# SECULARLY SMOOTHED DATA ON THE MINIMA AND MAXIMA OF SUNSPOT FREQUENCY

*(Research Note)*

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When I introduced the method of secular smoothing into the study of the variations of sunspot frequency (GLEISSBERG, 1944) I published a table containing the secularly smoothed epochs and ordinates of sunspot minima and maxima which I had deduced from the data published by BRUNNER in 1939. Since then, secular smoothing has proved to be one of the principal methods for investigating the properties of the 80-year cycle of solar activity (cf. RUBASHEV, 1964). The table mentioned above thus formed a starting-point for investigations on long-cycle variations of solar activity. Now, the table ought to be enlarged for two reasons: on the one side, two 11-year cycles more have elapsed in the meantime and, on the other side, the ordinates of five minima and four maxima between 1698 and 1745 were deduced (GLEISSBERG, 1960) from the annual means of sunspot-relative numbers 1700–1748 as published by WOLF (1868) and corrected by CHERNOSKY and HAGAN (1958). Thus it seems suitable to provide workers on solar activity with a new table of secularly smoothed data based upon all values of the epochs and ordinates of sunspot minima and maxima which are now available.

The epochs of the minima and maxima of sunspot frequency and their ordinates, i.e., the smallest and highest values of the smoothed monthly sunspot-relative numbers were taken from a table published by WALDMEIER (1961). The data given here in Tables I and II were added to the data taken from Waldmeier's table. Then secular smoothing was applied to all these data, i.e., to the epochs of minima and maxima during the period 1610–1964 and to their ordinates during the period 1698–1964.

TABLE I

Minima and Maxima of Sunspot Frequency, 1698–1750
(Ordinates from GLEISSBERG, 1960)

| Cycle no. | Minima | | Maxima | |
|---|---|---|---|---|
| | Epoch | Ordinate | Epoch | Ordinate |
| − 4 | 1698.0 | 0 | 1705.5 | 54 |
| − 3 | 1712.0 | 0 | 1718.2 | 60 |
| − 2 | 1723.5 | 11 | 1727.5 | 113 |
| − 1 | 1734.0 | 5 | 1738.7 | 112 |
| 0 | 1745.0 | 5 | 1750.3 | 92.6 |

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TABLE II

The Most Recent Sunspot Minimum (WALDMEIER, 1966)

| Cycle No. | Minimum | |
|---|---|---|
| | Epoch | Ordinate |
| 20 | 1964.7 | 9.6 |

It may be remembered that secular smoothing of a series of given quantities consists of forming moving averages of every four consecutive quantities and then taking the averages of every two consecutive averages; the same result is obtained by taking moving averages of every five consecutive quantities weighted so that half weight is given to the first and the last ones. Each secularly smoothed value belongs to the central of the five consecutive cycles whose quantities have been used for forming the average.

The secularly smoothed values of the epochs and ordinates of minima and maxima are given in Table III. Furthermore, the intervals between the secularly smoothed

TABLE III

Secularly Smoothed Minima and Maxima of Sunspot Frequency

| Cycle no. | Minima | | | Maxima | | |
|---|---|---|---|---|---|---|
| | Epoch | Ordinate | Interval (years) | Epoch | Ordinate | Interval (years) |
| − 10 | 1632.7 | | | 1638.1 | | |
| − 9 | 1644.1 | | 11.4 | 1649.8 | | 11.7 |
| − 8 | 1655.7 | | **11.6** | 1661.6 | | **11.8** |
| − 7 | 1666.9 | | 11.2 | 1672.8 | | 11.2 |
| − 6 | 1677.9 | | *11.0* | 1683.9 | | 11.1 |
| − 5 | 1689.0 | | 11.1 | 1695.0 | | 11.1 |
| − 4 | 1700.1 | | 11.1 | 1705.7 | | *10.7* |
| − 3 | 1711.3 | | 11.2 | 1716.8 | | 11.1 |
| − 2 | 1722.8 | *4.6* | **11.5** | 1728.1 | *89.6* | **11.3** |
| − 1 | 1734.0 | 6.3 | 11.2 | 1739.1 | 97.7 | 11.0 |
| 0 | 1744.8 | 7.4 | 10.8 | 1749.8 | 101.4 | 10.7 |
| 1 | 1755.4 | 7.7 | 10.6 | 1760.0 | 107.5 | 10.2 |
| 2 | 1765.5 | **8.5** | *10.1* | 1769.7 | 119.4 | *9.7* |
| 3 | 1775.9 | 8.4 | 10.4 | 1779.9 | **120.8** | 10.2 |
| 4 | 1786.8 | 6.4 | 10.9 | 1791.2 | 107.8 | 11.3 |
| 5 | 1798.2 | 4.1 | 11.4 | 1803.5 | 88.6 | 12.3 |
| 6 | 1810.4 | *2.9* | **12.2** | 1816.0 | *78.4* | **12.5** |
| 7 | 1822.2 | 3.6 | 11.8 | 1827.5 | 89.4 | 11.5 |
| 8 | 1833.5 | 4.9 | 11.3 | 1838.4 | 105.9 | 10.9 |
| 9 | 1844.7 | **5.9** | 11.2 | 1848.9 | **120.6** | *10.5* |
| 10 | 1855.8 | **5.9** | *11.1* | 1859.8 | 120.2 | 10.9 |
| 11 | 1867.2 | 4.6 | 11.4 | 1871.4 | 105.7 | 11.6 |
| 12 | 1878.6 | 3.8 | 11.4 | 1883.0 | 96.0 | 11.6 |
| 13 | 1890.1 | 3.3 | **11.5** | 1894.8 | 87.4 | **11.8** |
| 14 | 1901.5 | *3.2* | 11.4 | 1906.2 | *83.5* | 11.4 |
| 15 | 1912.7 | 3.5 | 11.2 | 1917.2 | 87.8 | 11.0 |
| 16 | 1923.5 | 3.9 | 10.8 | 1927.7 | 102.7 | 10.5 |
| 17 | 1933.9 | 4.8 | 10.4 | 1937.8 | 125.6 | 10.1 |
| 18 | 1944.1 | 5.5 | 10.2 | | | |

epochs of consecutive minima or maxima, respectively, are given. In order to call attention to the 80-year variations of the 11-year cycles, maximum values of the secularly smoothed ordinates and intervals in Table III are printed in bold type and minimum values in italics. It may appear somewhat doubtful whether the initial quantities in the third and sixth columns of Table III, i.e., the secularly smoothed ordinates of the minimum and maximum of Cycle – 2, which are printed in italics, really are minimum values in their columns or not. In fact, merely from the series of the secularly smoothed ordinates themselves it cannot be decided whether they attained minima in Cycle – 2 or not. But it can be seen from Table III that minimum and maximum values of the ordinates are closely connected with maximum and minimum values of the intervals. As, in Cycles 6 and 13, the maximum values of the intervals either coincide with minimum values of the ordinates or precede them by one cycle, we may conclude that the maximum values of the intervals in Cycle – 2 did not occur later than the corresponding minimum values of the ordinates, and this reasoning justifies the statement that the secularly smoothed ordinates of the sunspot minima and maxima attained minimum values in Cycle – 2. On the other hand, it is not yet possible to decide whether the final quantities in the third and sixth columns of Table III will be maximum values or not. In any case, it is obvious that, whereas the two earlier peaks in the series of the secularly smoothed ordinates of the sunspot maxima, which occurred in Cycles 3 and 9, have been of nearly equal height, the next peak will become considerably higher; for the most recent value of this ordinate, 125.6, already surpasses the two earlier maximum values, which amounted to 120.8 and 120.6, respectively. Thus the course of these ordinates in the near future deserves special attention.

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