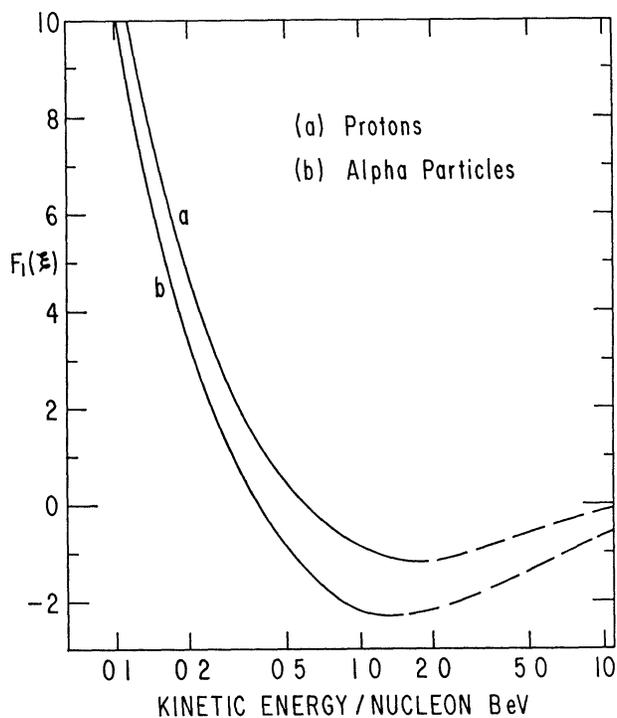

Fig. 4 —Function $F_1(\mathfrak{E})$ for protons and alpha particles in 1965. The current density is roughly proportional to $F_1(\mathfrak{E})$. When $\kappa \propto \exp (r/R)$, then S is not negligible for any value of R, if the energy is such that $F_1(\mathfrak{E}) \geq 3$.

alpha particles, we find that $F_2(x,X) \leq 0.3$ is required; this cannot be satisfied by any value of X (Fig. 3). However, we recall that η may be smaller than the value given by equation (22) by a factor of about 2 (Appendix A), and thus the alpha-particle observations are also probably consistent with the above range of R. We conclude that $R \approx 1$ a.u. and hence that the solar-cycle modulation of cosmic rays is largely confined to a region within a few astronomical units of the Sun.

This analysis sets both upper and lower limits on R, since, with a fixed value of κ at $r = 1$ a.u., small values of R imply substantial scattering close to the Sun, with the corresponding production of large streaming; with large values of R the scattering is more spread out, but the streaming becomes important at larger radial distances. These variations are reflected in the variations of $F_2(x,X)$ in Figure 3.

V. MODULATION OF COSMIC-RAY ELECTRONS

The theory which has been developed applies equally well to cosmic-ray electrons down to some lower limit of energy, provided that the appropriate modulation parameter Φ is used. In this section we shall apply our results to simultaneous electron and proton observations, again assuming κ_2 to be independent of t. Some further analytical development of the theory which is required is given in Appendix B.

Since the first experiments by Earl (1961) and Meyer and Vogt (1961a, b) the number of observations of cosmic-ray electrons has increased remarkably (Beedle and Webber 1968; Bleeker *et al.* 1968; L'Heureux and Meyer 1968; L'Heureux *et al.* 1968; Rubstov and Zatsepin 1968; Webber 1967, 1968; Webber and Chotkowski 1967). In general the statistical accuracy of the observations has been insufficient to justify a detailed analysis of the modulation. However, data for both protons and electrons in the energy range $100 \lesssim T \leq 3000$ MeV, obtained on balloon flights in mid-1965 ($=t_3$) and mid-1966 ($=t_4$) and presented by Webber (1967), allow modulation effects of the order of 50 per cent to be clearly distinguished.

No clear evidence of modulation of electrons was detected during this period by Bleeker *et al.* (1968) and L'Heureux and Meyer (1968). This discrepancy is one which must be resolved by the observers themselves; however, for what it is worth, we note that on the basis of any theory some modulation should have occurred, and indeed the results described by Webber (1967) are quite reasonable. Webber has given an analysis of these data in terms of convection-diffusion theory and energy-loss processes; we now interpret the measurements in terms of the present theory.

Since the changes in intensity are small, it is appropriate to use $\Delta J \simeq (\partial J/\partial \Phi)\Delta\Phi$, and $\partial J/\partial \Phi$ will be related, in the terms of the present theory, to $(\partial \log J/\partial \log T)$ obtained from the *locally observed* spectrum. In this way speculation about modulation occurring between Earth and outer space is avoided. The term $\Delta\Phi$, in this instance, is proportional to $\Delta\phi$. Thus in Appendix B it is shown that

$$\frac{\Delta J}{J} = \frac{1}{T}\left[-\frac{2(T + \mathfrak{E}_0)}{T + 2\mathfrak{E}_0} + \frac{\partial \log J}{\partial \log T}\right]\frac{|Z|eP}{\kappa_2(P)}\Delta\phi \ . \tag{23}$$

The component of the diffusion coefficient can be obtained from the magnetic-field power spectrum and is proportional to $P^2/P_{xx}(f_0)$ where f_0 is the cyclotron frequency and $P_{xx}(f)$ is the power-spectrum function (Jokipii 1968). A study of the available power spectra (Coleman 1966; Ness, Scearce, and Cantarano 1966; Siscoe *et al.* 1968) and analyses of these by Gloeckler and Jokipii (1966) and Webber (1968) have led us to expect that $P_{xx}(f) \propto (1/f)$ in the range of interest: $300 \text{ MV} \leq P \leq 10 \text{ BV}$. From this it follows that $\kappa \propto P$, and $\kappa_2 = P$ will be used in this analysis.

We have used Webber's intensity spectra for 1966 to obtain $\partial \log J/\partial \log T$ and have

used equation (23) with $\kappa_2 = P$ to evaluate $\log_e (1 + \Delta J/J)$ over a range of $\Delta\phi$ for both electrons and protons. These predicted values and Webber's observed values are shown in Figures 5 and 6. The computed values near $T = 10$ BeV have been determined for electrons by using the spectrum reported by Webber and Chotkowski (1967) and for protons by using the spectrum described by Webber (1968).

In the case of electrons excellent agreement is obtained down to $T = 400$ MeV with $\Delta\phi(r_e,t_4,t_3) \simeq -100$ MV, while for protons good agreement prevails down to 500 MeV with $\Delta\phi(r_e,t_4,t_3) \simeq -90$ MV. The small difference in $\Delta\phi$ for electrons and protons is unimportant in view of the accuracy of the observations, the uncertainty in the form of $\kappa_2(P)$, and the fact that the time intervals between observations of protons and electrons were not exactly the same.
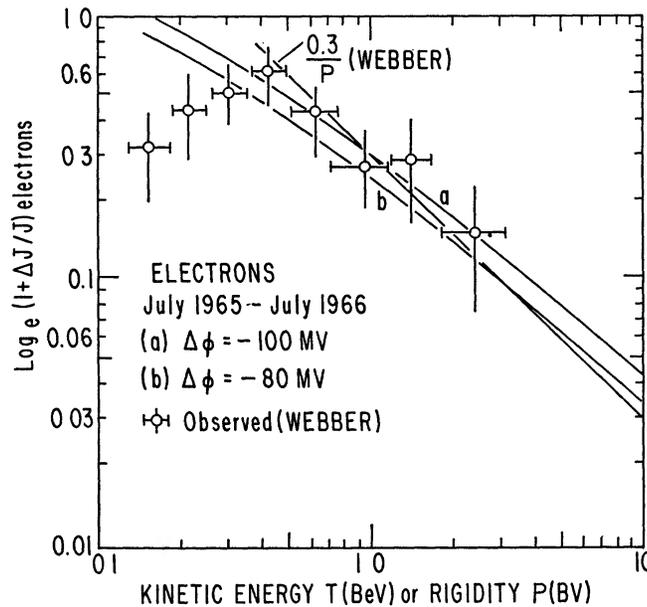

FIG. 5.—Observed values of $\log_e (1 + \Delta J/J)$ for electrons between July 1965 and June 1966 (after Webber 1967), together with values predicted from the local spectrum of 1966 and taking $\Delta\phi = -80$ MV and $\Delta\phi = -100$ MV.

Note again that $\kappa_2 = P$ is equivalent to having a heliocentric force field proportional to $|Z|$, thus giving the same energy loss for both electrons and protons. In the more general case, the energy loss Φ is a function of $|Z|$, $\mathfrak{E}$, and $\mathfrak{E}_0$.

According to the convection-diffusion theory, the ratio

$$\bar{R}(p/e) = \log (1 + \Delta J/J)_p / \log (1 + \Delta J/J)_e , \qquad (24)$$

where the subscripts p and e refer to protons and electrons, respectively, is $\beta_e\kappa_2(P_e)/\beta_p\kappa_2(P_p)$. For electrons and protons at the same rigidity, κ_2 cancels; $\bar{R} (p/e)$ is independent of the precise form of κ_2 and is thus a useful parameter for assessing the validity of the convection-diffusion model. Webber (1967) has pointed out that $\bar{R} (p/e)$ has very different forms for the convection-diffusion model and the heliocentric-electric-field or Liouville model (especially when plotted against rigidity) and that his observations favor the latter. This is to be expected from the theory given here.

An approximate expression for $\bar{R}$ (p/e) according to the present theory and for general κ_2 can be obtained immediately from equation (23), using $\log_e (1 + \Delta J/J) = \Delta J/J$ to first order:

$$\bar{R}(p/e) = \left\{\left[-\frac{2(T_p + \mathfrak{S}_{0p})}{T_p + 2\mathfrak{S}_{0p}} + \frac{\partial \log J_p}{\partial \log T_p}\right]\left[\frac{P_p}{T_p \kappa_2(P_p)}\right]\right\}$$

$$\div \left\{\left[-\frac{2(T_e + \mathfrak{S}_{0e})}{T_e + 2\mathfrak{S}_{0e}} + \frac{\partial \log J_e}{\partial \log T_e}\right]\left[\frac{P_e}{T_e \kappa_2(P_e)}\right]\right\} .$$

(25)

We note that κ_2 cancels in this case also if particles of the same rigidity are compared, and then $\bar{R}^*(p/e)$ is given in terms of the observed spectra; thus $\bar{R}(p/e)$ is also a useful parameter for assessing the validity of the present theory.
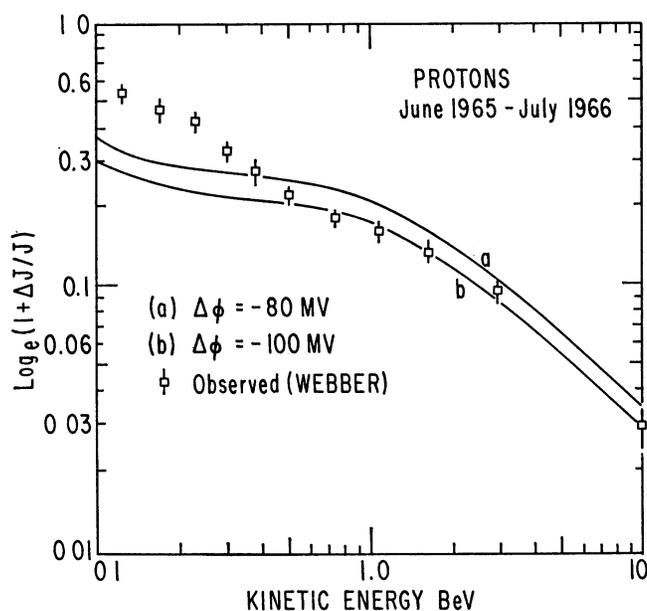

Fig. 6.—Observed values of $\log_e (1 + \Delta J/J)$ for protons between June 1965 and July 1966 (after Webber 1967), together with values predicted from the local spectrum of 1966 and taking $\Delta\phi = -80$ MV and $\Delta\phi = -100$ MV.

The values of $\bar{R}$ (p/e) determined for the convection-diffusion model and the present model are shown in Figure 7, together with the observed values of Webber (1967). At rigidities above about 1.2 BV there is no clear distinction between the two models, while at lower rigidities the experimental values lie much closer to the values given by the present theory. However, since this present analysis is not precise at kinetic energies below about 400–500 MeV for either electrons or protons, this correspondence cannot be used as strong evidence in its favor. We take it to show that convection-diffusion theory is not valid throughout the rigidity range below, say, 800 MV, and that the present theory appears to be better down to 300–400 MV. An adequate description of the modulation for energies less than a few hundred MeV per nucleon requires the solution of equations (1) and (2) including the effect of streaming.

The positron-negatron ratio (σ) might offer a convenient method of detecting energy changes due to solar modulation, provided that the unmodulated spectra of positrons

and negatrons are sufficiently different (Axford 1966). From equation (11) we see immediately that this ratio is given by

$$\sigma = \frac{J_+(r,\mathfrak{E})}{J_-(r,\mathfrak{E})} = \frac{J_+(\infty,\mathfrak{E}+\Phi)}{J_-(\infty,\mathfrak{E}+\Phi)}, \tag{26}$$

where J_+ and J_- are the differential intensities of positrons and negatrons, respectively. In particular, for the case $\kappa_2 = P$ and for small changes of intensity, the positron-negatron ratio at a given energy differs from that at infinity by a fractional amount:

$$\frac{\Delta\sigma}{\sigma} \simeq \left(\frac{1}{J_+}\frac{\partial J_+}{\partial\mathfrak{E}} - \frac{1}{J_-}\frac{\partial J_-}{\partial\mathfrak{E}}\right)_{r=\infty}\Phi. \tag{27}$$
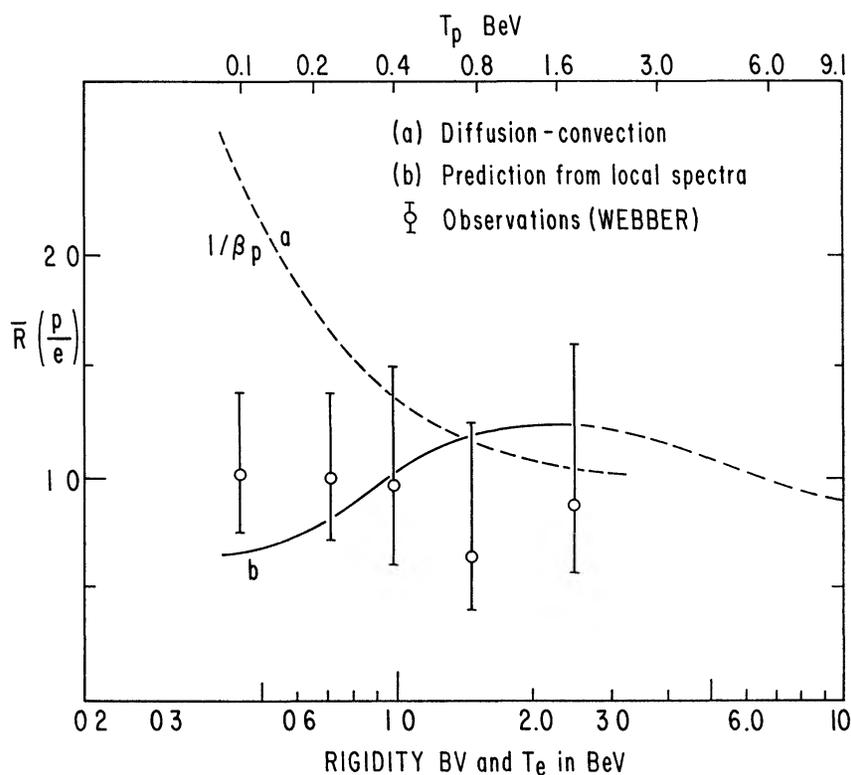

FIG. 7.—Observed values of $\bar{R}(p/e)$ between June 1965 and July 1966 given by Webber (1967), together with values predicted from the local spectra according to the present theory and those expected from convection-diffusion theory. (For the two points on the left, $Tp < 300$ MeV; hence there is some doubt about the validity of the assumption $\Delta J \ll J$ for photons.)

If it is assumed that $J_+(\infty,\mathfrak{E}) \propto \mathfrak{E}^{-\mu_+}$ and $J_-(\infty,\mathfrak{E}) \propto \mathfrak{E}^{-\mu_-}$ then $\Delta\sigma/\sigma \simeq (\mu_- - \mu_+)$ $(\Phi/\mathfrak{E})$. At energies of the order of 1 BeV, if $(\mu_- - \mu_+)$ is of the order of unity, one would expect the positron-negatron ratio to vary by several tens of per cent through the sunspot cycle. However, since this ratio is in any case probably quite small (no more than a few per cent in the energy range where the present theory is applicable), it might be very difficult to detect the predicted variation.

VI. CONCLUDING REMARKS

The integral presented here relates the cosmic-ray intensity at a given position and time to the intensity at infinity, through an energy-dependent modulation parameter

Φ and a formal application of Liouville's theorem. The parameter $\Phi(r,\mathfrak{E},Z,t)$ has been identified tentatively with the mean energy loss experienced by cosmic-ray particles in penetrating into the interplanetary region from the interstellar region.

The integral appears to describe quite adequately variations in differential intensity of protons and helium nuclei observed from 1963 through 1965 and electron and proton observations of mid-1965 and mid-1966, above kinetic energies of several hundred MeV per nucleon. If the result remains valid over an extended period of time, it will provide a useful description of the solar modulation in terms of the two quantities $\phi(r,t)$ and $\kappa_2(P)$, which are independent of the species. If conditions exist such that κ_2 is independent of time over a period of several years, then $\phi(r,t)$ is a single parameter describing the modulation.

The assumption $\kappa = \beta\kappa_1(r,t)\kappa_2(P,t)$ is the one most likely to invalidate this development; at present we have no adequate knowledge of the correct functional form of $\kappa(r,\mathfrak{E},Z,t)$. The assumption $\kappa_1 \propto \exp(r/R)$ made in the latter part of § IV is not used to obtain the modulation integral, but, by showing $0.8 \lesssim R \lesssim 1.6$ a.u., we use it to obtain some idea of the radial variation of κ and to illustrate some of the essential physical processes. In particular, it is shown that the modulation observed at the Earth depends upon conditions existing in the whole of the region between the outer boundary of the solar cavity and the Sun.

Because in each instance $\kappa_2 = P$ in the rigidity range of interest, the applications described in §§ III and V yield intensities identical with those obtained with an outwardly directed heliocentric force proportional to $|Z|$ (i.e., identical with the heliocentric-electric-field model applied to positively charged cosmic rays). This "explains" why that model is partially successful and also permits us to calculate the effective potential in terms of observable quantities and to set lower limits to the energy range in which the model is valid.

In particular, it is found (cf. §§ III and V) that $\Phi(r_e)$, the effective potential energy at Earth, decreased by approximately $80\,|Z|$ MeV between 1963 and 1965 and increased by approximately $90\,|Z|$ MeV between June 1965 and July 1966; calculations based on the estimate of R (cf. Appendix A) gave Φ at solar minimum as approximately $140\,|Z|$ MeV.

It will be necessary to use the more general result (eq. [6] or eq. [11]) when κ is not proportional to P and Φ is a function of energy. Evidence for or against the validity of this modulation formula can be provided by correlating, through the solar cycle, the form of $\kappa_2(P,t)$ predicted from the magnetic-field power spectrum, the modulation predicted by this analysis, and the observational data.

APPENDIX A

AN APPROXIMATE EXPRESSION FOR $\eta(r,\mathfrak{E})$

An approximate expression is to be obtained for $\eta(r,\mathfrak{E})$ defined by equation (20) and valid for the 1965 data of Figure 2 with $\kappa_2 = P$ and $\kappa_1 \propto \exp(r/R)$. In this case with $\zeta = (\mathfrak{E} - \mathfrak{E}_0)/Ze$; hence, making use of equation (6),

$$U - \frac{1}{3}\frac{\partial}{\partial T}(aTU) = -\frac{1}{3}(\mathfrak{E}^2 - \mathfrak{E}_0{}^2)^{3/2}\frac{\partial}{\partial \mathfrak{E}}\left\{H\left[\frac{\mathfrak{E} - \mathfrak{E}_0}{Ze} + \phi(r,t)\right]\right\}. \qquad (A1)$$

Thus the integral for S, given by equation (19), becomes

$$r^2 S = \frac{Ze}{9} \int_0^r \frac{s^2 V(s)^2}{\kappa_1(s)} \frac{\partial}{\partial \mathfrak{E}} \left((\mathfrak{E}^2 - \mathfrak{E}_0^2)^{3/2} H \frac{\partial}{\partial \mathfrak{E}} \left\{ \log_e H \left[\frac{\mathfrak{E} - \mathfrak{E}_0}{Ze} + \phi(s) \right] \right\} \right) ds . \quad (A2)$$

Assuming, and verifying later, that $0 < \phi < 400$ MV for $r > 0$, from the 1965 curve for $H(\zeta + \phi) = (4\pi/cZ^2e^2)J/P^2$ given in Figure 2, we deduce that, as $\mathfrak{E}$ varies over the range of $Ze\phi$, (a) $\partial \log_e H/\partial \mathfrak{E}$ is substantially constant; (b) H varies by a factor of 2–3, and $\partial H/\partial r > 0$. Thus equation (A2) can be written

$$r^2 S \approx \frac{Ze}{9} \frac{\partial \log_e H}{\partial \mathfrak{E}} \frac{\partial}{\partial \mathfrak{E}} [(\mathfrak{E}^2 - \mathfrak{E}_0^2)^{3/2} H] \int_0^r \frac{s^2 V(s)^2}{\kappa_1(s)} ds , \quad (A3)$$

with an error of a factor 2 or 3 at most. Making use of equation (A1) and rearranging, it follows that

$$\eta(r,\mathfrak{E}) \approx -\frac{Ze}{3\mathfrak{E}_0} \left(\frac{3\mathfrak{E}\mathfrak{E}_0}{\mathfrak{E}^2 - \mathfrak{E}_0^2} + \frac{\partial \log_e H}{\partial \mathfrak{E}} \right) \frac{1}{r^2 V(r)} \int_0^r \frac{s^2 V(s)^2}{\kappa_1(s)} ds , \quad (A4)$$

in which the magnitude of η is overestimated by a factor between 1 and 3. We may now obtain expression (21) for $\eta(r,\mathfrak{E})$ by writing $\kappa_1(r) = C_0 \exp (r/R)$, using $\kappa = 10^{22}$ cm² sec⁻¹ for protons at $r = 1$ a.u. and $T = 6$ BeV to determine C_0 (see Paper II), taking $V = 400$ km sec⁻¹, and integrating.

Finally, with the above parameters, integrating equation (8) yields

$$\phi(r) = 140(R/r_e) \exp[(r - r_e)/R] . \quad (A5)$$

If we take $0.8 \lesssim R \lesssim 1.6$ a.u. (see § IV), this equation shows that ϕ lies between 112 and 206 MV at Earth and between 390 and 420 MV at the Sun; this confirms the assumption about the range of ϕ used in deducing equation (A3). Since in this case $\Phi = |Z|e\phi$, these calculations immediately give estimates of the mean energy loss of the cosmic-ray particles, and ϕ is the equivalent electrical potential for positively charged particles.

APPENDIX B

THE CASE OF SMALL MODULATION CHANGES

When $\kappa_2 = \kappa_2(P)$, the r,t dependence of Φ is contained in $\phi(r,t)$. In this Appendix ΔJ is determined in terms of $\Delta\phi$ for the cases in which $|\Delta J/J| \ll 1$.

From equation (11), and to first order,

$$\Delta J = (\mathfrak{E}^2 - \mathfrak{E}_0^2) \frac{\partial}{\partial(\mathfrak{E} + \Phi)} \left[\frac{J(\infty, \mathfrak{E} + \Phi)}{(\mathfrak{E} + \Phi)^2 - \mathfrak{E}_0^2} \right] \Delta\Phi . \quad (B1)$$

Noting that Φ is a function of $\mathfrak{E}$, we then obtain

$$\frac{\partial}{\partial \mathfrak{E}} \left[\frac{J(\infty, \mathfrak{E} + \Phi)}{(\mathfrak{E} + \Phi)^2 - \mathfrak{E}_0^2} \right] = \frac{\partial}{\partial(\mathfrak{E} + \Phi)} \left[\frac{J(r, \mathfrak{E}, t)}{\mathfrak{E}^2 - \mathfrak{E}_0^2} \right] \left[1 + \left(\frac{\partial \Phi}{\partial \mathfrak{E}} \right)_{r,t} \right] . \quad (B2)$$

Combining equations (B1), (B2), and (11), we have, to first order,

$$\Delta J = (\mathfrak{E}^2 - \mathfrak{E}_0^2) \frac{\partial}{\partial \mathfrak{E}} \left[\frac{J(r, \mathfrak{E}, t)}{\mathfrak{E}^2 - \mathfrak{E}_0^2} \right] \frac{\Delta\Phi}{1 + (\partial\Phi/\partial\mathfrak{E})_{r,t}} , \quad (B3)$$

from which we obtain, after dividing by J and using T as an independent variable,

$$\frac{\Delta J}{J} = \frac{1}{T}\left[-\frac{2(T+\mathfrak{E}_0)}{T+2\mathfrak{E}_0} + \frac{\partial \log J}{\partial \log T}\right]\frac{\Delta \Phi}{1+(\partial \Phi/\partial \mathfrak{E})_{r,t}}. \tag{B4}$$

To complete the evaluation of equation (B4), the factor involving $\Delta \Phi$ and $(\partial \Phi/\partial \Phi \mathfrak{E})_{r,t}$ is determined in terms of $\Delta \phi$ and measurable quantities.

From definition (9),

$$\frac{\partial \psi(\zeta + \phi)}{\partial(\zeta + \phi)}(d\zeta + d\phi) = \left[1 + \left(\frac{\partial \Phi}{\partial \mathfrak{E}}\right)_\phi\right]d\mathfrak{E} + \left(\frac{\partial \Phi}{\partial \phi}\right)_\mathfrak{E}d\phi. \tag{B5}$$

Noting that ζ is a function of $\mathfrak{E}$ but not of ϕ, and equating coefficients, we find that

$$\left(\frac{\partial \Phi}{\partial \mathfrak{E}}\right)_\mathfrak{E} = \frac{\partial \psi(\zeta + \phi)}{\partial(\zeta + \phi)} = \left[1 + \left(\frac{\partial \Phi}{\partial \mathfrak{E}}\right)_\phi\right]\Big/\left(\frac{\partial \zeta}{\partial \mathfrak{E}}\right)_\phi. \tag{B6}$$

Then, since $(\partial \zeta/\partial \mathfrak{E}) = |Z|eP/\kappa_2(P)$, we obtain, to first order,

$$\frac{\Delta \Phi}{1+(\partial \Phi/\partial \mathfrak{E})_{r,t}} = \frac{(\partial \Phi/\partial \phi)_\mathfrak{E}}{1+(\partial \Phi/\partial \mathfrak{E})_\phi}\Delta \phi = \frac{|Z|eP}{\kappa_2(P)}\Delta \phi. \tag{B7}$$

The results (B4) and (B7) together yield equation (23).

Finally we note that, from the above, it can be shown readily that

$$(\partial \Phi/\partial \phi)_\mathfrak{E} = |Z|eP'/\kappa_2(P'), \tag{B8}$$

where $P' = P(\mathfrak{E} + \Phi)$ is the rigidity at energy $\mathfrak{E}' = \mathfrak{E} + \Phi$. This is an implicit differential equation relating Φ and ϕ which may be solved directly for Φ, thus eliminating the need to integrate the first of equations (8) and to evaluate the inverse function ψ. First noting that $\Phi = 0$ when $\phi = 0$, we see immediately from equation (B8) that, in order for Φ to be independent of $\mathfrak{E}$, it is necessary and sufficient that $\kappa_2 = P$.

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