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A MECHANISM FOR MASER ACTION OF OH MOLECULES IN INTERSTELLAR SPACE

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ABSTRACT

It is suggested that collisions between OH molecules and charged particles will discriminate between the different energy levels involved in the 18-cm wavelength transitions observed by radio astronomers, and hence could be responsible for a masering action on a vast scale. The basic atomic physics of the collision is analyzed, showing how this discrimination arises and what its consequences are in terms of observed microwave radiation. This simple collision process is then discussed in a kinetic theory context and results are derived which, though still highly idealized, have some point of contact with the real world. Finally, the general nature of the effect having been established, an attempt is made to show how it could be observed in interstellar space, and how it could be fitted to the OH observations in particular.

I. INTRODUCTION

It seems to be widely accepted that the anomalous emission features observed in interstellar space at the OH microwave frequencies (Weaver, Williams, Dieter, and Lum 1965; Weaver, Williams, and Dieter 1965; Weinreb, Meeks, Carter, Barrett, and Rogers 1965; Zuckerman, Lilley, and Penfield 1965; McGee, Robinson, Gardner, and Bolton 1965; Davies, de Jager, and Verschuur 1966; Barrett and Rogers 1966) must be the result of some kind of maser action. What is envisaged is that the population distribution of OH molecules among the four energy levels involved in the 18-cm wavelength transition is somehow inverted, so that ambient radiation is amplified by stimulated emission, in a manner exactly analogous to the laboratory maser. Such a hypothesis is attractive because it immediately explains some of the observed properties of these emission features: viz., (*a*) the high brightness temperatures; (*b*) the extreme narrowness of the lines; (*c*) the rapid fluctuations in intensity; and (*d*) the apparent smallness of the sources.

However, there are several other observations, equally remarkable but less easily explained, which must be accounted for. Most notably these are: (*a*) the intensity anomalies (that the line at 1665 Mc/s is almost invariably much stronger than the one at 1667 Mc/s); (*b*) the characteristic locations in interstellar space (generally) near the edges of H II regions; and (*c*) the fact that lines have been observed which are completely unpolarized, completely linearly polarized, or completely circularly polarized.

Because the OH molecule possesses an electric dipole moment, it responds to any applied electric fields by a very long-range interaction; and therefore it has long been believed (e.g., Barrett and Lilley 1957) that, at the extremely low densities in interstellar space, the population distribution among its various internal energy levels will be largely controlled by collisions with *charged* particles and by the prevailing density of electromagnetic radiation. Recently several authors (Cook 1966*a;* Perkins, Gold, and Salpeter 1966; Litvak, McWhorter, Meeks, and Zeiger 1966; Shklovskii 1966) have proposed that the interaction with electromagnetic radiation actually discriminates between the different energy levels involved in the 18-cm transitions; and have proceeded to show that the radiation fields likely to exist in some regions of interstellar space can, under certain conditions, produce an inversion in the OH population which is consistent, to a greater or lesser extent, with the properties outlined above.

It is argued here that collisions with charged particles also affect these levels asymmetrically, and this asymmetry again leads to a population inversion which can be consistent with all the observations above.

II. THE NATURE OF THE ASYMMETRY

That a collision between a charged particle and an OH molecule should discriminate between internal energy levels can be appreciated by the following semiclassical argument. Consider for simplicity a molecule possessing just two closely spaced energy levels, transitions between which are controlled by an electric dipole moment—e.g., the Λ doubled ground state of the OH molecule (with hyperfine splitting ignored). When a steady electric field is applied, an electric dipole will be induced in the molecule of such a sense that the energy of the upper level is increased and that of the lower level decreased. If the field is not uniform, a molecule in the upper level will experience a force toward regions of lower field intensity, while one in the lower level will experience a force toward regions of higher field intensity. Note that the *sign* of the field is not involved, only the sense of $\nabla|E|$. This of course is the basis of the ammonia maser.

Similarly, when a charged particle goes past such a molecule, it will induce an electric dipole of such a sign that a molecule in the upper level will be *repelled* by, and one in the lower level *attracted* to, the incident particle. Therefore the path followed by the charged particle will be different, depending on whether the molecule was initially in the upper or the lower state; and a closer examination will show that the scattering angle will be slightly greater in the latter case. Hence we expect the (elastic) scattering cross-section for collisions with a molecule in the lower state to be slightly greater than that with a molecule in the upper state. The difference is not very great; and can be seen, almost intuitively, to be of the order of:

(Energy separation of the two levels)/(Kinetic energy of the incident particle).

But the important point is that the *sign* of the charge on the incident particle is irrelevant, and one can expect the same qualitative result for scattering of either electrons or ions.

Unfortunately, so simple a picture as this, though it does demonstrate that some asymmetry might be expected to exist, offers little clue to what happens in an *inelastic* collision: and these are the ones which will dominate the charged-particle–OH-molecule interaction. This problem must be handled analytically.

The sixteen magnetic substates of the OH molecule which are involved in the 18-cm transition may be classified by three quantum numbers:

F, the total angular momentum,

m, the projection of F along some axis,

n, a quantum number which will be taken to be 0 for the lower level of the Λ doublet, and 1 for the upper level; n is actually a symmetry quantum number.

By the standard procedure of first Born approximation, as used in similar problems by Massey (1932) and Altshuler (1957), the cross-section for the reaction in which an *electron* of initial momentum $\hbar \boldsymbol{k}_0$ scatters off an OH molecule initially in an internal energy state $(n_0 F_0 m_0)$, coming off with momentum $\hbar \boldsymbol{k}$, and leaving the molecule in state (nFm), can be estimated to be

$$\sigma(n_0F_0m_0 \rightarrow nFm) = \frac{4\pi m_e^2 e^2}{\hbar^4 k_0^2} |\langle nFm|\boldsymbol{\mu}|n_0F_0m_0\rangle|^2 \left[\left(\frac{k^2+k_0^2}{2k_0^2}\log\frac{k+k_0}{|k-k_0|}-\frac{k}{k_0}\right)-\left(\frac{3k^2-k_0^2}{2k_0^2}\log\frac{k+k_0}{|k-k_0|}-3\frac{k}{k_0}\right)\cos^2\alpha\right], \tag{1}$$

where α is the angle between $\boldsymbol{k}_0$ and the (fixed) direction $\langle nFm|\boldsymbol{\mu}|n_0F_0m_0\rangle$.

This expression is accurate only so far as the first Born approximation is valid; but Altshuler has discussed this approximation in some detail, and for present purposes his most pertinent conclusion is that the energy dependence cannot change as more terms are included.

Now, the asymmetry mentioned above shows itself in this expression, not in the particle trajectories (which is a second-order effect), but more simply in the quantity k. By conservation of energy, the magnitude of k is given by

$$k^2 = k_0(1 \pm \xi) ,$$

where

$$\xi \equiv \frac{2m\Delta E}{\hbar^2 k_0^2} , \tag{2}$$

with the convention that the upper ambiguous sign refers to a downward transition, and the lower to an upward transition.

TABLE 1

TABULATION OF THE FUNCTION $f(n_0F_0m_0 \rightarrow nFm)$, DEFINED BY EQUATION (4), WITH THE VALUE LOG $(4/\xi) = 7$

m	$F_0=1$			$F_0=2$				
	$m_0=-1$	$m_0=0$	$m_0=+1$	$m_0=-2$	$m_0=-1$	$m_0=0$	$m_0=+1$	$m_0=+2$
$F=1$:								
−1 ..	$200 \mp 600\xi$	$600 \pm 350\xi$	....	$144 \pm 84\xi$	$24 \mp 72\xi$	$24 \pm 14\xi$	....	....
0...	$600 \pm 350\xi$	0	$600 \pm 350\xi$	....	$72 \pm 42\xi$	$32 \mp 96\xi$	$72 \pm 42\xi$	....
+1..	....	$600 \pm 350\xi$	$200 \mp 600\xi$	....	....	$24 \pm 14\xi$	$24 \mp 72\xi$	$144 \pm 84\xi$
$F=2$:								
−2..	$144 \pm 84\xi$	... . .	....	$288 \mp 864\xi$	$432 \pm 252\xi$	....	....	....
−1 .	$24 \mp 72\xi$	$72 \pm 42\xi$	... ..	$432 \pm 252\xi$	$72 \mp 216\xi$	$648 \pm 378\xi$	....	....
0..	$24 \pm 14\xi$	$32 \mp 96\xi$	$24 \pm 14\xi$	....	$648 \pm 378\xi$	0	$648 \pm 378\xi$	....
+1...	.. ..	$72 \pm 42\xi$	$24 \mp 72\xi$	....	....	$648 \pm 378\xi$	$72 \mp 216\xi$	$432 \pm 252\xi$
+2.	....	....	$144 \pm 84\xi$	....	....	....	$432 \pm 252\xi$	$288 \mp 864\xi$

Here ΔE is the (positive) difference in energy between the two levels of the Λ doublet, and is very nearly independent of F and m. Notice that ξ is the ratio of energy level separation to incident kinetic energy and is an extremely small quantity for most applications.

Choosing the axis of quantization along the z-axis and expanding the above expression to first order only in ξ, we obtain

$$\sigma_T(n_0F_0m_0 \rightarrow nFm) = \frac{4\pi m_e^2 e^2}{\hbar^4 k_0^2}\{[(1 \pm \tfrac{1}{2}\xi)\log(4/\xi) - 1][|\langle\mu_x\rangle|^2 + |\langle\mu_y\rangle|^2] + [\mp\xi\log(4/\xi) + (2 \pm \xi)]|\langle\mu_z\rangle|^2\} . \tag{3}$$

The individual matrix elements in these expressions can be reduced to a common factor, as described, for example, in Condon and Shortley (1964, p. 63); and the form of the individual cross-sections is most easily appreciated from Table 1, in which is tabulated the function $f(n_0F_0m_0 \rightarrow nFm)$, defined by

$$\sigma_T(n_0F_0m_0 \rightarrow nFm) \equiv \frac{\pi m_e^2 e^2 |\langle n|\mu|n_0\rangle|^2}{16\hbar^4 k_0^2} f(n_0F_0m_0 \rightarrow nFm) . \tag{4}$$

In this table too, the arithmetic simplification has been made of writing

$$\log (4/\xi) = 7\,, \tag{5}$$

which describes an incident electron velocity of ~50 km/sec; and values which will be invoked later on will not deviate greatly from this figure.

If one considers a volume filled with OH molecules which is constantly irradiated by an exactly parallel stream of monoenergetic charged particles, then, in the absence of any other mechanism to thermalize them, these molecules will acquire an *equilibrium distribution* among their various internal energy states determined by the equation: Number of molecules leaving state $(n_0F_0m_0)$ per unit time = Number of molecules entering state $(N_0F_0m_0)$ per unit time, or

$$N(n_0F_0m_0)\sum_{F,m} f(n_0F_0m_0\rightarrow nFm) = \sum_{F,m} N(nFm)\, f(nFm\rightarrow n_0F_0m_0)\,. \tag{6}$$

The solution of these equations, which can be verified by direct substitution, gives the relative distributions among the various states given in Table 2. In the terminology of

TABLE 2

EQUILIBRIUM DISTRIBUTION FROM A PARALLEL, MONOCHROMATIC INCIDENT BEAM

	$F=1$			$F=2$				
	$m=-1$	$m=0$	$m=+1$	$m=-2$	$m=-1$	$m=0$	$m=+1$	$m=+2$
Upper level	$1+1\ 33\xi$	$1-2\ 09\xi$	$1+1\ 33\xi$	$1+0\ 98\xi$	$1-0\ 60\xi$	$1-0\ 44\xi$	$1-0\ 06\xi$	$1+0\ 98\xi$
Lower level	$1-1\ 33\xi$	$1+2\ 09\xi$	$1-1\ 33\xi$	$1-0\ 98\xi$	$1+0\ 60\xi$	$1+0\ 44\xi$	$1+0\ 60\xi$	$1-0\ 98\xi$

Varshalovich (1965) the OH molecules have been aligned by exposure to the electron beam. The effect of this alignment can best be demonstrated by calculating the response of this population to incident microwave radiation. Since the absorption and emission of radiation is controlled by exactly the same matrix elements that have already appeared, one can say that, apart from irrelevant constants, the *net* probability that a photon of the frequency corresponding to the $F_0 \rightarrow F_1$ transition will be *absorbed* is proportional to

$$\Big\{\sum_{m} [\,N(0\,F_0m) - N(1\,F_1m)\,].\Big\} \qquad \sum_{m'} |\langle 1\,F_1m'\,|\,\mu_A\,|\,0\,F_0m\rangle|^2\,,$$

where μ_A is the component of $\boldsymbol{\mu}$ in the direction of polarization of the photon. This quantity, normalized to a single OH molecule, will be referred to hereafter as the net relative probability of absorption, and denoted by the symbol $p(\nu_{F_1F_0})$. Its value, for the present equilibrium, is tabulated in Table 3.

The quantity $p(\nu_{F_1F_0})$ appears in the equation for attenuation or growth of microwave radiation as it passes through a volume filled with OH molecules aligned thus, viz.,

$$I(\nu,x) = I(\nu,0)\,\exp\Big[-\frac{8\pi^3\nu}{\hbar c\Delta\nu}\,p(\nu)\int N\,dx\Big]\,, \tag{7}$$

and therefore, if the volume is optically thin, the depth of resulting absorption lines will be directly proportional to these numbers $p(\nu)$.

Notice particularly that for polarizations *parallel* to the axis of quantization, $p(1665)$ and $p(1667)$ are both negative, meaning that there will be net emission rather than ab-

TABLE 3

NET RELATIVE PROBABILITIES OF ABSORPTION OF 18-CM RADIATION FOR TWO DIFFERENT POLARIZATIONS EVALUATED FOR POPULATION DISTRIBUTIONS IN TABLE 2

Transition $F_0 \rightarrow F_1$	m	$N(0\,F_0\,m)$ $-N(1\,F\,1\,m)$ $\times\xi$	Polarization Parallel to z-Axis: $\sum_{m'} \lvert\langle 1\,F_1\,m' \lvert\mu_z\rvert 0\,F_0\,m\rangle\rvert^2 \times \frac{1}{16}\lvert\mu_{01}\rvert^2$	Polarization Parallel to z-Axis: Relative Probability of Absorption: $p(\nu) \times \frac{1}{256}\lvert\mu_{01}\rvert^2\xi$	Polarization Perpendicular to z-Axis: $\sum_{m'} \lvert\langle 1\,F_1\,m' \lvert\mu_x\rvert 0\,F_0\,m\rangle\rvert^2 \times \frac{1}{16}\lvert\mu_{01}\rvert^2$	Polarization Perpendicular to z-Axis: Relative Probability of Absorption: $p(\nu) \times \frac{1}{256}\lvert\mu_{01}\rvert^2\xi$
1→2 1612 Mc/s	−1	−0.73	3	+ 5.74	7	+ 2.48
	0	+2.53	4		6	
	+1	−0.73	3		7	
1→1 1665 Mc/s	−1	−2.66	25	−133.00	25	+35.75
	0	+4.09	0		50	
	+1	−2.66	25		25	
2→2 1667 Mc/s	−2	−1.96	36	−119.52	18	+42.48
	−1	+1.20	9		45	
	0	+0.88	0		54	
	+1	+1.20	9		45	
	+2	−1.96	36		18	
2→1 1720 Mc/s	−1	−0.73	3	+ 5.74	7	+ 2.48
	0	+2.53	4		6	
	+1	−0.73	3		7	

sorption. Therefore, for radiation perpendicular to the incident stream of electrons, a masering action can be expected for photons of one polarization. Furthermore, *the 1665 Mc/s line will be masered more strongly than the one at 1667 Mc/s.*

For radiation parallel to the axis of quantization only absorption can be expected; and what happens at any intermediate direction can be calculated by compounding these two directions. The result is shown in Figure 1.

At this stage too, it is possible to say something about the relative effectiveness of ions and electrons in controlling the population inversion. For incident particles of the same energy, ions rather than electrons will completely dominate the interaction because the collision cross-sections are bigger by a factor (mass of ion/mass of electron)—at least in first order Born approximation. However, for incident particles of the same *velocity*, the ion cross-section is still greater, though only by a factor ~2—actually by the ratio of

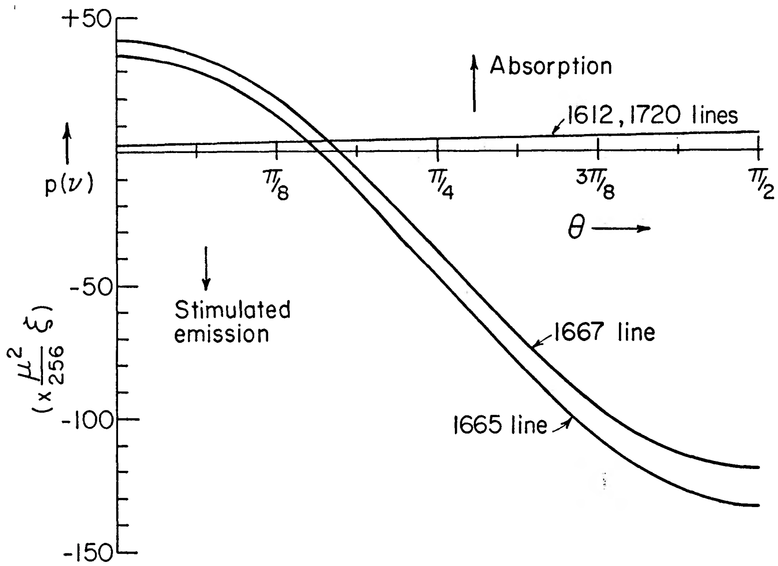

FIG. 1.—Plot of the relative probability of absorption against θ, the angle between the direction of streaming and the direction of observation.

log $(4/\xi_{\text{ion}})/\log(4/\xi_{\text{electron}})$. But the ions in this case will contribute nothing to the distribution *asymmetry*, since the asymmetry parameter ξ is proportional to m^{-1}. Hence it is possible to take account of scattering by both ions and electrons in equation (6); and to say that, if the OH molecules are bombarded by a beam of equal velocity plasma particles, the resulting asymmetry will be approximately the same as if it had been bombarded with a beam of electrons only, except that the effective value of ξ would be reduced by a factor $\frac{1}{3}$.

III. ALIGNMENT BY A THERMAL BEAM

A more realistic model that can be analyzed simply enough is that in which the OH molecules are exposed, not to a strictly parallel, monochromatic beam, but rather to a cloud of electrons that is moving with a streaming velocity v_0, and that has random motions within the cloud characteristic of temperature T. The electron velocities can then be described by a Maxwellian distribution, with a thermal velocity v_T, and an *effective* cross-section obtained, by averaging over all angles and velocities of the incident particles.

If one is content to restrict one's attention to very low temperatures, where the distribution is very sharply peaked in velocity space, then it is feasible to use the approximation defined by equation (2) and to assume that the quantity $\xi(v)$ does not change markedly over the range where the integrand contributes significantly. In this approximation the average cross-section becomes

$$\langle f(n_0F_0m_0 \rightarrow nFm)\rangle = [(6 \pm \tfrac{7}{2}\xi) - (4 \pm \tfrac{19}{2}\xi)\sin^2\phi]\,|\langle\mu_x\rangle|^2 + [(6 \pm \tfrac{7}{2}\xi)]\,|\langle\mu_y\rangle|^2 + [(6 \pm \tfrac{7}{2}\xi) - (4 \pm \tfrac{19}{2}\xi)\cos^2\phi]\,|\langle\mu_z\rangle|^2 ,$$

where

$$\sin^2\phi \equiv \frac{\operatorname{erf}(v_0/v_T) - [2v_0/(\pi v_T)^{1/2}]\,e^{-(v_0/v_T)^2}}{(v_0/v_T)^2 \operatorname{erf}(v_0/v_T)} , \tag{8}$$

again using the simplification of equation (5).

This problem can now be solved following the procedure of § II, and the resulting relative probabilities of absorption plotted as a function of the ratio v_T/v_0 (the degree of thermalization of the cloud). The results are shown in the unbroken parts of the curves in Figure 2.

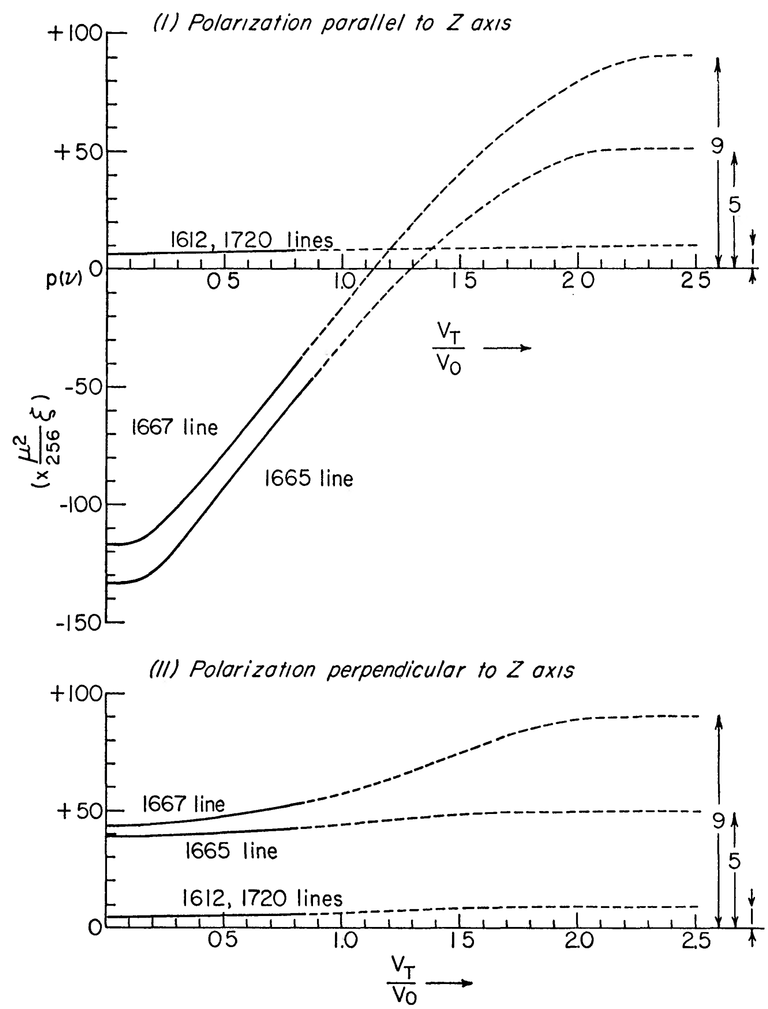

FIG. 2.—Plot of relative probability of absorption against the degree of thermalization (i.e., the ratio of thermal velocity to streaming velocity) for polarizations parallel to the axis of quantization and perpendicular to it.

However, when one seeks to discuss cases where the thermal velocity v_T is becoming comparable with the velocity of streaming v_0, then a significant number of collisions at extremely small relative velocities must be considered and the analytic form of the collision cross-section (eq. [1]) must be used, rather than the approximation defined by equation (3). Then contributions to the integrand in equation (8) from very low values of v are negligible. This is not to say that these cross-sections are accepted as being *physically* meaningful at very low impact velocities—clearly the Born approximation is meaningless there. What is being said is that retaining the closed analytic form introduces no logical inconsistency, and makes it possible to keep the distinction between the upward and downward transitions in a consistent manner.

One case can be solved analytically: the far limit where the velocity of streaming is zero and all the energy of the electrons is thermal. In this case the effective cross-section reduces to (with constant factors omitted)

$$\langle \sigma (n_0 F_0 m_0 \rightarrow nFm) \rangle = \{ |\langle \mu_x \rangle|^2 + |\langle \mu_y \rangle|^2 + |\langle \mu_z \rangle|^2 \} \times \int_0^\infty \frac{v^3 d\,v}{k_0{}^2} \log \frac{k + k_0}{|k - k_0|} \, e^{-(v/v_T)^2} . \quad (9)$$

It is easy to establish that the equilibrium population distribution associated with these cross-sections are (for all quantum numbers F_0, m_0, F_1, and m_1) is

$$\frac{N(1\ F_1 m_1)}{N(0\ F_0 m_0)} = e^{-(2\Delta E/m v_T{}^2)} = e^{-(\Delta E/kT)} , \quad (10)$$

which is exactly the Boltzmann distribution for the OH molecules in thermal equilibrium with the electrons to which they are exposed.

The over-all response of the system to radiation can now be extrapolated between two known limits, and the quantity $p(\nu)$ resulting from exposing the OH molecules to a cloud of electrons with constant average particle energy but in various stages of thermalization (i.e., for various values of the ratio v_T/v_0) can be predicted as shown in Figure 2.

It is seen that the possibility of maser action persists down to the point where thermal and streaming velocities are strictly comparable; while, when velocities are almost purely thermal, the relative absorption probabilities are in the "correct" ratios 1:5:9:1.

IV. MASER ACTION IN INTERSTELLAR SPACE

From the foregoing discussion it is clear that, if this effect is to be observed in any region of interstellar space, that region must satisfy some rather stringent requirements. There must be a sufficient number of electrons and positive ions in a free state, yet the local kinetic temperature must be quite low. Furthermore the region must be characterized by velocities of bulk motion which are capable of sustaining the inversion mechanism. These conditions are best fulfilled near the H II regions; not inside them where temperatures are typically $\sim 10^4$ ° K, but near their edges where temperatures are ~ 100° K. Indeed (see Matthews 1965) in the ionization shock fronts surrounding these regions, which are typically thin shells no more than ~ 0.5 light-years thick, ionized plasma is streaming radially outward at velocities of ~ 10 km/sec (the velocity with which the front itself is moving); yet the temperature is still quite low, specifically because the particle velocities are not random. It is only behind the front that velocities become randomized and the temperature rises to $\sim 10^4$ ° K.

The microwave observational evidence is that OH emission is indeed found in small clouds right on the edges of H II regions (Weaver, Williams, and Dieter 1965); and that these clouds themselves are moving with velocities ~ 50 km/sec. Since the thermal velocities associated with electrons at 100° K are also ~ 50 km/sec, it is possible for the relative velocity of OH molecules and charged particles to exceed this thermal velocity; and therefore, in just those regions of space where OH emission is observed, conditions are favorable for the present inversion mechanism to exhibit itself.

However, the simple picture of a cloud of OH molecules being irradiated by a beam of charged particles cannot very well apply. The *diffusion* cross-section (calculated by Altshuler 1957) for an electron colliding with OH molecules is $\sim 10^{-13}$ cm^2. So if the density of OH molecules is taken to be 10^{-2} cm^{-3}, a beam of electrons could travel through this cloud no further than 10^{15} cm ($\sim 10^{-3}$ light-years) before it became completely uncollimated by collisions with OH molecules alone. By the same token, neither is it useful to consider a cloud of OH molecules incident upon a (relatively) quiescent plasma. For the OH–H collision cross-section at these low energies would be at least $\sim 10^{-15}$ cm^2 (geometric), and in the absence of some external collimating mechanism, a stream of OH molecules could not retain its identity over distances greater than 10^{-3} light-years either.

However, Cook (1966*b*) has suggested that the bulk of the OH molecules which are observed are in fact formed *inside* the ionization shock front, when it encounters a cloud of denser material, like ice crystals or dust grains. There are several plausible processes by which this formation could occur. Cook's mechanism involves OH molecules being sputtered off ice crystals when they are struck by protons: for this mechanism he estimates a rate of production of $\sim 3 \times 10^{-2}$ cm^{-3} sec^{-1}. This general idea seems well suited to the present discussion. OH molecules will continually be formed throughout the front, and their effective lifetime will be the time taken for the molecule to suffer so many collisions that it loses the well-collimated (super-thermal) velocity of the material from which it was formed. This time is $\sim 10^9$ sec, and therefore the average density of coherent OH cannot be much greater than 10^{-2} cm^{-3}. But nonetheless the system is essentially steady. Charged particles are streaming predominantly in one direction (radially outward) and the (relevant) OH molecules are streaming in another (the direction of motion of the parent dust cloud); and in such a system the OH molecules will obviously be aligned in the manner described previously.

It has been pointed out by Perkins (1966) that infrared radiation of the frequency corresponding to transitions between the ground state of the OH molecule and the first rotational level will destroy a molecular alignment of the kind described here. However, the energy of this transition is greater than the electron energies under consideration, and the probability of exciting the first rotation level is very small. And even if it is excited, the resulting change of velocity of the OH molecule would be enough to insure that any infrared photon it subsequently emitted would have such a frequency shift that it would no longer have any appreciable interaction with the relevant "cloud" of OH molecules.

Actual details of how much radiation can be got out of this volume of population inversion depend on many things, but an *upper limit* will be set by the geometry. Referring to Figure 3, one sees that the inversion volume is a section of a thin spherical shell. The dimensions (based on the figures quoted by Westerhout [1958] for the region W3) are such that the longest optical path through this inversion is ~ 11 light-years. Hence the relevant inversion volume over which maser action might be expected to be coherent on a large scale, is vaguely a circular disk of radius ~ 5.5 light-years and thickness ~ 0.5 light-years. It will also be noted that, this volume being so much smaller in one dimension than in the other two, the most favorable circumstances for masering action will be when the axis of quantization (i.e., the direction of the relative motion of OH molecules and electrons) is parallel to this short dimension, i.e., radially into the H II region. Then when one observes radiation *perpendicular* to the axis of quantization, when most efficient masering is expected, one is looking through the longest possible masering path.

It is now possible to obtain order of magnitude estimates of various observable quantities characterizing the radiation emerging from this volume. The following numbers will be based on the most favorable possible circumstances: streaming velocities parallel to the short axis of the disk, the thermal velocities very low; and in the following calculations, these values are assumed:

i) Density of electrons, $N_e \sim 10^{-1}$ cm^{-3}
ii) Density of OH molecules $N_{\text{OH}} \sim 10^{-2}$ cm^{-3}
iii) Relative velocity of electrons and molecules $\sim$50 km/sec, i.e., $\xi \sim 10^{-3}$.

a) Amplification Length

If the matrix element used to calculate $p(\nu)$ from Table 3 is

$$|\langle\mu_{01}\rangle|^2 = 4.23 \times 10^{-37} \text{ erg cm}^3$$

(see Meyer and Meyer 1961; Powell and Lide 1965); the amplification length for the two strongest OH microwave lines (i.e., the distance a microwave beam must travel before it increases its intensity by a factor e—eq. [7]) is

$$L_A\ (1667) \sim 0.7 \text{ light-years}, \qquad L_A\ (1665) \sim 0.6 \text{ light-years}.$$

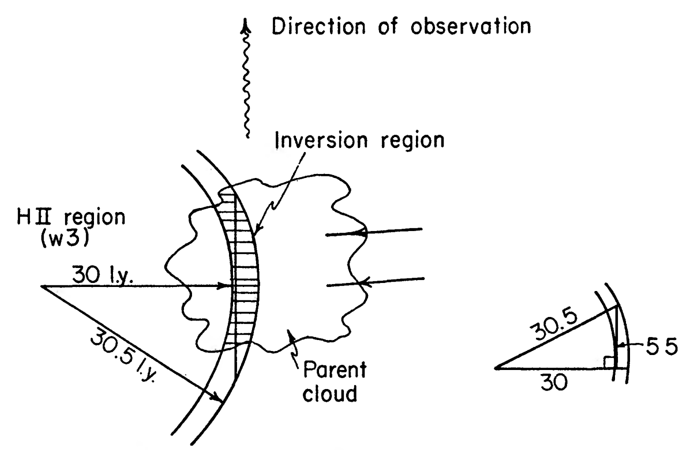

FIG. 3.—The geometry of the situation in which maser action might be expected

The time taken to build up the population inversion is of the order of the electron OH collision time, which can easily be estimated to be $\sim$0.1 year. This means that fluctuations in the masering population cannot occur much more quickly than this.

The most rapid intensity fluctuations observed in the OH emission lines are of the order of weeks (Dieter, Weaver, and Williams 1965).

b) Line Narrowing

The ratio between the "natural" width of a spectral line ($\Delta\nu$) and its width after it has been amplified by maser action ($\delta\nu$) is given by

$$\left(\frac{\delta\nu}{\Delta\nu}\right)^{-2} = \log\left[\frac{I(\nu,x)}{I(\nu,0)}\right]. \qquad (11)$$

The dimensions of the inversion volume are $\sim$11 light-years or 18 amplification lengths for the 1665 Mc/s line; hence it cannot be expected that the masering action described here will narrow this line by more than a factor 4, which means that no lines much narrower than $\sim$700 c/s can be expected.

The narrowest line so far observed is 600 c/s (Barrett and Rogers 1966).

c) Brightness Temperatures

Inside the inversion volume, the relative populations and local radiation density will change with time in a complicated non-linear relationship. In fact the system should be analyzed as a form of relaxation oscillation. However, following Perkins *et al.* (1966), it is possible to estimate the ***maximum*** radiation density, which occurs just before the population wipes itself out in one big flash. At this point the radiation density (u), which typically increases exponentially with time like exp $[(c/L_A)t]$, is balanced by the rate at which the population is being depleted, like exp $(-B\,u\,t)$—where B is the appropriate Einstein coefficient. Whence this formula for the maximum brightness temperature can be found:

$$u_{\max}=\frac{\nu^2 k\,(T_r)_{\max}}{c^3}\,d\Omega=\frac{c}{BL_A(\nu)}\,. \tag{12}$$

Furthermore, the radiation intensity pattern is extremely sharply peaked about the masering direction, and it is possible to estimate that the cloud will emit all of its radiation into a solid angle of only

$$d\Omega\sim 2\pi/10 \text{ sterad}\,, \tag{13}$$

so with the figures already derived,

$$[T_r\,(1665)]_{\max}\sim 2\times 10^4\,^\circ\,\mathrm{K}\,,\qquad [T_r\,(1667)]_{\max}\sim 1.5\times 10^4\,^\circ\,\mathrm{K}\,.$$

These figures can be converted to observable antenna temperatures by noting that the inversion disk subtends at Earth a solid angle of $\sim 3\times 10^{-7}$ sterad (using 2000 pc as the distance to W3); hence an antenna which sees 30′ would measure antenna temperatures of

$$[T_A(1665)]_{\max}\sim 300^\circ\,\mathrm{K}\,,\qquad [T_A(1667)]_{\max}\sim 230^\circ\,\mathrm{K}\,.$$

The largest antenna temperatures recorded have been: 160° (Barrett and Rogers 1966) for the 1665 Mc/s line and 25° (Williams, Dieter, and Weaver 1965) for the 1667 Mc/s line.

It is clear, of course, that the numbers calculated here overestimate the total emission which can be expected. Realistic temperatures will be lower than these. Notice, however, that, while one can expect brightness temperatures to be degraded as more realistic physical conditions are imposed, by the same token, as is shown by Figure 2, the 1667 line will be degraded much more rapidly than the 1665 line; and this too is in line with observation.

d) Apparent Size of the Source

While the emission volume presents to Earth a very long and thin surface area (11 × 0.5 light-years); radiation will not be seen uniformly over all this area, but will come mostly from the central portion. In fact it is possible to estimate the approximate length over which emergent radiation is at least half as intense as it is at maximum to be ~2.8 light-years; and so the *apparent* size of the source as seen from Earth is only (3 × 0.5 light-years) or (90″ × 15″).

This can be compared with the upper limit of 20″ measured by Cudaback, Read, and Rougoor 1966.

e) Polarizations

All the radiation emitted by this mechanism is plane-polarized (see Table 3), and so only those observations which detect plane polarized signals seem capable of being explained. The problem of accounting for lines which are unpolarized or circularly polarized is quite difficult. However, actual physical situations can be very complicated. Weaver, Williams, Dieter, and Lum (1965), for example, are able to locate four distinct clouds in the small region W3 C: and when it is remembered that the a priori probability of actually observing any one cloud is only $\frac{1}{20}$ (from eq. [13]), it is clear that as many as eighty clouds could be emitting from this same region. It is easy to see therefore that two clouds in the same line of sight could give an emission line which would be completely unpolarized. It is even possible to construct a geometry in which, in the presence of a strong magnetic field, an anomalous absorption line superimposed on an anomalous emission line would result in an emission line that is circularly polarized. However, while this idea could possibly explain the presence of an occasional circularly polarized line, it could not very well explain why the majority of observed lines should be so: and it is likely that one of the other mechanisms which have been advanced to explain circular polarization (e.g., that of Heer 1966) must be considered in conjunction with the present mechanisms.

V. CONCLUSION

One last point can be made. It has been argued here that at least some of the anomalous OH emission features could be the result of alignment of the OH molecule by charged particles. However, very little has been said that relates specifically to the OH molecule and that could not be said about any molecule which possesses a dipole moment. So too the conclusions should show a similarly wide applicability. Molecules like CH, SiH, and AlH should be observable *in anomalous emission* if their frequencies were well enough known. Similarly NH_3 would be a good candidate were it not for the fact that its relative abundance is (it is believed) too low, and that radio work at 1 mm is very difficult. But most promising of all is SH whose microwave frequencies are well known, and which has previously been estimated to be just on the verge of detectability (see Barrett 1964).

It is interesting to note on this point that the dipolar molecule CN has already been observed in unusually high concentrations near the H II region W1, and it shows anomalous absorption features in its optical spectrum (see Münch 1964).

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