

OBSERVATIONAL SPATIAL AND KINEMATIC
MODELS OF PLANETARY NEBULAE

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ABSTRACT

Models are presented for a number of planetary nebulae representing both the physical distribution of emitting material and the kinematics of this material. The models are based on an analysis of the line profiles derived from high-dispersion spectrograms of bright emission lines in the planetaries. The planetary nebulae studied are NGC 7662, 7009, 6826, 6818, 6572, 6210, 3242, 2392, and IC 3568 and 418. The spatial models include descriptions of the shell shapes, thicknesses, and inhomogeneities. The kinematic features describe the expansion velocities of different parts of the shells. It is shown that ellipsoidal shells show a velocity of expansion which increases with distance from the central star, implying that the shell is now ellipsoidal because the original velocity of ejection varied with direction. The radii at different parts of a shell do not appear directly proportional to the expansion velocities, however, and this fact is used together with an apparent relation between a shell's thickness and its expansion velocity to infer that a long-term pressure acted to eject the shell. Evidence is also presented that some planetaries consist of at least two independently ejected shells, with the outer shell being more massive.

I. INTRODUCTION

Although much knowledge concerning the sizes, distances, and histories of planetary nebulae has come from previous spectroscopic observations of these objects (Wilson 1950, 1958; Aller 1956), many assumptions have been made concerning their geometrical and kinematical structure. The analysis of observations for determining the masses and radii depends on the assumed form of the nebulae (Seaton 1966; O'Dell 1962), and the structure and internal kinematics are the primary evidence relating to a planetary's history. This history must be known before the mechanism that ejected the nebula can be established. It is therefore desirable to derive observational spatiokinematic models of planetary nebulae which will depict the structure and motions of the gases within the nebulae, especially since theoretical attempts are now being made to describe the structure and evolution of a typical object (Mathews 1966; Sofia and Hunter 1967). The most recently published observations concerned with this problem were those of Osterbrock, Miller, and Weedman (1966), who published high-dispersion emission-line profiles for several summer-sky planetaries. They then used these profiles to derive the distribution in radial velocity of the emitting gas in the apparent centers of the planetaries. The present paper carries similar techniques further to derive line profiles at several positions in a number of planetaries and uses these profiles to derive the spatiokinematic structures of these nebulae.

II. OBSERVATIONS

The objects observed were NGC 7662, 7009, 6826, 6818, 6572, 6210, 3242, 2392 and IC 3568 and 418, which are all bright enough and of suitable size for coudé observations and also symmetrical enough to validate a spatiokinematic model constructed from observations at only one slit orientation. All but IC 3568 were observed as described by Osterbrock *et al.* (1966) with the coudé spectrograph of the 100-inch Mount Wilson reflector. The spectrograms were taken with the slit through the central stars of the nebulae

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and, for elliptical objects, oriented along the major axes. For circular planetaries the slit was positioned east-west or north-south. In all cases, the image rotator was used during exposures to keep a nebula in the same orientation on the slit. IC 3568 is too far north for the 100-inch reflector, so it was observed with the coudé spectrograph on the 84-inch Kitt Peak reflector. This spectrograph was used with the $f/4.1$ camera, giving a dispersion of $8.9 \text{ \AA mm}^{-1}$ in the second-order blue when used with the 600 line mm^{-1} grating. $H\beta$, N1, and N2 were observed in the second order on IIaO plates. $H\alpha$ was observed simultaneously in the first order on IIaE plates. No image rotator was available for the 84-inch reflector, but IC 3568 is so exceptionally symmetric that the velocity structure should not depend on the slit orientation.

The emission-line images obtained for a given planetary cover the entire apparent diameter of the nebula and thus give the distribution of the emitting material along the cross-section of the nebula observed by the slit. The shape and width of the emission-line profile vary from point to point across the portion of the planetary seen through the slit, depending on the radial velocities of the emitting material. The lines are split with a red component arising from the receding far side of the expanding shell and a blue component arising from the approaching near side. Photometric calibration exposures were made on the 100-inch spectrograms as described by Osterbrock *et al.* (1966). For IC 3568 a spot sensitometer provided calibration exposures on plates that were developed simultaneously with the spectrograms. Filters were used to restrict the spots to wavelength regions near those of the emission lines, and the exposure times for the calibration plates were comparable to those for the spectrograms.

To obtain the emission-line profiles at different parts of the nebulae, the line images were traced on a microphotometer at a number of positions. The number varied from seven to twenty, depending on the length of the line (i.e., the apparent size of the nebula). Great care was taken to insure that the various tracings across different lines from the same object were made at the same positions. The analyzing slit of the microphotometer was typically about $1''.2$ long in terms of the plate scale, although the length varied somewhat depending on the nebula being traced. The plate scale is $15''.2 \text{ mm}^{-1}$ for the 100-inch spectrograms and $22''.6 \text{ mm}^{-1}$ for the 84-inch. The width of the microphotometer slit was such that the instrumental resolution, defined by the full width at half-maximum intensity of lines in the comparison spectrum, was about 5 km sec^{-1} for the 100-inch plates and 15 km sec^{-1} for the 84-inch plates. The slit width and length could not be made much smaller since the noise due to the finite size of the silver grains in the image would have prevented the determination of the true smooth-line profile. All tracings were made with the slit moving parallel to the direction of dispersion.

In most cases, a number of exposures of the same object were available so that the resultant line profiles were means of several independent tracings. Table 1 lists the objects, the number of positions traced on a line image, and the number of separate images traced for each line considered. The averaging of several profiles from the same point in a nebula was done by taking numerical averages of relevant parameters of a profile and by checking the final averaged profile by eye against the individual profiles which went into the average. The numerical-averaging procedure made it possible to estimate the errors involved in the profile shapes by noting the deviations of the individual profiles from the final average.

The output from the observation of any given nebula is therefore a set of line profiles which represents the distribution in velocity space of the emitting material at a number of points along the apparent major axis of the nebula. These various profiles are normalized to the most intense profile so that the relative amounts of total emission at any point considered can be determined. In most cases, only the distribution of H and [O III] is considered, since these usually provide the only emission lines bright enough for adequate tracings. Most of the models are based on the N1 and N2 lines of [O III] since these lines are generally the brightest and are blurred very little by thermal motions

because of the large mass of the O^{++} ion. Although bright, the H-lines are so blurred by thermal motions as to mask much of the velocity structure from non-thermal mass motions.

It is also apparent from the present observations as well as from the slit and slitless spectra of Wilson (1950) that the $[O\ III]$ emission occurs throughout most of the nebular regions in which hydrogen recombination emission occurs. Thus $[O\ III]$ emission would seem to be fairly general throughout the ionized part of the nebulae and therefore is an excellent tracer for the ionized gas. This similarity of the collisionally excited $[O\ III]$ and recombination H-line images is evidence that the changes in $[O\ III]$ emission primarily result from changes in density. The similarity in $[O\ III]$ and H emission is therefore taken as evidence that the emission decreases in the outer parts of the nebulae because the nebulae are density bounded. They can therefore be considered as optically thin to hydrogen ionizing radiation. Seaton (1966) estimates whether a planetary is optically thin or thick in the hydrogen continuum by noting the presence or absence of He II lines. He classifies NGC 7662, 7009, 6826, 3242, and 2392 as optically thin; NGC 6572 and 6818 as

TABLE 1
LIST OF LINES TRACED

Object	Positions Traced on a Line (No.)	[O III] Lines Traced (No.)	H-Lines Traced (No.)	Other Lines Traced (No.)
NGC 7662 ..	18	6	2	1 He II
7009 .	16	3	2	
6826	15	3	1	
6818 ..	12	1		
6572	9	4	2	1 [N II]
6210 .	9	5	2	
IC 3568	9	9	7	
NGC 3242	17	2		
2392*	20/17	3/4		
IC 418...	9	7	7	{ 5 [N II] 3 [O II]

* The notation gives the respective numbers for the east-west and north-south slit positions.

optically thicker than the above objects but still mostly ionized; and NGC 6210 and IC 3568 and 418 as optically thick. Since in both NGC 6210 and IC 3568 the distribution of H appears identical to that of $[O\ III]$, it is here assumed that most of the mass is also ionized for these objects, although this conclusion is not as certain as for those objects which Seaton also classifies as optically thin. Except for IC 418, the observed distributions of $[O\ III]$ emission are therefore taken as reflecting the distributions of matter in the nebulae and are used for deriving the total nebular structures.

III. SPATIOKINEMATIC MODELS

For the ellipsoidal nebulae, it is assumed that the objects are prolate spheroids (an ellipse rotated about its major axis). This is more than an assumption in some cases, since the approximate form of the nebular cross-section seen by the slit can often be deduced and comparison of this third nebular dimension with the two dimensions seen in direct photographs gives the true form of the nebula. As used in the remainder of this paper, the term "spheroid" therefore refers to a prolate spheroid. Basic information on the shape and orientation of a cross-section seen through the slit comes from the "tilt" of the emission-line image, i.e., the difference in radial velocity between the top and

bottom of a line. If a spheroidal shell of negligible thickness is expanding such that all motions are along radius vectors from the center, the line image will be tilted if an axis of the elliptical cross-section observed is not perpendicular to the line of sight (Wilson 1958). This tilt arises because the material at one apparent edge of the nebula, where the observer's line of sight passes tangent to the shell, has a component of approach toward the observer whereas the material at the opposite edge is receding.

The ratio of the tilt to the expansion velocity at the center of the nebula depends on the eccentricity and orientation of the cross-section. Since both of these quantities are unknown, the tilt alone does not lead to a unique determination of both. The magnitude of the tilt does set a lower limit on the possible eccentricity of the elliptical cross-section regardless of the orientation. For NGC 7662 this limit reinforces Wilson's (1958) conclusion from his multislit spectra that NGC 7662 is a prolate spheroid. Wilson also concluded that NGC 7009 is a prolate spheroid but has its major axis perpendicular to the line of sight so that the line images are not tilted. For three more objects considered here (NGC 6826, 6572, 6210) the lines are tilted, so the cross-sections observed must also be elliptical, and these ellipses are assumed to have axial ratios of 1.5, which is the ratio assigned to NGC 7662 and 7009 by Wilson. The magnitudes of the tilts then restrict the orientations of the major axes to a narrow range about an angle of 45° to the line of sight. Having adopted approximate shapes and orientations of the central cross-sections, the apparent axes of the nebulae expected in direct photographs if the objects are prolate spheroids can be calculated. In all cases the expected apparent axes agree with those that are observed. When they were available, the isophotometer tracings published by Aller (1956) were used to determine the apparent axes of the nebulae.

The assumption that the ellipsoidal nebulae are prolate spheroids is therefore compatible with all observations of the objects, although it is by no means proved; but, to the extent that the nebulae can be considered as symmetrical and homogeneous, it seems the best working hypothesis. Thus NGC 6818 and 3242, which appear ellipsoidal in direct photographs but whose line images show no tilt, are assumed to be prolate spheroids observed perpendicular to their major axes.

The locus of maximum emission density in an actual shell represents the idealized shell of negligible thickness found through the tilt analysis. A measure of the real shell's thickness can be determined by noting the widths perpendicular to the dispersion of the ends of the emission-line images, i.e., where the red and blue components merge. These end portions of the line images come from points in a nebula where the observer's line of sight passes tangent to the shell, and the material moving outward along the radius vector to the tangent point then represents the distribution of material through the shell. Once this distribution is found for the ends of the shell, it is applied to the rest of the shell by assuming the ratio of shell thickness to shell radius to be constant throughout the nebula. Such measures of shell thickness must be corrected for atmospheric seeing, since the true line image from the nebula oscillates up and down the slit (i.e., perpendicular to the dispersion) with an amplitude dependent on seeing. The shapes, thicknesses, and orientations of the shell cross-sections determined as described may not, of course, represent the mean shell structure. But the selection of the particular objects considered was based on the fairly homogeneous appearance of direct photographs and multislit spectra (Wilson 1958) of these objects. Thus to a reasonable approximation the nebulae should be well represented by the spheroids swept out when the derived cross-sections are rotated about their major axes. Although each nebula considered is discussed separately later in this paper, the summary of the parameters of the models is given in Table 2. All length units are seconds of arc since the distances of the objects are too uncertain to permit translation to true lengths.

Once a spatial model is determined, simple geometrical considerations can be used to derive the velocity distribution of material in the shell. The separations of the red and blue maxima in the line profiles correspond to the radial velocity separations of the far

TABLE 2
PARAMETERS FOR THE SPATIOKINEMATIC MODELS

Object	Inner-Shell-Axial Ratio	Outer-Shell-Axial Ratio	Inner-Shell-Major Axis*	Outer-Shell-Major Axis	Angle of Major Axis†	Inner-Shell Thickness‡	Outer-Shell Thickness	N_e (inner)/ N_e (outer)	M (outer)/ M (inner)	Velocity-Gradient Slope§ [km sec ⁻¹ (arc sec) ⁻¹]	Velocity-Gradient Intercept R_0
NGC 7662...	3:2	1.1.1	16".5	28".2	50°	1".6	3".2	3.3	2.2	6.2	1".5
7009	3:2	..	27.7		90	2.0	..	..	..	7.1	6.9
6826.	3:2	..	13.9	..	40	4.3	..	2.6	3.5	2.5	3.4
6818.	3:2	..	17.3	..	90		..	..	..	5.9	0.8
6572.	3:2	..	7.9	..	40		..	..	..	3.4	0.0
6210	3:2	..	11.9	..	50	4.1	..	..	..	5.4	1.0
IC 3568.	1:1	..	..	..	..		..	..	..		
NGC 3242.	3:2	1.1:1	18.0	39.1	90	2.3	2.6	1.5	8.5	6.6	3.0
2392.	3:1.1	1:1	37.2	37.2	0	5.9	3.6	2.0	2.4		
IC 418.	1:1									1.8	0.0

* Length of the major axis defined as the distance along the axis between positions of maximum density in the shell. § Slope s of the expansion velocity gradient from $V_X = s(R - R_0)$, where V_X is the expansion velocity of a point in the shell, R is the radius to the point, and R_0 is the radius at which the expansion velocity is zero.
† The acute angle between the major axis and the observer's line of sight.
‡ The shell thickness is the distance through the shell between points of half-maximum emission density, measured along the major axis.

and near sides of the densest portion of an expanding shell. Since this separation may be determined at various points across the face of a nebula, the velocity of expansion of the shell can be determined at these points. In each spheroidal nebula it was found that the velocity of expansion increases as the radial coordinate increases. Since this increase appears approximately linear within the observational error, the slope and intercept of the velocity-radius gradient are listed in Table 2. The observed relations are shown in Figures 1 and 3.

The velocity-radius gradient derived as described applies only to the maximum-density locus in the shell, so it does not apply to the local velocity structure along a single radius through the shell. Here the data involved are the velocity structure within the individual components (red and blue) of the split-line profiles. In all cases where these components are clearly separated, the observed component shapes differ very little from Gaussians within the errors of grain noise. To simplify the analysis, it was therefore assumed that each component of a profile is a Gaussian with the same half-maximum intensity width as the observed width in the actual profile. Then the number of emitting ions per radial-velocity interval in the nebular shell is determined by a Gaussian distribution.

To determine the velocity width of the true distribution of emitters in the shell, the observed component widths had to be corrected for several artificial broadening mechanisms. These included broadening due to (1) the finite length of the microphotometer slit, (2) the resolution of the spectrograph and microphotometer, and (3) seeing and guiding effects. The magnitudes of these various broadening mechanisms vary from object to object, depending on line-image shape and size. Fortunately, however, the conclusions are not overdependent on the corrections since the artificial broadening on the whole is only about 10 per cent of the intrinsic component widths.

NGC 7662.—Due to its apparent size, symmetry, and high expansion velocity, NGC 7662 is the most suitable of the objects considered for a spatiokinematic analysis. Fortunately it appears to be representative of a large number of planetaries in its spheroidal structure and the apparent presence of a faint outer shell surrounding the bright inner one. On long exposures (three hours or more) the N1 and N2 lines clearly show that there are at least two shells which are spatially and kinematically separate. Typical lines are pictured by Osterbrock *et al.* (1966). The brighter inner shell produces a symmetrical, tilted line whose two components merge smoothly at top and bottom. The fainter, outer shell line merges with the inner line in the central part of the nebula so that the velocity structure of the outer shell is apparent only toward the edge of the nebula. The outer shell appears to expand at approximately the same velocity as the inner shell and is symmetrical about the inner shell. Wilson's (1958) multislit spectra of NGC 7662 led him to conclude that the inner shell is a spheroid of axial ratio 1.5 with the major axis at about 45° to the observer. The line tilt in the present observations is too great to arise from a radially expanding shell of axial ratio less than 1.5 regardless of the orientation. That is, if the observed cross-section of the nebula is more rounded than this, no angle of orientation could produce the observed line tilt. The shell could have a higher axial ratio, although the derived lower limit agrees well with Wilson's conclusions about the shape of the shell. In addition, those planetaries which do appear ellipsoidal rarely have an axial ratio greater than 1.5. If this ratio is therefore adopted for NGC 7662, the observed tilt arises if the acute angle between the major axis of the shell and the observer's line of sight is 50° . The shell at this angle would have an apparent axial ratio which agrees with that taken from isophotometer tracings by Aller (1956) of slitless images of this nebula.

To determine the thickness of the inner shell and the distribution of emission within it, the line images were traced along their longest axis, nearly perpendicular to the dispersion, with a very short microphotometer slit. The average distribution of emission through the shell at the two observed edges appears Gaussian with full width at half-

maximum emission density of $1''.6$. Due to seeing effects which are larger than the thickness of the shell, the actual shape of the distribution cannot be determined, although the observed distribution after being blurred by seeing is indeed Gaussian, with a mean width of $2''.6$. The adopted intrinsic width of $1''.6$ depends on the assumption of $2''.0$ seeing, and poorer mean seeing would imply an even narrower shell. The $2''.0$ value for seeing is the measured width of the central-star continuum in another planetary observed on a night of typical seeing. No continuum is visible on the spectrograms of NGC 7662.

Using the line profiles to calculate emission measures from different parts of the shell and using the spatial model to correct for path-length differences, an estimate was made of the degree of inhomogeneity in the observed part of the inner shell. The results show that from point to point within the shell the average variation of the emission density from the mean is ± 23 per cent. For an isothermal, optically thin shell in which the emission density is always proportional to N_e^2 , this implies typical variations of ± 11 per cent in the electron density.

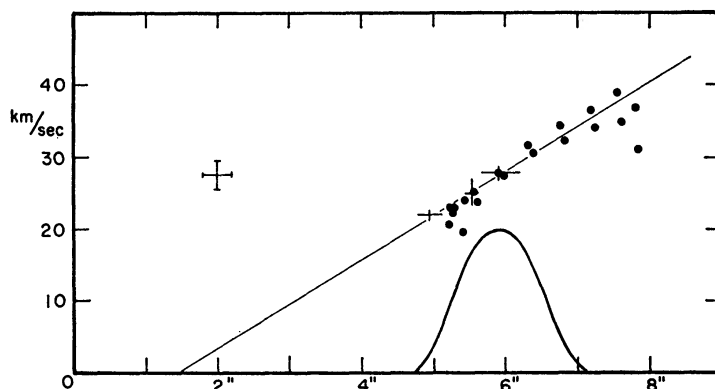


FIG. 1.—Expansion velocity gradient in NGC 7662. Solid line is the expansion velocity of the nebular shell as a function of the shell radius. Observed points are shown by crosses and filled circles, which are explained in the text. Central error bar shows estimated mean error for filled circles. Curve at bottom is the normalized distribution of [O III] emission through that part of shell between observer and center of nebula.

The emission line from the outer shell shows very little tilt, implying that the outer shell is nearly spherical. If the major axes of the inner and outer shells are parallel, the axial ratio for the outer shell derived from the tilt is 1.03. The distribution of emission through the outer shell can be estimated only from the observed emission measures and the geometry of the adopted model. It is clear, however, that the nebula is a true two-shell structure with the emission density reaching a primary maximum, decreasing, and again rising to a secondary maximum. If the changes in emission arise only because of density changes in the nebula, the outer shell is one-third as dense as the inner but 2.2 times as massive. This compares well with Aller's (1957) estimate that the density ratio of the two shells is 11:4.

A kinematic model of the nebula was derived only for the inner shell since its spatial model is much more secure. The expansion velocities determined for a number of positions in the densest part of the shell are shown as filled circles in Figure 1. These points are from different parts of the ellipsoidal shell and are therefore at different distances from the center of the nebula. As shown by Figure 1, those parts of the shell that are most distant from the center have the greatest velocity, and their distance is nearly proportional to their velocity. This implies that the current shape of the nebula reflects the fact that some parts of the shell received a greater outward velocity than did other

parts during the ejection process, and the present shape of the shell represents the directional distribution of momentum given the shell.

The lengthwise tracings of the line images show that the maximum-density shells of H, He⁺, and O⁺⁺ are of slightly different sizes. Since the three shells also show different expansion velocities, these velocities may be used with the different radii of the respective density maxima to derive the expansion velocity as a function of distance along a single radius vector. The He II emission is closest to the center, H is next, and [O III] is the most distant. The three crosses in Figure 1 are located in this way, assuming that the H and He⁺ shells have the same shape and orientation as the O⁺⁺ shell. These crosses, whose sizes represent the mean deviations of the several measured lines, give the different expansion velocities within the cross-section of the shell shown by the curve at the bottom of Figure 1. This curve, taken from the spatial model, is the normalized distribution of emitting O⁺⁺ ions in that part of the shell which the observer looks through at the center of the nebula. The velocity gradient defined by the three crosses is shown, and this gradient indicates that an observer looking through the [O III] shell would see the emitting material distributed in radial velocity, since the inner edge of the shell has a lower expansion velocity than the outer edge. The magnitude of this difference indicates that the half-maximum width of either component of the emission-line profile from the apparent nebular center should be 8.0 ± 1.0 km sec⁻¹ if there were no thermal motions. After correcting the observed profile width for the instrumental and seeing broadening and removing the thermal Doppler width corresponding to 12500° K (O'Dell 1963), the observed width due to non-thermal motions is 10.0 ± 1.0 km sec⁻¹. Since the observed width is within the error limits of the predicted width due to the expansion velocity gradient, there is no evidence that turbulent mass motions exist within the shell on a scale comparable to the systematic differences in expansion velocity.

The fact that the velocity gradient through the shell (*crosses*) and the velocity-distance relation for the entire shell (*filled circles*) have the same slope in Figure 1 may simply be an interesting coincidence. It seems unlikely that the same mechanism responsible for ejecting different portions of the shell at different velocities was also responsible for establishing the velocity gradient within the shell. Theoretical models (Mathews 1966; Sofia and Hunter 1967) indicate that the latter occurred as the shell evolved and the material in it was redistributed by thermal and radiative forces.

As can be seen from Figure 1, the difference in radius between the [O III] emission maximum and the H emission maximum is small compared to the shell thickness. Thus the statement that the distribution of [O III] represents the ionized material in the nebula remains a good approximation. Since the [O III] emission comes from a shell whose thickness appears the same as the shell of H emission, the line broadening due to the expansion velocity gradient should be identical for both. Then the only difference in the line profiles will arise from the different thermal motions. By a comparison of the observed profile widths, the non-thermal motions are canceled and the thermal Doppler width derived for the ions. This width corresponds to a kinetic temperature for the emitting ions of $14000^\circ \pm 3000^\circ$ K (mean deviation), which is consistent with the value of 12500° K for the electron temperature derived from line intensity ratios (O'Dell 1963).

NGC 7009.—From multislit observations Wilson (1958) concluded that the inner shell of NGC 7009 is a spheroid of axial ratio 1.5 with its major axis perpendicular to the line of sight. This conclusion is supported by Aller's (1956) isophotes, which give an apparent axial ratio of 1.5, and the emission lines considered in this paper, which show no detectable tilt. The faint outer shell contributed nothing to the emission lines and is therefore neglected in this analysis. The [O III] lines and the H-lines are not nearly as symmetrical and regular as those from NGC 7662. The velocity separation between the split components does not vary as regularly along the line, and the red and blue components do not smoothly merge at the ends of the lines. Because of these irregularities it was not possible to make lengthwise tracings of the lines, so the thicknesses of the ends of the

lines where the components merge had to be measured visually on a measuring engine. The estimated half-maximum thickness of the emitting shell at the ends of the major axis is $2''.0$, which compares to the major axis length of $27''.6$. The shape of the distribution of material through the shell could not be determined accurately due to the inhomogeneities of the observed cross-section. These inhomogeneities, however, show a distinct symmetry. The model in Figure 2 is divided into zones, and in each zone is a square whose area represents the $[\text{O III}]$ emission density in that zone as calculated from the observed emission measures. Since the inhomogeneities are similar in $[\text{O III}]$ and H , the differences in emission probably represent true differences in mass density. The symmetry of the density variations about a line through the nebular center is apparent from Figure 2. When we assume that the emission variations are caused only by mass

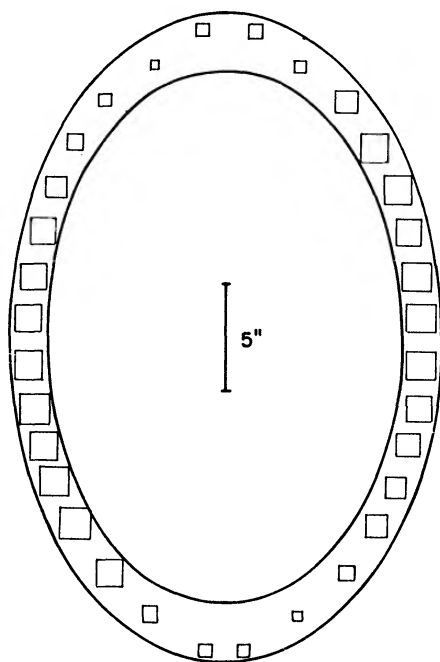


FIG. 2.—Distribution of emission in NGC 7009. Ellipses outline shell of nebula, and areas of squares represent emission densities in shell.

density changes in the shell, the densest portion has three times the mass density of the least-dense portion. The calculations of emission density come from the emission measures using path lengths taken from the model outlined in Figure 2. If the model is wrong, some of the variation in emission measures could be due to path-length changes rather than density changes, but the symmetry of these changes would still remain. It thus appears that despite the inhomogeneities the portion of the shell observed has nearly zero net momentum in a reference frame centered on the central star. NGC 7009 shows the diametric symmetry of the inhomogeneities in a particularly striking way, but similar symmetric variations of smaller amplitude were also noted in NGC 7662, 3242, and 6572.

According to Wilson (1958), multislit spectra of NGC 7009 indicate a definite increase of expansion velocity with radius for this shell. This conclusion is quantitatively verified here. The calculated expansion velocities of various parts of the shell are shown in Figure 3 as a function of shell radius. Here, as in NGC 7662, those parts of the shell most distant from the center have the greatest expansion velocity. There is, however, a substantially steeper gradient in this case, and the shell's radius does not vary in direct proportion to

its expansion velocity. Also there is too much scatter for the points to represent a smooth velocity gradient.

The kinetic temperature derived by comparing widths of the [O III] and H profiles is $8500^\circ \pm 1000^\circ$ K, which is substantially lower than the electron temperature of 13900° K derived from line-intensity ratios (Liller and Aller 1954). The determination here is of high weight since it is based on a comparison of line widths at nineteen different positions on the line images. Here, as in NGC 7662, the velocity and intensity irregularities among the profiles are similar in [O III] and H, so these two ions should be emitting from essentially the same region. The high temperature measured from the line-intensity ratios may therefore represent certain hot, bright spots which are not typical of the whole nebula.

NGC 3242.—This planetary is quite similar in appearance both in direct photographs and in spectrograms to NGC 7662. The very slight tilt observed in the emission lines indicates that this planetary is viewed almost perpendicular to its major axis, so the axial ratio for the shell was taken from Aller's (1956) isophotes of slitless images. For the inner shell this ratio is 1.5 and for the outer shell, 1.1. The inner and outer shells appear kinematically separate since the slight emission-line tilts for the two shells are in opposite directions. The inner-shell thickness and the diameters of the two shells were estimated visually with a measuring engine. The line profiles were used to derive the distribution of material through the outer shell by noting the variation in emission measures along different lines of sight. The maximum emission density in the outer shell was in this way found to be approximately one-half the inner-shell maximum, but the outer shell must contain about 8 times the mass of the inner shell if the emission-density changes are due only to mass-density changes. With this assumption the emission measures from various parts of the shell were also used to estimate the variations in mass density within the shell. These mass densities fluctuate about the mean density with an amplitude of ± 15 per cent.

The derived velocity structure of the inner shell is shown in Figure 3, where the observed values represent the expansion velocities of different parts of the inner shell. The gradient is similar to that in NGC 7662.

NGC 6818.—Because of its low surface brightness, only one very faint N I line was measurable on the spectrograms of this nebula. The faint line is very similar to the inner-shell emission line in NGC 7662, although it is slightly more distorted by irregular motions like those in NGC 7009. The line is not tilted, so the observer must be looking perpendicular to a nebular axis. Drawings by Curtis (1918) provide the only measure of the apparent axial ratio for the inner shell, which appears very close to the 1.5 ratio observed for NGC 7662, 7009, and 3242. Using this model, along with the line splitting along the measured image, the expansion velocity as a function of shell radius was calculated for several points around the shell and is shown in Figure 3.

NGC 6210.—The observations of NGC 6210 were made with the slit east-west across the nebula since this object has no distinct major axis. The line images are tilted, however, indicating that the shell is indeed spheroidal. It was assumed that the shell has an axial ratio of 1.5 and that the slit was approximately aligned with the major axis.

The velocity splitting between the near and far sides of the shell varies smoothly along the line image, implying an orderly kinematical structure. Because of the small apparent size of the nebula, there is substantial error in the determination of the expansion velocity as a function of shell radius. Thus the distribution shown in Figure 3 has more scatter than those for the nebulae considered previously. The gradient is taken as shown, so that it includes the velocity-distance point at the apparent center of the nebula (*cross*), since this point may be located most securely.

Assuming that this gradient also defines the expansion velocity gradient along any radius, as in NGC 7662, we may derive the distribution of emitting material as a function of radius from the line profiles if there are no turbulent mass motions within the shell.

After correcting for broadening due to seeing, resolution, and a temperature of 11000°K (Liller and Aller 1954), the observed profile from the center of the nebula has red and blue components with a mean half-width of 17 km sec^{-1} . This is taken as the velocity broadening introduced by the gradient through the shell, and this value applied to the slope of the gradient gives the thickness of the shell listed in Table 2.

NGC 6826.—This planetary appears almost spherical, but the emission lines are tilted substantially. Since the shell must therefore be spheroidal, it was also taken to have an

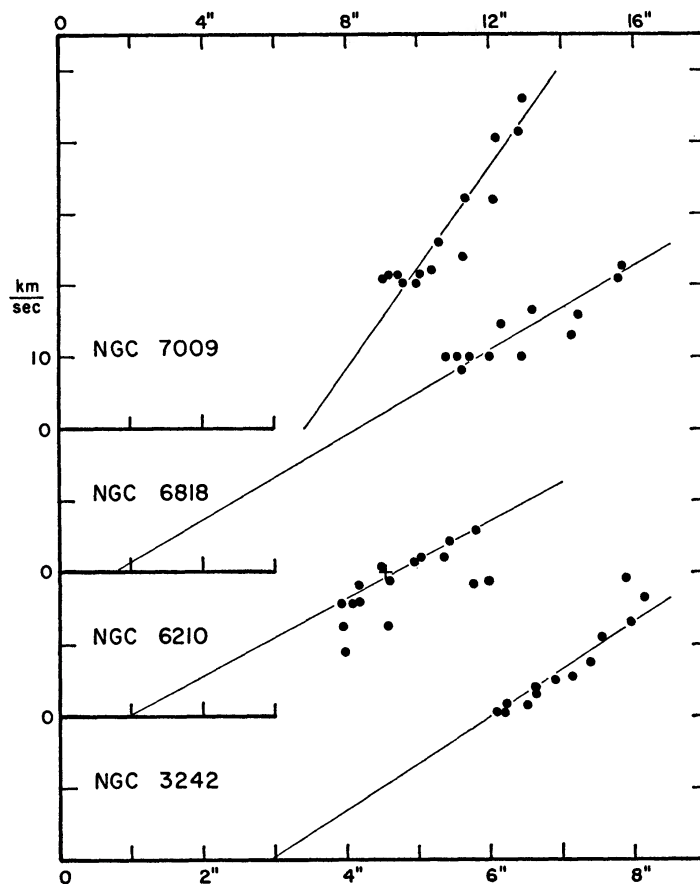


FIG. 3.—Expansion velocity gradients in four planetaries. Solid lines show expansion velocities as a function of radius for nebular shells. Bottom distance scale refers to all objects except NGC 7009, for which top scale is used. Filled circles are observed points and are described in text. Grid lines on vertical axis are separated by 10 km sec^{-1} for all objects.

axial ratio of 1.5. It would then have to be observed at an angle of about 40° to produce the tilt and at this angle would have an apparent axial ratio of 1.3, comparing favorably to the 1.2 ratio in the drawings by Curtis. The velocity splitting of the lines is small in $[\text{O III}]$ and undetectable in the H-lines. It is therefore impossible to measure directly the shell thickness. Fortunately, the line profiles indicate that the observed portion of the shell has no density irregularities, so the variation of emission measure over the face of the nebula can be used to derive a shell model. With the shape and orientation defined above, a constant-density shell whose thickness is 0.63 of the radius to its midpoint produces the apparent variation in emission measure that best fits the observations. This thick-shell model is consistent with the fact that the line image shows splitting over very little of its length. The emission measure in the outer parts of the nebula decreases

smoothly but more slowly than the model predicts. Also the heavily exposed line images show a faint but broad border outlining the distinct inner portion. It seems, therefore, that there is definitely an outer shell in NGC 6826, although it cannot be shown to be kinematically separate from the inner shell. It may have been ejected earlier or just leaked out of the inner shell. If this outer shell is of constant density and is symmetric about the inner shell, its mass density is 0.4 that of the inner shell but its total mass is 3.5 times as great. These estimates are also based on the assumption that variations in emission density are due only to variations of mass density.

The only information on the velocity gradient through the shell comes from assuming that the line profile at the apparent center of the nebula is broadened only by the expansion velocity gradient after corrections for resolution and a temperature of 11500°K (O'Dell 1963). This seems a justified assumption since the lines show little structure with no evidence of irregular motions. Also the full-velocity width of the inner shell line over its full length may be explained by the values calculated from expansion of the adopted shell model. The velocity gradient through the shell is therefore derived by matching the inner edge, midpoint, and outer edge of the shell model to the inner half-maximum intensity, maximum intensity, and outer half-maximum intensity velocity points on the line profile from the apparent center of the nebula.

NGC 2392.—This object has the highest-known apparent expansion velocity of any planetary nebula (Wilson 1950). The emission lines show very intricate, ill-defined structure which prevents a detailed analysis of the kinematics. However, the lines obtained with the slit both east-west and north-south across the roughly circular nebula show that the object consists of two kinematically and physically independent shells. The inner shell has the high expansion velocity (about 50 km sec^{-1}) and a chaotic structure, but the outer shell produces a reasonably well-defined line resembling that which would be produced by a spherical shell expanding at about 25 km sec^{-1} . There is no significant line tilt from the outer shell, and its diameter is the same in both north-south and east-west directions. Yet the velocity structure expected from the outer shell is not present in the central portion of the nebula where the emission lines show evidence only of a shell expanding at about 50 km sec^{-1} . This implies that the inner, rapidly expanding shell reaches the outer shell in the apparent center of the nebula and forces the outer shell to move with the inner shell's velocity. The nebula thus appears to have a spheroidal inner shell immersed within a spherical outer shell with the inner shell's major axis being at least as long as the outer shell's diameter. The implication that the observer is looking along the major axis of a spheroidal inner shell is also supported by the inner shell's velocity structure. The material at its apparent edges is spread over a very large range of velocities, up to 100 km sec^{-1} . There must then be a great velocity difference between the near and far sides, even when the observer looks close to the edge of the shell, but this circumstance can arise in a radially expanding spheroid only if the shell's major axis is much longer than its minor axis. Finally, since the lines are not tilted, the observer must be looking along an axis. This model of the nebula is consistent with Aller's (1956) isophotometer tracings, which show circular inner and outer rings with the inner ring being very bright and fairly homogeneous, as would be the case when looking down the major axis of a spheroidal shell. If this model is correct, the high apparent expansion velocity of NGC 2392 does not really make it unusual, since several of the spheroidal nebulae would show such high velocities if they were viewed along their major axes (see Table 2). Because of the irregularities in the lines, no significant information could be obtained concerning any expansion velocity gradient that may exist in this planetary.

NGC 6572.—This small, bright nebula has [N II] lines that appear to arise in a shell. However, the [O III], H, and [Ne III] lines all show a structure indicating that they do not arise from a distinct shell but rather from a spheroid filled in at the center with emitting material. The orientation of the nebula was found from the tilt of the [N II] lines, whose emitting shell was assumed to have an axial ratio of 1.5. Since the center of the nebula is

so filled with emission, the only way the distribution of the [O III] emission could be derived was by determining an expansion velocity gradient and using the distribution in velocity of the line profiles to deduce the spatial distribution of emission. The expansion velocity gradient was determined by measuring the expansion velocities and angular sizes of shells of [Ne III], [O III], and [N II]. This gradient and the derived distribution of emission along a line of sight to the center of the nebula are shown in Figure 4.

IC 3568.—This is the most symmetrical and uniform planetary visible in the Northern Hemisphere. Unfortunately, it is located too far north for observation with the 100-inch reflector, so Wilson (1950, 1958) was not able to include it in his studies of planetaries. Using direct photographs taken with the 200-inch reflector, Wilson and Aller (Aller 1956) attempted to deduce the distribution with radius of the emission, but they could not derive this distribution near the center of the nebula due to the light of the central

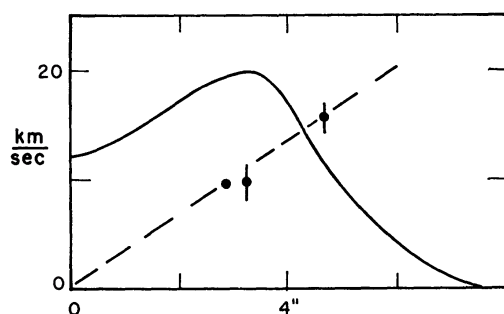


FIG. 4

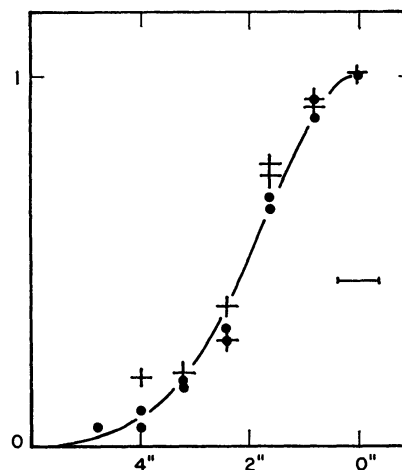


FIG. 5

FIG. 4.—Spatiokinematic structure of NGC 6572. Expansion velocity gradient is shown as dashed line through observed points. Solid curve is normalized distribution of [O III] emission along radius pointing to observer.

FIG. 5.—Apparent distribution of emission in IC 3568. Filled circles are observed normalized emission measures for [O III] shown as a function of the apparent nebular radius. Crosses represent observed H emission measures. Horizontal error bar is the estimated positional error for observed points, and solid curve is the expected distribution using model described in text.

star. Because of its obvious interest and the lack of previous coudé observations, IC 3568 was observed with the 84-inch reflector at Kitt Peak (see § I). This instrument was not equipped with an image rotator, although this presumably makes no difference since the apparent spherical symmetry implies that the object should look the same spatially and kinematically at any slit orientation. In fact, spectrograms exposed at different times of the night and thus at different slit-position angles are not detectably different. Since the slit was placed along a nebular diameter, the variation of emission measure along this diameter could be determined from the profiles found for different parts of the emission lines. The emission measure is shown in Figure 5 as a function of the apparent radius. Within the errors of microphotometer slit length and placement, there is no difference between the distribution of [O III] and H. The solid curve in Figure 5 would be the observed variation with seeing corrections included if the emission densities of [O III] and H were maximum at the center of the nebula and decreased with radius as $\exp[-(r/1''.85)^2]$. Even though the emission distribution within the nebula may not actually be Gaussian, it must be peaked at the center if a spherically symmetric model is to fit the observations. The planetary therefore cannot be regarded as a shell.

Only the [O III] lines and the H-lines are visible on the spectrograms, and in no case is there any indication of line splitting. The limited resolution would mask any splitting of 16 km sec^{-1} or less, so we can only state that there is no splitting greater than this. The half-width of the central profile after correction for instrumental resolution is $33.5 \pm 3.8 \text{ km sec}^{-1}$. It is not possible uniquely to derive the kinematics of the nebula, but an expansion velocity ranging from zero at the center to 17 km sec^{-1} at the radius of half-maximum emission density and continuing to increase linearly would explain the observed profiles. It thus appears that IC 3568 is expanding but as yet has no visible shell structure.

IC 418.—This planetary is a nearly circular, quite symmetrical object which appears to have a simple structure. Because of its smooth appearance, Wilson and Aller (1951) extensively studied its monochromatic images on slitless spectra to determine the distribution with radius of the emitting material. Assuming spherical symmetry, they showed in results summarized by Aller (1956) that the distribution of H emission as a shell within the nebula is not the same as the distribution for [O III]. The light of the central star interfered with their analysis of the [O III] emission, which seemed to peak in the nebular center. The present study uses spectrograms taken with the slit both north-south and east-west across the nebula. The observed variation of emission measure with apparent radius for the [O III] emission can be well explained if the [O III] emission density within the nebula varies as $\exp[-(r/4''.6)^2]$ along a nebular radius r . Our observed emission measures for hydrogen can be fit by the shell model of Wilson and Aller. Present observations of [O II] and [N II] emission lines show that there is no detectable difference between the location of maximum emission for these ions and for hydrogen. This result, also consistent with that of Wilson and Aller, is fortunate since the velocity of expansion of the [O II] and [N II] emission shells can be found from the line profiles. This velocity (9.5 km sec^{-1}) can then be considered as the expansion of the H emission shell for which the lines are so spread by thermal broadening that the splitting is masked. There is no splitting observed in the [O III] lines, which is consistent with the derived distribution of the [O III] emission density if the expansion velocity gradient goes to zero at the nebular center. IC 418 thus has a distinct anomaly not seen in any other object considered in this paper in that the H and [O III] emission regions do not coincide. This separation may mean that IC 418 is optically thick (see § II).

IV. SUMMARY

The five planetaries NGC 7662, 6818, 7009, 6210, and 3242 all appear very similar in both shape and kinematical properties, particularly if only the inner shells are considered. In all these objects the velocity of expansion of the maximum-density locus in the shell increases with increasing distance from the nebular center, so this seems to be the general explanation for the shape of ellipsoidal nebulae. If the expansion velocities were directly proportional to the distance from the center, they would reflect the different impulses given to different portions of the shell at the instant of ejection. The current shape of the shell would then represent the initial directional distribution of momentum given the shell. To a close approximation, this is the case as seen from the derived velocity gradients. For all five objects, however, the gradient extrapolates to zero velocity at a point in the nebula between the shell and the center. This implies that the velocity of the faster-moving parts of the shell relative to the slower-moving parts has increased in the time since the ejection. It therefore seems that whatever mechanism imparted momentum to the shell did not do so instantaneously but acted over a time significant compared to the age of the nebula and modified the relative expansion velocities of different parts of the shell.

We find an apparent relation between the thickness of a shell and its expansion velocity for the objects considered in this paper. The thickness is defined as the width between positions of half-maximum emission density for the [O III] shell. The points in Figure

6 show the maximum expansion velocity of the shell as a function of the ratio of shell thickness to shell radius for the inner shell of each planetary except IC 418 and NGC 6818. IC 418 is not plotted because the [O III] and H shells are not the same, and NGC 6818 is omitted because no measure could be made of its shell thickness. The point with zero expansion velocity is IC 3568, which has no shell structure. The value of 2 for the ratio is found by approximating the nebula as a homogeneous sphere, i.e., a shell with a vanishingly small central hole. As can be seen from Figure 6, the shell thickness in units of the radius decreases as the expansion velocity increases.

If all of the planetaries in Figure 6 are similar objects seen at different stages of their evolution, the trend of the points indicates that the nebulae begin as a sphere of material with a low velocity of expansion. The material is then accelerated outward, and the shell's expansion velocity and radius increase while its thickness changes very little. As the nebula evolves, it moves to the upper left in Figure 6. In order for the shell thickness to remain nearly constant, there must be a continuous pressure acting on the inside of the shell to balance the gas and radiation pressure within the shell tending to

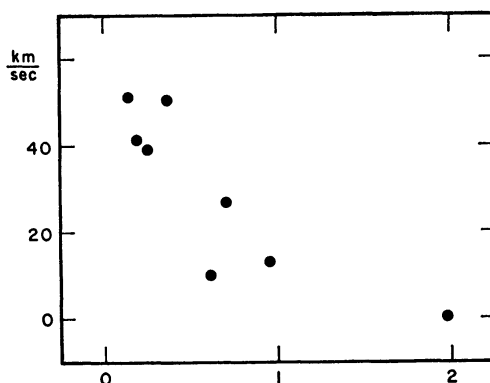


FIG. 6.—Observed relation between shell thickness and expansion velocity. Filled circles show maximum expansion velocity as function of ratio (shell thickness)/(shell radius) for eight planetaries. Points are derived from observed spatiokinematic models.

thicken it. This continuous pressure would also provide the acceleration of the shell and, if it varied directionally, would produce the ellipsoidal nebulae with the velocity-radius gradients described earlier.

Previous calculations of the radii of planetaries support the evolutionary sequence here outlined. For the four objects in Figure 6 with the greatest expansion velocities Seaton (1966) derives a mean radius of 0.14 pc, but the mean radius of the four objects with lower expansion velocities is 0.06 pc. The latter mean is obtained by considering IC 418 as a low-expansion planetary in place of IC 3568, for which Seaton lists no radius.

The absolute sizes of planetaries must be considered with caution, however, since these sizes are valid only on a statistical basis. When calculating sizes and masses (O'Dell 1962, Seaton 1966) the assumption has always been made that all planetaries are similar in mass and evolutionary behavior. These assumptions can best be checked astrophysically by using the methods outlined by Seaton (1966), which require accurate knowledge of the electron densities in planetaries. But, until improved observational knowledge is available, the true masses, sizes, and evolutionary characteristics of planetaries will remain uncertain. It does appear from the sample of objects considered in this paper that the numerous planetaries appearing as reasonably homogeneous ellipsoids are very similar in their spatiokinematic structure, so they were probably formed by the same sort of disruptive event. This still by no means implies that all planetary nebulae

are identical in origin since the planetaries picked for observation were the simplest ones observable. It should be noted that in no case was a true shell nebula found to be spherical. Thus this approximation, while desirable for theoretical calculations, is difficult to justify observationally. The kinematical structure of the observed nebulae implies that the ellipsoidal form of planetaries is directly related to their origin, so that a valid theory of shell ejection must include a variation with direction of the momentum transferred to the shell.

Most of the work summarized in this paper was done as part of a Ph.D. dissertation submitted to the University of Wisconsin. I am grateful to Professor Donald E. Osterbrock, who not only provided consultation and suggestions but also loaned me spectrograms without which this work could not have been done. Thanks are due the personnel of the Mount Wilson and Palomar Observatories for their hospitality during the observing runs and to the California Institute of Technology for use of their microphotometer. I also wish to thank the Kitt Peak National Observatory for observing time on the 84-inch reflector and the Space Astronomy Laboratory of the University of Wisconsin for computer time. Financial support for study during the period of this research was provided by the NASA Traineeship program.

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