

PROPERTIES OF SOME OLD PLANETARY NEBULAE

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ABSTRACT

Eighty-six objects identified on the original plates taken for the Palomar Sky Survey are provisionally classed as planetary nebulae. Most of these objects are large and faint and are probably at an advanced stage in their evolution as planetary nebulae. On the assumption that the nebulae are optically thin and all of the same mass, distances are calculated for them by a variation of the method used by Shklovsky. The galactic distribution of the nebulae studied suggests that they have a disk population, a result in agreement with earlier studies. Comparison of the magnitudes of the nebulae and their central stars leads to estimates of lower limits to the temperatures of the stars, and hence of lower limits to their bolometric corrections. Most of the stars have very high temperatures, and in general they seem to lie near the extreme left end of the horizontal branch on the Hertzsprung-Russell diagram. Many of the stars have dimensions of white dwarfs. The evidence for evolution of the stars as their nebular shells expand is critically examined.

I. INTRODUCTION

Among the many new objects discovered on the photographs taken for the *National Geographic Society-Palomar Observatory Sky Survey*, eighty-six provisionally classed as planetary nebulae are described here. A. G. Wilson first identified about half of these objects; the rest were found by R. G. Harrington, R. Minkowski, and the writer. Since the publication of a preliminary finding list of seventy-three of these nebulae (Abell 1955), several more have been discovered; more reliable positions and other data for all of the nebulae are given here. The present list of eighty-six objects includes two entries in the *Index Catalogue* that were not there identified as planetary nebulae, five objects in a list of new planetary nebulae that was subsequently published by Kohoutek (1962*b*, 1963), and at least one object that is a radio source and thus which may be a supernova remnant. All eighty-six objects are included in this investigation, however, and are described as "planetary nebulae," with cognizance of the fact that one or two of them may be improperly identified.

The new planetary nebulae comprise a class of objects of low surface brightness and large angular diameter. Except for a very few that are nearby but that are too heavily obscured to have been discovered on the earlier visual or photographic surveys, these new nebulae are intrinsically large and faint. Since planetary nebulae must decrease in surface brightness as they enlarge about their parent stars, these objects are probably at an advanced stage in their evolution as planetary nebulae.

The 48-inch Schmidt telescope, the camera used for the Sky Survey, is ideally suited for the detection of faint extended objects. Those planetary nebulae that lie north of $\delta = -33^\circ$, that have surface brightnesses brighter than 26.0 (photographic) or 24.5 (photored) mag/sec², and with angular diameters greater than 10'' are probably nearly completely identified. Objects smaller than 10'' in diameter appear too nearly stellar on the direct Schmidt photographs to be identified readily; those small nebulae with apparent surface brightness too low to be visible on the various objective prism surveys remain undiscovered.

II. OBSERVATIONAL DATA

a) *The Sky Survey*

Most of the observational data used in the analysis described here are obtained from the original blue and red photographs taken for the *National Geographic Society-Palomar*

Observatory Sky Survey. The Survey covers the sky from the north celestial pole to declination -27° on 879 pairs of plates—one blue-sensitive and one red-sensitive. Detailed descriptions of the Survey have been given elsewhere (Wilson 1952; California Institute of Technology 1954; Abell 1958). However, those features that are pertinent to the investigation of the planetary nebulae are summarized in Table 1. A “southern extension” of the regular survey covers the declination zone from -27° to -33° . Exposure times on plates in this zone are about 20 per cent less than for the rest of the Survey because of the greater sky brightness only 25° above the southern horizon at Palomar. The shorter exposures and greater atmospheric extinction result in the southern extension photographs having a limiting magnitude of 0.3 to 0.4 less than for the average of the main survey. Four of the eighty-six nebulae described here are in the southern extension zone.

TABLE 1
SALIENT FEATURES OF THE PALOMAR SKY SURVEY

	Blue	Red
Field.....	6°6×6°6 1 mm=67''	
Scale.....		
Limiting stellar magnitude....	21.1 (m_{vg})	20.0 (m_{vr})
Limiting surface brightness (mag/sec ²).....	>26.5	>25.0
Emulsion	103a-O	103a-E
Filter		Red Plexiglas
Spectral range	$\lambda\lambda$ 3500–5000	$\lambda\lambda$ 6200–6700
Approximate effective wave- length.....	λ 4050	λ 6440

b) Photometry

Projected photographic and photoreduced surface brightnesses of the various portions of the nebular images are estimated by comparing them with spots of various densities on film strips exposed with a sensitometer. The spots on the film strips have been calibrated by comparing them to the densities of extrafocal star images in Selected Areas 57 and 61 photographed with the 48-inch Schmidt telescope under identical conditions, on identical emulsions, and with identical exposure and development as for the Survey photographs. The density of a photographic image is sensitive to the contrast of the emulsion used and to the processing procedure, but the development procedure was so well standardized for the Survey plates that the extreme range of photographic contrast is only $1.5 < \gamma < 2.0$. The background density does vary somewhat among the Survey plates because of variations in sky brightness from night to night, ranging as much as 0.2 (diffuse) density units from the mean (and from the background densities on the extrafocal plates of Selected Areas 57 and 61 with which the step scales were calibrated). An elementary analysis shows, however, that this range in plate contrast and background density can lead to errors in surface brightness of at most 0.3 mag/sec². Since the characteristic curves for the emulsions used are available and the background densities are all known, one could, in principle, correct for the effects of contrast and background variations. The corrections would almost always be smaller than the statistical errors in the surface-brightness estimates, however, so they are not made.

The accuracy of the surface-brightness estimates does vary with the sizes and structures of the nebular images. The brightnesses of extended uniform objects can be determined

with considerable precision—easily to 0.1 mag/sec² if the densities are not too near the threshold. Over many of the nebulae, however, there are continuous fluctuations in surface brightness with amplitudes of one or two tenths of a magnitude, and some nebulae show considerable internal structure with different parts at slightly different brightnesses. Whenever the surface brightness over a nebula varies by as much as 0.3 or 0.4 mag/sec² the nebula is divided into sections, and the brightness of each section is estimated separately. A typical example is a nebula that appears as a homogeneous ring of one brightness with a central “hole” of somewhat lower brightness. Within a given section of a nebula, a surface-brightness estimate is expected to be reliable to 0.4 mag/sec² and to have a standard error of 0.1 mag/sec².

From the surface brightnesses of the various portions of a nebula, and from the projected area, in square seconds of arc, of each portion, the total or integrated magnitude is readily obtained. Sometimes, however, a nebula is visible only on the red plate, in which case only a lower limit to the blue magnitude can be determined. Portions of some other nebulae are too faint to record on one or both plates—for example, the “holes” in the centers of “ring” nebulae are often of undetectable surface brightness. In such objects, upper and lower limits to the nebular magnitudes are found. The fainter limits are based on the assumption that the invisible parts of a nebula give off no light at all, and the brighter on the assumption that the invisible parts have surface brightnesses at the limit of detection (26.5 mag/sec² in the blue, 25 mag/sec² in the red).

In sixty-five of the nebulae, the central stars are identified with varying degrees of certainty. The identification of a star as the actual nucleus is considered “almost certain” if it is at or near the center of the projected nebular image and is very blue. If a star is so faint that even a crude estimate of its color can not be obtained from the Survey plates, or if the star does not appear to be very blue (many of the nebulae and nuclei are highly reddened), an identification is still considered “almost certain” if the star is very symmetrically located in the nebular image and if there is no other star in the field that can reasonably be the nucleus. There were originally fifty-five such “almost certain” identifications, but subsequent photoelectric three-color observations have shown that at least one of these stars (Nebula 14) is probably a foreground object (see § IV). An additional ten stars have been provisionally identified as planetary nuclei, but their identification is classed only as “probable.”

The photographic and photoreduced magnitudes for the central stars are estimates made with a step scale or “flyspanker” calibrated against images of stars on Survey photographs containing Selected Area 57. Experience has shown that magnitudes obtained in this way from stellar images on the Survey plates, after correction for atmospheric extinction, have standard errors of 0.2 to 0.3 mag. In addition, photoelectric measures in *UBV* are given for thirty stars observed at the 60-inch and 100-inch telescopes at Mount Wilson. Photoelectric and photographic observations of stars in Selected Area 68 lead to the following approximate color equations relating the *UBV* system to the photographic-photoreduced system of the Sky Survey:

$$P = V + 1.2(B - V) - 0.3; \quad P - R = 1.8(B - V) - 0.4. \quad (1)$$

Photographic and photoreduced magnitudes obtained with equations (1) for the thirty stars observed photoelectrically yield additional calibration of the stellar step scale. The standard error in the magnitude of a star measured with the step scale is found to be under 0.3 mag., except for stars within a magnitude of the plate limit where the uncertainty is closer to 0.5 mag. The differences between the computed photographic and photoreduced magnitudes and those obtained with the step scale are as much as 0.5 mag., however, for a few of even the brighter stars. Part or all of these larger differences may

be due to an intrinsic variability of those central stars that is suggested by the photoelectric data (see § IV).

The photoelectric observations are, of course, reduced and corrected for extinction in the usual manner. All magnitudes obtained directly from the survey plates (both of nebulae and stars) are corrected for extinction by assuming that the extinction in magnitudes is $-0.25 \sec z \log T$, where z is the zenith distance at mid-exposure and T is the atmospheric extinction at the zenith. Pettit's (1940) data on the atmospheric transmission at the Mount Wilson zenith is assumed to be appropriate to Palomar; adopted values of T for the blue and red Survey plates are 0.735 and 0.925, respectively, appropriate to effective wavelengths of λ 4050 and λ 6440.

c) Observed Properties of the Nebulae and Stars

The observed properties of the nebulae and their central stars are summarized in Tables 2 and 3.

The positions given for the nebulae (in Table 2), accurate to $1'$, are those of the central stars when they can be identified; otherwise the positions refer to the centers of the nebular images, as determined by eye estimate. Where positions given here differ from those of the preliminary finding list (Abell 1955), these should be considered revisions. The galactic coordinates are on the International Astronomical Union's new system (l^{II} and b^{II}). The maximum and minimum diameters of the nebular images visible on the red Survey plates are given in seconds of arc. These diameters are accurate to about $4''$.

Morphological descriptions of the nebulae are given by letter codes. A capital letter is used to describe a nebula that is regular and symmetrical, while a lower-case letter denotes a nebula of the same general appearance but with marked deviation from perfect regularity or symmetry. The code is as follows: A or a denotes a homogeneous disk; B or b , a homogeneous ring; C or c , a ring with bright spots or regions; D or d , a ring of variable thickness; E or e , a ring with gaps (incomplete ring); and F or f , double or multiple rings. Nebulae whose appearances do not fit into these categories are described in the notes. Several of the nebular images in the red Sky Survey plates are reproduced in Figures 1, 2A, 2B, 2C, and 2D.

Table 3 summarizes the photometric data: the surface brightness of the brightest portion of each nebula in mag/sec^2 ; the integrated nebular magnitude; the (UBV) magnitude and colors, if available, of each central star, and the number, n , of photoelectric observations; and, if photoelectric observations of the star have not been obtained, its photographic and photored magnitudes, estimated with the calibrated flyspanker.

III. DERIVED CHARACTERISTICS OF THE NEBULAE

a) Method of Determination of Distances

The distance of each nebula is derived from its photometric and geometrical data by assuming a mass for the part of it that is ionized. The principle appears to have been applied first by Minkowski and Aller (1954) to estimate the distance to the Owl Nebula, and various variations of the idea have been applied extensively more recently by Shklovsky (1956), by Kohoutek (1960, 1961, 1962*a*), and by O'Dell (1962, 1963). The energy, L , emitted per second by the nebula in any particular recombination line is proportional to the product of the ion and electron densities (and hence to the square of the gas density) times the volume of the emitting region; that is,

$$L = \text{constant} \frac{M^2}{V}, \quad (2)$$

where M is the mass of the emitting gas and V is its volume. But L is also proportional to the product of the observed flux in the given line, f , and the square of the distance

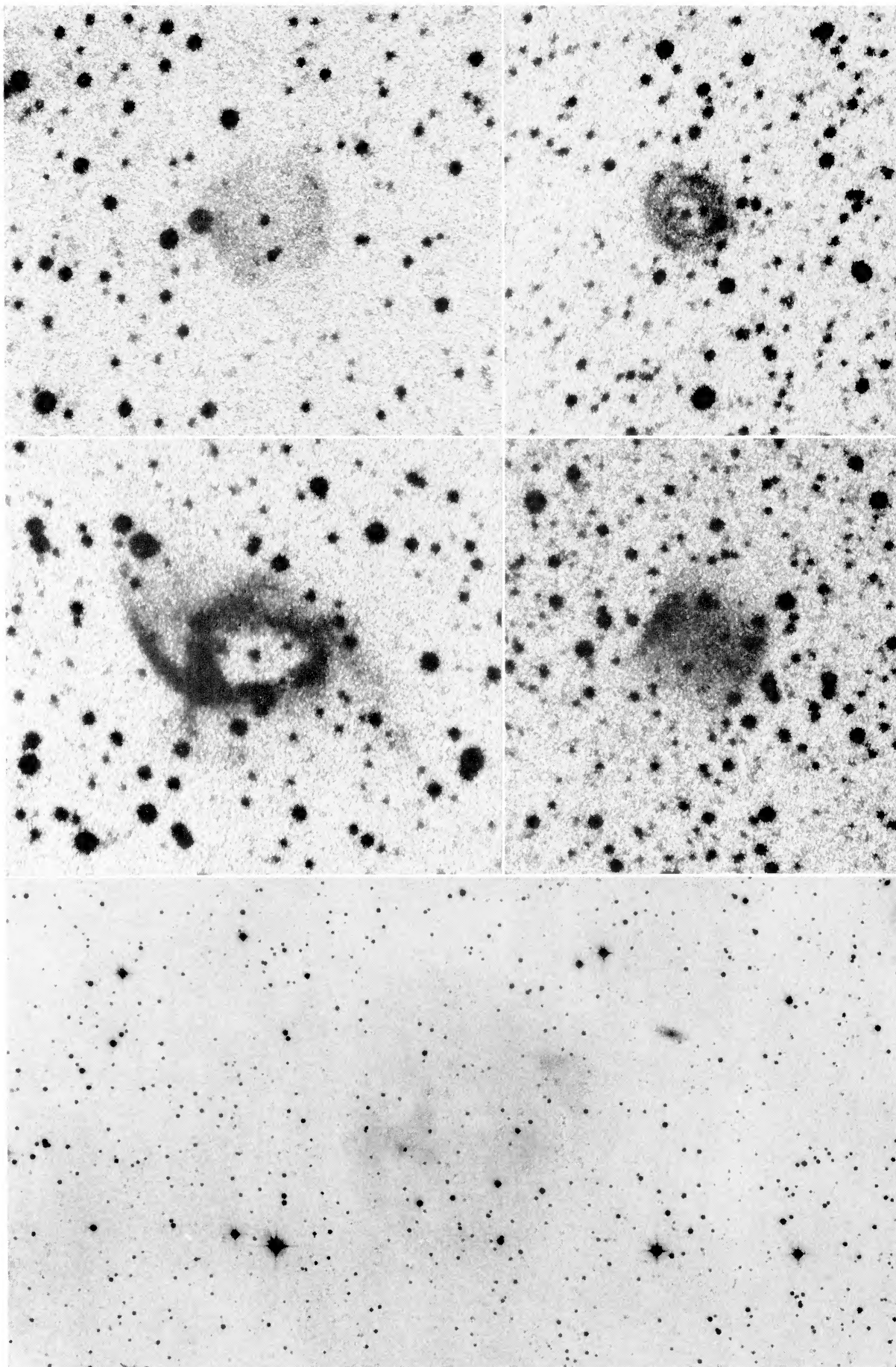


FIG. 1.—*Upper left:* Nebula 3 (scale, 1 mm = 3"0); *upper right:* Nebula 48 (1 mm = 3"0); *middle left:* Nebula 79 (1 mm = 2"5); *middle right:* Nebula 55 (1 mm = 2"9); *bottom:* Nebula 7 (1 mm = 17"). North at top, east to left in all reproductions. Source: National Geographic Society–Palomar Observatory Sky Survey.

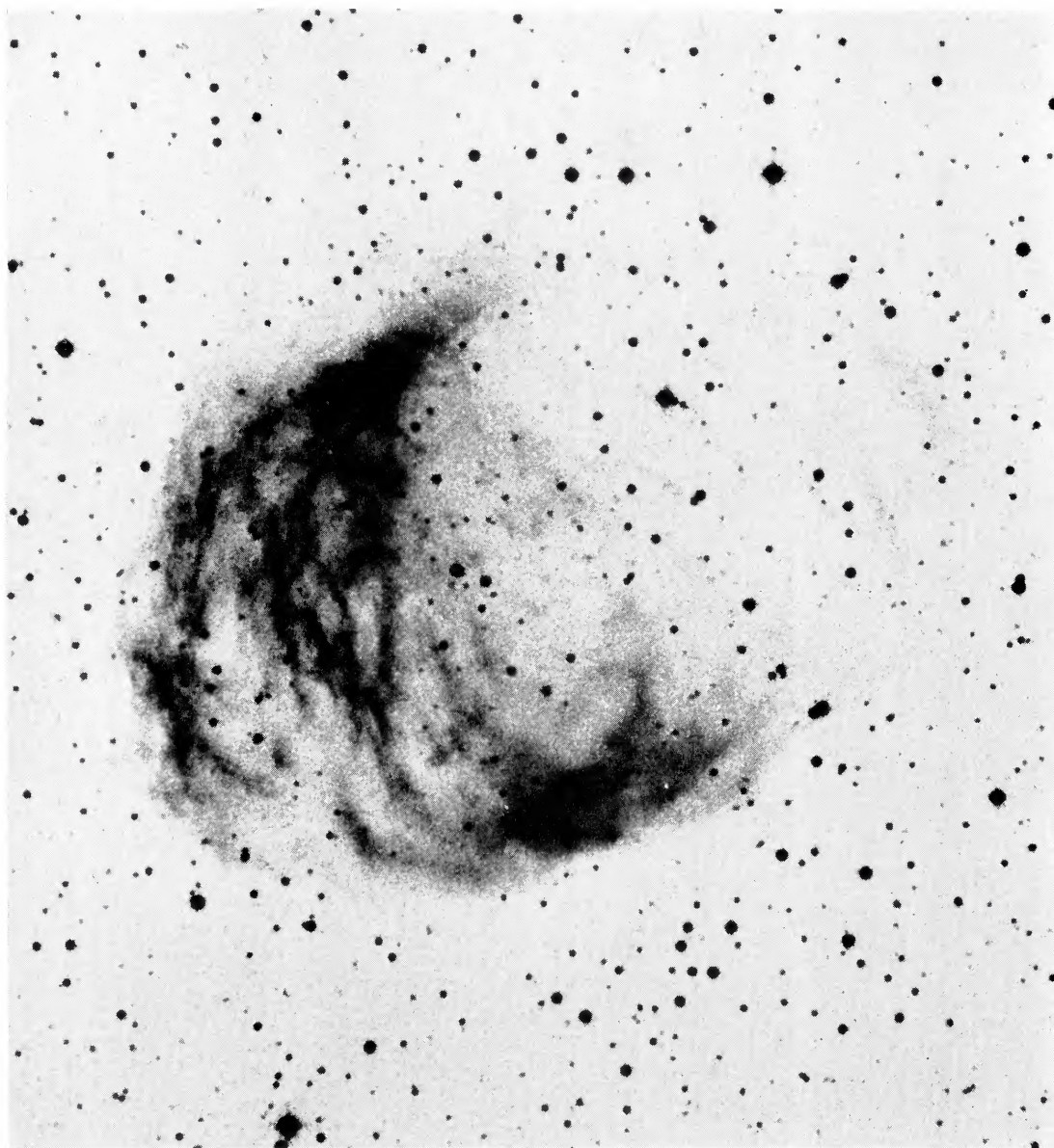


FIG. 2A.—Nebula 21. Scale, 1 mm = 7".2. Orientation and source as Fig. 1

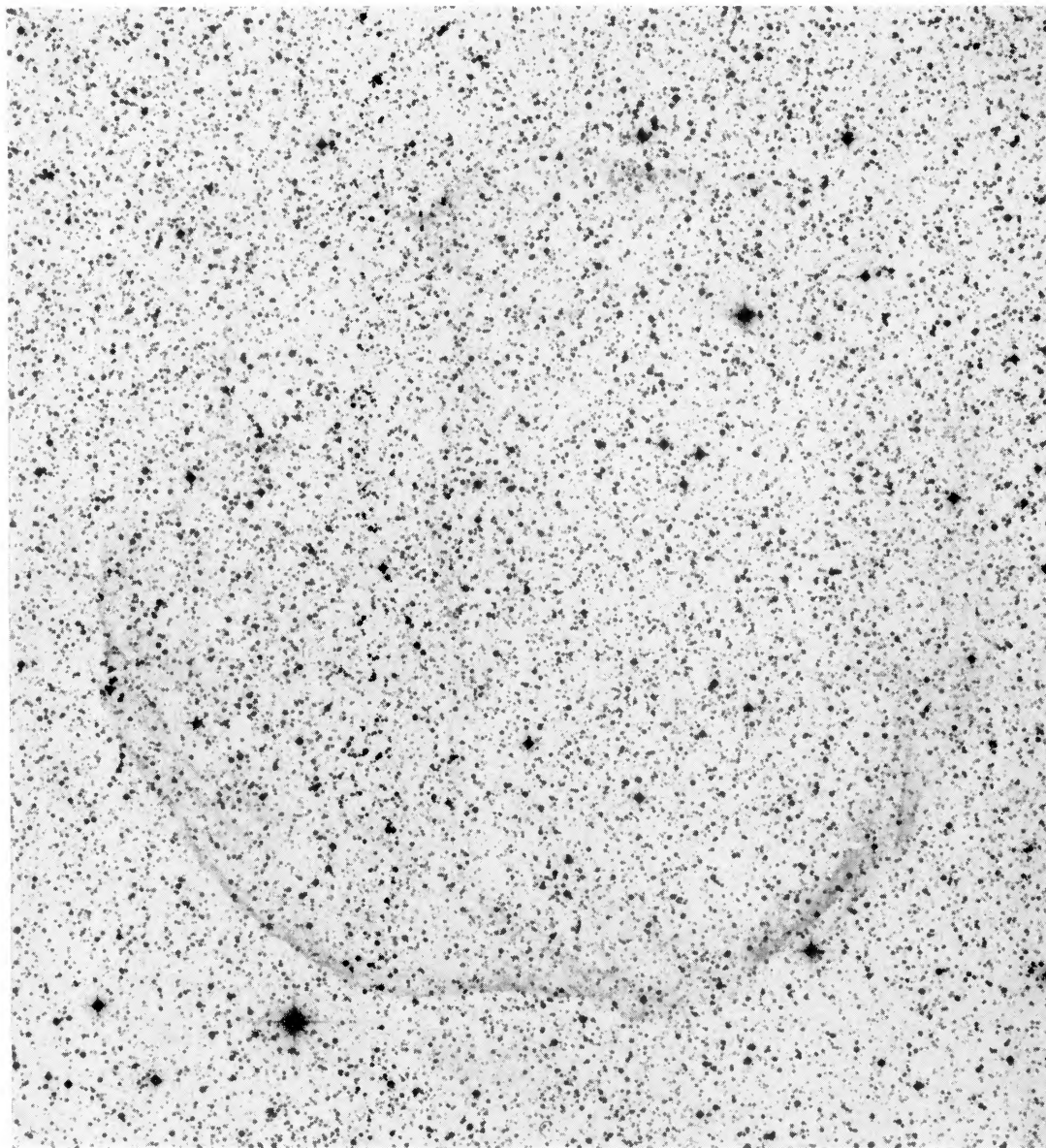


FIG. 2*B*.—Nebula 85. Scale, 1 mm = 19". Orientation and source as Fig. 1

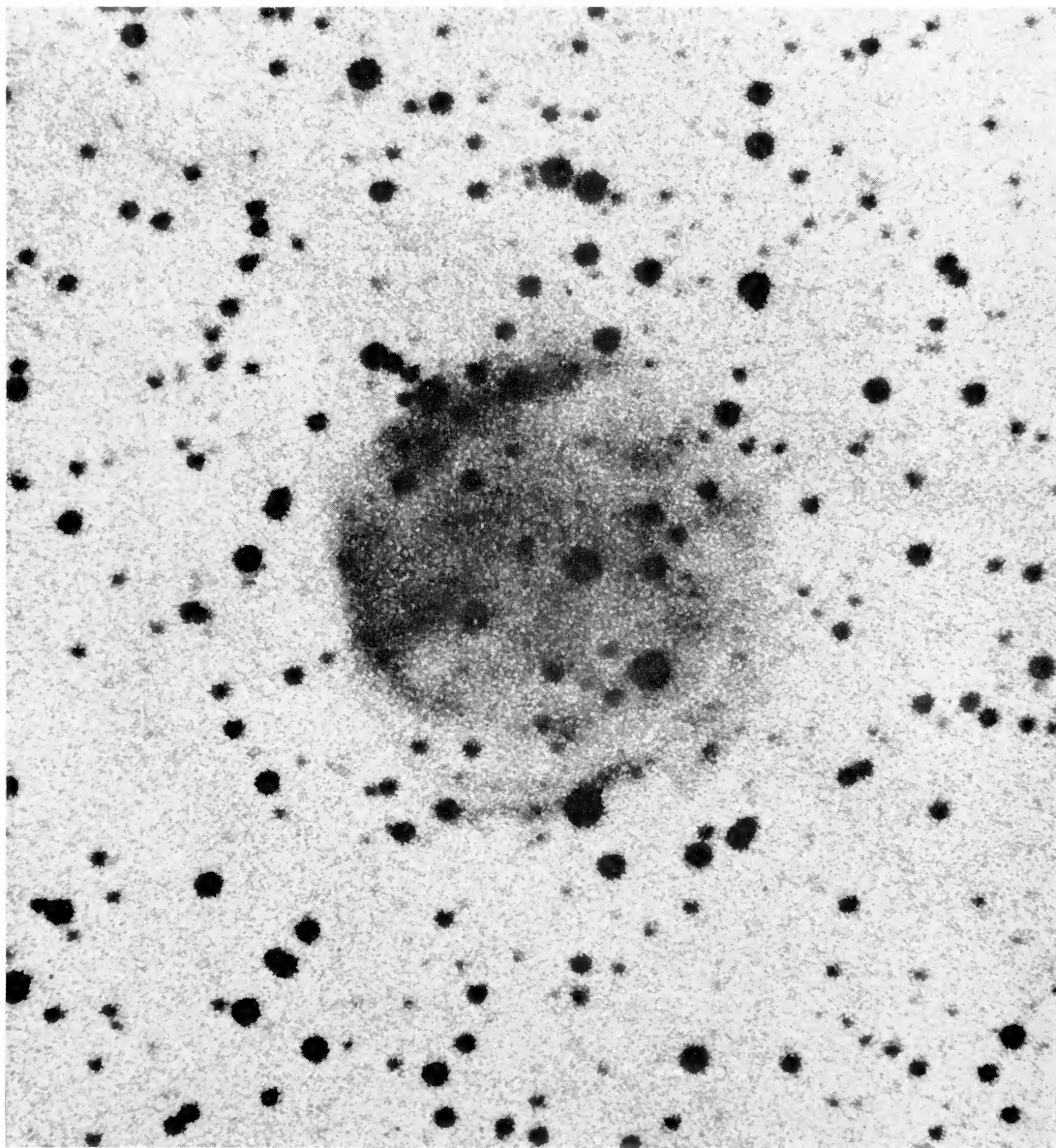


FIG. 2C.—Nebula 71. Scale, 1 mm = 2".8. Orientation and source as Fig. 1

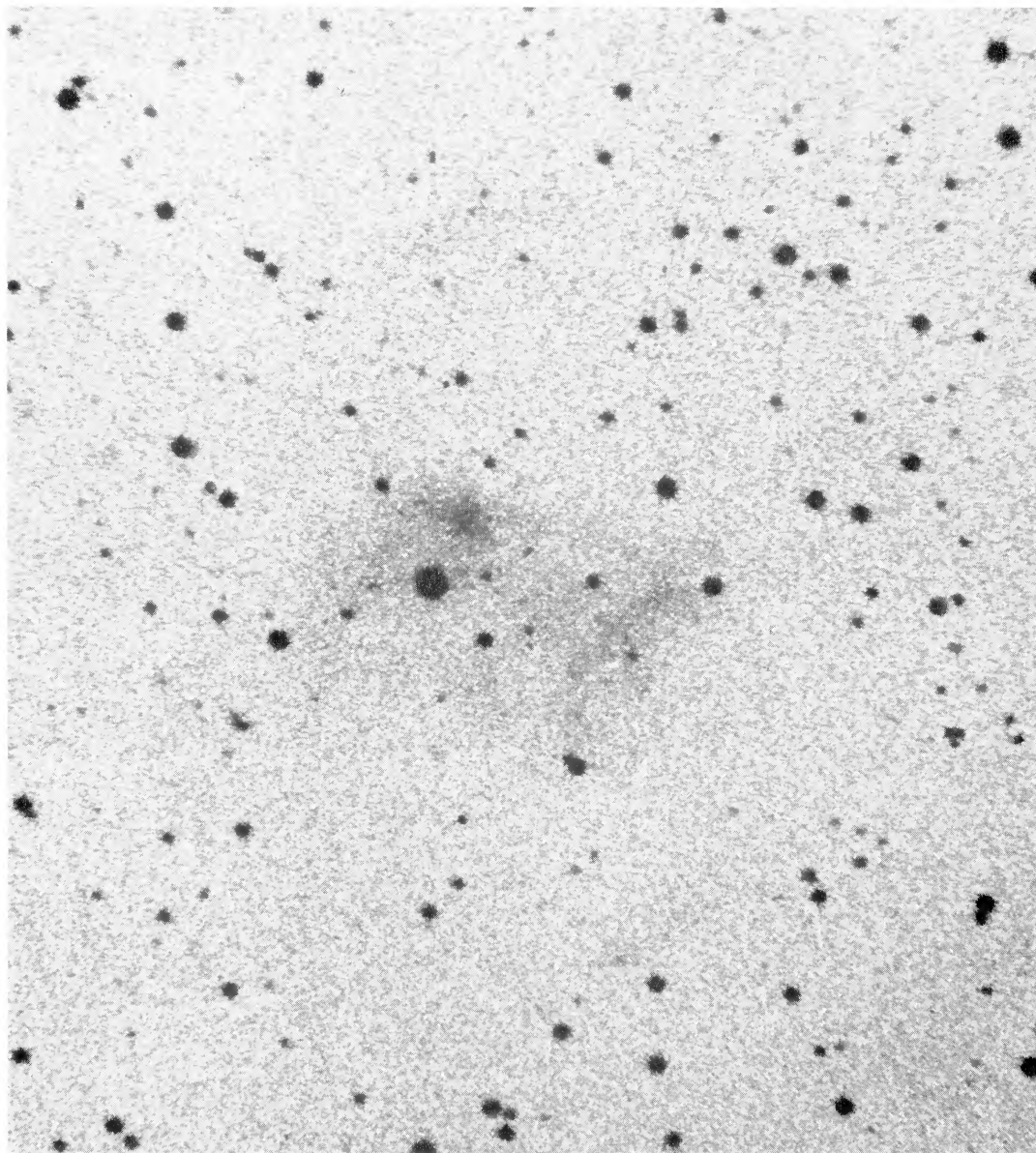


FIG. 2*D*.—Nebula 22. Scale, 1 mm = 2".9. Orientation and source as Fig. 1

TABLE 2
THE PLANETARY NEBULAE INVESTIGATED

No	$\alpha(1950)$	$\delta(1950)$	l^{II}	b^{II}	DIAMETER (sec)		DESCRIP- TION	NOTES
					Max	Min		
1	0 ^h 9 ^m 9	+68°54'	119°5	+ 6°6	47	47	<i>a</i>	
2	0 42 7	+57 41	122 2	- 4 9	32	28	<i>B</i>	
3	2 8 4	+63 55	131 6	+ 2 6	60	60	<i>D</i>	
4	2 42 2	+42 20	144 3	-15 5	20	20	<i>B</i>	
5	2 48 8	+50 24	141 7	- 7 8	134	121	<i>e</i>	
6	2 54 7	+64 18	136 1	+ 4 9	188	174	<i>D</i>	
7	5 0 9	-15 40	215 6	-30 8	871	670	<i>e</i>	
8	5 3 2	+39 4	167 0	- 1 0	60	60	<i>B</i>	
9	5 25 6	+36 0	172 1	+ 0 8	40	34	<i>D</i>	
10	5 29 1	+ 6 53	197 3	-14 2	34	34	.	1, 2
11	5 34 6	+ 8 14	196 8	-12 4	34	30	<i>d</i>	
12	5 59 6	+ 9 39	198 7	- 6 3	37	37	<i>b</i>	
13	6 2 2	+ 3 57	204 0	- 8 5	174	134	<i>d</i>	
14	6 8 4	+11 47	197 9	- 3 4	40	27	<i>D</i>	
15	6 25 0	-25 21	233 5	-16 3	34	34	<i>B</i>	
16	6 39 3	+61 50	153 8	+22 8	141	141	<i>A</i>	
17	6 46 2	- 9 29	221 1	- 5 0	54	34	<i>e</i>	
18	6 53 7	- 2 49	216 0	- 0 2	80	67	<i>e</i>	
19	6 57 1	+14 41	200 8	+ 8 4	67	67	<i>B</i>	
20	7 20 4	+ 1 52	215 0	+ 7 8	67	67	<i>B</i>	
21	7 26 2	+13 21	205 1	+14 2	744	509	.	3, 4
22	7 33 5	+ 2 49	215 6	+11 2	261	261	.	5, 6
23	7 41 4	-34 38	249 4	- 5 5	54	54	<i>D</i>	
24	7 49 1	+ 3 8	217 2	+14 8	375	335	<i>f</i>	
25	8 4 2	- 2 44	224 4	+15 3	188	147	<i>e</i>	7
26	8 7 0	-32 31	250 4	+ 0 1	40	40	<i>a</i>	
27	8 29 8	-31 55	252 6	+ 4 4	47	40	<i>a</i>	8
28	8 37 7	+58 24	158 8	+37 2	268	268	<i>B</i>	
29	8 38 0	-20 43	244 6	+12 6	482	335	<i>e</i>	
30	8 44 0	+18 4	208 6	+33 3	127	127	<i>B</i>	
31	8 51 5	+ 9 6	219 1	+31 3	1005	938	.	9
32	9 13 8	+ 4 6	227 5	+33 8	134	134	<i>A</i>	
33	9 36 6	- 2 34	238 0	+34 9	268	268	<i>a</i>	
34	9 43 2	-12 56	248 7	+29 6	281	268	<i>E</i>	
35	12 50 9	-22 36	303 6	+40 0	938	636	<i>a</i>	
36	13 37 9	-19 38	318 4	+41 5	478	281	.	10
37	14 1 7	-16 59	326 8	+42 2	47	47	<i>B</i>	11
38	16 20 1	-31 38	347 0	+12 5	154	94	<i>f</i>	12
39	16 25 5	+28 1	47 0	+42 5	174	174	<i>B</i>	
40	16 45 6	-20 56	359 2	+15 1	34	34	<i>B</i>	
41	17 26 2	-15 11	9 7	+10 5	20	17	<i>E</i>	
42	17 28 8	- 8 17	16 0	+13 6	60	60	<i>B</i>	
43	17 51 2	+10 38	36 1	+17 6	80	80	<i>F</i>	
44	18 27 2	-16 46	15 6	- 3 0	67	47	<i>e</i>	
45	18 27 5	-11 39	20 2	- 0 7	302	281	<i>b</i>	
46	18 29 2	+26 53	55 4	+16 0	67	60	<i>B</i>	
47	18 32 8	- 0 16	30 9	+ 3 5	17	15	<i>D</i>	
48	18 40 2	- 3 16	29 1	+ 0 4	40	40	<i>F</i>	
49	18 50 8	- 6 33	27 4	- 3 4	44	44	<i>B</i>	
50	18 58 0	+48 24	78 5	+18 7	27	27	<i>B</i>	
51	18 58 1	-18 17	17 6	-10 2	67	67	<i>B</i>	
52	19 2 3	+17 53	50 4	+ 5 3	37	37	<i>E</i>	
53	19 4 3	+ 6 19	40 4	- 0 5	31	31	<i>B</i>	
54	19 6 6	+22 54	55 4	+ 6 7	67	47	<i>E</i>	
55	19 7 9	- 2 26	33 0	- 5 3	54	54	.	13
56	19 10 6	+ 2 48	38 0	- 3 5	188	174	<i>C</i>	
57	19 15 0	+25 32	58 6	+ 6 2	40	34	.	14
58	19 15 8	+ 1 42	37 6	- 5 2	44	36	<i>D</i>	

TABLE 2—Continued

No.	$\alpha(1950)$	$\delta(1950)$	μ^{II}	δ^{II}	DIAMETER (sec)		DESCRIP- TION	NOTES
					Max.	Min.		
59	19 ^h 16 ^m 5	+19°28'	53°4	+ 3°0	94	80	<i>e</i>	
60	19 16 5	−12 22	25 0	−11 7	74	74	<i>E</i>	
61	19 17 7	+46 9	77 7	+14 8	201	201	<i>E</i>	
62	19 31 0	+10 31	47 2	− 4 3	161	161	<i>E</i>	
63	19 39 9	+16 58	53 9	− 3 0	40	40	<i>A</i>	
64	19 43 1	+ 5 27	44 2	− 9 4	40	34	<i>E</i>	
65	19 43 6	−23 16	17 3	−22 0	134	74	<i>a</i>	
66	19 54 6	−21 45	19 8	−23 8	295	241	...	15
67	19 55 9	+ 2 55	43 5	−13 4	67	67	<i>C</i>	
68	19 58 0	+21 35	60 0	− 4 3	40	37	<i>D</i>	
69	20 18 1	+38 15	76 4	+ 1 2	23	20	<i>C</i>	
70	20 28 9	− 7 16	38 1	−25 5	44	40	<i>c</i>	
71	20 30 8	+47 11	85 0	+ 4 5	168	147	<i>a</i>	
72	20 47 7	+13 22	59 8	−18 7	134	121	<i>d</i>	
73	20 55 2	+57 15	95 3	+ 7 8	80	67	<i>C</i>	
74	21 14 6	+23 57	72 7	−17 1	871	791	<i>a</i>	
75	21 25 2	+62 40	101 8	+ 8 7	67	47	<i>f</i>	
76	21 27 5	− 3 1	50 8	−36 1	13	13	<i>E</i>	
77	21 30 6	+55 40	97 5	+ 3 2	80	54	...	16
78	21 33 4	+31 28	81 3	−14 9	107	107	<i>f</i>	
79	22 24 4	+54 35	103 0	− 2 3	121	60	...	17
80	22 32 7	+52 11	102 8	− 5 0	161	114	<i>c</i>	
81	22 42 2	+80 11	117 6	+19 0	34	34	<i>B</i>	18
82	23 43 4	+56 47	114 1	− 4 7	94	94	<i>a</i>	
83	23 44 3	+54 28	113 6	− 7 0	54	54	<i>B</i>	
84	23 45 2	+51 7	112 9	−10 2	147	114	<i>c</i>	
85	23 56 6	+62 11	116 9	+ 0 2	2077	2077	<i>c</i>	19
86	23 59 0	+70 26	118 8	+ 8 2	70	70	<i>a</i>	

NOTES TO TABLE 2

1. K1-7 (Kohoutek 1963).
2. Nebula appears as a disk with bright stripes.
3. Listed as probable supernova remnant by Vorontsov-Velyaminov (1961).
4. Appears as overlapping rings and filaments.
5. K1-11 (Kohoutek 1963).
6. Appears as a small parallelogram surrounded by a large ring.
7. K1-13 (Kohoutek 1963).
8. K1-1 (Kohoutek 1962*b*).
9. Appears as a disk with superposed filaments.
10. Appears as a disk with extended filaments.
11. IC 972.
12. K1-3 (Kohoutek 1962*b*).
13. Appears as a rectangle with bright edges.
14. Appears as a disk with several sections of different brightness and extended filaments.
15. Appears as a ring with an extended filament.
16. Appears as an ellipse with several sections of different brightness.
17. Appears as a ring with extended filaments and luminous areas.
18. IC 1454.
19. The center of the extended radio source CTB-1 (Wilson and Bolton 1960) is 0°4 east of the nebula; it may be a supernova remnant.

TABLE 3
PHOTOMETRIC DATA FOR THE NEBULAE

No.	SURFACE BR. (mag/sec ²)		INTEGRATED NEB. MAG.		CENTRAL STARS						NOTES
	<i>m</i> _{pg}	<i>m</i> _{pr}	<i>m</i> _{pg}	<i>m</i> _{pr}	Photoelectric				Photographic		
					<i>V</i>	<i>B</i> − <i>V</i>	<i>U</i> − <i>B</i>	<i>n</i>	<i>m</i> _{pg}	<i>m</i> _{pr}	
1...	25 6	22 2	18 1–18 5	14 7	.	.	...	.	19 9	19 2	1
2 ..	23 3	21 2	16.3	14 1	. .	.	.	.	19 8	19 6	
3 ..	26 7	21 8	18 0–18 4	13 3	.	.	.	.	18 1	16 6	
4 ..	22 7	20 4	16 7	14 3	.	.	.	.	19 4	>20	2
5 ..	>26 5	22 3	>16 0	12 7	.	.	.	.	20 3	19 2	1
6 ..	>26 5	22 7	>15.5	12 6	.	.	.	.	19 0	17 9	
7 ..	>26 5	23 8	12 2–14 3	10 4–10 5	15 45	−0 33	−1 35	4	.	.	
8 ..	24 8	21 4	16 6	14 0	.	.	.	.	19 7	>19 6	2
9 ..	>26 5	22 7	>18 9	15.3	.	.	.	.	20 8	>20	1
10 ..	22 6	19 9	15 2	12 7	.	.	.	.	19 5	.	2
11...	24 0	22 6	17 1	15 7	.	.	.	.	.	.	3
12 ..	21 5	≤19 2	13 9	≤11 7	.	.	.	.	19 1	17 7	2
13 ..	>26 5	21 3	>16 0	11 5	19 87	+ 19	−1 00	2	.	.	4
14 ..	>26 5	21 2	>18 2	14 0	15 24	+ 51	−0 01	5	.	.	5
15 ..	23 6	21 4	16 3	14 1	15 72	− 31	−1 24	2	.	.	
16 ..	26 4	23 3	15 9	12 8	.	.	.	.	17 7	18 4	
17 ..	>26 5	22 5	>18 5	14 8	.	.	.	.	19 9	18 4	
18 ..	26 1	21 5	17.5	12 8	.	.	.	.	.	.	3
19 ..	25 5	23 3	16 9–17 1	14 8	.	.	.	.	.	.	3
20 ..	25 0	22 3	16 3	13 6	16 56	− 27	−1 12	4	.	.	
21 ..	24 3	20 5	11 2–11 4	7 7	15 99	− 32	−1 26	4	.	.	
22 ..	24 7	20 8	14 6–16 2	12 2	19 59	− 35	−1 17	1	.	.	
23 ..	.	21 3	.	13 1	.	.	.	.	.	.	3, 6
24 ..	25 7	21 1	13 6	9 3	17 18	− 21	−1 15	1	.	.	
25 ..	25 4	22 5	15 4	13 0	18 94	− 14	−1 15	2	.	.	
26 ..	25 9	22 1	18 1	14 3	.	.	.	.	.	.	3
27..	>26 4	20 4	>17 9	13.0	.	.	.	.	.	.	3
28 ..	>26 5	24 7	>14.6	13 0–13 7	.	.	.	.	15 9	16 6	
29 ..	26 4	22 2	13 6–15 0	10 5–10 7	18 35	− 23	−1 08	1	.	.	
30 ..	25 6	23 5	15.6	13 5	14 30	− 07	−1 03	4	.	.	
31 ...	26 0	23 4	11 6–12 7	9 4	15 51	− 31	−1 28	3	.	.	
32 ..	>26 5	24 2	>16 1	13 8	.	.	.	.	19 4	18 4	1, 2
33 ..	25 3	23 0	13 4	11 1	15 54	− 16	−1 15	2	.	.	
34 ..	25 0	22 2	14 5	12 0	16 32	− 26	−1 02	2	.	.	
35 ..	25 6	22 9	12 0	9 7	.	.	.	.	.	.	3
36 ..	25 0	22 8	13 0	10 6	11 51	− 33	−1 23	4	.	.	
37 ...	22 8	20 6	14 9	12 7	.	.	.	.	18 0	17 1	
38 ..	>26 4	21 4	>15 7	11 7	.	.	.	.	.	.	3
39 ..	24 3	22 7	13.7	12 1	15 60	− 33	−1 23	7	.	.	
40 ..	26 0	22 6	16 7–16 9	13 4	.	.	.	.	19 2	.	2
41 ..	22 6	19 5	17.2	13 9	15 95	+ 11	−0 87	3	.	.	
42 ..	26 3	23 1	17 7–17 9	14 6	.	.	.	.	19 8	.	2
43 ..	23 5	21 4	14 7	12 7	14 71	− 18	−1 12	8	.	.	
44 ..	25 3	20 6	17.4	12 6	.	.	.	.	.	.	3
45 ..	>26 5	23 3	>14 3	11 5	.	.	.	.	20 1	18 6	1
46 ..	24 3	21 9	15 6	13 2	15 07	− 19	−1 11	8	.	.	
47 ..	>26 5	21 6	>20 9	16 2	.	.	...	.	>20 3	19 2	2
48 ..	>26 5	20 6	>18 7	13 3	.	.	.	.	.	.	3
49 ..	24 6	21 1	16 7	13 2	.	.	.	.	.	.	3
50 ..	21 7	19 8	15 0	13 1	.	.	.	.	19 4	.	2
51 ..	24 2	21 8	15 4	13 0	15 42	−0 12	−1 04	4	.	.	
52 ..	23 6	21 5	16 5	13 6	.	.	.	.	17 7	17 4	
53 ..	24 0	19 5	16 9	12 3	.	.	.	.	20 3	19 6	2
54 ..	25 6	22 7	17 1	14 5	.	.	.	.	.	.	3
55 ..	23 8	20 3	15 4	12 0	.		...	..	19 9	19 2	1

TABLE 3—*Continued*

No	SURFACE BR (mag/sec) ²		INTEGRATED NEB MAG		CENTRAL STARS						NOTES
					Photoelectric				Photographic		
	<i>m</i> _{pg}	<i>m</i> _{pr}	<i>m</i> _{pg}	<i>m</i> _{pr}	<i>V</i>	<i>B</i> − <i>V</i>	<i>U</i> − <i>B</i>	<i>n</i>	<i>m</i> _{pg}	<i>m</i> _{pr}	
56	>26 5	22 2	>15 5	12 3–12 4				..	..	..	3
57	24 4	21 8	17 5	14 8–14 9	17 66	+0 20	−0 89	2	..	..	3
58	>26 5	22 2	>18.8	14 7				..	..	..	3
59	25 5	20 3	16 8–17 7	12 2				..	18 0	19 2	3
60	24 6	22 7	15 7	13 7				..	..	..	4
61	25 4	22 6	14 4	12 2	17 39	− 34	−1 23	2	..	..	3
62	25 2	21 6	14 8	10 3				..	..	..	3
63	24 8	21 8	17 1	14 0	14 67	+ 38	−0 70	6	..	..	2
64	22 3	20 6	15 3	13 3				..	18 1	16 9	3
65	24 9	22 4	15.2	12 7	15 90	+ 09	−0 98	4	..	..	2
66	25 9	22 4	14 7–15 1	11 5	17 39	+ 32	−0 87	4	..	..	3
67	24 5	22 4	16 0	14 2				..	19 1		4
68	23 9	21 1	16 6	13 8				..	19 6	19 0	3
69	>26 5	20 0	>20 1	14 3				..	18 6	17 4	4
70	20 6	19 5	14.3	12 7				..	..	..	3
71	25 5	20 6	15 1–15 2	10 2	18 95	+ 37	−0 66	3	..	..	3
72	24 3	22 5	14 6	12 8	16 12	− 33	−1 24	6	..	..	2
73	>26 5	22 2	>17 4	13 7				..	..	..	3
74	>26 5	24 3	>12.2	10 0	17 11	− 20	−1 19	5	..	..	3
75	24 3	21 4	17 0–17 1	14 2				..	17 2	17 2	3
76	20 9	19 7	15.6	14 3				..	16 3	15 4	3
77	24 3	19 6	16 3–16 5	11 5				..	..	..	3
78	23 9	22 4	16 0–16 1	14 3	13 25	− 21	−1 18	6	..	..	2
79	24 3	19 6	15 6–15 9	11 1				..	18 4	16 3	2
80	25 2	20 7	15 2	11 3				..	19 7	19 6	1, 2
81	21 9	19 7	14 8	12 6				..	18 6	..	2
82	24 4	21 3	15 2	12 0				..	..	..	3
83	25 9	23 2	17 6	14 9				..	>19 7	..	2
84	23 7	20 3	14.4	11 0	18 49	+0 18	−1 00	2	..	..	3
85	25 7	22 4	10 1–12 5	7 6				..	..	..	3
86	25 4	22 1	16 7	13 8				..	..	..	3

NOTES TO TABLE 3

1. Star identification is uncertain.
2. The color of the central star either cannot be obtained from the photographic observations or is especially uncertain.
3. The central star has not been identified.
4. The identification of the central star is probable but not certain.
5. The star near the center of the image of Nebula 14 has the colors of an F7 V star; it is probably a foreground object.
6. A red plate only is available

to the nebula. Therefore, L can be eliminated in equation (2), which results in the following expression for the distance:

$$r = \text{constant} \frac{M^{2/5}}{v^{1/5} f^{1/5}}, \quad (3)$$

where v is the "angular volume," that is, the ratio of V to the cube of r . It should be noted that the derived distance is rather insensitive to the assumed mass and is very insensitive to the observed values of v and f .

Available evidence suggests that a typical mass for a planetary nebula is about 0.2

solar mass (e.g., see Goldberg and Aller 1943; O'Dell 1962; Osterbrock 1964). Since the nebulae in the present study are all large and faint, they are probably optically thin—that is, completely ionized. Thus M in equation (3) was assigned the value 0.2 solar mass for each nebula. Of course the mass may vary considerably from one nebula to another. On the other hand, planetary nebulae have a “disk” distribution in the Galaxy (Minkowski and Abell 1963; Perek 1963) and thus belong to Population II. According to current ideas of stellar evolution, therefore, an extreme upper limit to the mass of a planetary nebula shell should be the order of 1 solar mass; even in this extreme, the distances derived with equation (3) would be underestimated by less than a factor of 2. If any of the planetaries is optically thick, or has a much smaller mass than the adopted value, its distance is overestimated; however, the most interesting derived properties of the nebulae and their central stars are based on upper limits to their distances (see § IV).

Because the spectral response of the red Survey plates is peaked very near $H\alpha$, the photored magnitudes of the nebulae are used to estimate their flux in $H\alpha$. The constant in equation (2), therefore, is evaluated directly for the first Balmer line. Various investigators (Searle 1958; Burgess 1958; Pengelly 1964) have calculated hydrogen line intensities in optically thin (case A) and optically thick (case B) interstellar gas. The analysis reported here was completed before the publication of Pengelly's most recent evaluation, and is based on the calculation of Burgess. He gives for the energy emitted in $H\alpha$ per unit volume and unit time from an optically thin nebula at an electron temperature of 10^4 °K, $2.0 \times 10^{-25} N_e N_p$ erg cm $^{-3}$ sec $^{-1}$. Pengelly's intensity for $H\alpha$ is greater than Burgess' by only 18 per cent for case A and 11 per cent for case B. Because the derived distances of the nebulae vary only with the fifth root of the adopted intensity for $H\alpha$, the difference between Burgess' and Pengelly's values is trivial for our purposes. If the line centers have very high optical depths so that case B would better represent the conditions in the nebulae, the $H\alpha$ intensity should be increased by a factor of about 1.6, while if the electron temperature were actually 2×10^4 °K, it should be reduced by about a factor of 2. In any case the adopted $H\alpha$ intensity can hardly be in error by more than a factor of 2, and even this extreme error would affect the derived distances by only about 15 per cent.

The planetary nebulae in this study are assumed to be 64 per cent hydrogen, by weight, and the free electrons are assumed to be contributed by hydrogen and singly ionized helium. Recent data (Aller 1964) suggest a slightly lower hydrogen abundance, but not enough lower to change appreciably the results given here.

b) Application of Observed Data

The photored magnitudes should be corrected for interstellar absorption before the $H\alpha$ fluxes are derived from them. The absorption can be estimated from the reddening of the central stars of those nebulae in which the stars can be identified. The stars that were observed photoelectrically are found to have colors of hot black bodies or reddened hot black bodies (see § IV). The color excesses for those stars (and nebulae) are therefore found by moving them along calculated reddening lines for black bodies in the three-color diagram to the locus of unreddened black bodies. The unreddened $B - V$ colors of these stars average about -0.3 , which corresponds to a $m_{pg} - m_{pr}$ color index of -1.0 . This value is used to estimate the reddening for the stars which are not observed photoelectrically. In either case, total to selective absorption is taken from Whitford's (1948) reddening-curve.

Interstellar absorption cannot be estimated for nebulae in which central stars have not been identified. Distances calculated for them, therefore, are based on photored magnitudes uncorrected for absorption and are probably too large. Moreover, the reddening determined from the photographic observations alone are not very reliable for stars near the Sky Survey photographic limit ($m_{pg} > 19$). It should be noted, however, that the large faint planetary nebulae studied here all lie at smaller distances than that of the galactic center. A rather extreme value for total absorption is 4 mag. in the photo-

red. Even if an absorption correction of this magnitude were ignored, the distance of the nebula would be overestimated by scarcely a factor of 2. Indeed, if we were to ignore all corrections for absorption and then assume that the true distances of the nebulae were 70 per cent of their calculated ones, the error introduced in distance would probably never exceed 30 per cent. Consequently, distances to the nebulae have been calculated both with absorption corrections (when available) and without.

In the system defined by the sensitivity-curve of the red-plate-filter combination used for the Sky Survey (California Institute of Technology 1954) the apparent and absolute photored magnitudes of the Sun are -27.1 and $+4.4$, respectively. From Pettit's (1940) spectral-energy distribution of sunlight received outside the Earth's atmosphere, it is found that an absolute photored magnitude of $+4.4$ corresponds to a photored flux of 2.0×10^{32} ergs sec $^{-1}$.

The photored light of the nebula is not due entirely to H α , but is contaminated by light from the [N II] lines at λ 6548 and λ 6584. White's (1952) measures of the relative strengths of the [N II] lines and H α in twenty-two planetary nebulae are used to estimate a correction to the photored flux to reduce it to the H α flux, f . A more recent study of the strength of [N II] in 103 nebulae has been carried out by Henize (1965). Henize's results are in excellent agreement with those of White, and therefore have not been incorporated in the present analysis. From White's data, mean values of 0.43 and 4.3 are adopted for the ratios [N II] $_{6584}$ /H α and [N II] $_{6584}$ /[N II] $_{6548}$, respectively. Actually, White found considerable variation in the strengths of the [N II] lines, the ratio [N II] $_{6584}$ /H α varying among the twenty-two nebulae from 0 to 3.3. Moreover, White selected nebulae of relatively high surface brightness, where the relative strengths of H α and the [N II] lines could be systematically different from the corresponding values in the nebulae studied here. Fortunately, however, a derived distance is very insensitive to f ; even if, say, 80 per cent of the photored flux of a nebula were in [N II], the corresponding error in its derived distance would be only 29 per cent.

The measured dimensions of the projected nebular images on the Sky Survey give their angular volumes, v . There is, of course, some uncertainty regarding the three-dimensional form of a nebula. Those which appear as homogeneous disks or ellipsoids are assumed to be really spheres or ellipsoids of revolution with the axes of revolution along the minor axes of the projected ellipses. Nebulae that appear as rings are assumed to be shells, the surfaces of which are either spherical or ellipsoids of revolution. In the latter case, if a projected ring is of variable thickness but with an axis of symmetry, the axes of revolution of the oblate-spheroidal-bounding surfaces are assumed to pass through the thinnest parts of the shell. Filamentary extensions, when they exist, are usually taken to be thin shells. Nebulae 21, 55, the inner part of 22, and the outer extension of 66, are assumed to be cylindrical tubes. Despite the ambiguity of the true shape of a nebula, the eccentricities of the elliptical boundaries of the nebular images are generally small, so the calculated volumes are probably not appreciably in error. A more serious error arises if the nebular light comes largely from small dense filaments that are not resolved in the photographic images, in which case a much smaller value of v should be used in equation (3). Because v enters to the $\frac{1}{3}$ power, however, even if most of the emitted light comes from only one-tenth of the observed image, the distance is overestimated by only 60 per cent. Derived distances, in other words, are rather insensitive to even large fluctuations in the density of the nebular material.

c) Characteristics of the Nebulae

Transformation to convenient units leads to the following form of equation (3), from which the distances of the nebulae are calculated:

$$\log r = 3.240 + 0.08 m_{pr} - 0.2 \log v, \quad (4)$$

where v is expressed in cubic seconds of arc. Distances are given with m_{pr} both corrected and uncorrected for interstellar absorption. The results are in Table 4. Also in the table

TABLE 4
DERIVED PROPERTIES OF THE NEBULAE

No.	VOLUME (10 ⁶ sec ³)	Δm_{pr}	DISTANCE (pc)		DIAMETER (pc)	N_e	M_{pr}
			With Corr	No Corr.			
1	0 0540	2 0	2038	2945	0 46	136	+1 2
2	0 0154	1 4	2619	3389	0 41	225	+0 6
3	0 0993	3 0	1160	2015	0 34	402	-0 0
4	0 00373	0 0	4670	4670	0 45	164	+1 0
5	0 709	2 5	768	1218	0 50	193	+0 8
6	0 656	2 5	766	1214	0 70	211	+0 7
7	259	0 3	236	250	1 00	18	+3 3
8	0 0528	1 3	2047	2601	0 60	137	+1 1
9	0 0217	2 2	2632	3947	0 51	157	+1 0
10	0 0197			2494	(0 41)		
11	0 0172			4453	(0 73)		
12	0 0257	2 9	1153	1967	0 21	1580	-1 5
13	0 701	1 6	729	978	0 61	229	+0 6
14	0 0214			3115	(0 60)		
15	0 0184	0 0	3270	3270	0 54	96	+1 5
16	1 46	0 4	997	1073	0 68	43	+2 4
17	0 0352	3 0	1880	3267	0 49	265	+0 4
18	0 0935			1860	(0 72)		
19	0 123			2543	(0 83)		
20	0 147	0 0	1968	1968	0 64	55	+2 1
21	98 2	0 0	181	181	0 65	107	+1 4
22	1 24	0 0	993	993	1 26	51	+2 2
23	0 0571			2169	(0 57)		
24	24 1	0 0	321	321	0 58	78	+1 8
25	0 704	0 5	1175	1289	1 07	54	+2 2
26	0 0340			3002	(0 58)		
27	0 0306			2413	(0 55)		
28	5 83	0 4	892	961	1 16	15	+3 6
29	29 7	0 0	399	399	0 93	33	+2 7
30	0 811	0 3	1300	1374	0 80	35	+2 6
31	306	0 1	193	197	0 94	28	+2 9
32	1 26	2 4	854	1329	0 55	79	+1 7
33	10 1	0 4	495	533	0 64	51	+2 2
34	1 4	0 0	991	991	1 35	61	+2 0
35	151			240	(1 09)		
36	21 5	0 0	418	418	0 97	40	+2 5
37	0 0439	2 3	1390	2124	0 32	527	-0 3
38	1 09			930	(0 69)		
39	1 67	0 0	918	918	0 77	48	+2 3
40	0 0129			3085	(0 51)		
41	0 00223	0 6	4307	4810	0 42	350	+0 1
42	0 105			2533	(0 74)		
43	0 174	0 2	1554	1613	0 60	95	+1 5
44	0 0343			2190	(0 71)		
45	9 56	3 0	334	580	0 49	175	+0 9
46	0 137	0 0	1854	1854	0 60	71	+1 9
47	0 00174	2 5	4869	7717	0 40	309	+0 3
48	0 0236			2686	(0 52)		
49	0 0390			2385	(0 51)		
50	0 00761			3246	(0 42)		
51	0 147	0 0	1762	1762	0 57	77	+1 8
52	0 0257	1 6	2079	2791	0 37	270	+0 4
53	0 0151	2 0	1690	2443	0 25	853	-0 8
54	0 0707			2690	(0 87)		
55	0 0529	2 0	1244	1799	0 33	610	-0 5
56	1 73			964	(0 88)		
57	0 0158	1 3	3077	3910	0 60	135	+1 2
58	0 0237			3472	(0 74)		
59	0 0977			1651	(0 75)		

TABLE 4—*Continued*

No	VOLUME (10^6 sec^3)	Δm_{pr}	DISTANCE (pc)		DIAMETER (pc)	N_e	M_{pr}
			With Corr	No Corr			
60	0 182	0 0	1922	1922	0 69	48	+2 3
61	0 394	0 0	1249	1249	1 22	81	+1 7
62	1 26	..	...	698	(0 54)	...	..
63	0 0340	1 5	2154	2840	0 42	183	+0 8
64	0 0212	2 6	1700	2745	0 33	597	-0 4
65	0 693	0 5	1116	1223	0 72	65	+2 0
66	8 29	1 9	421	597	0 60	101	+1 5
67	0 153	..	...	2181	(0 71)	...	..
68	0 0189	1 9	2170	3080	0 42	322	+0 2
69	0 00430	..	...	4539	(0 51)	...	..
70	0 0299	2 6	1420	2293	0 30	725	-0 7
71	2 17	1 2	493	615	0 40	240	+0 5
72	0 639	0 0	1266	1266	0 82	48	+2 3
73	0 166	..	...	1956	(0 76)	...	..
74	314	0 4	203	219	0 86	24	+3 1
75	0 0945	1 2	1926	2402	0 63	92	+1 6
76	0 00124	..	...	5821	(0 37)	...	..
77	0 181	2 3	839	1282	0 33	580	-0 4
78	0 616	0 2	1621	1682	0 84	24	+3 0
79	0 238	3 7	571	1128	0 33	1406	-1 4
80	1 06	1 3	683	868	0 53	184	+0 8
81	0 0172	..	...	2514	(0 41)	...	..
82	0 432	..	...	1182	(0 54)	...	..
83	0 0794	..	...	2830	(0 74)	...	..
84	0 823	1 6	643	864	0 46	283	+0 4
85	2750	..	...	91	(0 92)	...	..
86	0 183	..	...	1956	(0 66)	...	..

are the adopted volumes of the nebulae, the adopted values for interstellar (photored) absorption (when it can be estimated), the maximum nebular diameters in parsecs, electron densities derived on the basis of the assumed composition described in § IIIa, and the absolute photored magnitudes of the nebulae. The diameters correspond to the distances found when absorption corrections were made, except when the absorption cannot be estimated, in which cases they are given in parentheses.

d) Comparison with Others

To compare the distances obtained by estimating $H\alpha$ fluxes from the red Sky Survey plates with those obtained by other investigators, we have selected fourteen well-studied nebulae, all of which are large and faint and probably optically thin. Dimensions and surface brightnesses of these nebulae, measured on the Survey plates exactly as was done with the objects discovered on the Survey, enable the calculation of their distances with equation (4). When the central stars can be identified and the reddening estimated, corrections are applied for interstellar absorption. Both corrected (when available) and uncorrected distances are given in Table 5, along with the distances obtained by several other investigators. The photored magnitudes, estimated absorptions, and adopted volumes used in the calculations are also given.

Berman's (1937) distances are based on a statistical correlation between the absolute magnitudes of the nebulae and the differences in apparent magnitudes between the central stars and the nebulae. Berman's correlation is valid at best for optically thick nebulae. It is clear that these fourteen large faint objects that are probably optically

thin have far lower luminosities than Berman derived from his formula; thus it is understandable that his distances are systematically too large. Shklovsky (1956) and O'Dell (1962) both derived distances by a procedure similar to the one used here. Shklovsky utilized published values of photographic magnitudes of the nebulae, and his distance determinations had to be empirically calibrated. O'Dell used measured $H\beta$ fluxes. The writer's distances, in the mean, are 80 per cent of O'Dell's and 127 per cent of Shklovsky's, which appears to be satisfactory agreement, especially when the inherent uncertainties in the calibration of different photometric systems and in the correction for space absorption are considered.

From the distances he obtained from $G\beta$ fluxes, O'Dell calibrated a formula for obtaining distances also from integrated photographic magnitudes. With his calibration,

TABLE 5
COMPARISON OF DISTANCES OBTAINED FOR 14 SELECTED NEBULAE

NGC OR IC	m_{pr}	ABSORPTION (Δm_{pr})	VOLUME (10^4 sec^3)	DISTANCE (pc)				
				Berman	Shklovsky	O'Dell	Abell	
							Corrected for Absorption	Uncor- rected
246	9.7		854	1410	220	430		426
650-1 ..	9.1		79 7	4570	820	537		613
I 289 ..	11.9	2 4	2 57	3350	1100	1710	1312	2041
1501 .	10.7	0 6	9 77	3390	880	1450	1121	1252
2610 .	11.9	0 0	4 33	8750	1300	2340	1838	1838
3587 .	8.6	0 8	247	3630	440	498	385	446
I 1295 .	11.6	0 0	37 0	<3000?	900	1550	1132	1132
6772 .	10.7	1 2	16 8	13200	1200	1690	900	1123
6781..	9.0	1 7	7 70	3280	630	1010	702	960
6804 .	11.2	2 2	5 26	3600	1300	1850	1036	1554
6842 .	12.5	1 9	7 76	3530	1200	1800	1287	1827
6894..	11.2		4 96	6310	1400	2040		1572
7008 .	10.0	1 8	3 04	3800	630	1040	998	1390
7293 . .	6.3	0 8	6650	1050	50	137	130	151

he obtained distances to the same eighty-six nebulae listed in Table 4, using the photographic magnitudes given in Table 3, which were supplied to him by the writer prior to publication. The ratio of distances given in Table 4 to those found by O'Dell (1963) have considerable statistical scatter, as one might expect because the photographic spectral region contains many emission lines, the relative intensities of which vary greatly from one nebula to another. The mean ratio, on the other hand, is 0.795, almost exactly the same as the mean ratio of Abell to O'Dell distances given in Table 5. This circumstance speaks well for the consistency of O'Dell's calibration procedures.

Various other investigators have also derived distances to some of the nebulae listed in Table 5. In particular, distances to six of the nebulae derived by R. Minkowski (Aller 1965) are in good agreement with those found by the writer.

e) *Distribution of the Nebulae in the Galaxy*

The distribution of the nebulae in the sky (not shown here for the sake of brevity) exhibits essentially the same galactic concentration as that of all known planetary nebulae (Minkowski and Abell 1953). The distribution projected on the galactic plane of the fifty-six nebulae for which absorption was estimated shows no apparent tendency

for the nebulae to lie along the local spiral arm, a result consistent with findings by Perek (1963). Again, their association with the older "disk" population is suggested.

IV. PROPERTIES OF THE CENTRAL STARS

In sixty-four of the planetary nebulae studied the central stars are identified with fair certainty, and the photometric data for these stars are given in Table 2. Seven of the central stars, however, are visible only on the blue Sky Survey plates and are too faint to obtain accurate colors photoelectrically; consequently, for these stars and nebulae absorption estimates could not be made and the intrinsic properties of the stars could not be computed reliably. An eighth star (Nebula 14) has the color of an F7 V star, and is probably a foreground object. These eight stars are eliminated in the following discussion. The properties of these nuclei of planetary nebulae have already been summarized (Abell 1964); fuller details are given here.

a) The Absolute Magnitudes and Colors of the Stars

The absolute magnitudes of the fifty-six stars, based on the distances calculated which include corrections for interstellar absorption, are given in Table 6. Absolute visual magnitudes, M_v , are given for the stars observed photoelectrically, and absolute photographic magnitudes, M_{pg} , are given for the stars observed only on the Sky Survey plates. The absolute magnitudes range from 1 to 10, but a large part of this range will be seen to be due to a temperature range; the bolometric magnitudes have far less scatter.

Intrinsic colors of black bodies have been recomputed by the writer, using the spectral scans of twenty-five stars by Code (1960) and Melbourne (1960), the (*UBV*) response functions published by Johnson and Morgan (1951), and a re-evaluation of the extinction produced by the Earth's atmosphere. For the hotter black bodies the results agree with those of Arp (1961) within the accuracy of his graphical integration except for small effects that arise because of improvements in the way the writer has handled atmospheric extinction. Reddening lines for black bodies on the three-color diagram have also been computed; these are based on Whitford's (1948) reddening-curve, but with very slight adjustments so that it predicts the correct reddening line for O stars as observed by Hiltner and Johnson (1956). The locus of black bodies of various temperatures on the three-color diagram is shown in Figure 3; also shown are reddening lines for black bodies of two different temperatures.

The thirty stars that have been observed photoelectrically have also been plotted on the three-color diagram of Figure 3. It is seen that these stars can be represented well by very hot black bodies or reddened hot black bodies. Arp (1961), to whom the writer had supplied colors of some of these stars, has also called attention to their similarity to hot black bodies. The spread on the three-color diagram of stars hotter than 20000°K is so small that temperatures of stars that radiate like black bodies cannot be determined accurately from their (*UBV*) colors; however, minimum temperatures can be derived for the stars by the method described in § IVb.

Finally, it should be noted that some of the stars may be variable in magnitude and color. Repeated photoelectric measures sometimes showed differences (up to 0.5 mag.) that are large compared to expected errors in photometry. As yet, none of the stars has been observed extensively enough to ascertain whether such variations are continuing or periodic. Stars which are particularly suspected of being variable are those of Nebulae Nos. 14, 20, 30, 36, 39, 41, 46, 51, 61, 63, 74, and 78.

According to Vorontsov-Velyaminov (1960), the nucleus of IC 4997 is probably variable. Other recent evidence for the variability of the nucleus of a planetary nebula is presented by Kohoutek (1964). He reports the identification of a planetary at $\alpha = 8^{\text{h}}55^{\text{m}}7^{\text{s}}$; $\delta = -28^\circ45'$ which appears on the overlap of two southern extension Sky Survey prints. The original plates were taken one day apart in March, 1956. Kohoutek estimates the photographic magnitude of the central star to be 15.9 on March 9 and

TABLE 6
DERIVED PROPERTIES OF THE CENTRAL STARS

No.	M_v	M_{pg}	$\log T_{\min} (^{\circ} \text{K})$	$\log(L/L_{\odot})_{\min}$	$(R/R_{\odot})_{\max}$
1 .		+4 7	4 72	1 55	0 071
2 .		5 1	4 85	1 68	.046
3 .		2 3	4 60	2 26	.29
4 .		6 1	4 88	1 39	.029
5 .		6 3	5 00	1 62	.021
6 .		5 0	4 83	1 70	.053
7 .	+8 26		4 85	0 62	.014
8 .		5 7	4 87	1 47	.034
9 .		4 7	4 75	1 61	.068
12 .		3 5	4 92	2 53	.087
13 .	8 58		5 34	1 94	.0065
15 .	3 15		4 49	1 87	.31
16 .		7 0	4 87	0 97	.019
17 .		3 0	4 63	2 00	.19
20 .	5 09		4 60	1 33	.099
21 .	9 70		5 40	1 65	.0036
22 .	9 60		5 23	1 21	.0047
24 .	9 64		5 30	1 39	.0041
25 .	7 98		4 96	1 06	.014
28 .		5 4	4 56	0 91	.073
29 .	10 34		5 27	0 99	.0031
30 .	3 31		4 41	1 64	.33
31 .	8 95		5 00	0 79	.0081
32 .		5 3	4 74	1 31	.051
33 .	6 59		4 75	1 05	.036
34 .	6 34		4 75	1 15	.040
36 .	3 40		4 43	1 65	.30
37 .		3 1	4 71	2 13	.15
39 .	5 79		4 66	1 17	.062
41 .	1 98		4 49	2 34	.51
43 .	3 56		4 51	1 74	.24
45 .		7 0	5 11	1 66	.014
46 .	3 73		4 50	1 64	.22
47 .		2 3	4 57	2 18	.30
51 .	4 19		4 54	1 57	.17
52 .		3 2	4 65	1.96	.16
53 .		5 5	5 14	2 34	.026
55 .		5 7	5 12	2 20	.024
57 .	3 55		4 55	1 83	.22
60 .		6 6	4 85	1 10	.023
61 .	6 91		4 87	1 01	.020
63 .	1 12		4 39	2 44	.94
64 .		2 1	4 63	2 35	.28
65 .	4 99		4 60	1 37	.10
66 .	6 89		4 90	1.33	.025
68 .		4 4	4 81	1 88	.069
70 .		3 0	4 75	2 27	.14
71 .	9 05		5 40	1 91	.0048
72 .	5 61		4 64	1 20	.071
74 .	10 06		5 16	0 82	.0042
75 .		3 6	4 57	1 66	.16
77 .		2 5	4 66	2 30	.23
78 .	1 94		4 31	1 99	.80
79 .		2 8	4 81	2 52	.15
80 .		8 1	5 32	1 80	.0061
84 .	7 49		5 17	1 89	0 013

17.4 on March 10. The obvious variability of the star is easily verified by inspection of either the red or blue Survey prints.

b) Minimum Temperatures of the Stars

The temperature of an optically thick nebula can be estimated by the well-known method described by Zanstra at the March 6, 1926, meeting of the American Physical Society (1926, 1927). The method was also apparently independently suggested by Menzel later the same year (1926). Since every Balmer quantum emitted by the nebula must have resulted originally from the ionization of a hydrogen atom by a Lyman quantum from the star, the observed nebular flux in the Balmer lines is an indirect measure of the flux from the star at wavelengths shorter than the Lyman limit. Comparison of the

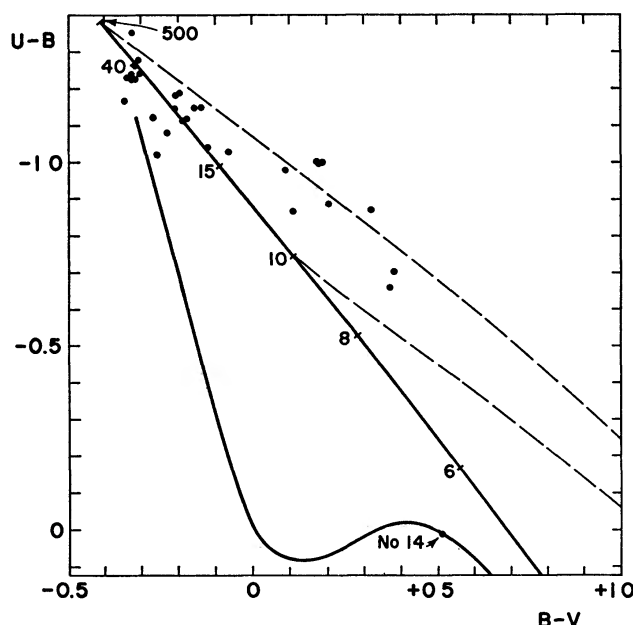


FIG. 3.—The distribution of the central stars on the three-color diagram

Balmer flux with the stellar magnitude, therefore, is a comparison of the stellar flux in the ultraviolet and visible regions, which leads to a temperature estimate. Specifically, a Zanstra (or Menzel) temperature is the temperature of a black body that emits the same ratio of the number of photons in the Lyman continuum to the number in the observed photographic spectral region as the star does.

In the present application, the photored magnitudes of a nebula and its central star are compared to obtain the stellar temperature. It is assumed that 59 per cent of the Balmer quanta are in $H\alpha$. However, because the nebulae investigated here are probably all optically thin, a photored nebular magnitude underestimates the ultraviolet flux from the star, and actual temperatures are expected to be higher than the ones derived. These minimum temperatures are given in Table 6. The temperatures, of course, are independent of interstellar absorption. O'Dell (1963) has derived minimum temperatures for most of these same stars from the photographic magnitudes supplied him by the writer; while we expect the photographic magnitude data to be less consistent than the photored, O'Dell's results are, nevertheless, statistically in satisfactory agreement with those given in Table 6.

It is evident that most of these central stars are extremely hot; even the minimum temperatures range to more than 200000°K . Greenstein (Greenstein and Minkowski

1959, 1964) has obtained spectra of some of the central stars and confirms their high temperatures. One star (Nebula 78) shows stellar emission lines, including a strong pair of O VI lines. The spectrum of the nucleus of NGC 246, described by Aller (1948) also has O VI in emission; perhaps the nucleus of NGC 246 can be considered a prototype of the stars described here.

c) *Luminosities and Radii of the Stars*

Bolometric corrections to photographic and photovisual magnitudes were evaluated numerically for black bodies of various temperatures. For corrections to our photographic magnitudes, the response function of the blue plate-Schmidt correcting lens combination was used (California Institute of Technology 1954), and for corrections to visual magnitudes Johnson's (Johnson and Morgan 1951) *V* response function was used. Thus minimum bolometric corrections were obtained for the central stars of the nebulae from their minimum temperatures. These bolometric corrections, applied to the derived absolute photographic or visual magnitudes for the stars, lead to lower limits to the stellar luminosities, which are given in Table 6.

Radii of stars of known effective temperatures and luminosities can be found from Stefan's law. Here we have only lower limits to the temperatures and luminosities of the central stars of the planetary nebulae. Even for the very hottest stars, however, the factor by which the luminosity is underestimated can vary at most as the third power of the factor by which the temperature is too low. For cooler stars, the underestimate in luminosity is less sensitive to the temperature. Therefore, the ratio $L_{\min}/T_{\min}^4$ is an upper limit to the ratio L/T^4 , and values of radii derived from

$$R_{\max} = [L_{\min}/4\pi\sigma T_{\min}^4]^{1/2} \quad (5)$$

are *upper limits* to the radii of the central stars. The maximum radii derived from equation (5) are given in Table 6, in units of the solar radius. Most of the stars are seen to be very small and many have the dimensions of white dwarfs, (Abell 1961, 1964), a result in agreement with the findings of O'Dell (1963) and others. It is interesting to note that Menzel (1926) arrived at a similar conclusion nearly four decades ago.

The central stars investigated are plotted on a section of the luminosity-temperature diagram in Figure 4. Filled circles represent stars observed photoelectrically, and open circles represent those observed photographically. The central stars of the several well-known planetary nebulae that were observed to compare the distance scale used here to those of others (§ III d) are also plotted on Figure 4 as crosses. The solid lines are loci of constant radii. Since only minimum temperatures and luminosities of the stars are known, dashed lines with arrows are shown to indicate how the points representing stars move on the diagram as the stellar temperatures are increased. It should be noted that the stars observed photographically seem to be systematically more luminous than those observed photoelectrically. The former are also systematically more distant (compare Tables 4 and 6). At large distances in the galactic plane, only those stars of greater-than-average luminosity can be identified, and because of interstellar absorption, they actually appear fainter than many of the nearer, less luminous objects. Stars of most of the more distant planetaries, therefore, could not be observed photoelectrically from Mount Wilson.

V. EVOLUTION OF THE PLANETARY NEBULAE AND CENTRAL STARS

O'Dell (1963) presents evidence suggesting rapid evolution of the central stars as the nebular shells expand, in the sense that the stars evolve to lower luminosities and radii. It is therefore of interest to investigate whether the nebulae and central stars in the present study indicate any evolutionary patterns.

In Figure 5, the logarithms of the diameters of the nebulae (in parsecs) are plotted

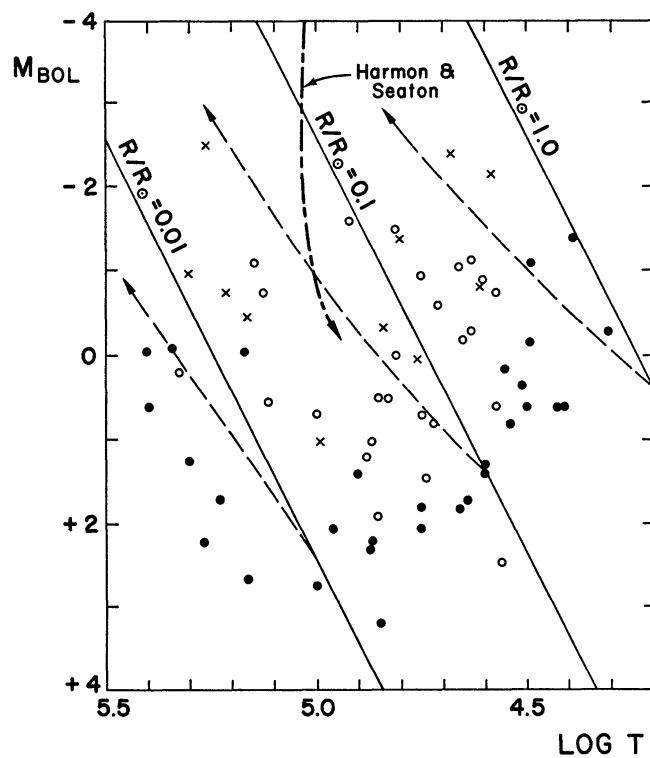


FIG. 4.—The central stars of the nebulae on a portion of the luminosity-temperature diagram; *crosses*—large, faint NGC objects; *filled circles*—objects observed photoelectrically; *open circles*—objects observed only photographically.

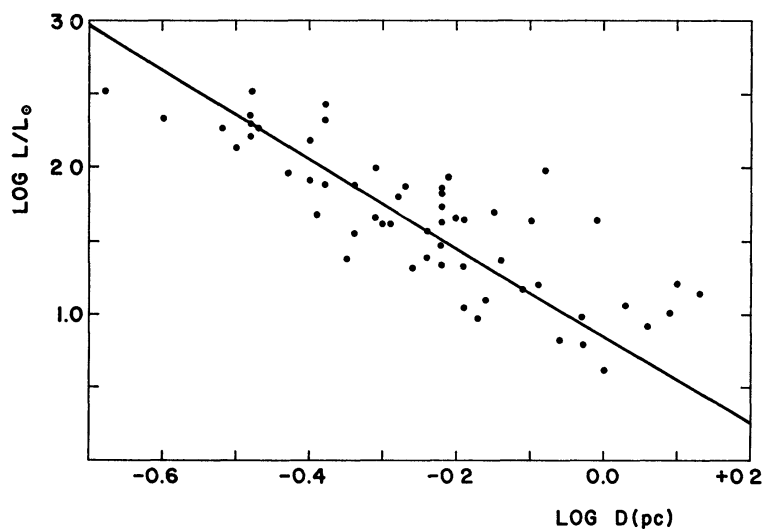


FIG. 5.—The derived relation between the lower limits to the luminosities of the central stars and the nebular diameters.

against the logarithms of the lower limits to the luminosities of the central stars (in solar units). Although a strong correlation is found, it does not necessarily suggest evolution, because, as the nebulae are probably optically thin, we expect their diameters to be correlated with the derived luminosities of their central stars. The ultraviolet energy (of wavelength less than the Lyman limit) required to keep ionized a sphere of gas of mass M , diameter D , and uniform density ρ , is proportional to $\rho^2(\frac{1}{6}\pi D^3)$, which in turn is proportional to M^2/D^3 . Thus if we idealize our nebulae by homogeneous spheres and assume that all have the same mass, the ultraviolet (less than $\lambda 912$) luminosity we would derive for their central stars from the nebular photoreduced magnitudes (mostly due to $H\alpha$) should be proportional to $1/D^3$. With stars of such high temperature as those in

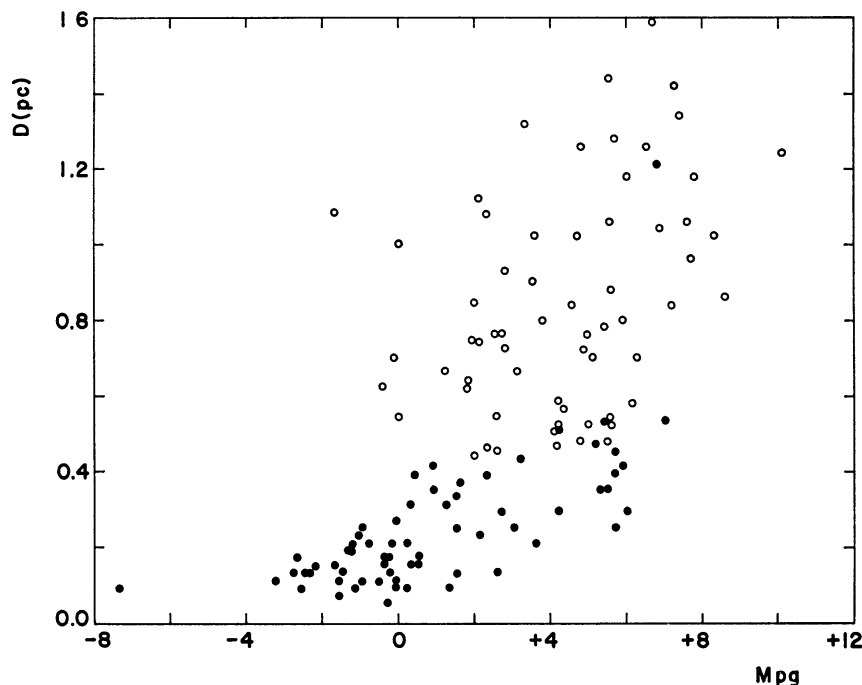


FIG. 6.—The relation between the diameters of the nebulae and the absolute photographic magnitudes of their central stars; *open circles*—objects discovered on the Survey; *filled circles*—objects investigated by O'Dell.

the present study, most of the luminosity is at wavelengths less than the Lyman limit; therefore, we would expect the following approximate correlation between the derived stellar luminosities and the nebular diameters:

$$\log \frac{L}{L_{\odot}} = \text{constant} - 3 \log D. \quad (6)$$

The straight line in Figure 5 has a slope of -3 . If the luminosities of the nuclear stars and diameters of the gas shells in the nebulae studied by O'Dell (1963) are similarly plotted, they are found to show this same correlation.

It is clear that the central stars show a considerable range in temperature, or in luminosity, or in both, but since only lower limits to temperatures or luminosities are available, it is not possible from this study to find which, if either, is correlated with nebular diameter. The nebular diameters themselves strongly influence the derived temperatures.

The absolute photographic magnitudes of the stars, on the other hand, do not depend on their derived minimum temperatures. The plot of the nebular diameters against stellar absolute magnitudes for the nebulae in the present study (*open circles* in Fig. 6)

does not show very convincing correlation. The slight tendency that may exist for the intrinsically brighter stars to be associated with the smaller, brighter nebulae, however, is far more definite when we include the sample of nebulae of higher average surface brightness investigated by O'Dell (*small filled circles* in Fig. 6). A loose correlation between nebular diameter and the absolute magnitudes of the central stars may therefore exist and may be due to evolution of the stars. On the other hand, a possible selection effect could be partly or wholly responsible for the apparent correlation. If central stars of relatively higher luminosity are less common than those of lower luminosity, we would expect to find most of the brighter stars at relatively larger distances; because distant nebulae are more obscured by interstellar matter, they are identified only if they are intrinsically small and bright.

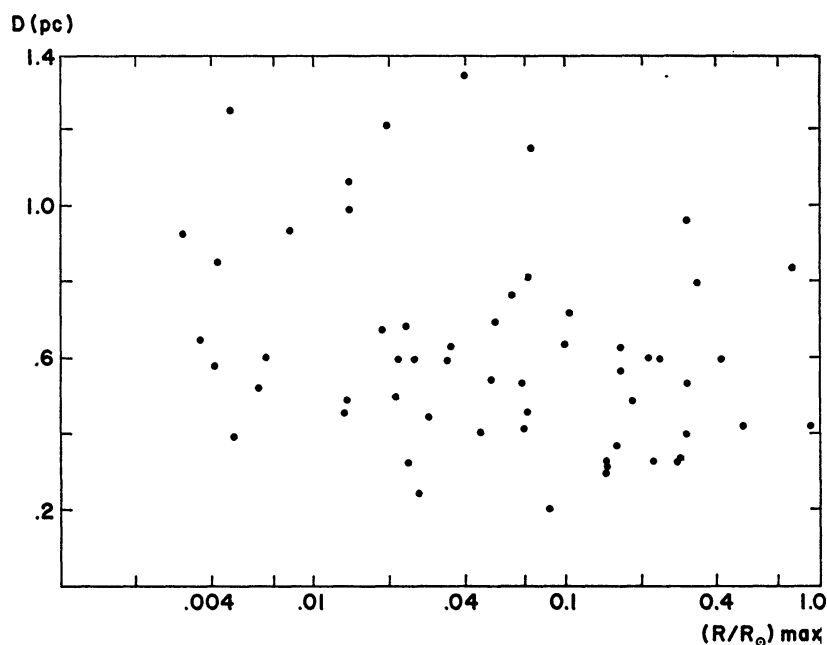


FIG. 7.—Plot of the nebular diameters against derived upper limits to the radii of their central stars

The diameters of the nebular shells in the present study are plotted against the upper limits to the stellar radii in Figure 7. It is apparent that there is no strong correlation, other than a possible deficiency of large nebulae with large stars. This slight correlation could also be due to the selection effect described in the last paragraph. Unless stellar diameters were strongly correlated with temperature, stars of larger than average luminosity must have, statistically, larger than average radii. These stars would be expected to be identified only in association with those smaller, brighter nebulae that can be observed at large distances through considerable interstellar absorption.

These comments are not intended to discredit the observations favoring evolution of the nuclei of planetary nebulae. On the contrary, we might well expect rapid evolution of a star so soon ($\sim 10^4$ years) after the ejection of 0.2 solar mass of stellar material. We mean here only to warn against too naïve an interpretation of the results of the study of this particular sample of planetary nebulae discovered on the Palomar Sky Survey.

Harman and Seaton (1964) have also suggested an evolutionary sequence for planetary nebulae nuclei. The dashed line in Figure 4 shows their proposed evolutionary path on the luminosity-temperature plane. The observations reported here neither support nor contradict the Harman-Seaton evolutionary theory.

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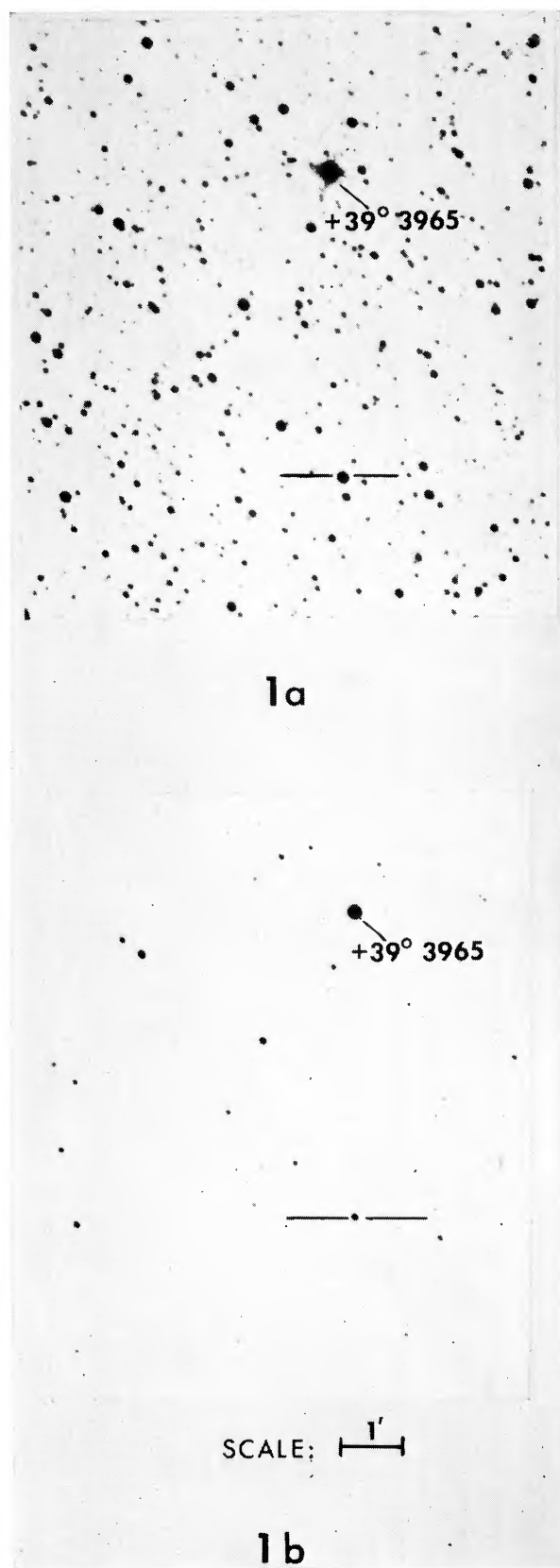


FIG. 1.—The field of MH α 328-116. The upper chart (a) is from a Kodak 103a-D+GG11 plate-filter combination; the lower chart (b) is from a Kodak 1N+RG8 infrared combination. North is at the top; east to the left.