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The Sungrazing Comet Group

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The principal effects of planetary perturbations on the orbit of a sungrazing comet are derived, both from a simplified general theory and by direct numerical integration of typical trajectories. New orbit determinations are made for the three most recently observed members of the Kreutz group: comets 1965 VIII, 1963 V, and 1945 VII. For comet 1965 VIII the results are obtained in terms of both general relativity theory and the Brans–Dicke theory, but the observations are insufficient to enable one to decide which theory is correct. Some remarks are made about the less certain members of the comet group, and evidence is provided for dividing the comets into two distinct subgroups. Studies on the past motions of comets 1882 II and 1965 VIII reveal it as virtually certain that these comets separated from each other at their previous perihelion passage, and it seems possible that they were then observed as the comet of 1106. Not all the members of each subgroup can be explained so simply, but it is probable that the subsidiary separations took place not more than two revolutions ago. The two subgroups can presumably be explained by a similar split some 10 to 20 revolutions ago, although a few other possible explanations are touched upon in the final section. An ephemeris is provided for use in searching for further members of the group.

I. INTRODUCTION

SUNGRAZING comets have puzzled man for almost three centuries. Ever since it was realized that the great comet of December 1680 had skimmed by the solar surface at a mere 200 000 km astronomers have been faced with the problem as to how such comets can survive. In addition, they have speculated as to which of the cometary apparitions on record were earlier manifestations of which particular sungrazing comet.

Halley was strongly of the belief that the 1680 comet was the return of one seen in February 1106. As is well known, later researches have indicated that this was not the case. The 1106 comet was also considered as identical with the sungrazing comet 1843 I. Another contender was a comet observed by the Greeks about the year −371. The most probable candidates for earlier appearances of comet 1843 I were thought to be three or four comets seen during the last third of the seventeenth century (not the 1680 comet), but which had not been followed well enough for their orbits to be determined with much reliability. It is entertaining to look through the astronomical literature of the mid-nineteenth century as various people put forth their suggestions regarding the length of the period of 1843 I, the values ranging from a fairly respectable 175 yr down to an utterly impossible 7 yr.

Direct calculations from the observations of comet 1843 I showed that the period was probably not shorter than 400 or 500 yr. Nevertheless, when comet 1880 I was found moving in almost precisely the same orbit, proponents of a 35- to 40-yr revolution period had a further heyday. Other astronomers suggested that the period was being drastically reduced at each perihelion passage by friction in a resisting medium surrounding the sun. This notion seemed to be confirmed when comet 1882 II appeared so soon afterward, also traveling in a very similar orbit, but the observations left no doubt that this comet had a period of several centuries.

Thus it was finally agreed that there were a number of comets moving in essentially the same sungrazing orbit. The concept of comet groups had been discussed in some detail by Clausen and Hoek, although Kirkwood (1880) was apparently the first to suggest that the sungrazing comets formed such a group. Kirkwood remarked that 1843 I and 1880 I might both be fragments of the comet of −371. The Greek historian Ephorus had claimed to have seen that comet split in two, although this must be regarded as rather doubtful evidence, for he might well have manufactured it as support for the ancient theory that comets were formed when two stars passed close to each other—and thus that they ceased to exist when the stars separated again. A dramatic confirmation of Kirkwood's general ideas was provided, however, when the nucleus of comet 1882 II broke up into four or five discrete fragments. A few years later a probable further member of the group was observed—comet 1887 I.

The sungrazing comet group was studied very extensively by Kreutz (1888, 1891, 1901). He made new determinations of the orbits of 1843 I, 1880 I, 1887 I, and four of the nuclei of 1882 II, reaffirming that there was no reason for believing that any of these comets had a period as short as 40 yr. It seemed probable that "Tewfik," the comet seen only during the total solar eclipse in May 1882, also belonged to the system. Kreutz felt it likely that comets 1668 and 1702*a* were members of the group, but that comets 1689 and 1695 were not. The eclipse comet of 1893 and the object found by Pogson in 1872 during a search for Biela's comet can almost certainly be excluded from membership. Kreutz presumed that the members of the group had been formed when some primordial comet had split up at perihelion, and he indicated that even the comet of 1680, which otherwise had rather different orbital elements, might be related in the same general way.

Since Kreutz made his study three further members of the group have appeared—comets 1945 VII (du

TABLE I. Orbital elements of the Kreutz group of comets (equinox 1950.0).

Comet	T (UT)	ω	Ω	i	q (a.u.)	e	P (yr)	L	B
1668	Feb. 28.08	109°.81	2°.52	144°.38	0.066604	1.0	...	248°.61	+33°.23
1843 I	Feb. 27.91	82.64	2.83	144.35	0.005527	0.999914	512	281.86	+35.31
1880 I	Jan. 28.12	86.25	7.08	144.66	0.005494	1.0	...	281.68	+35.25
1882 II	Sept. 17.72	69.59	346.96	142.00	0.007751	0.999907	761	282.24	+35.24
1887 I	Jan. 11.63	58.35	325.50	128.47	0.009665	1.0	...	280.24	+41.79
1945 VII	Dec. 28.01	50.93	321.69	137.02	0.006305	1.0	...	289.73	+31.96
1963 V	Aug. 23.92	85.82	6.77	144.52	0.005161	0.999952	1111	281.90	+35.37
1965 VIII	Oct. 21.18	69.03	346.25	141.85	0.007761	0.999918	929	282.24	+35.21

Toit), 1963 V (Pereyra), and 1965 VIII (Ikeya-Seki). The second and third of these were observed more thoroughly than any of the earlier members, with the possible exception of comet 1882 II. It is thus appropriate now to reinvestigate the group in the hope that these recent members can shed more light on its composition and evolution.

II. EMPIRICAL REMARKS

In Table I we summarize the orbital elements of the eight generally acknowledged members of the Kreutz sungrazing comet group. The times of perihelion passage T, arguments of perihelion ω, longitudes of the ascending node Ω, inclinations i, perihelion distances q, and eccentricities e have been taken from the usual catalogues (Porter 1961; Marsden 1966). Further columns give the revolution periods P and the longitudes and latitudes of perihelion L, B. The figures for comets 1882 II and 1965 VIII are those of the principal nucleus. The orbits of comets 1668, 1887 I, and 1945 VII are not well determined, and only for 1882 II are e and P reasonably certain.

It can scarcely be doubted that the comets are fragments of one original comet. The fragmentation may of course have taken place randomly over several revolution periods. It is nevertheless of interest to investigate how the various members might have evolved, and for this reason we enumerate here a few points that seem to describe the system. Some of these remarks will subsequently be found to have no dynamical foundation, but they do serve as a guide in formulating a tentative hypothesis for the origin of the comet group. Specifically, we may note the following:

(1) The times of the appearances of the comets are distributed in a highly nonuniform manner. There seem to be three distinct clusters of comets—one in the late seventeenth century, a second in the nineteenth century, and a third in progress at the present time. The strong concentrations in the 1880's and 1960's are particularly noteworthy.

(2) The positions of the perihelion points of the five well-defined orbits are all contained in a very localized region: $L=282\overset{\circ}{.}0\pm0\overset{\circ}{.}3$, $B=+35\overset{\circ}{.}3\pm0\overset{\circ}{.}1$. The deviations of the figures for the other comets are probably due to uncertainties in the orbit determinations.

(3) The values of ω, Ω, i, and q differ quite markedly and seem to be correlated with the position of Jupiter at the time of perihelion passage.

(4) The elements ω, Ω, i, and q for the individual nuclei of the comets observed to split are almost precisely the same. The derived revolution periods for adjacent nuclei differ by a century or more.

The peculiar distribution of the times of appearance is even more striking if one includes the comets 1689, 1695, 1702*a*, and the 1882 eclipse comet. Although Kreutz did not regard the first two of these as belonging to the group, his dismissal of them seems somewhat hasty. We discuss these comets further in Sec. VII. In any case, what are the chances that these comets would appear in the right part of the sky at the right time of the year, and yet were *not* members of the group? Of all the other bright comets seen during the last three centuries only 1948 XI might have been mistaken for a member if it had been observed as imperfectly as these were. The records mention a bright comet observed in Ceylon in February 1666. It is generally supposed that the year intended was 1668, although Lynn (1888) suggested that 1666 might indeed have been correct, and that this was a further member of the group. This can hardly be regarded as independent evidence for clustering, and neither can the report of a strange object observed in Scotland in December 1882 (Botley 1967). Nevertheless, there does seem to be a complete absence of suspected members between 1702 and 1843 and again between 1887 and 1945.

Because of the way their orbits are oriented, the members of the system are best observed in the southern hemisphere, especially if perihelion occurs in December or January. It is certainly possible that some members came during the eighteenth century, but that they were not seen owing to the lack of astronomers in southern latitudes. On the other hand, it is improbable that the paucity of southern observers contributed to the lapse between 1887 and 1945, for several comet hunters were active in the southern hemisphere during this interval: indeed, during the 1920's they were more successful than the northern hunters.

The real difficulty with these comets is that one at perihelion between mid-May and mid-August will undoubtedly be missed, unless it can be seen in daylight. The geometry is such that the comet would approach and leave the sun from behind, and it would never be seen in a dark sky. It is to be expected that one member out of four passes unobserved on this account. The number missed may in fact be as great as one out of two, for of the known and suspected members, only Tewfik was at perihelion between late February and late August. Such selectivity should not, of course, affect the over-all distribution of the rich and barren periods. If Tewfik indeed belongs to the group, it is remarkable that it should have been at perihelion within a few hours of the only time it could have been seen, during the eclipse on 1882 May 17. One cannot help wondering how many other members escaped detection by coming to perihelion between then and September, when the great comet appeared.

The orbital elements of comets 1882 II and 1965 VIII are so similar that suggestions have been made (if only informally) that the two comets were identical. The orbits of comets 1880 I and 1963 V are also strikingly similar. During the 83-yr period that elapsed between the appearances of the members of each pair Jupiter made almost exactly seven revolutions about the sun. At the times of the perihelion passages of the 1882–1965 pair the longitudes of Jupiter differed by only 2°, and at the perihelion passages of the other pair they differed by 10°. This suggests that we might try representing the orbital elements of the comets by expressions of the form $a+b\sin(\lambda_1'+\beta)$, where λ_1' is the mean longitude of Jupiter when the comets are at perihelion and a, b and β are constants. The least-squares solutions for the elements of the five principal members yield the following:

$$\begin{aligned}
\omega &= 67\overset{\circ}{.}16 + 19\overset{\circ}{.}42 \sin(\lambda_1' + 99\overset{\circ}{.}60),\\
\Omega &= 343\overset{\circ}{.}99 + 23\overset{\circ}{.}60 \sin(\lambda_1' + 99\overset{\circ}{.}59),\\
i &= 141\overset{\circ}{.}92 + 2\overset{\circ}{.}82 \sin(\lambda_1' + 105\overset{\circ}{.}88),\\
q &= 0.007795 - 0.002610 \sin(\lambda_1' + 104\overset{\circ}{.}81)\ \text{a.u.}
\end{aligned} \tag{1}$$

Although these solutions are obtained as the result of solving sets of five equations in three unknowns, because of the similarity of the values of λ_1' for comets 1882 II and 1965 VIII, and for 1880 I and 1963 V, they effectively arise from sets of only three equations. The period of the argument could thus be *any* aliquot part of 83 yr, and not just the period of Jupiter's revolution. No other fraction has any reasonable physical significance, however. Furthermore, Jupiter's period gives a somewhat better fit than any other: the residuals are given in Table II. We include also the residuals for the three other orbits and remark that no other period fits these orbits even remotely. This is our basis for point (3) above.

With regard to point (4), let us note that the longitudes of the ascending nodes, for example, of the four nuclei of comet 1882 II, agree to within $0\overset{\circ}{.}005$ (Kreutz 1891). The agreement may not really be quite as close as this, for the orbits were determined on the assumption that the four fragments coincided with each other before breakup. Differential forces acting at breakup would be smoothed out. On the other hand, the difference of almost $0\overset{\circ}{.}03$ between the values of Ω for the components of comet 1965 VIII (Sekanina 1966) could well be an overestimate. This time the nuclei were considered as completely independent, but the necessity of using a short arc of observation would make the results less determinate. In any case, the differences between the values of Ω for the individual components are very considerably smaller than the dispersion of the figures in Table I, even if the three less certain orbits are omitted. An analogous situation exists with the other elements.

From our empirical remarks we can deduce a working hypothesis. We assume that the fragments of the primordial comet initially had identical values of ω, Ω, i, and q (and T). If the breakup were indeed like those of comets 1882 II and 1965 VIII, the components would have periods differing by a century or so. We associate these components with the three clusters noted in point (1). It would have been difficult to distinguish fragments with periods differing by only a few years (as would be necessary for explaining all the individual comets at their next returns), although Andrews (1965) has commented that one of the nuclei of comet 1965 VIII was possibly a triple complex. The differences among the orbits of the returning comets would presumably be explained by planetary perturbations, particularly by Jupiter. The breakup probably took place at the previous perihelion passage of the comets, during the twelfth century, or the clustering would not have been so pronounced. Anyway, the observed members could not have separated from each other too long ago, for there is some evidence (cf. the short-period comets Biela, Brooks 2, and Taylor) that when a comet has split all but perhaps the most substantial fragments do not survive for many more revolutions.

Unfortunately, such a simple picture as this is untenable, as we find in the following section. It can still serve as a basis for our model, however, and we try to describe the evolution of the sungrazing comet system with the very minimum of modification.

III. PLANETARY PERTURBATIONS

The heliocentric distance of a member of the sungrazing comet group ranges from 0.005 to about 200 a.u. We should consider whether to take for the unperturbed motion of the comet an elliptical orbit about the sun or one about the center of mass of the sun and planets. Obviously, the comet spends a very great deal of time moving essentially in an ellipse about the center of mass of the solar system. When it is observed, however, it is traveling very much under the influence of the sun alone. Because of the moderately high orbital inclination of the comet, the direct attractions of the planets are small; it is impossible for these

TABLE II. Residuals from expressions giving the variations with Jupiter's longitude [Eq. (1)].

Comet	$\Delta\omega$	$\Delta\Omega$	Δi	Δq (a.u.)
1668	(+30°57	+3°83	+ 0°95	+0.060241)
1843 I	+ 0.07	+0.11	+ 0.01	−0.000063
1880 I	− 0.32	−0.50	− 0.05	+0.000284
1882 II	− 0.02	−0.01	+ 0.03	+0.000048
1887 I	(+ 7.31	+1.11	−11.30	−0.000156)
1945 VII	(+ 1.62	−0.61	− 2.35	−0.003785)
1963 V	+ 0.28	+0.44	+ 0.04	−0.000252
1965 VIII	− 0.02	−0.04	− 0.04	−0.000017

sungrazing comets to approach Jupiter within about 3 a.u., for example.

Thus we take for the unperturbed solution the usual heliocentric elliptical motion. It will suffice to consider as the only perturbation the indirect attraction of Jupiter, and to assume that Jupiter has a circular orbit in the plane of the ecliptic. The equations for the variation of arbitrary constants then yield

$$\begin{aligned}
dz/dt&=2nz\varphi\alpha^2m'(S\eta\cos u-C\sin u),\\
dq/dt&=nz^{-1}\eta e^{-1}\alpha^2m'\{S[\cos u(\delta^2\varphi-1)+e]\\
&\qquad+C\eta\sin u(1-\delta^2\eta^{-2}\varphi)\},\\
di/dt&=n\eta^{-1}cs\alpha^2m'[S'(\cos u-e)-C'\eta\sin u],\\
d\sigma/dt&=n\alpha^2m'\{S\eta\sin u(1+\eta^2e^{-1}\varphi\cos u)\\
&\qquad+C[2(\cos u-e)-\eta^2e^{-1}(1+\varphi\sin^2u)]\},\\
d\chi/dt&=n\alpha^2m'[S\sin u(1-\eta^2e^{-1}\varphi\cos u)\\
&\qquad+C\eta e^{-1}(1+\varphi\sin^2u)],\\
d\Omega/dt&=\tfrac{1}{2}n\eta^{-1}\alpha^2m'[S'\eta\sin u+C'(\cos u-e)],\\
d\omega/dt&=(s^2-c^2)(d\Omega/dt)+d\chi/dt,\\
d^2\rho/dt^2&=\tfrac{3}{2}nz^{-1}(dz/dt),
\end{aligned}\tag{2}$$

where

$$\begin{aligned}
S&=c^2\sin(\lambda'-\omega-\Omega)-s^2\sin(\lambda'+\omega-\Omega),\\
C&=c^2\cos(\lambda'-\omega-\Omega)+s^2\cos(\lambda'+\omega-\Omega),\\
S'&=\sin(\lambda'-\omega-\Omega)+\sin(\lambda'+\omega-\Omega),\\
C'&=\cos(\lambda'-\omega-\Omega)-\cos(\lambda'+\omega-\Omega),
\end{aligned}\tag{3}$$

with

$$\eta=(1-e^2)^{\frac{1}{2}},\quad \delta=1-e,\quad c=\cos\tfrac{1}{2}i,$$
$$s=\sin\tfrac{1}{2}i,\quad \varphi=(1-e\cos u)^{-1}.$$

n is the mean motion of the comet, z the reciprocal of the semimajor axis, u the eccentric anomaly, α the ratio of the semimajor axis of the comet's orbit to the radius of Jupiter's orbit, and m' is Jupiter's mass. σ, ρ, and χ are auxiliary quantities, the mean anomaly being given by $l=\sigma+\rho$. λ' is the mean longitude of Jupiter, and taking the epoch ($t=0$) at the time of a perihelion passage of the comet, we have

$$\lambda'=\nu(u-e\sin u)+\lambda_0',\tag{4}$$

where λ_0' is the mean longitude of Jupiter at the epoch and ν is the ratio of the mean motions of Jupiter and the comet.

Since the comets are observed only near perihelion, it is sufficient to form the definite integrals of the equations of motion between one perihelion passage and the next, and everything except for u may be regarded as constant. The equations may be integrated between the epoch and the time of the following aphelion passage in terms of the Anger and Lommel–Weber functions

$$\begin{aligned}
\mathbf{J}_\tau(e\nu)&=\frac{1}{\pi}\int_0^\pi\cos(\tau u-e\nu\sin u)du,\\
\mathbf{E}_\tau(e\nu)&=\frac{1}{\pi}\int_0^\pi\sin(\tau u-e\nu\sin u)du,
\end{aligned}\tag{5}$$

with τ having the values $\nu-2$, $\nu-1$, ν, $\nu+1$, and $\nu+2$. We remark that, if τ is an integer, then $\mathbf{J}_\tau=J_\tau$ (the Bessel function). These functions may be expressed in terms of $\mathbf{J}_\nu$, $\mathbf{E}_\nu$, and

$$\begin{aligned}
\mathbf{J}_\nu'&=[d/d(e\nu)]\mathbf{J}_\nu,\\
\mathbf{E}_\nu'&=[d/d(e\nu)]\mathbf{E}_\nu,
\end{aligned}\tag{6}$$

although instead of $\mathbf{J}_\nu$ and $\mathbf{E}_\nu$ we find it more convenient to take

$$\begin{aligned}
\hat{\mathbf{J}}_\nu&=(1/e\nu)[\mathbf{J}_\nu-(\sin\nu\pi/\nu\pi)],\\
\hat{\mathbf{E}}_\nu&=(1/e\nu)\{\mathbf{E}_\nu-[(1-\cos\nu\pi)/\nu\pi]\};
\end{aligned}\tag{7}$$

the argument is understood to be $e\nu$.

In order to integrate the equations from the aphelion to the following perihelion we adopt that perihelion as the epoch and take the integrals over the range $(-\pi, 0)$, noting that the Anger functions are even and the Lommel–Weber functions are odd. We denote the mean longitude of Jupiter at the second perihelion passage by λ_1' and remark that

$$\lambda_1'=\lambda_0'+2\nu\pi.\tag{8}$$

By adding the integrals over the two ranges we obtain, after a moderate amount of algebra, the perturbations Δz, Δq, $\cdots$ between the two perihelion passages as

$$\begin{aligned}
\begin{Bmatrix} z^{-1}\Delta z\\ ze\eta^{-1}\Delta q\\ \eta\Delta i\end{Bmatrix}
=\pi\alpha^2m'\Bigg[&\begin{Bmatrix} c^2\\ -c^2\\ cs\end{Bmatrix}\{(E_p-\eta F_p)[\cos(\lambda_1'-\omega-\Omega)-\cos(\lambda_0'-\omega-\Omega)]\\
&-(J_p-\eta K_p)[\sin(\lambda_1'-\omega-\Omega)+\sin(\lambda_0'-\omega-\Omega)]\}+\begin{Bmatrix} s^2\\ s^2\\ cs\end{Bmatrix}\{(E_p+\eta F_p)[\cos(\lambda_1'+\omega-\Omega)-\cos(\lambda_0'+\omega-\Omega)]\\
&-(J_p+\eta K_p)[\sin(\lambda_1'+\omega-\Omega)+\sin(\lambda_0'+\omega-\Omega)]\}\Bigg],
\end{aligned}\tag{9}$$

$$\begin{Bmatrix} e^{-2}\Delta\sigma \\ e^{-2}\Delta\chi \\ \eta\Delta\Omega \end{Bmatrix} = \pi\alpha^2 m' \left[\begin{Bmatrix} c^2 \\ -c^2 \\ -1 \end{Bmatrix} \{(E_p-\eta F_p)[\sin(\lambda_1'-\omega-\Omega)-\sin(\lambda_0'-\omega-\Omega)] \right.$$

$$+(J_p-\eta K_p)[\cos(\lambda_1'-\omega-\Omega)+\cos(\lambda_0'-\omega-\Omega)]\} + \begin{Bmatrix} s^2 \\ s^2 \\ 1 \end{Bmatrix} \{(E_p+\eta F_p)[\sin(\lambda_1'+\omega-\Omega)-\sin(\lambda_0'+\omega-\Omega)]$$

$$\left. +(J_p+\eta K_p)[\cos(\lambda_1'+\omega-\Omega)+\cos(\lambda_0'+\omega-\Omega)]\} \right],$$

$$\Delta\omega = (s^2-c^2)\Delta\Omega + \Delta\chi,$$
$$\Delta l = \tfrac{3}{2}\pi z^{-1}\Delta z + \Delta\sigma,$$

where the E_p and F_p take on the following values:

$$\begin{aligned}
E_z &= -2\mathbf{E}_\nu', \\
F_z &= 2\nu\hat{\mathbf{E}}_\nu, \\
E_q &= -\nu^{-1}\mathbf{E}_\nu' + (1-e)^2\nu\hat{\mathbf{E}}_\nu - (\nu\pi)^{-1}(1-e), \\
F_q &= -(1-e)(1+e)^{-1}\mathbf{E}_\nu' + \hat{\mathbf{E}}_\nu, \\
E_i &= -\nu^{-1}\mathbf{E}_\nu' - (\nu\pi)^{-1}(1-e), \\
F_i &= \hat{\mathbf{E}}_\nu, \\
E_\sigma &= -(1-3e^2)\nu^{-1}\mathbf{E}_\nu' + (1-e^2)^2\nu\hat{\mathbf{E}}_\nu \\
&\qquad - (\nu\pi)^{-1}(1-e)(1+3e), \qquad (10) \\
F_\sigma &= -(1-e^2)\mathbf{E}_\nu' + (1-2e^2)\hat{\mathbf{E}}_\nu, \\
E_\chi &= (1-e^2)\mathbf{E}_\nu' - \hat{\mathbf{E}}_\nu, \\
F_\chi &= \nu^{-1}\mathbf{E}_\nu' - (1-e^2)\nu\hat{\mathbf{E}}_\nu + (\nu\pi)^{-1}(1+2e), \\
E_\Omega &= -\tfrac{1}{2}\nu^{-1}\mathbf{E}_\nu' - \tfrac{1}{2}(\nu\pi)^{-1}(1-e), \\
F_\Omega &= \tfrac{1}{2}\hat{\mathbf{E}}_\nu.
\end{aligned}$$

The corresponding J_p and K_p may be obtained by replacing $\mathbf{E}_\nu'$ by $\mathbf{J}_\nu'$, $\hat{\mathbf{E}}_\nu$ by $\hat{\mathbf{J}}_\nu$, and omitting the terms not factored by either $\mathbf{E}_\nu'$ or $\hat{\mathbf{E}}_\nu$.

Equations (9) could be simplified somewhat by making further use of Eq. (8). In the case of the sungrazing comets, however, it is convenient to retain the equations in the present form, for the coefficients are not too dependent upon ν, which has a value of about 60 but is not known very precisely. If the equations were simplified, the corresponding coefficients might depend very much on how ν differed from an integer, for example. No tables exist of the Anger and Lommel–Weber functions for $e \simeq 1$ and $\nu \simeq 60$, and the conventional methods for calculating these functions (Watson 1922, Chap. 8) are not very satisfactory. The most convenient way to obtain the functions seems to be by direct numerical integration of Eq. (5).

We assume that at the first perihelion passage the comet broke up into fragments in which the elements ω, Ω, i, and q were identical. Substituting the numerical values for comet 1882 II into Eq. (9), we find that the orbital elements of the fragments at their next returns to perihelion will be given, approximately, by

$$\begin{aligned}
\omega &= \bar{\omega} + 1\overset{\circ}{.}1 \quad \sin(\lambda_1'+\beta), \\
\Omega &= \bar{\Omega} + 1\overset{\circ}{.}4 \quad \sin(\lambda_1'+\beta), \qquad (11) \\
i &= \bar{i} + 0\overset{\circ}{.}3 \quad \sin(\lambda_1'+\beta), \\
q &= \bar{q} - 0.00039 \sin(\lambda_1'+\beta-90^\circ) \text{ a.u.}
\end{aligned}$$

The contributions from λ_0' (which is the same for each comet) have been absorbed into the constant parts $\bar{\omega}$, $\bar{\Omega}$, $\bar{i}$, and $\bar{q}$. β is a constant phase angle, the precise value being immaterial.

These expressions are of the same form as Eq. (1) for the observed members of the Kreutz group. But the amplitudes are far too small! The only way in which the theoretical values could be significantly increased is to suppose that η (appearing in the denominators of $\Delta\omega$, $\Delta\Omega$, and Δi) and z (in that of Δq) are very much smaller than they are thought to be, but this is no real solution to the dilemma. In any case, q is 90° out of phase with the other elements.

As a check on the theory, numerical integrations have been made of trajectories of sungrazing comets with identical starting values of ω, Ω, i, and q, and considering the complete attractions of the planets Jupiter to Pluto. A typical calculation resulted in expressions similar to Eq. (11), with the amplitudes of the variations of the elements being $1\overset{\circ}{.}6$, $2\overset{\circ}{.}1$, $0\overset{\circ}{.}4$, and 0.00046 a.u., respectively. On account of other terms, due to Saturn in particular, the total variation of the elements from one perihelion passage to the next could be somewhat larger, but the extreme amplitude is certainly not more than twice these amounts.

The line of apsides is scarcely affected by the perturbations, and this is consistent with the point (2) of Sec. II. Although there are significant perturbations in the line of nodes they are considerably smaller than the differences among the orbits of the various members of the Kreutz group. Point (3) of Sec. II must thus be completely discarded.

IV. COMET 1965 VIII (1965f IKEYA-SEKI)

This most recent of the sungrazing comets was discovered by two Japanese amateur astronomers, K. Ikeya and T. Seki, within 15 min of each other on 1965 September 18.8. In Hydra, and about magnitude 8 at discovery, the comet brightened considerably as it approached the sun. The comet was visible in broad daylight around the time of perihelion passage on October 21.2, and emission lines due to iron, nickel, chromium, potassium, ionized calcium, and other elements were observed in the spectrum. During the following few weeks the comet was a fine object in the

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TABLE III. Palomar observations of comet 1965 VIII.

1965/66 UT	α_{1950}	δ_{1950}	Nucleus
Dec. 31.37778	$7^h38^m31^s.88$	$-37°37'10''.2$	A
	7 38 32.67	−37 36 21.0	B
Jan. 14.28785	6 43 12.66	−32 42 37.0	A
14.32674	6 43 05.05	−32 41 43.5	A
	6 43 07.97	−32 40 58.7	B

predawn sky, the tail attaining a length of perhaps 30°. A second nucleus was observed, initially by H. Pohn, U. S. Geological Survey, on November 4.5. The most systematic series of positional observations was made by Z. M. Pereyra at the Córdoba Observatory. Not only did this observer obtain the first and last published precise positions (on September 21.4 and December 24.2), but he also obtained the last before perihelion (on October 16.4) and the first afterward (October 28.3).

A few later plates were taken by G. A. Tammann with the 48-in. Schmidt at Palomar Mountain. They have been measured and reduced by the writer, and the results are given in Table III. The images of the comet on 1966 January 14, the last time the comet was definitely photographed, were not particularly good. Possible images of the comet were recorded on Baker-Nunn films at Smithsonian observing stations as late as February 12, but photographs taken around that time through large telescopes failed to show the comet. Evidently the comet faded very much more rapidly than had been expected.

The first set of orbital elements in Table IV has been determined from 119 observations made between 1965 September 21 and 1966 January 14. Seventy-five of the observations were made before perihelion, and after the comet split the positions of the principal nucleus (nucleus A) were used. The mean residual is $\pm 1''.2$, both before and after perihelion. The errors quoted are mean ones. Perturbations by all nine planets were taken into account, an integration step of 0.001 day being used around perihelion passage.

The second set of elements refers to nucleus B, of which there are 26 satisfactory observations from 1965 November 12 to 1966 January 14. To improve the determinacy of the solution these observations were combined with the 75 pre-perihelion observations of the single mass. The mean residual is $\pm 1''.4$ ($1''.3$ before perihelion and $1''.9$ afterward).

It is important to discuss the effect of general relativity on the motion of the comet. Kustaanheimo and Lehti (1963) concluded that, if relativity is ignored, the period of a sungrazing comet will be greatly underestimated, by 10% for the example they considered and by 20% for a typical member of the Kreutz group. Their conclusion was based on the assumption that the observations were made around perihelion; for none of the observations of 1965 VIII, however, was the true anomaly smaller than about 160° in absolute value. Wright (1967) has criticized the papers by Kustaanheimo and Lehti and by Geisler and McVittie (1965) on theoretical grounds. The only correct procedure is to compare the actual observations of the comet with the rigorous equations of relativity theory. Such a calculation was made for comet 1882 II by Hufnagel (1919), who found that the observations could be satisfied equally well whether relativity is included or not. Inclusion of the relativistic terms indicated that e and P should be decreased from the Newtonian values of 0.9999078±0.0000002 (m.e.) and 771.8±3.0 yr (as given by Kreutz) to 0.9999068 and 760.9 yr.

The relativistic equations of motion may be written

$$\ddot{x}+\mu x r^{-3}=\partial R/\partial x+Mx+N\dot{x}, \tag{12}$$

where μ is the product of the constant of gravitation and the mass of the sun, R denotes the planetary disturbing function, $x\rightarrow y$, z with $r^2=x^2+y^2+z^2$, and dots denote differentiation with respect to the time. If the Schwarzschild metric is used, then we have, to the first order of $\gamma=\mu c^{-2}$ (c being the velocity of light),

$$\begin{aligned} M&=\gamma r^{-3}(2\mu r^{-1}-2v^2+3\dot{r}^2), \\ N&=2\gamma\dot{r}r^{-2}, \end{aligned} \tag{13}$$

where $v^2=\dot{x}^2+\dot{y}^2+\dot{z}^2$ and $r\dot{r}=x\dot{x}+y\dot{y}+z\dot{z}$. Very nearly the same result would be obtained numerically if the isotropic metric is used, in which case

$$\begin{aligned} M&=\gamma r^{-3}(4\mu r^{-1}-v^2), \\ N&=4\gamma\dot{r}r^{-2}. \end{aligned} \tag{14}$$

The third set of elements in Table IV was calculated from the 119 observations of comet 1965 VIII used

TABLE IV. Orbital elements for comet 1965 VIII (Epoch 1965 Oct. 7.0 ET).

	Nucleus A (nonrelativistic)		Nucleus B (nonrelativistic)		Nucleus A (relativistic)	Nucleus A (Brans–Dicke)
T (1965 Oct. ET)	21.183689	±0.000049	21.182917	+0.000072	21.183679	21.183673
$\omega°$	69.05030	±0.00048	69.03587	±0.00069	69.05018	69.04936
$\Omega°$ 1950.0	346.29699	±0.00060	346.28381	±0.00089	346.29735	346.29668
$i°$	141.85767	±0.00014	141.85465	±0.00018	141.85770	141.85763
q (a.u.)	0.00778521	±0.00000020	0.00777780	±0.00000027	0.00778572	0.00778606
e	0.99991511	±0.00000013	0.99992496	±0.00000020	0.99991521	0.99991528
z (a.u.$^{-1}$)	+0.010904	±0.000017	+0.009648	±0.000026	+0.010891	+0.010881
P (yr)	878.3	±2.1	1055.2	±4.3	879.9	881.0

TABLE V. Orbital elements for comet 1963 V (Epoch 1963 Oct. 18.0 ET).

	Present investigation		Sekanina (1967a)	
T (1963 Aug. ET)	23.95630	± 0.00491	23.95785	± 0.00620
ω°	86.1576	± 0.0335	86.1568	± 0.0413
Ω° 1950.0	7.2391	± 0.0489	7.2461	± 0.0604
i°	144.5757	± 0.0055	144.5765	± 0.0067
q (a.u.)	0.0050644	± 0.0000146	0.0050534	± 0.0000183
e	0.99994566	± 0.00000055	0.9999442	± 0.0000007
z (a.u.$^{-1}$)	+0.010729	± 0.000141	+0.011048	± 0.000138
P (yr)	900	±17	861	±16

TABLE VI. Orbital elements for comet 1945 VII.

Obs. (1945 Dec.)	11, 12, 13	12, 13, 14	13, 14, 15	11, 13, 15	11, 12, 13, 15	
T (1945 Dec. ET)	27.9603	27.9865	27.9834	27.9662	27.96524	±0.00104
ω°	65.977	84.887	56.481	72.658	72.0627	±0.8875
Ω° 1950.0	342.495	7.819	329.749	351.296	350.5027	±1.1777
i°	141.120	141.504	139.150	141.906	141.8669	±0.0591
q (a.u.)	0.007295	0.008481	0.006918	0.007550	0.0075159	±0.0000494

for the first set, but including the effect of general relativity. The mean errors are exactly the same as for the first set.

It is also of interest to study the orbit of comet 1965 VIII in the framework of the theory by Brans and Dicke (1961). Taking their parameter $\omega=6$, it follows that, to a high degree of approximation,

$$M=\tfrac{1}{8}\gamma r^{-3}(30\mu r^{-1}-7v^2), \qquad (15)$$
$$N=(15/4)\gamma\dot{r}r^{-2}.$$

In addition, in order to explain the anomalous motion of Mercury's perihelion completely, it is necessary to assume that the sun is an oblate spheroid with dynamical form factor $J_2\simeq 2.7\times 10^{-5}$. It has recently been claimed that such an oblateness has been directly observed (Dicke and Goldenberg 1967). The final orbit in Table IV results from the inclusion of the Brans–Dicke and the solar oblateness perturbations, and again the mean errors are the same as before.

It is not possible to choose between Einstein's general relativity and the Brans–Dicke theory on the basis of the observations of comet 1965 VIII. Both theories satisfy the observations equally well—as indeed does a completely Newtonian theory.

V. COMET 1963 V (1963*e* PEREYRA)

This member was discovered, also in Hydra, by Z. M. Pereyra at the Córdoba Observatory on 1963 September 14.4. At discovery the comet was of magnitude 2 and had a 12° tail, but it faded rather rapidly. The comet had been at perihelion on August 24.0 and could not have been picked up earlier. Positional measurements of the comet were made at a handful of observatories until late October, and then only by Elizabeth Roemer at the U. S. Naval Observatory's Flagstaff station until December 18.5. On November 9 this observer reported a possible secondary nucleus $0\overset{'}{.}1$ from the first (Roemer 1965).

The first set of orbital elements in Table V has been determined from 33 observations made between September 15 and December 18, the mean residual being $\pm 1\overset{''}{.}3$. Perturbations by all nine planets were included. For comparison we give also the orbit calculated by Sekanina (1967a), who used 24 observations during the same time interval and found the mean positional residual to be $\pm 2\overset{''}{.}8$.

VI. COMET 1945 VII (1945*f* OR *g* DU TOIT)

This comet was discovered by D. du Toit at the Boyden Observatory on 1945 December 11.0. It was a magnitude 7 object in Triangulum Australe. Five rough observations, made at the Boyden Observatory on five consecutive nights, were reported. Three orbits were determined by Cunningham (1946), who felt that the second of them, that given in Table I, was the most probable one. Unfortunately, no further observations were obtained, and the question as to whether the comet was indeed a member of the Kreutz group was open to some doubt.

The five Boyden plates were measured and reduced in 1952 by A. G. Mowbray. In Table VI we give a number of parabolic orbits, based on various combinations of three observations, and the residuals are given in Table VII. The dispersion of the results is considerably smaller than was found by Cunningham, and there can be little doubt now that this comet is a member of the sungrazing group.

It seems likely that the December 14 observation is inferior, and the final set of elements has been obtained by least-squares from the other four observations. The corresponding longitude and latitude of perihelion are $L=282\overset{\circ}{.}87$, $B=+35\overset{\circ}{.}98$, in good agreement with the more reliable of the figures in Table I. The in-

1967AJ.....72.1170M

TABLE VII. Residuals for comet 1945 VII.

Obs. (1945 Dec.)	11, 12, 13		12, 13, 14		13, 14, 15		11, 13, 15		11, 12, 13, 15	
1945 Dec.	$\Delta\alpha\cos\delta$	$\Delta\delta$	$\Delta\alpha\cos\delta$	$\Delta\delta$	$\Delta\alpha\cos\delta$	$\Delta\delta$	$\Delta\alpha\cos\delta$	$\Delta\delta$	$\Delta\alpha\cos\delta$	$\Delta\delta$
11.0	− 0″.4	+ 1″.3	−41″.1	− 4″.9	+160″.9	+54″.3	− 0″.9	+ 2″.9	−1″.9	+1″.2
12.0	0.0	0.0	− 1.0	+ 2.3	+ 65.7	+28.5	+ 6.0	+ 2.9	+4.0	+0.4
13.1	− 0.7	+ 1.2	0.0	0.0	+ 1.6	− 2.6	0.0	0.0	−1.7	−2.9
14.1	+27.8	+29.3	− 1.4	+ 2.0	0.0	0.0	+14.0	+15.9	...	...
15.1	+33.8	+38.4	−50.9	−45.7	+ 2.0	− 2.2	− 1.9	+ 2.3	−0.3	+1.2

dividual elements differ considerably from those given for this comet in Table I, and thus they do not conform with the empirical relations involving the position of Jupiter [Eq. (1)]. The new elements are rather similar to those of comets 1882 II and 1965 VIII.

VII. EARLIER MEMBERS OF THE KREUTZ GROUP

The earlier members and possible members of the sungrazing comet group were discussed so thoroughly by Kreutz that little can be added. A few remarks concerning the less certain members may be in order, however.

A. Comet 1668

The most careful observations of this comet were made by Gottignies at Goa daily from 1668 March 9.6 to 17.6 (UT). The elements in Table I are the set II by Kreutz (1901), and they fit the observations on March 9 and 17 exactly. The residuals of the observations on the intervening days show a strong systematic trend, to a maximum of 2°.4 on March 12, the sum of the squares of the residuals [*vv*] on the nine days being 21.5 (square degrees).

Kreutz also gives the residuals from the orbit of comet 1843 I for different assumed values of T. For the best choice ($T=1668$ March 1.43) the maximum residual is 2°.0 (on March 9) and [*vv*] is only 8.5. A similar computation using the orbit of comet 1882 II gives [*vv*]$=16.7$.

The orbit by Kreutz which satisfies these nine observations best is his set VI, for which the maximum residual is 0°.9 and [*vv*]$=4.2$. This is not a sungrazing orbit at all, for $q=0.30$ a.u.

B. Comet 1689

The observations and orbit of this comet are very uncertain indeed. The generally accepted orbit by Holetschek (1892) has $q=0.06$ a.u., but it is otherwise unlike those of the sungrazing group. An orbit by Peirce (1843) is quite similar to that of the not then discovered 1882 II, and even that calculated by Pingré (1784), long before any members of the group had been recognized, shows some resemblance. Kreutz felt it improbable that this comet belonged to the group, but noted that the orbit of comet 1882 II (with $T=1689$ December 2) seemed to agree with the observations more closely than did that of 1843 I.

C. Comet 1695

Kreutz also considered this comet not to be a member of the group. The most satisfactory observations were made from a ship in the Arabian Sea, and the best of these were made from 1695 November 6.1 to 17.1. The generally accepted elements (the set IIIa by Kreutz) leave residuals of 3°.8 on November 7 and 17 and a total [*vv*] of 37.6 for the observations during this 11-day interval.

The orbit of comet 1843 I, with $T=1695$ October 23.1, satisfies these observations rather better, with a maximum residual of 2°.1 and [*vv*]$=12.0$. This orbit leaves residuals of 6° for some of the observations during the previous week, however, whereas the largest residual from set IIIa is only some 3°. The fit from the orbit of 1882 II is less satisfactory.

D. Comet 1702*a*

The observations of this comet were so rough that no independent orbit has ever been calculated. The tail was seen in South Africa as early as February 20, although the only tolerably useful observations were obtained on February 27, 28 and March 2. Kreutz found that the observations could be represented reasonably well by both the 1843 I and 1882 II orbits, but that the latter gave a slightly better fit (with $T=1702$ February 15.1).

E. Tewfik

Kreutz found that the position obtained by Trépied (1882) was within 0°.07 of the path of comet 1843 I. Perihelion would have occurred some five hours later ($T=1882$ May 17.5). The tail was curved much as would be expected for a comet very close to the sun and rapidly approaching perihelion. The position determined by Abney and Schuster (1884) differs from Trépied's by some 0°.17 and is within 0°.01 of the 1843 I orbit. The deviations of the positions from the path of comet 1882 II are 0°.43 and 0°.35, respectively.

F. Comet 1887 I

This comet was first seen in South Africa on 1887 January 18.8, and some two dozen rough observations were made at various stations in the southern hemisphere between January 21.5 and 30.5. By February 1 the comet had completely disappeared. The comet was

TABLE VIII. Orbital elements for comet 1887 I.

	Oppenheim (1889)	Alternative orbit
T (1887 Jan. UT)	11.8377	10.591 ±0.198
ω°	65.367	69.16 ±1.98
Ω° 1950.0	340.512	314.84 ±0.91
i°	137.630	118.05 ±0.96
q (a.u.)	0.005485	0.04316±0.00642

TABLE IX. Members of the two subgroups (identified by the dates of perihelion passage).

Subgroup I	Subgroup II
1668 Mar. 1?	1689 Dec. 2??
1695 Oct. 23?	1702 Feb. 15??
1843 Feb. 27	1882 Sept. 17
1880 Jan. 28	1945 Dec. 27
1882 May 17?	1965 Oct. 21
1887 Jan. 11??	
1963 Aug. 23	

unusual in that it did not appear to have anything that could be interpreted as a head, and what positions were obtained are consequently very discordant. The orbital elements in Table I are the set C by Kreutz. They were obtained from four normal places, and for what appear to be the 14 most reliable observations $[vv]=0.48$ (square degrees). Set C seems to be rather better than set A, which is very similar to the orbit determined by Oppenheim (1889) and given in Table VIII. Oppenheim's orbit has $[vv]=0.80$, but the position of the perihelion ($L=282^{\circ}.17$, $B=+37^{\circ}.67$) agrees more closely with the figures for the other members of the group.

Both the Kreutz set C and Oppenheim's orbit indicate a possible systematic trend in the residuals on January 26 and 27, to 0°.2 for set C and to 0°.4 for Oppenheim's orbit. The second set of elements in Table VIII was calculated in order to smooth out this trend, and it satisfies six observations at Adelaide and the Cape from January 22 to 29 within 0°.08. In this case $L=263^{\circ}.83$, $B=+55^{\circ}.56$.

Attempts have also been made to fit orbits in which the direction of perihelion agrees with that of the sungrazing group. For the 14 most reliable observations the 1843 I orbit (with $T=1887$ January 11.82) gives $[vv]=4.1$. All the residuals from January 21 to 28 were between 0°.4 and 0°.7 and in the same general direction. The fits to the 1880 I and 1882 II orbits were worse (with $[vv]=5.3$ and 17.7, respectively).

Bearing in mind the fact that the observations of comet 1887 I should be better than those of the seventeenth century comets, we may conclude that there is more evidence for believing that this comet is *not* a member of the group than for any of the other comets discussed in this section.

VIII. DISCUSSION

Table I indicates that the longitudes of the ascending nodes of the orbits of the members of the sungrazing group are spread over a range of more than 40°. In Sec. II we implicitly assumed that the values of Ω were distributed fairly uniformly over this range. The five accurately determined orbits seem to fall into two distinct subgroups, in which the lines of nodes are separated by some 20°, and there are corresponding differences among the other elements. The results of Secs. VI and VII tend to confirm this separation into two subgroups, rather than that the nodes of the orbits are distributed at random. We tentatively assign the members and possible members to the two subgroups in the manner shown in Table IX. Presumably, the members of each subgroup are physically related more closely to each other than to the members of the other subgroup. The likelihood that there are in fact two subgroups has also been pointed out by Hasegawa (1966) and by Kresák (1966).

We have integrated the motion of nucleus A of comet 1965 VIII back over its most recent revolution about the sun. Owing to the perturbations by the planets, the actual period between the two perihelion passages was some 30 yr shorter than the osculating value of the period in 1965. For the time of the previous perihelion we obtained September 1116, March 1115 or March 1114, according to whether we started from the first, the third, or the fourth orbits in Table IV. Any additional forces that may have been acting at perihelion in 1965 (and presumably some would have been required to split the comet) were ironed out in the orbit calculations, so the true uncertainty in the time of the previous perihelion must be quite a bit larger than the two years one deduces from the mean error in Table IV.

Nevertheless, the above dates are remarkably close to February 1106, and the comet observed that month obviously had a very small perihelion distance. On February 4 (or 5) the comet was seen in Europe for most of the day as a bright star very close to the sun (some records even call it a comet). In Constantinople and Palestine on February 7 it was in the southwestern sky after sunset, with the tail stretching eastward to the south of Orion. This observation of the tail has been the principal evidence against the supposed identity with the 1680 comet. It shows that the 1106 comet was south of the ecliptic, whereas the 1680 comet would have passed to the north of the ecliptic very soon after perihelion and remained there until about a month before the following perihelion. Observations in the second half of February mention that the 1106 comet moved around to the west and northwest, with the tail stretching toward the northeast and visible until midnight. It remained in this general position for 25 days. This northern declination would, of course, preclude any association with the Kreutz group.

Ho Peng Yoke (1962) has summarized the oriental observations of the 1106 comet. In Korea the comet was

Table X. Comets 1882 II and 1965 VIII.

	1882 II in 1882 −1965 VIII in 1965	1882 II in 1115 −1965 VIII in 1115	1965 VIII nucleus A −1965 VIII nucleus B
$\Delta\omega^\circ$	+0.536	+0.013	+0.014
$\Delta\Omega^\circ$	+0.662	+0.019	+0.013
Δi°	+0.147	+0.004	+0.003
Δq (a.u.)	−0.0000345	+0.0000060	+0.0000074

Table XI. Comets 1843 I and 1963 V.

	1843 I in 1843 −1963 V in 1963	1843 I in 1057 −1963 V in 1057
$\Delta\omega^\circ$	−3.35	−1.62
$\Delta\Omega^\circ$	−4.21	−2.14
Δi°	−0.19	−0.14
Δq (a.u.)	+0.000425	+0.000371

seen first on February 9 in the southwestern sky, and it remained visible for over a month. The Japanese records also mention the comet first on February 9 and state that it had a tail 100 ft (degrees?) long. It appeared in Cetus, with its rays (tail) pointing eastward. During the following nights it moved eastward and its brightness gradually diminished. It disappeared after more than 30 days.

Neither of these records mentions any westward or northward motion, so there is some contradiction with the accounts mentioned in the previous paragraph. The *Anglo-Saxon Chronicle* speaks of the comet on February 16 as in the south*east*, with the tail pointing towards the northwest, and this must definitely be wrong.

The Chinese wrote that the comet was seen in the west on February 10. It was said to point obliquely to the northeast and to penetrate from the fifteenth lunar mansion (Andromeda, Pisces), through the sixteenth, seventeenth, and eighteenth, to the nineteenth (Taurus). The word "point" has been interpreted by some authorities as meaning "traverse," presumably over a period of weeks. It is likely that "point" is indeed meant, and that the position of the tail on the one night is being described. It would be impossible for the tail to pass through the lunar mansions mentioned, and the report presumably refers merely to the *longitudes* of these lunar mansions. Many of the Chinese cometary records specifically refer to the longitudes of the lunar mansions, much as the western records often gave just ecliptic longitudes, omitting the latitudes. Taken in conjunction with the Korean and Japanese records, as well as the observation on February 7, we deduce that the tail was considerably south of the lunar mansions. A tail pointing obliquely to the northeast could mean one generally in the southern part of the sky, but curving somewhat upward to the left.

Since the date derived for the previous perihelion passage of comet 1965 VIII is so close to 1106, it would be most desirable to study the original sources describing the 1106 comet in order to remove the discrepancies among the various observations. Perhaps the northern observations refer to an aurora? Of all the comets reported during the eleventh and twelfth centuries, this seems by far the most promising candidate for the previous appearance of comet 1965 VIII.

We have also traced back the motion of the principal nucleus of comet 1882 II. The nonrelativistic elements by Kreutz (1891) gave the previous perihelion passage as April 1138, while the relativistic elements by Hufnagel (1919) gave November 1149. Kreutz discussed the possibility that the 1106 comet was the previous appearance of comet 1882 II. He concluded that the daytime and nighttime observations in 1106 referred to two different objects, but that there was a good chance that the former was indeed comet 1882 II. Kreutz was unaware of the Korean and Japanese records. The calculated date of the previous perihelion is not in such good agreement as in the case of comet 1965 VIII. It is to be expected that the calculated date would be more uncertain for 1882 II. The observations of this comet were visual, and even the best ones were more discordant than the best photographic observations of comet 1965 VIII. Furthermore, the neglected nongravitational forces may have had a greater effect, for one of the subsidiary nuclei was on the sunward side of the principal nucleus, and the situation was further complicated by an anomalous extension towards the sun and a luminous "sheath" that seemed to surround the whole head of the comet.

We can investigate whether comets 1882 II and 1965 VIII could be fragments of the same comet, regardless as to whether that comet appeared in 1106 or not. In the first column of Table X are given the differences between the elements of the two comets at the time they were observed. By increasing the orbital eccentricity of 1882 II very slightly, to about 0.9999097, we can force the previous perihelion passage to coincide with that calculated for comet 1965 VIII. The second column in Table X gives the differences between the elements for the two comets at that time, the elements themselves being very similar to those in Table I. The very small differences are of precisely the same order as those between the individual nuclei of comet 1965 VIII, given in the third column. The differences depend only weakly on the time of perihelion passage, and essentially the same result would be obtained if one assumed any date during the first half of the twelfth century. The agreement between the elements is so close that we may regard it as virtually proven that the two comets were indeed one at their previous approach to the sun. The other members of subgroup II may also have originated at the same time—or perhaps at the next previous perihelion passage, which would have taken place about the fourth century A.D.

For the subgroup I comets the revolution period is tolerably well determined only in the case of comet

Table XII. Search ephemeris.

$t-T$ (days) r (a.u.)	−50 1.49	−40 1.28	−30 1.06	−20 0.80	−10 0.50	+10 0.50	+20 0.80	+30 1.06	+40 1.28	+50 1.49	$t-T$ r (a.u.)
Jan. 1	5 10 −52.2 0.93	4 31 −60.9 0.79	2.38 −71.4 0.67	21 58 −69.6 0.62	19 48 −48.4 0.68	17 39 −46.8 0.64	15 24 −69.4 0.56	10 24 −70.0 0.61	8 39 −57.9 0.73	8 04 −48.7 0.88	Jan. 1
11	4 16 −47.2 0.98	3 27 −53.0 0.84	1 52 −58.6 0.72	23 16 −57.2 0.67	21 04 −43.4 0.72	19 14 −47.1 0.61	19 26 −76.6 0.52	6 39 −77.1 0.57	6 52 −59.9 0.70	6 54 −49.3 0.85	11
21	3 43 −40.6 1.06	2 58 −44.1 0.92	1 46 −47.0 0.81	0 00 −46.0 0.74	22 05 −37.3 0.76	20 46 −43.8 0.61	23 12 −66.1 0.52	3 39 −65.8 0.58	5 15 −54.1 0.71	5 49 −45.4 0.86	21
31	3 27 −33.9 1.49	2 48 −36.0 1.02	1 52 −37.4 0.90	0 32 −36.6 0.82	22 53 −31.0 0.82	22 06 −37.6 0.62	0 31 −51.1 0.56	3 01 −50.3 0.63	4 23 −44.3 0.76	5 04 −38.7 0.92	31
Feb. 10	3 22 −28.0 1.28	2 49 −29.0 1.14	2 03 −29.6 1.01	0 59 −28.8 0.92	23 35 −25.0 0.89	23 10 −30.1 0.66	1 15 −37.9 0.63	2 58 −37.5 0.72	4 01 −34.3 0.86	4 39 −31.4 1.01	Feb. 10
20	3 23 −22.7 1.40	2 55 −23.2 1.26	2 16 −23.2 1.13	1 23 −22.3 1.02	0 11 −19.4 0.95	0 03 −22.6 0.71	1 48 −27.3 0.72	3 05 −27.4 0.83	3 55 −26.1 0.97	4 28 −24.5 1.12	20
Mar. 2	3 29 −18.3 1.53	3 04 −18.2 1.38	2 31 −17.9 1.24	1 47 −16.9 1.11	0 43 −14.4 1.02	0 47 −15.8 0.78	2 16 −19.1 0.82	3 17 −19.7 0.94	3 58 −19.4 1.09	4 26 −18.8 1.25	Mar. 2
12	3 37 −14.5 1.65	3 16 −14.1 1.50	2 48 −13.5 1.35	2 10 −12.3 1.21	1 14 −10.0 1.09	1 26 − 9.9 0.85	2 42 −12.6 0.93	3 32 −13.6 1.07	4 06 −14.0 1.22	4 29 −14.1 1.38	12
22	3.47 −11.2 1.77	3 29 −10.6 1.61	3 05 − 9.8 1.46	2 32 − 8.4 1.30	1 43 − 6.0 1.15	2 01 − 4.8 0.92	3 06 − 7.4 1.04	3 48 − 8.9 1.19	4 16 − 9.7 1.35	4 36 −10.2 1.52	22
Apr. 1	3 59 − 8.4 1.88	3 43 − 7.6 1.72	3 23 − 6.6 1.55	2 54 − 5.0 1.38	2 11 − 2.4 1.21	2 34 − 0.5 0.99	3 30 − 3.3 1.14	4 05 − 5.1 1.31	4 29 − 6.2 1.48	4 46 − 7.0 1.65	Apr. 1
11	4 13 − 6.0 1.99	3 59 − 5.1 1.82	3 41 − 3.9 1.64	3 16 − 2.2 1.46	2 38 + 0.7 1.27	3 06 + 3.1 1.07	3 54 0.0 1.24	4 23 − 2.0 1.42	4 43 − 3.3 1.60	4 58 − 4.3 1.78	11
21	4 27 − 4.1 2.08	4 15 − 3.0 1.91	3 59 − 1.7 1.72	3 38 + 0.3 1.53	3 06 + 3.3 1.31	3 36 + 6.0 1.14	4 17 + 2.7 1.33	4 42 + 0.5 1.53	4 58 − 1.1 1.71	5 11 − 2.2 1.89	21
May 1	4 41 − 2.4 2.16	4 31 − 1.3 1.98	4 18 + 0.2 1.79	4 00 + 2.3 1.58	3 33 + 5.6 1.36	4 06 + 8.4 1.20	4 40 + 4.8 1.42	5 00 + 2.4 1.62	5 14 + 0.8 1.82	5 24 − 0.5 2.00	May 1
11	4.57 − 1.1 2.23	4 48 + 0.1 2.05	···	···	···	4 35 +10.3 1.26	5 03 + 6.4 1.50	5 20 + 3.9 1.71	5 31 + 2.2 1.91	5 39 + 0.9 2.10	11
21	···	···	···	···	···	5 04 +11.6 1.32	5 26 + 7.6 1.56	5 39 + 5.1 1.79	5 48 + 3.3 1.99	5 54 + 1.9 2.18	21
31	···	···	···	···	···	5 32 +12.5 1.36	5 49 + 8.4 1.62	5 59 + 5.8 1.85	6 05 + 4.0 2.06	6 10 + 2.6 2.26	31
Aug. 9	7 21 − 1.7 2.23	7 27 − 0.6 2.04	···	···	···	···	···	···	···	···	Aug. 9
19	7 36 − 3.2 2.16	7 43 − 2.1 1.97	7 53 − 0.7 1.76	8 08 + 1.3 1.54	8 33 + 4.6 1.29	···	···	···	···	···	19

Table XII (*continued*).

$t-T$ (days) r (a.u.)	−50 1.49	−40 1.28	−30 1.06	−20 0.80	−10 0.50	+10 0.50	+20 0.80	+30 1.06	+40 1.28	+50 1.49	$t-T$ r (a.u.)
Aug. 29	7 51 − 4.9 2.08	8 00 − 4.0 1.89	8 12 − 2.8 1.69	8 30 − 0.9 1.48	9 00 + 2.1 1.24	...	...	...	8 40 − 1.7 2.06	8 31 − 2.6 2.25	Aug. 29
Sept. 8	8 04 − 7.0 1.99	8 15 − 6.3 1.80	8 30 − 5.2 1.61	8 52 − 3.6 1.40	9 28 − 0.9 1.19	9 58 + 1.0 1.37	9 29 − 1.2 1.60	9 10 − 2.6 1.80	8 56 − 3.6 2.00	8 45 − 4.4 2.18	Sept. 8
18	8 17 − 9.5 1.88	8 30 − 8.9 1.70	8 48 − 8.1 1.52	9 14 − 6.7 1.32	9 57 − 4.4 1.13	10 23 − 1.8 1.32	9 49 − 3.7 1.54	9 27 − 5.0 1.73	9 11 − 5.8 1.92	8 59 − 6.4 2.09	18
28	8 29 −12.3 1.77	8 44 −12.0 1.59	9 05 −11.4 1.42	9 36 −10.4 1.23	10 26 − 8.3 1.06	10 49 − 5.0 1.27	10 10 − 6.6 1.46	9 45 − 7.6 1.64	9 26 − 8.3 1.82	9 12 − 8.7 1.99	28
Oct. 8	8 39 −15.6 1.65	8 57 −15.6 1.48	9 22 −15.4 1.31	9 59 −14.7 1.14	10 57 −12.9 0.99	11 16 − 8.5 1.21	10 31 − 9.8 1.38	10 02 −10.6 1.55	9 40 −11.0 1.72	9 24 −11.3 1.88	Oct. 8
18	8 47 −19.5 1.53	9 08 −19.9 1.36	9 38 −20.1 1.20	10 22 −19.9 1.04	11 30 −17.9 0.93	11 43 −12.2 1.14	10 52 −13.5 1.29	10 18 −14.0 1.45	9 53 −14.2 1.60	9 34 −14.3 1.76	18
28	8 52 −24.0 1.40	9 17 −24.9 1.24	9 52 −25.8 1.08	10 46 −25.9 0.95	12 06 −23.6 0.86	12 13 −16.3 1.08	11 14 −17.5 1.19	10 34 −17.9 1.33	10 05 −17.8 1.48	9 43 −17.6 1.63	28
Nov. 7	8 52 −29.2 1.28	9 21 −30.9 1.12	10 05 −32.6 0.97	11 12 −33.3 0.85	12 48 −29.9 0.80	12 44 −20.8 1.00	11 36 −22.2 1.09	10 49 −22.3 1.22	10 15 −21.9 1.36	9 49 −21.4 1.50	Nov. 7
17	8 46 −35.3 1.49	9 20 −38.1 1.01	10 14 −41.0 0.87	11 42 −42.1 0.76	13 38 −36.5 0.74	13 19 −25.6 0.93	11 59 −27.7 0.98	11 02 −27.6 1.09	10 22 −26.7 1.22	9 52 −25.7 1.37	17
27	8 28 −41.8 1.05	9 07 −46.5 0.90	10.17 −51.3 0.77	12 19 −52.7 0.69	14 39 −42.9 0.70	13 58 −30.7 0.85	12 25 −34.2 0.88	11 14 −34.0 0.97	10 24 −32.4 1.09	9 50 −30.6 1.23	27
Dec. 7	7 54 −48.1 0.98	8 31 −55.3 0.83	10 01 −63.8 0.70	13 19 −64.9 0.63	15 55 −48.2 0.67	14 46 −36.0 0.78	12 54 −42.0 0.77	11 22 −41.8 0.85	10 20 −39.1 0.97	9 39 −36.0 1.11	Dec. 7
17	6 57 −52.7 0.93	7 15 −62.3 0.78	8 30 −76.1 0.66	15 51 −76.4 0.60	17 27 −51.0 0.66	15 44 −41.2 0.72	13 31 −51.6 0.68	11 22 −51.6 0.74	10 01 −46.7 0.86	9 16 −41.8 1.00	17
27	5 45 −53.5 0.92	5 22 −63.3 0.78	3 52 −77.0 0.66	20 38 −75.1 0.60	19 04 −50.1 0.67	16 57 −45.4 0.66	14 31 −63.2 0.59	10 59 −63.6 0.65	9 17 −54.6 0.76	8 34 −47.0 0.91	27

At each entry are given the right ascension (in hours and minutes, equinox 1950.0), the declination (in degrees, equinox 1950.0) and the geocentric distance (in a.u.). At $t-T=0$, the radius vector $r<0.01$ (a.u.), and the position is very close to the sun.

1963 V. Tracing back the first orbit in Table V, we find the date of the previous perihelion as July 1057. The orbit by Sekanina (1967a) indicates July 1101. The planetary perturbations cause the true period for this comet to be a few years longer than the osculating value in 1963. The 1106 comet can, of course, be invoked as a previous appearance of this comet; and indeed, it is by far the most suitable candidate, although the comet of November 1075 is another possibility. The previous perihelion could not have been between 1049 and 1053, for example, because the perihelion distance then would have been smaller than the solar radius.

Although the direct determination of the orbit of comet 1843 I yielded $P=512$ yr, Kreutz also gives an orbit in which the period was assumed to be 800 yr, and he remarks that such a value does no great injustice to the observations. By adjusting the eccentricity a little more, to about 0.9999357, we can arrange that the date of the previous perihelion agrees with the July 1057 date obtained for comet 1963 V. In Table XI are given the differences corresponding to those in Table X for comets 1882 II and 1965 VIII. In this case, it does not seem to be possible to reconcile the differences with breakup in the manner described. The differences between the values of Ω, for example, are certainly diminished, and one might surmise that the two comets originated in a breakup at the next previous perihelion passage, although the perihelion distance of comet 1963 V seems to be persistently smaller than that of comet 1843 I. The other members of subgroup I presumably originated at one or the other of these passages.

The final problem is to explain the differences between the orbits of the two subgroups. This may obviously be done by supposing that, following a breakup of the primordial comet at some perihelion passage at least five revolutions ago, and probably between 10 and 20 revolutions ago, only two of the fragments consistently survived. By the cumulative action of the planets, the lines of nodes and perihelion distances of the two fragments differed more and more until, two revolutions ago at the most, the two fragments could become the parents of the several members of the two subgroups.

This picture is not too greatly different from that suggested in Sec. II. We have strictly adhered to points (2) and (4) of that section. Point (3) has been dismissed as fortuitous, and point (1) must be largely fortuitous too. We have made use of the further point, considered there, that only the more substantial fragments of a split comet seem to survive for a long time. Except for comet 1882 II, all the members of the Kreutz group seem to have faded out surprisingly quickly. It may well happen that only the two principal nuclei of 1882 II, and possibly also comet 1843 I, will ever return.

IX. SEARCHES FOR FURTHER MEMBERS

It is obviously desirable that as many of these sungrazing comets be observed as possible. Table XII gives a perennial ephemeris for members of the group within 50 days of perihelion at any time. Any member should be within some 3° of the positions tabulated. Needless to say, it would be preferable to hunt for members on the inward branch, but the outward branch should also be searched for comets that may have been too faint on the way in. One should particularly search the post-perihelion region in September, for comets found there would have been too close to the sun in angular distance for observation before perihelion.

A total solar eclipse provides an excellent opportunity for picking up sungrazing comets. Comets have been found at four of the 65 or so eclipses during the past century (at two of the last 13 eclipses), although three of them could not have belonged to the Kreutz group.

X. SPECULATION

The writer feels that the simple process described in Sec. VIII for the evolution of the Kreutz group of comets is the most probable one. Harwit (1967) has discussed how the progenitor could have come to have a revolution period of only some 1000 yr. We do not agree with his remark that the differences among the known members of the group cannot be explained by planetary perturbations. Sekanina (1967b) has attempted to explain the comets in terms of the collision of two protocomets at a distance of about 1 a.u. from the sun. Again, the present writer feels that this process is unnecessarily complicated.

The velocities of separation of the nuclei of comets 1882 II and 1965 VIII were not more than some 15 m/sec and possibly even as small as 1 m/sec. Such velocity differences seem to be typical of all comets that have split, even at great distances from the sun (Stefanik 1966). One could always explain the differences between the orbits of the subgroups by a separation at this velocity near aphelion. Although most of the comets observed to split have done so for no obvious reason, one really does require an explanation when the velocity of separation is some 20% of the velocity of the comet itself! A collision with some asteroidal object at 200 a.u. from the sun and 100 a.u. above the ecliptic plane, even though it would only have to happen once, is scarcely worthy of serious consideration.

In order to explain the two subgroups by a direct separation at perihelion, the velocity differences would again have to be some 20% of the comet's velocity, amounting to about 100 km/sec. In October and November 1882 several astronomers reported seeing cometlike objects some degrees from the main comet 1882 II. It is probable that they had separated from the main comet in some manner. The first and best documented account of such objects was by Schmidt (1882). If one supposes that the objects left the main comet at perihelion, they would have been moving in orbits with $e \simeq 1.09$, and the velocity of separation must have been about 5 km/sec. None of the comet clouds persisted for more than two or three days. But comet 1882 II passed relatively far from the sun's surface: A comet like 1963 V, or one which practically brushed by the sun, could have had material torn away in a much more violent manner, and some of this material might have persisted for a considerable time in the form of separate comets.

Machinery such as this is too powerful merely for explaining the two subgroups, however. It should be reserved for such questions as the possible relation with other comets of small perihelion distance. Kresák (1957) discussed the distribution of the perihelion points and orbit poles of comets with $q < 0.5$ a.u. He found that, for $q < 0.25$ and omitting the Kreutz group, the perihelia cluster around $L = 276°$, $B = -25°$ (around $L = 278°$, $B = -2°$ if the Kreutz group is included). For $0.25 < q < 0.50$, they cluster around $L = 258°$, $B = +54°$. Is there perhaps a possibility that some of these comets are distantly related to the Kreutz group? Öpik (1966) has also made this suggestion. Comet 1887 I might indeed have to be explained in this way.

In particular, we have the 1680 comet, for which

$L = 271\overset{\circ}{.}31$, $B = -8\overset{\circ}{.}17$. As Kreutz remarked, the orbits of comets 1680 and 1882 II pass within 0.0005 a.u. of each other. As one final, wild, speculation, we suggest that, if the comet of -371 were indeed observed to split, then the situation must have been very much more violent than it was for comets 1882 II and 1965 VIII: Might not Ephorus perhaps have witnessed the separation of the 1680 comet from the Kreutz group?

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REFERENCES

Abney, W., and Schuster, A. 1884, *Phil. Trans. Roy. Soc. London* **175**, 262.
Andrews, A. D. 1965, *Intern. Astron. Union Circ.* No. 1937.
Botley, C. M. 1967, *Sky and Telescope* **33**, 84.
Brans, C., and Dicke, R. H. 1961, *Phys. Rev.* **124**, 925.
Cunningham, L. E. 1946, *Harvard Announc. Card* No. 733.
Dicke, R. H., and Goldenberg, H. M. 1967, *Phys. Rev. Letters* **18**, 313.
Geisler, P. A., and McVittie, G. C. 1965, *Astron. J.* **70**, 14.
Harwit, M. 1967, *Cornell-Sydney Univ. Astron. Cent. Rept.* No. 64.
Hasegawa, I. 1966, *The Heavens* (February), 31.
Holetschek, J. 1892, *Astron. Nachr.* **129**, 325.
Ho Peng Yoke. 1962, *Vistas in Astronomy*, A. Beer, Ed. (Pergamon Press, New York), Vol. 5.
Hufnagel, L. 1919, *Astron. Nachr.* **209**, 17.
Kirkwood, D. 1880, *Observatory* **3**, 590.
Kresák, L. 1957, *Contrib. Astron. Obs. Skalnaté Pleso* **2**, 19.
——. 1966, *Bull. Astron. Inst. Czech.* **17**, 188.
Kreutz, H. 1888, *Publ. Sternw. Kiel* No. 3.
——. 1891, *ibid.* No. 6.
——. 1901, *Astron. Abhandl.* **1**, No. 1.
Kustaanheimo, P., and Lehti, R. 1963, *Astron. J.* **68**, 392.
Lynn, W. T. 1888, *Observatory* **11**, 375.
Marsden, B. G. 1966, *Mem. Brit. Astron. Assoc.* **40**, No. 2.
Öpik, E. 1966, *Irish Astron. J.* **7**, 141.
Oppenheim, H. 1889, *Astron. Nachr.* **121**, 337.
Peirce, B. 1843, *ibid.* **20**, 396.
Pingré, A. G. 1784, *Cométographie* **2**, 102 (Paris).
Porter, J. G. 1961, *Mem. Brit. Astron. Assoc.* **39**, No. 3.
Roemer, E. 1965, *Astron. J.* **70**, 397.
Schmidt, J. F. J. 1882, *Astron. Nachr.* **103**, 305.
Sekanina, Z. 1966, *Intern. Astron. Union Circ.* No. 1949.
——. 1967a, *ibid.* No. 1993.
——. 1967b, *Publ. Astron. Inst. Charles Univ. Prague* (to be published).
Stefanik, R. P. 1966, *Mem. Soc. Roy. Sci. Liège* (Sér. 5) **12**, 29.
Trépied, C. 1882, *Compt. Rend.* **95**, 898.
Watson, G. N. 1922, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, Cambridge, England).
Wright, J. P. 1967, Paper presented at meeting of the American Astronomical Society, Williams Bay, Wisconsin.